Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.4-Improper/31-1.1.4.3-e-x-^m-a-x^j+b-x^k-^p-c+d-x^n-^q

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [298]. This is test number [31].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (298)	0.00 (0)
Mathematica	99.33 (296)	0.67 (2)
Fricas	92.95 (277)	7.05 (21)
Maple	92.28 (275)	7.72 (23)
Giac	76.17 (227)	23.83 (71)
Maxima	71.14 (212)	28.86 (86)
Mupad	66.11 (197)	33.89 (101)
Sympy	42.28 (126)	57.72 (172)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

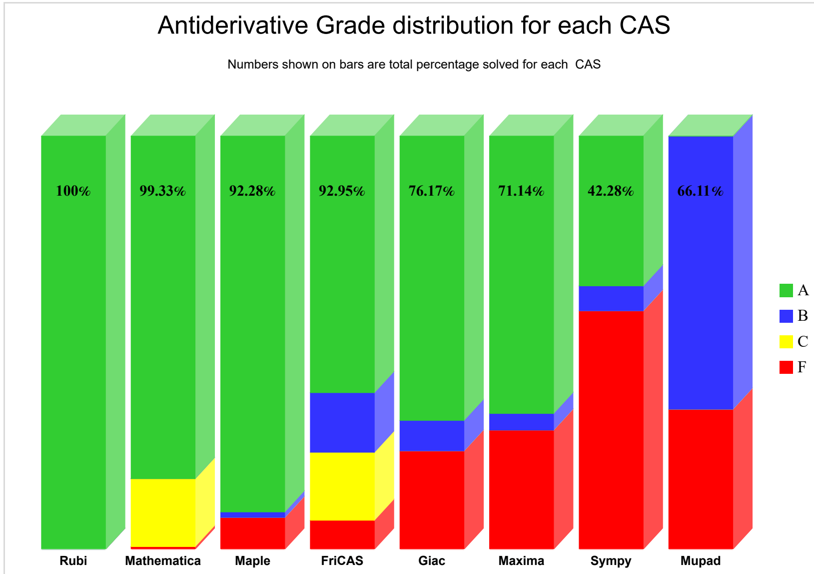
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

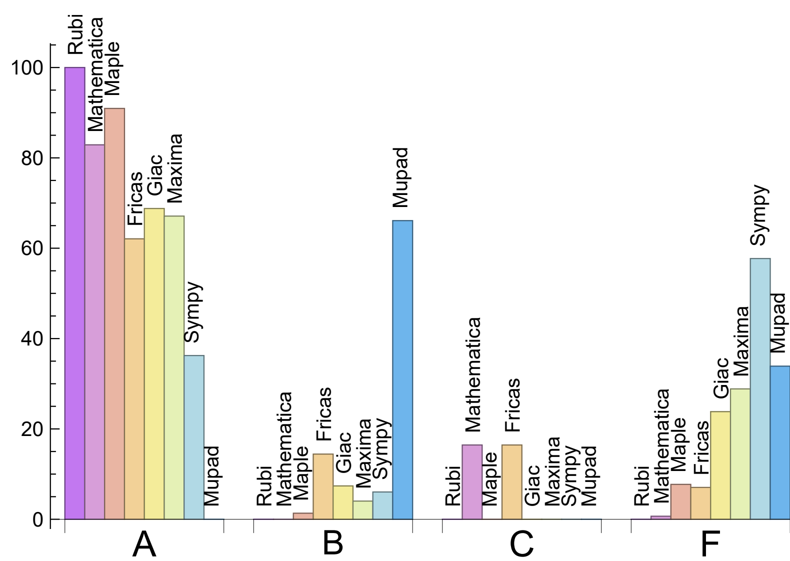
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Maple	90.94	1.34	0.00	7.72
Mathematica	82.89	0.00	16.44	0.67
Giac	68.79	7.38	0.00	23.83
Maxima	67.11	4.03	0.00	28.86
Fricas	62.08	14.43	16.44	7.05
Sympy	36.24	6.04	0.00	57.72
Mupad	N/A	66.11	0.00	33.89

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	23	100.00 %	0.00 %	0.00 %
Fricas	21	100.00 %	0.00 %	0.00 %
Giac	71	100.00 %	0.00 %	0.00 %
Maxima	86	100.00 %	0.00 %	0.00 %
Sympy	172	70.35 %	20.93 %	8.72 %
Mupad	101	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

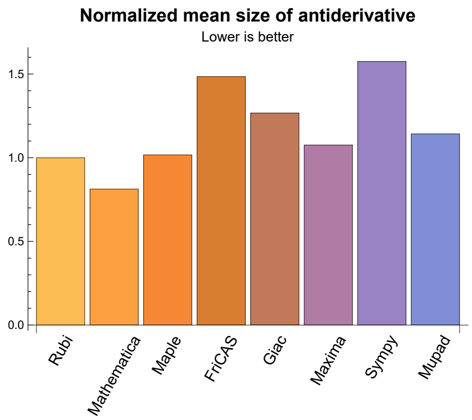
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

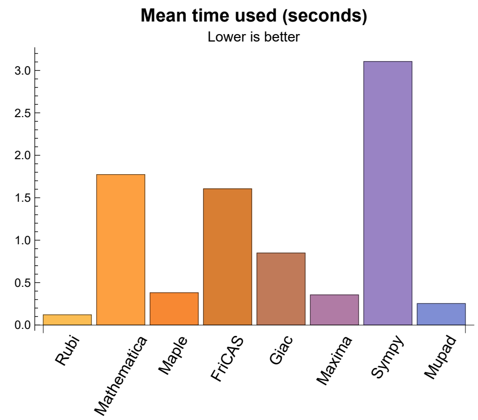
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	145.03	1.00	103.50	1.00
Mathematica	1.77	96.34	0.81	87.00	0.90
Maple	0.38	146.49	1.02	99.00	0.94
Maxima	0.36	124.24	1.08	84.00	0.99
Fricas	1.61	224.38	1.48	106.00	1.10
Sympy	3.11	123.29	1.57	80.00	1.13
Giac	0.85	145.46	1.27	103.00	1.04
Mupad	0.25	151.83	1.14	76.00	0.96

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0.0
0.5
1.0
1.5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

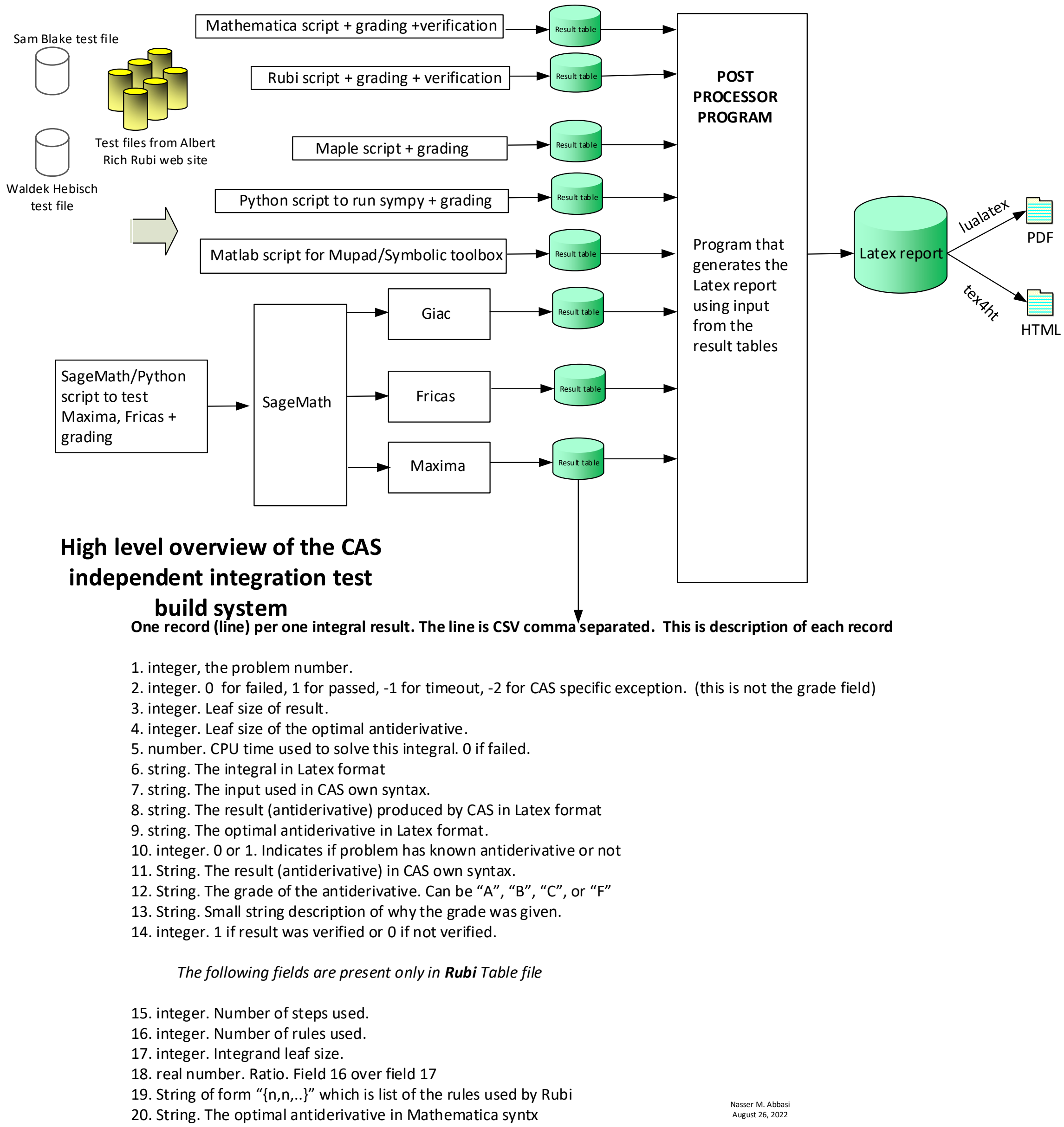
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298 }

B grade: { }

C grade: { 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268 }

F grade: { 284, 292 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 271, 278, 280, 281 }

B grade: { 126, 269, 270, 279 }

C grade: { }

F grade: { 272, 273, 274, 275, 276, 277, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 106, 109, 110, 111, 112, 119, 120, 121, 122, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 269, 270, 271, 275, 278, 280, 281 }

B grade: { 91, 96, 107, 108, 113, 114, 115, 116, 117, 118, 279, 283 }

C grade: { }

F grade: { 103, 104, 105, 123, 124, 125, 126, 127, 128, 129, 142, 143, 144, 156, 157, 158, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 276, 277, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 275, 278, 279, 280, 281 }

B grade: { 84, 86, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 269, 270, 271, 283 }

C grade: { 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268 }

F grade: { 272, 273, 274, 276, 277, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 46, 47, 48, 50, 53, 55, 57, 58, 59, 61, 62, 63, 64, 65, 67, 68, 69, 71, 73, 74, 75, 76, 78, 79, 80, 81, 82, 84, 86, 87, 88, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 186, 187, 188, 189, 190, 191, 192, 278, 279, 280, 281 }

B grade: { 28, 43, 45, 49, 51, 52, 54, 56, 60, 66, 70, 72, 77, 83, 85, 269, 270, 271 }

C grade: { }

F grade: { 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 183, 184, 185, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 275, 276, 277, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 278, 279, 280, 281 }

B grade: { 28, 95, 96, 97, 98, 99, 113, 114, 115, 116, 117, 118, 135, 136, 137, 150, 151, 152, 269, 270, 271, 283 }

C grade: { }

F grade: { 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 275, 276, 277, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 97, 98, 99, 100, 101, 102, 103, 108, 114, 115, 116, 117, 118, 119, 120, 121, 122, 133, 134, 135, 136, 137, 138, 139, 140, 141, 148, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 269, 270, 271, 278, 279, 280, 281 }

C grade: { }

F grade: { 94, 95, 104, 105, 106, 107, 109, 110, 111, 112, 113, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 142, 143, 144, 145, 146, 147, 156, 157, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 275, 276, 277, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.018	0.005	0.163	0.285	2.913	0.007	1.457	0.179

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.028	0.006	0.206	0.286	3.156	0.007	1.656	0.059

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.016	0.004	0.134	0.299	2.928	0.006	1.149	0.040

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.025	0.005	0.117	0.645	2.434	0.006	0.695	0.059

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.013	0.004	0.168	0.309	1.431	0.006	0.773	0.060

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	28	25	27	30	26
N.S.	1	1.00	1.00	0.97	0.97	0.86	0.93	1.03	0.90
time (sec)	N/A	0.019	0.007	0.088	0.273	1.595	0.032	0.655	0.061

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	24	28	20	23	24
N.S.	1	1.00	1.00	0.92	0.92	1.08	0.77	0.88	0.92
time (sec)	N/A	0.016	0.007	0.026	0.278	1.140	0.030	0.743	0.073

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	28	30	26	42	25
N.S.	1	1.00	1.00	0.90	0.97	1.03	0.90	1.45	0.86
time (sec)	N/A	0.019	0.008	0.034	0.270	1.334	0.075	0.629	0.042

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	27	25	26	29	27	28	26
N.S.	1	1.00	1.04	0.96	1.00	1.12	1.04	1.08	1.00
time (sec)	N/A	0.016	0.009	0.030	0.295	1.549	0.087	0.620	0.057

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	26	30	31	29	39	29
N.S.	1	1.00	1.07	0.90	1.03	1.07	1.00	1.34	1.00
time (sec)	N/A	0.019	0.013	0.031	0.272	2.139	0.178	0.685	0.074

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	28	29	29	32	31	29
N.S.	1	1.00	1.06	0.90	0.94	0.94	1.03	1.00	0.94
time (sec)	N/A	0.015	0.008	0.023	0.276	3.694	0.179	0.594	0.038

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	56	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.034	0.007	0.430	0.283	2.604	0.011	0.620	0.085

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	53	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	0.96	0.96	0.93
time (sec)	N/A	0.051	0.006	0.400	0.282	1.806	0.011	0.650	0.047

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	56	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.026	0.007	0.425	0.268	1.246	0.011	1.137	0.049

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	52	51	51	53	53	51
N.S.	1	1.00	1.21	1.24	1.21	1.21	1.26	1.26	1.21
time (sec)	N/A	0.049	0.009	0.524	0.288	1.907	0.012	1.838	0.045

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.022	0.007	0.501	0.282	2.498	0.011	0.883	0.047

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	51	51	52	49	49	53	48
N.S.	1	1.00	1.19	1.19	1.21	1.14	1.14	1.23	1.12
time (sec)	N/A	0.027	0.012	0.361	0.275	2.061	0.047	0.926	0.042

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	49	48	53	48	48	48
N.S.	1	1.00	1.00	1.02	1.00	1.10	1.00	1.00	1.00
time (sec)	N/A	0.025	0.013	0.351	0.282	2.100	0.044	0.804	0.051

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	48	52	54	48	70	48
N.S.	1	1.00	0.96	0.94	1.02	1.06	0.94	1.37	0.94
time (sec)	N/A	0.036	0.020	0.403	0.289	1.042	0.099	0.717	0.047

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	50	52	51	50	50
N.S.	1	1.00	1.04	0.96	1.04	1.08	1.06	1.04	1.04
time (sec)	N/A	0.025	0.014	0.359	0.280	1.706	0.109	0.602	0.049

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	46	54	55	51	72	51
N.S.	1	1.00	0.98	0.90	1.06	1.08	1.00	1.41	1.00
time (sec)	N/A	0.031	0.025	0.364	0.278	1.566	0.267	0.613	0.081

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	51	53	54	53	50
N.S.	1	1.00	1.00	0.94	1.06	1.10	1.12	1.10	1.04
time (sec)	N/A	0.028	0.015	0.358	0.281	1.316	0.311	1.120	0.068

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	46	55	55	56	66	51
N.S.	1	1.00	1.04	0.90	1.08	1.08	1.10	1.29	1.00
time (sec)	N/A	0.032	0.019	0.360	0.284	1.639	0.533	1.786	0.091

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	59	48	53	53	58	55	52
N.S.	1	1.00	1.11	0.91	1.00	1.00	1.09	1.04	0.98
time (sec)	N/A	0.025	0.013	0.363	0.281	1.687	0.573	1.603	0.038

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	76	73	73	80	77	69
N.S.	1	1.00	1.00	1.01	0.97	0.97	1.07	1.03	0.92
time (sec)	N/A	0.044	0.010	0.568	0.274	1.540	0.014	0.937	0.060

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	69	76	73	73	82	77	69
N.S.	1	1.00	1.01	1.12	1.07	1.07	1.21	1.13	1.01
time (sec)	N/A	0.095	0.013	0.401	0.273	1.838	0.016	0.927	0.030

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	76	73	73	82	77	69
N.S.	1	1.00	1.00	1.01	0.97	0.97	1.09	1.03	0.92
time (sec)	N/A	0.036	0.008	0.413	0.287	1.431	0.014	1.442	0.030

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	69	76	73	73	80	77	69
N.S.	1	1.00	1.64	1.81	1.74	1.74	1.90	1.83	1.64
time (sec)	N/A	0.047	0.013	0.391	0.289	1.397	0.015	1.380	0.031

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	70	70	76	73	65
N.S.	1	1.00	1.00	1.04	1.00	1.00	1.09	1.04	0.93
time (sec)	N/A	0.031	0.008	0.492	0.279	1.386	0.014	1.084	0.031

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	71	76	74	71	80	78	67
N.S.	1	1.00	1.18	1.27	1.23	1.18	1.33	1.30	1.12
time (sec)	N/A	0.034	0.014	0.418	0.282	1.574	0.063	0.707	0.035

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	71	69	75	68	70	65
N.S.	1	1.00	1.00	1.09	1.06	1.15	1.05	1.08	1.00
time (sec)	N/A	0.030	0.015	0.369	0.280	1.476	0.059	0.771	0.033

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	73	73	74	77	78	97	67
N.S.	1	1.00	1.03	1.03	1.04	1.08	1.10	1.37	0.94
time (sec)	N/A	0.052	0.020	0.368	0.290	1.704	0.117	0.683	0.039

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	71	70	73	75	75	74	68
N.S.	1	1.00	1.03	1.01	1.06	1.09	1.09	1.07	0.99
time (sec)	N/A	0.033	0.017	0.375	0.273	1.678	0.126	0.706	0.057

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	73	68	76	76	75	98	76
N.S.	1	1.00	1.01	0.94	1.06	1.06	1.04	1.36	1.06
time (sec)	N/A	0.049	0.021	0.460	0.276	1.550	0.301	0.822	0.074

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	64	73	75	78	75	73
N.S.	1	1.00	1.00	0.94	1.07	1.10	1.15	1.10	1.07
time (sec)	N/A	0.033	0.017	0.377	0.278	1.419	0.336	0.754	0.057

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	64	77	77	78	99	75
N.S.	1	1.00	1.00	0.90	1.08	1.08	1.10	1.39	1.06
time (sec)	N/A	0.042	0.026	0.379	0.281	1.703	0.679	0.693	0.087

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	63	73	75	80	77	71
N.S.	1	1.00	1.00	0.95	1.11	1.14	1.21	1.17	1.08
time (sec)	N/A	0.033	0.020	0.361	0.273	1.375	0.796	0.615	0.080

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	77	64	77	77	82	90	75
N.S.	1	1.00	1.22	1.02	1.22	1.22	1.30	1.43	1.19
time (sec)	N/A	0.031	0.025	0.369	0.304	1.772	1.328	0.571	0.103

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	66	75	75	83	79	74
N.S.	1	1.00	1.00	0.90	1.03	1.03	1.14	1.08	1.01
time (sec)	N/A	0.035	0.021	0.463	0.279	1.438	2.457	0.671	0.070

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	78	66	75	75	83	79	76
N.S.	1	1.00	1.59	1.35	1.53	1.53	1.69	1.61	1.55
time (sec)	N/A	0.026	0.013	0.409	0.280	1.465	3.546	0.636	0.071

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	123	124	274	204	133	144
N.S.	1	1.00	1.00	1.03	1.04	2.30	1.71	1.12	1.21
time (sec)	N/A	0.060	0.053	0.461	0.487	1.451	0.223	0.609	0.171

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	92	98	97	98	94	101	100
N.S.	1	1.00	0.96	1.02	1.01	1.02	0.98	1.05	1.04
time (sec)	N/A	0.087	0.025	0.376	0.277	1.505	0.176	0.633	0.058

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	99	100	228	180	108	118
N.S.	1	1.00	1.00	1.01	1.02	2.33	1.84	1.10	1.20
time (sec)	N/A	0.047	0.043	0.411	0.493	1.318	0.204	0.601	0.043

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	74	74	75	70	77	76
N.S.	1	1.00	0.95	0.99	0.99	1.00	0.93	1.03	1.01
time (sec)	N/A	0.064	0.020	0.378	0.295	1.613	0.163	0.639	0.087

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	75	78	178	153	85	96
N.S.	1	1.00	1.00	0.97	1.01	2.31	1.99	1.10	1.25
time (sec)	N/A	0.042	0.035	0.398	0.492	1.482	0.185	0.621	0.066

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	47	50	50	51	46	52	52
N.S.	1	1.00	0.87	0.93	0.93	0.94	0.85	0.96	0.96
time (sec)	N/A	0.046	0.015	0.434	0.274	1.538	0.144	0.663	0.074

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	51	53	129	90	57	70
N.S.	1	1.00	0.98	0.88	0.91	2.22	1.55	0.98	1.21
time (sec)	N/A	0.031	0.027	0.391	0.501	1.547	0.167	0.660	0.107

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	32	31	30	27	32	31
N.S.	1	1.00	0.89	0.91	0.89	0.86	0.77	0.91	0.89
time (sec)	N/A	0.027	0.008	0.373	0.277	1.594	0.121	0.890	0.056

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	34	99	82	34	31
N.S.	1	1.00	1.00	0.85	0.85	2.48	2.05	0.85	0.78
time (sec)	N/A	0.018	0.016	0.422	0.493	1.843	0.136	0.683	0.054

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	35	32	26	34	32
N.S.	1	1.00	1.00	0.97	1.03	0.94	0.76	1.00	0.94
time (sec)	N/A	0.029	0.009	0.379	0.289	2.194	0.380	0.638	0.085

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	36	105	82	36	35
N.S.	1	1.00	1.00	0.88	0.86	2.50	1.95	0.86	0.83
time (sec)	N/A	0.021	0.018	0.413	0.490	2.029	0.162	0.641	0.092

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	39	51	103	75	38	33
N.S.	1	1.00	1.00	0.95	1.24	2.51	1.83	0.93	0.80
time (sec)	N/A	0.022	0.018	0.500	0.510	1.621	0.170	0.848	0.145

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	46	48	47	41	71	46
N.S.	1	1.00	1.00	0.94	0.98	0.96	0.84	1.45	0.94
time (sec)	N/A	0.038	0.016	0.438	0.296	0.808	0.387	0.852	0.137

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	60	54	56	135	129	57	53
N.S.	1	1.00	0.98	0.89	0.92	2.21	2.11	0.93	0.87
time (sec)	N/A	0.032	0.037	0.412	0.495	1.774	0.205	0.686	0.106

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	64	70	73	61	100	70
N.S.	1	1.00	1.00	0.91	1.00	1.04	0.87	1.43	1.00
time (sec)	N/A	0.050	0.021	0.385	0.276	0.924	0.444	0.781	0.137

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	74	79	184	163	81	70
N.S.	1	1.00	1.00	0.95	1.01	2.36	2.09	1.04	0.90
time (sec)	N/A	0.042	0.038	0.421	0.503	1.710	0.246	0.845	0.106

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	96	86	96	98	88	126	92
N.S.	1	1.00	1.04	0.93	1.04	1.07	0.96	1.37	1.00
time (sec)	N/A	0.063	0.027	0.382	0.275	0.955	0.482	0.682	0.153

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	134	123	136	350	238	139	203
N.S.	1	1.00	1.01	0.92	1.02	2.63	1.79	1.05	1.53
time (sec)	N/A	0.110	0.070	0.415	0.496	2.080	0.412	0.634	0.104

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	103	107	148	104	135	121
N.S.	1	1.00	0.89	0.98	1.02	1.41	0.99	1.29	1.15
time (sec)	N/A	0.095	0.047	0.383	0.267	1.088	0.393	0.736	0.097

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	111	100	112	298	211	115	141
N.S.	1	1.00	1.01	0.91	1.02	2.71	1.92	1.05	1.28
time (sec)	N/A	0.084	0.057	0.394	0.509	1.840	0.384	0.885	0.085

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	76	82	121	78	106	86
N.S.	1	1.00	0.87	0.92	0.99	1.46	0.94	1.28	1.04
time (sec)	N/A	0.071	0.037	0.410	0.277	1.307	0.359	0.816	0.072

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	75	85	240	129	88	104
N.S.	1	1.00	1.00	0.84	0.96	2.70	1.45	0.99	1.17
time (sec)	N/A	0.061	0.049	0.398	0.498	1.631	0.347	0.699	0.110

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	59	60	81	56	70	62
N.S.	1	1.00	0.82	0.97	0.98	1.33	0.92	1.15	1.02
time (sec)	N/A	0.048	0.025	0.376	0.275	1.405	0.312	1.375	0.116

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	57	61	208	114	59	59
N.S.	1	1.00	1.00	0.84	0.90	3.06	1.68	0.87	0.87
time (sec)	N/A	0.040	0.036	0.390	0.496	1.726	0.279	1.474	0.126

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	38	40	44	36	37	37
N.S.	1	1.00	1.00	0.93	0.98	1.07	0.88	0.90	0.90
time (sec)	N/A	0.033	0.009	0.375	0.270	1.514	0.181	1.325	0.086

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	57	182	112	57	51
N.S.	1	1.00	1.00	0.90	0.90	2.89	1.78	0.90	0.81
time (sec)	N/A	0.023	0.032	0.419	0.488	1.889	0.206	0.957	0.120

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	48	51	70	46	52	47
N.S.	1	1.00	0.90	0.94	1.00	1.37	0.90	1.02	0.92
time (sec)	N/A	0.040	0.021	0.371	0.274	1.483	0.208	1.201	0.159

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	62	63	210	114	62	63
N.S.	1	1.00	1.00	0.89	0.90	3.00	1.63	0.89	0.90
time (sec)	N/A	0.048	0.023	0.405	0.487	1.866	0.252	1.625	0.129

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	64	76	76	117	70	80	78
N.S.	1	1.00	0.88	1.04	1.04	1.60	0.96	1.10	1.07
time (sec)	N/A	0.057	0.036	0.383	0.277	2.019	0.493	1.410	0.147

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	78	93	250	184	85	83
N.S.	1	1.00	1.00	0.87	1.03	2.78	2.04	0.94	0.92
time (sec)	N/A	0.084	0.049	0.474	0.498	2.164	0.302	1.681	0.137

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	85	96	106	154	100	150	100
N.S.	1	1.00	0.88	0.99	1.09	1.59	1.03	1.55	1.03
time (sec)	N/A	0.074	0.066	0.447	0.276	2.102	0.558	0.849	0.138

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	112	99	119	308	218	112	104
N.S.	1	1.00	1.01	0.89	1.07	2.77	1.96	1.01	0.94
time (sec)	N/A	0.135	0.061	0.424	0.501	1.371	0.339	0.775	0.158

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	133	119	147	416	252	138	177
N.S.	1	1.00	0.95	0.85	1.05	2.97	1.80	0.99	1.26
time (sec)	N/A	0.155	0.070	0.424	0.499	2.234	0.765	0.829	0.120

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	94	102	116	179	119	132	118
N.S.	1	1.00	0.85	0.92	1.05	1.61	1.07	1.19	1.06
time (sec)	N/A	0.098	0.043	0.429	0.280	1.457	0.802	0.833	0.118

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	113	95	120	358	214	111	138
N.S.	1	1.00	0.96	0.81	1.02	3.03	1.81	0.94	1.17
time (sec)	N/A	0.109	0.058	0.417	0.485	2.379	0.705	0.700	0.078

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	92	85	94	142	94	93	95
N.S.	1	1.00	1.03	0.96	1.06	1.60	1.06	1.04	1.07
time (sec)	N/A	0.071	0.026	0.395	0.286	1.195	0.707	0.823	0.091

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	92	77	94	328	194	80	92
N.S.	1	1.00	0.97	0.81	0.99	3.45	2.04	0.84	0.97
time (sec)	N/A	0.072	0.049	0.409	0.503	2.070	0.589	0.989	0.152

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	61	72	89	70	55	70
N.S.	1	1.00	0.96	0.91	1.07	1.33	1.04	0.82	1.04
time (sec)	N/A	0.054	0.017	0.393	0.267	1.629	0.489	1.394	0.108

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	83	76	92	301	155	78	82
N.S.	1	1.00	0.92	0.84	1.02	3.34	1.72	0.87	0.91
time (sec)	N/A	0.048	0.057	0.436	0.501	2.268	0.385	1.343	0.146

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	39	42	42	42	28	44
N.S.	1	1.00	0.94	1.22	1.31	1.31	1.31	0.88	1.38
time (sec)	N/A	0.020	0.010	0.366	0.274	1.261	0.265	1.122	0.076

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	84	77	92	300	150	78	82
N.S.	1	1.00	0.91	0.84	1.00	3.26	1.63	0.85	0.89
time (sec)	N/A	0.031	0.041	0.481	0.500	2.115	0.294	2.368	0.140

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	59	63	77	119	75	76	71
N.S.	1	1.00	0.87	0.93	1.13	1.75	1.10	1.12	1.04
time (sec)	N/A	0.049	0.033	0.380	0.278	1.684	0.291	1.713	0.178

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	82	96	324	194	82	113
N.S.	1	1.00	1.00	0.85	1.00	3.38	2.02	0.85	1.18
time (sec)	N/A	0.076	0.042	0.405	0.503	1.653	0.350	1.053	0.176

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	86	102	109	197	107	105	107
N.S.	1	1.00	0.89	1.05	1.12	2.03	1.10	1.08	1.10
time (sec)	N/A	0.086	0.043	0.385	0.270	1.795	0.586	1.356	0.152

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	119	98	128	368	226	108	114
N.S.	1	1.00	1.02	0.84	1.09	3.15	1.93	0.92	0.97
time (sec)	N/A	0.123	0.049	0.428	0.486	1.276	0.399	1.016	0.179

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	108	123	137	229	136	132	131
N.S.	1	1.00	0.89	1.02	1.13	1.89	1.12	1.09	1.08
time (sec)	N/A	0.094	0.055	0.391	0.291	1.594	0.650	1.081	0.174

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	140	119	154	426	260	135	135
N.S.	1	1.00	1.00	0.85	1.10	3.04	1.86	0.96	0.96
time (sec)	N/A	0.224	0.051	0.408	0.502	1.492	0.440	0.939	0.195

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	135	143	170	267	165	201	155
N.S.	1	1.00	0.91	0.97	1.15	1.80	1.11	1.36	1.05
time (sec)	N/A	0.125	0.085	0.388	0.289	1.380	0.698	0.839	0.185

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	187	290	321	368	0	245	289
N.S.	1	1.00	0.86	1.33	1.47	1.69	0.00	1.12	1.33
time (sec)	N/A	0.246	0.289	0.410	0.298	1.922	0.000	1.044	1.470

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	168	248	273	321	0	211	233
N.S.	1	1.00	0.93	1.37	1.51	1.77	0.00	1.17	1.29
time (sec)	N/A	0.217	0.253	0.382	0.285	2.723	0.000	0.770	0.886

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	145	206	225	272	0	177	177
N.S.	1	1.00	1.16	1.65	1.80	2.18	0.00	1.42	1.42
time (sec)	N/A	0.133	0.204	0.384	0.293	1.823	0.000	1.181	0.736

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	123	164	177	223	0	140	140
N.S.	1	1.00	1.15	1.53	1.65	2.08	0.00	1.31	1.31
time (sec)	N/A	0.102	0.150	0.393	0.287	1.858	0.000	0.899	0.752

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	97	124	128	172	0	103	117
N.S.	1	1.00	0.97	1.24	1.28	1.72	0.00	1.03	1.17
time (sec)	N/A	0.130	0.111	0.432	0.288	2.039	0.000	1.129	0.609

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	132	105	161	0	92	-1
N.S.	1	1.00	0.93	1.36	1.08	1.66	0.00	0.95	-0.01
time (sec)	N/A	0.135	0.119	0.379	0.278	1.801	0.000	1.695	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	99	109	96	160	0	163	-1
N.S.	1	1.00	1.24	1.36	1.20	2.00	0.00	2.04	-0.01
time (sec)	N/A	0.125	0.105	0.390	0.281	1.212	0.000	1.291	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	44	48	111	59	0	250	113
N.S.	1	1.00	0.72	0.79	1.82	0.97	0.00	4.10	1.85
time (sec)	N/A	0.102	0.097	0.388	0.298	2.784	0.000	1.080	0.469

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	66	70	161	85	0	310	160
N.S.	1	1.00	0.69	0.73	1.68	0.89	0.00	3.23	1.67
time (sec)	N/A	0.138	0.117	0.383	0.278	1.820	0.000	1.411	0.676

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	88	94	209	109	0	370	210
N.S.	1	1.00	0.66	0.71	1.57	0.82	0.00	2.78	1.58
time (sec)	N/A	0.160	0.147	0.380	0.288	1.289	0.000	1.742	1.039

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	110	118	257	133	0	430	260
N.S.	1	1.00	0.65	0.69	1.51	0.78	0.00	2.53	1.53
time (sec)	N/A	0.198	0.164	0.396	0.304	1.743	0.000	2.196	1.406

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	82	91	106	106	0	140	103
N.S.	1	1.00	0.63	0.69	0.81	0.81	0.00	1.07	0.79
time (sec)	N/A	0.145	0.055	0.406	0.289	1.335	0.000	0.627	0.252

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	64	67	83	82	0	105	83
N.S.	1	1.00	0.68	0.71	0.88	0.87	0.00	1.12	0.88
time (sec)	N/A	0.108	0.042	0.407	0.284	1.499	0.000	0.578	0.191

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	41	45	51	57	0	72	60
N.S.	1	1.00	0.67	0.74	0.84	0.93	0.00	1.18	0.98
time (sec)	N/A	0.013	0.026	0.381	0.286	1.887	0.000	0.475	0.153

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	84	85	0	159	0	116	99
N.S.	1	1.00	1.08	1.09	0.00	2.04	0.00	1.49	1.27
time (sec)	N/A	0.099	0.082	0.389	0.000	2.443	0.000	0.589	0.498

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	94	135	0	169	0	76	-1
N.S.	1	1.00	0.94	1.35	0.00	1.69	0.00	0.76	-0.01
time (sec)	N/A	0.120	0.100	0.394	0.000	1.390	0.000	0.542	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	106	175	0	198	0	132	-1
N.S.	1	1.00	1.03	1.70	0.00	1.92	0.00	1.28	-0.01
time (sec)	N/A	0.104	0.133	0.399	0.000	1.616	0.000	0.559	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	209	328	363	418	0	280	-1
N.S.	1	1.00	0.94	1.47	1.63	1.87	0.00	1.26	-0.00
time (sec)	N/A	0.256	0.352	0.440	0.294	1.541	0.000	0.568	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	191	286	315	369	0	246	-1
N.S.	1	1.00	1.14	1.71	1.89	2.21	0.00	1.47	-0.01
time (sec)	N/A	0.170	0.303	0.388	0.285	2.227	0.000	0.873	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	165	244	267	316	0	207	236
N.S.	1	1.00	1.11	1.65	1.80	2.14	0.00	1.40	1.59
time (sec)	N/A	0.129	0.221	0.375	0.296	1.240	0.000	0.650	1.011

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	149	202	216	275	0	178	-1
N.S.	1	1.00	1.03	1.40	1.50	1.91	0.00	1.24	-0.01
time (sec)	N/A	0.170	0.200	0.373	0.284	1.776	0.000	0.571	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	124	162	168	224	0	142	-1
N.S.	1	1.00	0.91	1.18	1.23	1.64	0.00	1.04	-0.01
time (sec)	N/A	0.194	0.180	0.378	0.290	1.109	0.000	0.608	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	108	174	148	209	0	126	-1
N.S.	1	1.00	0.84	1.36	1.16	1.63	0.00	0.98	-0.01
time (sec)	N/A	0.176	0.191	0.373	0.298	2.269	0.000	0.614	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	112	219	167	189	0	225	-1
N.S.	1	1.00	0.82	1.61	1.23	1.39	0.00	1.65	-0.01
time (sec)	N/A	0.177	0.203	0.380	0.300	1.073	0.000	0.865	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	110	153	177	207	0	254	-1
N.S.	1	1.00	1.06	1.47	1.70	1.99	0.00	2.44	-0.01
time (sec)	N/A	0.161	0.165	0.380	0.285	1.858	0.000	1.201	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	44	48	193	82	0	370	156
N.S.	1	1.00	0.72	0.79	3.16	1.34	0.00	6.07	2.56
time (sec)	N/A	0.113	0.156	0.503	0.287	1.132	0.000	1.866	1.031

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	66	70	241	109	0	430	206
N.S.	1	1.00	0.69	0.73	2.51	1.14	0.00	4.48	2.15
time (sec)	N/A	0.150	0.184	0.378	0.288	1.753	0.000	2.775	1.439

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	89	94	289	134	0	490	256
N.S.	1	1.00	0.67	0.71	2.17	1.01	0.00	3.68	1.92
time (sec)	N/A	0.186	0.226	0.395	0.298	1.501	0.000	2.562	1.925

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	110	118	337	157	0	550	306
N.S.	1	1.00	0.65	0.69	1.98	0.92	0.00	3.24	1.80
time (sec)	N/A	0.217	0.260	0.389	0.287	1.816	0.000	2.740	2.503

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	132	142	385	181	0	582	356
N.S.	1	1.00	0.64	0.69	1.86	0.87	0.00	2.81	1.72
time (sec)	N/A	0.241	0.320	0.687	0.292	1.609	0.000	5.364	3.210

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	113	115	150	154	0	175	143
N.S.	1	1.00	0.67	0.68	0.89	0.92	0.00	1.04	0.85
time (sec)	N/A	0.198	0.076	0.392	0.289	1.503	0.000	0.874	0.349

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	94	91	128	131	0	140	124
N.S.	1	1.00	0.72	0.69	0.98	1.00	0.00	1.07	0.95
time (sec)	N/A	0.163	0.065	0.368	0.297	1.578	0.000	1.062	0.292

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	70	67	105	106	0	105	103
N.S.	1	1.00	0.73	0.70	1.09	1.10	0.00	1.09	1.07
time (sec)	N/A	0.048	0.048	0.395	0.288	1.588	0.000	1.499	0.255

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	41	45	80	80	0	72	83
N.S.	1	1.00	0.67	0.74	1.31	1.31	0.00	1.18	1.36
time (sec)	N/A	0.107	0.032	0.363	0.277	1.579	0.000	1.342	0.229

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	108	99	0	206	0	140	-1
N.S.	1	1.00	1.06	0.97	0.00	2.02	0.00	1.37	-0.01
time (sec)	N/A	0.142	0.119	0.360	0.000	1.603	0.000	1.524	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	109	172	0	195	0	115	-1
N.S.	1	1.00	0.82	1.29	0.00	1.47	0.00	0.86	-0.01
time (sec)	N/A	0.150	0.136	0.396	0.000	1.712	0.000	1.191	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	114	213	0	217	0	145	-1
N.S.	1	1.00	0.84	1.58	0.00	1.61	0.00	1.07	-0.01
time (sec)	N/A	0.159	0.154	0.383	0.000	1.448	0.000	0.972	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	132	259	0	250	0	175	-1
N.S.	1	1.00	0.94	1.85	0.00	1.79	0.00	1.25	-0.01
time (sec)	N/A	0.145	0.212	0.399	0.000	1.675	0.000	1.058	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	154	302	0	299	0	214	-1
N.S.	1	1.00	0.87	1.71	0.00	1.69	0.00	1.21	-0.01
time (sec)	N/A	0.180	0.267	0.394	0.000	1.473	0.000	1.250	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	172	344	0	345	0	234	-1
N.S.	1	1.00	0.80	1.61	0.00	1.61	0.00	1.09	-0.00
time (sec)	N/A	0.226	0.335	0.398	0.000	1.436	0.000	0.943	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	198	386	0	393	0	294	-1
N.S.	1	1.00	0.79	1.54	0.00	1.57	0.00	1.17	-0.00
time (sec)	N/A	0.258	0.363	0.405	0.000	3.391	0.000	1.214	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	147	211	231	275	0	184	-1
N.S.	1	1.00	0.84	1.20	1.31	1.56	0.00	1.05	-0.01
time (sec)	N/A	0.205	0.179	0.394	0.287	2.423	0.000	0.694	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	125	169	183	226	0	150	-1
N.S.	1	1.00	0.90	1.22	1.32	1.63	0.00	1.08	-0.01
time (sec)	N/A	0.178	0.147	0.443	0.283	1.414	0.000	0.705	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	99	127	134	177	0	112	-1
N.S.	1	1.00	1.19	1.53	1.61	2.13	0.00	1.35	-0.01
time (sec)	N/A	0.112	0.104	0.392	0.288	1.669	0.000	0.616	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	82	88	88	131	0	77	89
N.S.	1	1.00	1.24	1.33	1.33	1.98	0.00	1.17	1.35
time (sec)	N/A	0.085	0.063	0.401	0.306	0.925	0.000	0.929	0.814

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	77	67	56	136	0	66	57
N.S.	1	1.00	1.35	1.18	0.98	2.39	0.00	1.16	1.00
time (sec)	N/A	0.095	0.063	0.384	0.312	1.670	0.000	0.564	0.509

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	43	47	70	38	0	124	39
N.S.	1	1.00	0.70	0.77	1.15	0.62	0.00	2.03	0.64
time (sec)	N/A	0.105	0.075	0.389	0.277	0.908	0.000	0.662	0.197

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	64	70	119	62	0	180	62
N.S.	1	1.00	0.67	0.73	1.24	0.65	0.00	1.88	0.65
time (sec)	N/A	0.136	0.097	0.378	0.338	1.667	0.000	0.760	0.251

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	89	94	167	86	0	236	121
N.S.	1	1.00	0.67	0.71	1.26	0.65	0.00	1.77	0.91
time (sec)	N/A	0.167	0.118	0.387	0.406	1.280	0.000	1.081	0.301

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	110	118	215	110	0	292	156
N.S.	1	1.00	0.65	0.69	1.26	0.65	0.00	1.72	0.92
time (sec)	N/A	0.193	0.141	0.390	0.275	1.740	0.000	1.443	0.322

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	85	89	106	83	0	130	87
N.S.	1	1.00	0.65	0.68	0.81	0.63	0.00	0.99	0.66
time (sec)	N/A	0.155	0.057	0.373	0.283	1.018	0.000	0.474	0.265

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	63	65	83	59	0	98	64
N.S.	1	1.00	0.67	0.69	0.88	0.63	0.00	1.04	0.68
time (sec)	N/A	0.116	0.042	0.377	0.285	1.377	0.000	0.515	0.228

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	40	42	50	36	0	67	41
N.S.	1	1.00	0.68	0.71	0.85	0.61	0.00	1.14	0.69
time (sec)	N/A	0.085	0.029	0.372	0.288	0.882	0.000	0.480	0.187

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	73	72	0	138	0	80	-1
N.S.	1	1.00	1.33	1.31	0.00	2.51	0.00	1.45	-0.02
time (sec)	N/A	0.014	0.046	0.372	0.000	1.291	0.000	0.475	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	86	105	0	152	0	66	-1
N.S.	1	1.00	1.26	1.54	0.00	2.24	0.00	0.97	-0.01
time (sec)	N/A	0.074	0.081	0.523	0.000	1.723	0.000	0.500	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	104	146	0	199	0	125	-1
N.S.	1	1.00	1.01	1.42	0.00	1.93	0.00	1.21	-0.01
time (sec)	N/A	0.103	0.120	0.445	0.000	1.041	0.000	0.473	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	139	166	237	340	0	183	-1
N.S.	1	1.00	0.76	0.90	1.29	1.85	0.00	0.99	-0.01
time (sec)	N/A	0.217	0.192	0.388	0.283	1.063	0.000	0.475	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	114	140	187	289	0	145	-1
N.S.	1	1.00	0.78	0.95	1.27	1.97	0.00	0.99	-0.01
time (sec)	N/A	0.180	0.155	0.391	0.281	1.205	0.000	0.465	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	90	114	138	230	0	104	-1
N.S.	1	1.00	0.80	1.02	1.23	2.05	0.00	0.93	-0.01
time (sec)	N/A	0.153	0.121	0.395	0.281	2.558	0.000	0.428	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	77	75	79	188	0	70	78
N.S.	1	1.00	1.15	1.12	1.18	2.81	0.00	1.04	1.16
time (sec)	N/A	0.109	0.079	0.367	0.287	2.740	0.000	0.439	0.633

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	47	65	49	0	65	53
N.S.	1	1.00	1.00	1.27	1.76	1.32	0.00	1.76	1.43
time (sec)	N/A	0.078	0.067	0.388	0.287	1.862	0.000	0.462	0.181

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	66	112	72	0	189	70
N.S.	1	1.00	0.97	1.00	1.70	1.09	0.00	2.86	1.06
time (sec)	N/A	0.104	0.100	0.387	0.284	1.388	0.000	0.669	0.274

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	85	94	160	98	0	302	95
N.S.	1	1.00	0.84	0.93	1.58	0.97	0.00	2.99	0.94
time (sec)	N/A	0.140	0.131	0.392	0.288	1.798	0.000	1.258	0.424

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	108	118	208	121	0	415	173
N.S.	1	1.00	0.78	0.86	1.51	0.88	0.00	3.01	1.25
time (sec)	N/A	0.173	0.158	0.391	0.282	0.997	0.000	2.027	0.559

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	82	91	82	93	0	145	92
N.S.	1	1.00	0.59	0.65	0.59	0.67	0.00	1.04	0.66
time (sec)	N/A	0.162	0.059	0.385	0.280	1.839	0.000	0.837	0.397

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	60	66	59	68	0	107	67
N.S.	1	1.00	0.58	0.63	0.57	0.65	0.00	1.03	0.64
time (sec)	N/A	0.131	0.044	0.376	0.287	1.476	0.000	1.392	0.280

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	35	44	39	45	0	63	44
N.S.	1	1.00	0.51	0.64	0.57	0.65	0.00	0.91	0.64
time (sec)	N/A	0.095	0.030	0.379	0.281	1.563	0.000	1.236	0.210

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	73	79	0	199	0	109	-1
N.S.	1	1.00	1.14	1.23	0.00	3.11	0.00	1.70	-0.02
time (sec)	N/A	0.087	0.067	0.534	0.000	1.655	0.000	0.925	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	96	130	0	260	0	107	-1
N.S.	1	1.00	0.68	0.92	0.00	1.83	0.00	0.75	-0.01
time (sec)	N/A	0.057	0.110	0.447	0.000	1.635	0.000	0.637	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	117	157	0	315	0	149	89
N.S.	1	1.00	0.85	1.15	0.00	2.30	0.00	1.09	0.65
time (sec)	N/A	0.124	0.160	0.397	0.000	1.178	0.000	0.519	1.263

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.014	0.024	0.142	0.268	1.362	1.088	0.464	0.056

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.014	0.026	0.135	0.264	2.259	0.731	0.469	0.106

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	28	27	32	46	29	31
N.S.	1	1.00	1.05	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.014	0.021	0.126	0.263	2.496	0.472	0.469	0.044

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	37	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	0.95	0.74	0.79
time (sec)	N/A	0.015	0.022	0.136	0.267	1.879	1.117	0.452	0.042

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.015	0.019	0.130	0.268	1.748	0.269	0.456	0.044

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	30	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.77	1.18	0.74	0.79
time (sec)	N/A	0.015	0.019	0.128	0.268	1.869	0.317	0.462	0.043

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	28	27	29	44	29	31
N.S.	1	1.00	0.95	0.76	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.015	0.022	0.129	0.266	1.630	0.367	0.569	0.042

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	30	27	29	44	29	31
N.S.	1	1.00	0.95	0.81	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.014	0.024	0.059	0.268	1.929	0.541	0.570	0.103

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.026	0.036	0.349	0.267	1.871	2.236	0.486	0.130

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.026	0.034	0.362	0.272	1.605	1.567	0.443	0.050

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.027	0.033	0.354	0.267	1.548	1.097	0.465	0.048

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	66	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.05	0.84	0.81
time (sec)	N/A	0.026	0.032	0.387	0.266	1.682	1.781	0.460	0.049

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.027	0.032	0.359	0.264	1.744	0.742	0.444	0.048

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.027	0.032	0.352	0.270	1.627	0.861	0.468	0.048

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.029	0.034	0.357	0.263	1.550	1.018	0.428	0.049

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	54	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.86	1.27	0.84	0.81
time (sec)	N/A	0.026	0.031	0.350	0.266	1.938	1.316	0.435	0.051

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	76	73	78	114	77	69
N.S.	1	1.00	1.00	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.035	0.072	0.358	0.267	1.828	3.957	0.463	0.102

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	76	73	78	114	77	69
N.S.	1	1.00	1.00	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.034	0.070	0.365	0.266	2.192	2.920	0.441	0.033

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	97	76	73	78	114	77	69
N.S.	1	1.00	1.14	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.033	0.047	0.363	0.266	1.428	2.192	0.470	0.035

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	83	76	73	78	95	77	69
N.S.	1	1.00	0.98	0.89	0.86	0.92	1.12	0.91	0.81
time (sec)	N/A	0.034	0.048	0.354	0.321	1.832	2.649	0.428	0.033

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	83	76	73	78	114	77	69
N.S.	1	1.00	0.98	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.034	0.048	0.365	0.322	1.352	1.627	0.477	0.033

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	83	76	73	78	114	77	69
N.S.	1	1.00	0.98	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.034	0.043	0.352	0.265	1.691	1.883	0.433	0.033

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	83	76	73	78	114	77	69
N.S.	1	1.00	0.98	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.033	0.044	0.354	0.274	1.574	2.020	0.464	0.036

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	83	76	73	78	114	77	69
N.S.	1	1.00	0.98	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.034	0.041	0.358	0.297	3.025	2.391	0.446	0.033

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	172	165	237	920	0	298	115
N.S.	1	1.00	0.62	0.59	0.85	3.31	0.00	1.07	0.41
time (sec)	N/A	0.188	0.216	0.446	0.490	2.488	0.000	0.486	0.227

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	173	164	259	714	0	298	788
N.S.	1	1.00	0.63	0.59	0.94	2.59	0.00	1.08	2.86
time (sec)	N/A	0.173	0.233	0.376	0.493	2.033	0.000	0.441	0.252

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	152	142	214	899	0	264	92
N.S.	1	1.00	0.59	0.55	0.83	3.50	0.00	1.03	0.36
time (sec)	N/A	0.144	0.172	0.417	0.534	2.845	0.000	0.428	0.236

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	150	141	235	660	275	263	789
N.S.	1	1.00	0.59	0.55	0.92	2.59	1.08	1.03	3.09
time (sec)	N/A	0.141	0.171	0.389	0.514	2.486	104.003	0.438	0.275

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	135	124	194	834	303	251	71
N.S.	1	1.00	0.57	0.52	0.82	3.52	1.28	1.06	0.30
time (sec)	N/A	0.124	0.154	0.396	0.520	2.083	38.972	0.442	0.153

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	134	127	218	645	238	251	739
N.S.	1	1.00	0.57	0.54	0.93	2.74	1.01	1.07	3.14
time (sec)	N/A	0.126	0.151	0.377	0.519	2.515	16.399	0.432	0.292

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	135	127	194	843	309	251	71
N.S.	1	1.00	0.57	0.54	0.83	3.59	1.31	1.07	0.30
time (sec)	N/A	0.131	0.195	0.410	0.536	2.966	8.918	0.440	0.222

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	136	124	218	653	257	251	811
N.S.	1	1.00	0.57	0.52	0.92	2.76	1.08	1.06	3.42
time (sec)	N/A	0.129	0.201	0.400	0.500	2.938	14.198	0.428	0.301

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	152	140	213	883	258	268	90
N.S.	1	1.00	0.60	0.55	0.84	3.46	1.01	1.05	0.35
time (sec)	N/A	0.145	0.199	0.397	0.532	2.499	71.031	0.510	0.226

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	154	141	247	707	303	257	555
N.S.	1	1.00	0.60	0.55	0.96	2.75	1.18	1.00	2.16
time (sec)	N/A	0.147	0.210	0.397	0.494	2.657	70.478	0.520	0.320

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	174	158	237	931	0	291	107
N.S.	1	1.00	0.63	0.57	0.86	3.37	0.00	1.05	0.39
time (sec)	N/A	0.167	0.247	0.402	0.512	2.462	0.000	0.472	0.234

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	176	160	276	734	0	291	563
N.S.	1	1.00	0.63	0.58	0.99	2.64	0.00	1.05	2.03
time (sec)	N/A	0.164	0.250	0.400	0.546	2.716	0.000	0.595	0.351

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	206	196	298	804	0	335	857
N.S.	1	1.00	0.62	0.59	0.90	2.42	0.00	1.01	2.58
time (sec)	N/A	0.185	0.535	0.553	0.522	2.631	0.000	0.443	0.219

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	184	171	247	989	0	299	127
N.S.	1	1.00	0.59	0.55	0.80	3.19	0.00	0.96	0.41
time (sec)	N/A	0.165	0.509	0.504	0.493	2.789	0.000	0.434	0.171

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	183	171	271	748	0	298	823
N.S.	1	1.00	0.59	0.55	0.87	2.41	0.00	0.96	2.65
time (sec)	N/A	0.163	0.509	0.408	0.537	2.041	0.000	0.438	0.259

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	162	153	223	925	0	283	106
N.S.	1	1.00	0.56	0.53	0.77	3.20	0.00	0.98	0.37
time (sec)	N/A	0.148	0.512	0.413	0.486	1.691	0.000	0.468	0.247

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	161	152	250	725	0	283	744
N.S.	1	1.00	0.56	0.53	0.87	2.51	0.00	0.98	2.57
time (sec)	N/A	0.150	0.498	0.407	0.529	1.925	0.000	0.450	0.212

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	152	146	217	912	0	273	91
N.S.	1	1.00	0.58	0.56	0.83	3.49	0.00	1.05	0.35
time (sec)	N/A	0.130	0.469	0.386	0.547	1.900	0.000	0.505	0.229

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	151	146	241	717	0	273	750
N.S.	1	1.00	0.58	0.56	0.92	2.75	0.00	1.05	2.87
time (sec)	N/A	0.131	0.456	0.369	0.520	2.058	0.000	0.453	0.331

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	162	153	222	920	0	278	104
N.S.	1	1.00	0.57	0.54	0.78	3.24	0.00	0.98	0.37
time (sec)	N/A	0.149	0.513	0.398	0.537	1.756	0.000	0.448	0.232

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	165	153	251	741	0	283	859
N.S.	1	1.00	0.57	0.53	0.87	2.56	0.00	0.98	2.97
time (sec)	N/A	0.157	0.514	0.415	0.516	2.181	0.000	0.452	0.372

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	187	170	250	974	0	303	121
N.S.	1	1.00	0.60	0.55	0.81	3.14	0.00	0.98	0.39
time (sec)	N/A	0.165	0.516	0.398	0.529	2.295	0.000	0.466	0.246

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	187	170	286	795	0	292	595
N.S.	1	1.00	0.60	0.55	0.92	2.56	0.00	0.94	1.92
time (sec)	N/A	0.170	0.535	0.402	0.520	2.087	0.000	0.447	0.399

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	209	190	276	1024	0	328	142
N.S.	1	1.00	0.63	0.57	0.83	3.08	0.00	0.99	0.43
time (sec)	N/A	0.188	0.580	0.409	0.513	2.697	0.000	0.476	0.200

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	204	191	306	817	0	321	865
N.S.	1	1.00	0.59	0.56	0.89	2.38	0.00	0.94	2.52
time (sec)	N/A	0.186	0.528	0.406	0.548	2.471	0.000	0.472	0.249

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	184	173	256	993	0	304	138
N.S.	1	1.00	0.57	0.54	0.80	3.08	0.00	0.94	0.43
time (sec)	N/A	0.171	0.581	0.528	0.491	2.441	0.000	0.461	0.261

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	184	172	283	793	0	304	760
N.S.	1	1.00	0.57	0.53	0.88	2.46	0.00	0.94	2.36
time (sec)	N/A	0.178	0.573	0.472	0.514	2.560	0.000	0.477	0.228

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	171	166	251	990	0	293	122
N.S.	1	1.00	0.58	0.57	0.86	3.38	0.00	1.00	0.42
time (sec)	N/A	0.153	0.592	0.380	0.512	2.173	0.000	0.470	0.174

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	173	167	280	806	0	298	799
N.S.	1	1.00	0.58	0.56	0.94	2.70	0.00	1.00	2.68
time (sec)	N/A	0.192	0.579	0.367	0.521	2.239	0.000	0.472	0.383

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	175	168	253	1005	0	298	124
N.S.	1	1.00	0.59	0.56	0.85	3.37	0.00	1.00	0.42
time (sec)	N/A	0.316	0.595	0.374	0.550	2.262	0.000	0.447	0.165

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	172	166	276	793	0	293	780
N.S.	1	1.00	0.59	0.57	0.94	2.71	0.00	1.00	2.66
time (sec)	N/A	0.332	0.593	0.372	0.536	3.292	0.000	0.507	0.392

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	187	173	255	988	0	300	133
N.S.	1	1.00	0.59	0.55	0.81	3.13	0.00	0.95	0.42
time (sec)	N/A	0.278	0.640	0.411	0.524	2.740	0.000	0.465	0.261

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	187	173	285	809	0	304	888
N.S.	1	1.00	0.58	0.54	0.89	2.51	0.00	0.94	2.76
time (sec)	N/A	0.177	0.538	0.408	0.503	2.615	0.000	0.563	0.446

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	209	190	285	1043	0	326	152
N.S.	1	1.00	0.61	0.55	0.83	3.04	0.00	0.95	0.44
time (sec)	N/A	0.194	0.554	0.418	0.534	2.262	0.000	0.541	0.194

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	210	190	321	864	0	315	626
N.S.	1	1.00	0.61	0.55	0.94	2.52	0.00	0.92	1.83
time (sec)	N/A	0.199	0.560	0.413	0.538	2.756	0.000	0.617	0.449

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	231	210	311	1093	0	351	173
N.S.	1	1.00	0.63	0.58	0.85	2.99	0.00	0.96	0.47
time (sec)	N/A	0.214	0.594	0.416	0.547	2.755	0.000	0.550	0.289

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	231	210	353	894	0	351	639
N.S.	1	1.00	0.63	0.58	0.97	2.45	0.00	0.96	1.75
time (sec)	N/A	0.216	0.584	0.412	0.545	2.153	0.000	0.512	0.507

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	136	307	0	123	0	0	-1
N.S.	1	1.00	0.56	1.26	0.00	0.51	0.00	0.00	-0.00
time (sec)	N/A	0.250	10.134	0.422	0.000	0.332	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	111	446	0	103	0	0	-1
N.S.	1	1.00	0.30	1.21	0.00	0.28	0.00	0.00	-0.00
time (sec)	N/A	0.307	10.116	0.510	0.000	0.325	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	111	283	0	98	0	0	-1
N.S.	1	1.00	0.54	1.39	0.00	0.48	0.00	0.00	-0.00
time (sec)	N/A	0.204	10.087	0.434	0.000	0.768	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	94	422	0	77	0	0	-1
N.S.	1	1.00	0.29	1.29	0.00	0.24	0.00	0.00	-0.00
time (sec)	N/A	0.263	10.072	0.384	0.000	0.543	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	94	257	0	74	0	0	-1
N.S.	1	1.00	0.57	1.56	0.00	0.45	0.00	0.00	-0.01
time (sec)	N/A	0.170	10.044	0.383	0.000	0.524	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	97	399	0	71	0	0	-1
N.S.	1	1.00	0.30	1.24	0.00	0.22	0.00	0.00	-0.00
time (sec)	N/A	0.246	9.145	0.389	0.000	0.450	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	97	239	0	61	0	0	-1
N.S.	1	1.00	0.60	1.47	0.00	0.37	0.00	0.00	-0.01
time (sec)	N/A	0.166	9.371	0.381	0.000	0.608	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	96	422	0	76	0	0	-1
N.S.	1	1.00	0.29	1.29	0.00	0.23	0.00	0.00	-0.00
time (sec)	N/A	0.249	10.032	0.392	0.000	0.525	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	98	255	0	71	0	0	-1
N.S.	1	1.00	0.59	1.53	0.00	0.43	0.00	0.00	-0.01
time (sec)	N/A	0.170	10.042	0.391	0.000	0.583	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	99	452	0	103	0	0	-1
N.S.	1	1.00	0.27	1.22	0.00	0.28	0.00	0.00	-0.00
time (sec)	N/A	0.294	10.046	0.391	0.000	0.552	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	98	283	0	96	0	0	-1
N.S.	1	1.00	0.48	1.39	0.00	0.47	0.00	0.00	-0.00
time (sec)	N/A	0.204	10.054	0.392	0.000	0.736	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	486	160	518	0	175	0	0	-1
N.S.	1	1.00	0.33	1.07	0.00	0.36	0.00	0.00	-0.00
time (sec)	N/A	0.457	10.161	0.397	0.000	0.789	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	160	355	0	170	0	0	-1
N.S.	1	1.00	0.50	1.11	0.00	0.53	0.00	0.00	-0.00
time (sec)	N/A	0.335	10.148	0.392	0.000	0.990	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	138	494	0	150	0	0	-1
N.S.	1	1.00	0.31	1.11	0.00	0.34	0.00	0.00	-0.00
time (sec)	N/A	0.407	10.134	0.380	0.000	0.569	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	138	331	0	147	0	0	-1
N.S.	1	1.00	0.49	1.17	0.00	0.52	0.00	0.00	-0.00
time (sec)	N/A	0.296	10.130	0.534	0.000	0.962	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	115	470	0	127	0	0	-1
N.S.	1	1.00	0.28	1.15	0.00	0.31	0.00	0.00	-0.00
time (sec)	N/A	0.367	10.121	0.420	0.000	0.738	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	115	307	0	121	0	0	-1
N.S.	1	1.00	0.48	1.28	0.00	0.51	0.00	0.00	-0.00
time (sec)	N/A	0.260	10.120	0.385	0.000	0.539	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	98	446	0	102	0	0	-1
N.S.	1	1.00	0.27	1.21	0.00	0.28	0.00	0.00	-0.00
time (sec)	N/A	0.314	10.071	0.386	0.000	0.714	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	97	283	0	97	0	0	-1
N.S.	1	1.00	0.48	1.41	0.00	0.48	0.00	0.00	-0.00
time (sec)	N/A	0.218	10.047	0.381	0.000	0.810	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	85	429	0	94	0	0	-1
N.S.	1	1.00	0.24	1.21	0.00	0.26	0.00	0.00	-0.00
time (sec)	N/A	0.312	10.050	0.385	0.000	0.777	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	101	260	0	86	0	0	-1
N.S.	1	1.00	0.50	1.30	0.00	0.43	0.00	0.00	-0.00
time (sec)	N/A	0.214	10.053	0.394	0.000	0.627	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	99	427	0	81	0	0	-1
N.S.	1	1.00	0.28	1.21	0.00	0.23	0.00	0.00	-0.00
time (sec)	N/A	0.323	10.038	0.398	0.000	0.657	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	101	254	0	75	0	0	-1
N.S.	1	1.00	0.50	1.25	0.00	0.37	0.00	0.00	-0.00
time (sec)	N/A	0.223	10.043	0.390	0.000	0.421	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	100	452	0	102	0	0	-1
N.S.	1	1.00	0.27	1.24	0.00	0.28	0.00	0.00	-0.00
time (sec)	N/A	0.307	10.037	0.399	0.000	0.591	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	143	298	0	122	0	0	-1
N.S.	1	1.00	0.59	1.23	0.00	0.50	0.00	0.00	-0.00
time (sec)	N/A	0.256	10.131	0.396	0.000	0.619	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	122	437	0	102	0	0	-1
N.S.	1	1.00	0.33	1.18	0.00	0.28	0.00	0.00	-0.00
time (sec)	N/A	0.314	10.106	0.391	0.000	0.650	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	122	274	0	99	0	0	-1
N.S.	1	1.00	0.60	1.34	0.00	0.49	0.00	0.00	-0.00
time (sec)	N/A	0.210	10.107	0.373	0.000	0.518	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	97	413	0	79	0	0	-1
N.S.	1	1.00	0.29	1.25	0.00	0.24	0.00	0.00	-0.00
time (sec)	N/A	0.266	10.103	0.504	0.000	0.532	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	97	248	0	73	0	0	-1
N.S.	1	1.00	0.58	1.49	0.00	0.44	0.00	0.00	-0.01
time (sec)	N/A	0.169	10.095	0.435	0.000	0.674	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	81	378	0	55	0	0	-1
N.S.	1	1.00	0.28	1.29	0.00	0.19	0.00	0.00	-0.00
time (sec)	N/A	0.219	10.065	0.379	0.000	0.413	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	80	216	0	51	0	0	-1
N.S.	1	1.00	0.62	1.66	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.140	10.061	0.383	0.000	0.327	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	82	377	0	62	0	0	-1
N.S.	1	1.00	0.29	1.34	0.00	0.22	0.00	0.00	-0.00
time (sec)	N/A	0.219	10.046	0.394	0.000	0.423	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	82	219	0	57	0	0	-1
N.S.	1	1.00	0.63	1.67	0.00	0.44	0.00	0.00	-0.01
time (sec)	N/A	0.140	10.046	0.392	0.000	0.462	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	83	413	0	76	0	0	-1
N.S.	1	1.00	0.25	1.24	0.00	0.23	0.00	0.00	-0.00
time (sec)	N/A	0.283	10.041	0.395	0.000	0.525	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	85	247	0	69	0	0	-1
N.S.	1	1.00	0.51	1.48	0.00	0.41	0.00	0.00	-0.01
time (sec)	N/A	0.167	10.051	0.407	0.000	0.326	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	84	443	0	104	0	0	-1
N.S.	1	1.00	0.23	1.20	0.00	0.28	0.00	0.00	-0.00
time (sec)	N/A	0.315	10.043	0.420	0.000	0.350	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	84	274	0	96	0	0	-1
N.S.	1	1.00	0.41	1.34	0.00	0.47	0.00	0.00	-0.00
time (sec)	N/A	0.217	10.057	0.414	0.000	0.558	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	134	281	0	156	0	0	-1
N.S.	1	1.00	0.53	1.12	0.00	0.62	0.00	0.00	-0.00
time (sec)	N/A	0.266	10.130	0.437	0.000	0.692	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	110	420	0	134	0	0	-1
N.S.	1	1.00	0.29	1.11	0.00	0.36	0.00	0.00	-0.00
time (sec)	N/A	0.324	10.112	0.428	0.000	0.620	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	110	255	0	129	0	0	-1
N.S.	1	1.00	0.51	1.19	0.00	0.60	0.00	0.00	-0.00
time (sec)	N/A	0.218	10.122	0.409	0.000	0.710	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	85	394	0	106	0	0	-1
N.S.	1	1.00	0.25	1.16	0.00	0.31	0.00	0.00	-0.00
time (sec)	N/A	0.279	10.098	0.523	0.000	0.698	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	86	230	0	102	0	0	-1
N.S.	1	1.00	0.48	1.29	0.00	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.184	10.096	0.477	0.000	0.415	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	78	388	0	98	0	0	-1
N.S.	1	1.00	0.26	1.30	0.00	0.33	0.00	0.00	-0.00
time (sec)	N/A	0.227	10.070	0.378	0.000	0.557	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	76	222	0	90	0	0	-1
N.S.	1	1.00	0.55	1.62	0.00	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.147	10.043	0.372	0.000	0.256	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	77	392	0	115	0	0	-1
N.S.	1	1.00	0.24	1.23	0.00	0.36	0.00	0.00	-0.00
time (sec)	N/A	0.269	10.038	0.408	0.000	0.677	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	92	235	0	111	0	0	-1
N.S.	1	1.00	0.55	1.41	0.00	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.173	10.052	0.398	0.000	0.509	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	79	420	0	134	0	0	-1
N.S.	1	1.00	0.21	1.14	0.00	0.36	0.00	0.00	-0.00
time (sec)	N/A	0.318	10.046	0.409	0.000	0.337	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	79	254	0	126	0	0	-1
N.S.	1	1.00	0.39	1.25	0.00	0.62	0.00	0.00	-0.00
time (sec)	N/A	0.219	10.046	0.405	0.000	0.272	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	79	450	0	163	0	0	-1
N.S.	1	1.00	0.20	1.11	0.00	0.40	0.00	0.00	-0.00
time (sec)	N/A	0.360	10.038	0.424	0.000	0.576	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	89	474	129	381	2077	603	291
N.S.	1	1.00	0.93	4.94	1.34	3.97	21.64	6.28	3.03
time (sec)	N/A	0.049	0.114	0.375	0.307	2.417	1.065	1.411	0.477

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	262	91	217	1051	340	179
N.S.	1	1.00	0.93	3.69	1.28	3.06	14.80	4.79	2.52
time (sec)	N/A	0.037	0.075	0.363	0.281	1.902	0.616	1.342	0.340

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	55	53	94	415	149	97
N.S.	1	1.00	0.93	1.22	1.18	2.09	9.22	3.31	2.16
time (sec)	N/A	0.019	0.052	0.034	0.274	2.460	0.330	1.035	0.247

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	55	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.075	0.067	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	92	131	0	0	0	0	0	-1
N.S.	1	0.94	1.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.206	0.054	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	126	135	0	0	0	0	0	-1
N.S.	1	0.90	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.117	0.069	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	63	0	112	140	0	0	-1
N.S.	1	1.00	0.66	0.00	1.18	1.47	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.089	0.196	0.375	2.015	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	111	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.147	1.554	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	210	0	0	0	0	0	-1
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.239	0.190	0.155	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	17	17	13
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.89	0.89	0.68
time (sec)	N/A	0.019	0.004	0.393	0.493	1.510	0.031	0.531	0.171

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	36	35	11	10	16	11
N.S.	1	1.00	1.00	2.40	2.33	0.73	0.67	1.07	0.73
time (sec)	N/A	0.015	0.004	0.407	0.516	1.743	0.038	0.612	0.153

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	16	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.94	0.76
time (sec)	N/A	0.017	0.004	0.405	0.492	1.642	0.046	0.500	0.139

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	18	10	16	14
N.S.	1	1.00	1.00	0.94	0.88	1.12	0.62	1.00	0.88
time (sec)	N/A	0.010	0.003	0.423	0.298	2.225	0.027	0.497	0.129

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.093	0.018	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	0	37	43	0	48	-1
N.S.	1	1.00	1.00	0.00	2.06	2.39	0.00	2.67	-0.06
time (sec)	N/A	0.026	0.279	0.187	0.332	1.721	0.000	0.662	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.175	0.054	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	157	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.214	0.076	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	119	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.160	0.059	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.123	0.055	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.131	0.063	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.140	0.066	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	112	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.198	0.070	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	184	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.115	0.256	0.087	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.154	0.056	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	179	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.165	0.237	0.063	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	120	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.185	0.061	0.000	0.000	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.109	0.064	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	88	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.172	0.067	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	125	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.169	0.071	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	182	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.181	0.294	0.081	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [283] had the largest ratio of [39]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	22	0.091
2	A	4	3	1.00	20	0.150
3	A	3	2	1.00	19	0.105
4	A	4	3	1.00	22	0.136
5	A	3	2	1.00	22	0.091
6	A	4	3	1.00	22	0.136
7	A	3	2	1.00	22	0.091
8	A	4	3	1.00	22	0.136
9	A	3	2	1.00	22	0.091
10	A	4	3	1.00	22	0.136
11	A	3	2	1.00	22	0.091
12	A	3	2	1.00	21	0.095
13	A	4	3	1.00	24	0.125
14	A	3	2	1.00	24	0.083
15	A	4	3	1.00	24	0.125
16	A	3	2	1.00	24	0.083
17	A	5	4	1.00	24	0.167
18	A	3	2	1.00	24	0.083
19	A	4	3	1.00	24	0.125
20	A	3	2	1.00	24	0.083
21	A	4	3	1.00	24	0.125
22	A	3	2	1.00	24	0.083
23	A	4	3	1.00	24	0.125
24	A	3	2	1.00	24	0.083
25	A	3	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	4	3	1.00	24	0.125
27	A	3	2	1.00	24	0.083
28	A	4	3	1.00	24	0.125
29	A	3	2	1.00	24	0.083
30	A	5	4	1.00	24	0.167
31	A	3	2	1.00	24	0.083
32	A	4	3	1.00	24	0.125
33	A	3	2	1.00	24	0.083
34	A	4	3	1.00	24	0.125
35	A	3	2	1.00	24	0.083
36	A	4	3	1.00	24	0.125
37	A	3	2	1.00	24	0.083
38	A	5	4	1.00	24	0.167
39	A	3	2	1.00	24	0.083
40	A	4	4	1.00	24	0.167
41	A	5	4	1.00	24	0.167
42	A	4	3	1.00	24	0.125
43	A	5	4	1.00	24	0.167
44	A	4	3	1.00	24	0.125
45	A	5	4	1.00	24	0.167
46	A	4	3	1.00	24	0.125
47	A	4	4	1.00	24	0.167
48	A	4	3	1.00	24	0.125
49	A	3	3	1.00	24	0.125
50	A	4	3	1.00	22	0.136
51	A	3	3	1.00	21	0.143
52	A	3	3	1.00	22	0.136
53	A	4	3	1.00	24	0.125
54	A	4	4	1.00	24	0.167
55	A	4	3	1.00	24	0.125
56	A	5	4	1.00	24	0.167
57	A	4	3	1.00	24	0.125
58	A	5	4	1.00	24	0.167
59	A	4	3	1.00	24	0.125
60	A	5	4	1.00	24	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	3	1.00	24	0.125
62	A	5	4	1.00	24	0.167
63	A	4	3	1.00	24	0.125
64	A	4	4	1.00	24	0.167
65	A	4	3	1.00	24	0.125
66	A	3	3	1.00	24	0.125
67	A	4	3	1.00	24	0.125
68	A	4	4	1.00	24	0.167
69	A	4	3	1.00	22	0.136
70	A	5	4	1.00	21	0.190
71	A	4	3	1.00	24	0.125
72	A	5	4	1.00	24	0.167
73	A	6	5	1.00	24	0.208
74	A	4	3	1.00	24	0.125
75	A	6	5	1.00	24	0.208
76	A	4	3	1.00	24	0.125
77	A	5	5	1.00	24	0.208
78	A	4	3	1.00	24	0.125
79	A	4	4	1.00	24	0.167
80	A	3	3	1.00	24	0.125
81	A	4	4	1.00	24	0.167
82	A	4	3	1.00	24	0.125
83	A	5	4	1.00	24	0.167
84	A	4	3	1.00	24	0.125
85	A	6	5	1.00	24	0.208
86	A	4	3	1.00	22	0.136
87	A	6	5	1.00	21	0.238
88	A	4	3	1.00	24	0.125
89	A	8	7	1.00	26	0.269
90	A	7	7	1.00	26	0.269
91	A	5	5	1.00	26	0.192
92	A	5	5	1.00	24	0.208
93	A	5	5	1.00	26	0.192
94	A	5	5	1.00	26	0.192
95	A	5	5	1.00	26	0.192

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.00	26	0.115
97	A	4	4	1.00	26	0.154
98	A	5	4	1.00	26	0.154
99	A	6	4	1.00	26	0.154
100	A	4	3	1.00	26	0.115
101	A	3	3	1.00	26	0.115
102	A	2	2	1.00	23	0.087
103	A	4	4	1.00	26	0.154
104	A	4	4	1.00	26	0.154
105	A	4	4	1.00	26	0.154
106	A	8	7	1.00	26	0.269
107	A	6	5	1.00	26	0.192
108	A	6	5	1.00	24	0.208
109	A	6	6	1.00	26	0.231
110	A	6	6	1.00	26	0.231
111	A	6	5	1.00	26	0.192
112	A	6	6	1.00	26	0.231
113	A	6	5	1.00	26	0.192
114	A	3	3	1.00	26	0.115
115	A	4	4	1.00	26	0.154
116	A	5	4	1.00	26	0.154
117	A	6	4	1.00	26	0.154
118	A	7	4	1.00	26	0.154
119	A	5	4	1.00	26	0.154
120	A	4	4	1.00	26	0.154
121	A	3	3	1.00	23	0.130
122	A	2	2	1.00	26	0.077
123	A	5	4	1.00	26	0.154
124	A	5	4	1.00	26	0.154
125	A	5	5	1.00	26	0.192
126	A	5	4	1.00	26	0.154
127	A	6	5	1.00	26	0.192
128	A	7	5	1.00	26	0.192
129	A	8	5	1.00	26	0.192
130	A	7	6	1.00	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	6	6	1.00	26	0.231
132	A	4	4	1.00	26	0.154
133	A	4	4	1.00	24	0.167
134	A	4	4	1.00	26	0.154
135	A	3	3	1.00	26	0.115
136	A	4	4	1.00	26	0.154
137	A	5	4	1.00	26	0.154
138	A	6	4	1.00	26	0.154
139	A	4	3	1.00	26	0.115
140	A	3	3	1.00	26	0.115
141	A	2	2	1.00	26	0.077
142	A	3	3	1.00	23	0.130
143	A	3	3	1.00	26	0.115
144	A	4	4	1.00	26	0.154
145	A	7	6	1.00	26	0.231
146	A	6	6	1.00	26	0.231
147	A	5	5	1.00	26	0.192
148	A	4	4	1.00	26	0.154
149	A	2	2	1.00	24	0.083
150	A	3	3	1.00	26	0.115
151	A	4	4	1.00	26	0.154
152	A	5	4	1.00	26	0.154
153	A	4	3	1.00	26	0.115
154	A	3	3	1.00	26	0.115
155	A	2	2	1.00	26	0.077
156	A	3	3	1.00	26	0.115
157	A	5	5	1.00	23	0.217
158	A	5	5	1.00	26	0.192
159	A	3	2	1.00	24	0.083
160	A	3	2	1.00	24	0.083
161	A	3	2	1.00	24	0.083
162	A	3	2	1.00	24	0.083
163	A	3	2	1.00	24	0.083
164	A	3	2	1.00	24	0.083
165	A	3	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	3	2	1.00	24	0.083
167	A	3	2	1.00	26	0.077
168	A	3	2	1.00	26	0.077
169	A	3	2	1.00	26	0.077
170	A	3	2	1.00	26	0.077
171	A	3	2	1.00	26	0.077
172	A	3	2	1.00	26	0.077
173	A	3	2	1.00	26	0.077
174	A	3	2	1.00	26	0.077
175	A	3	2	1.00	26	0.077
176	A	3	2	1.00	26	0.077
177	A	3	2	1.00	26	0.077
178	A	3	2	1.00	26	0.077
179	A	3	2	1.00	26	0.077
180	A	3	2	1.00	26	0.077
181	A	3	2	1.00	26	0.077
182	A	3	2	1.00	26	0.077
183	A	14	10	1.00	26	0.385
184	A	14	10	1.00	26	0.385
185	A	13	10	1.00	26	0.385
186	A	13	10	1.00	26	0.385
187	A	12	9	1.00	26	0.346
188	A	12	9	1.00	26	0.346
189	A	12	9	1.00	26	0.346
190	A	12	9	1.00	26	0.346
191	A	13	10	1.00	26	0.385
192	A	13	10	1.00	26	0.385
193	A	14	10	1.00	26	0.385
194	A	14	10	1.00	26	0.385
195	A	15	10	1.00	26	0.385
196	A	14	10	1.00	26	0.385
197	A	14	10	1.00	26	0.385
198	A	13	10	1.00	26	0.385
199	A	13	10	1.00	26	0.385
200	A	12	9	1.00	26	0.346

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	12	9	1.00	26	0.346
202	A	13	10	1.00	26	0.385
203	A	13	10	1.00	26	0.385
204	A	14	10	1.00	26	0.385
205	A	14	10	1.00	26	0.385
206	A	15	10	1.00	26	0.385
207	A	15	11	1.00	26	0.423
208	A	14	11	1.00	26	0.423
209	A	14	11	1.00	26	0.423
210	A	13	10	1.00	26	0.385
211	A	13	10	1.00	26	0.385
212	A	13	10	1.00	26	0.385
213	A	13	10	1.00	26	0.385
214	A	14	11	1.00	26	0.423
215	A	14	11	1.00	26	0.423
216	A	15	11	1.00	26	0.423
217	A	15	11	1.00	26	0.423
218	A	16	11	1.00	26	0.423
219	A	16	11	1.00	26	0.423
220	A	7	6	1.00	28	0.214
221	A	8	8	1.00	28	0.286
222	A	6	6	1.00	28	0.214
223	A	7	7	1.00	28	0.250
224	A	5	5	1.00	28	0.179
225	A	7	7	1.00	28	0.250
226	A	5	5	1.00	28	0.179
227	A	7	7	1.00	28	0.250
228	A	5	5	1.00	28	0.179
229	A	8	8	1.00	28	0.286
230	A	6	6	1.00	28	0.214
231	A	11	8	1.00	28	0.286
232	A	9	6	1.00	28	0.214
233	A	10	8	1.00	28	0.286
234	A	8	6	1.00	28	0.214
235	A	9	8	1.00	28	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	7	6	1.00	28	0.214
237	A	8	7	1.00	28	0.250
238	A	6	5	1.00	28	0.179
239	A	8	7	1.00	28	0.250
240	A	6	5	1.00	28	0.179
241	A	8	8	1.00	28	0.286
242	A	6	6	1.00	28	0.214
243	A	8	7	1.00	28	0.250
244	A	7	5	1.00	28	0.179
245	A	8	7	1.00	28	0.250
246	A	6	5	1.00	28	0.179
247	A	7	7	1.00	28	0.250
248	A	5	5	1.00	28	0.179
249	A	6	6	1.00	28	0.214
250	A	4	4	1.00	28	0.143
251	A	6	6	1.00	28	0.214
252	A	4	4	1.00	28	0.143
253	A	7	7	1.00	28	0.250
254	A	5	5	1.00	28	0.179
255	A	8	7	1.00	28	0.250
256	A	6	5	1.00	28	0.179
257	A	7	5	1.00	28	0.179
258	A	8	7	1.00	28	0.250
259	A	6	5	1.00	28	0.179
260	A	7	7	1.00	28	0.250
261	A	5	5	1.00	28	0.179
262	A	6	6	1.00	28	0.214
263	A	4	4	1.00	28	0.143
264	A	7	7	1.00	28	0.250
265	A	5	5	1.00	28	0.179
266	A	8	8	1.00	28	0.286
267	A	6	6	1.00	28	0.214
268	A	9	8	1.00	28	0.286
269	A	3	2	1.00	24	0.083
270	A	3	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	3	2	1.00	22	0.091
272	A	3	3	1.00	24	0.125
273	A	3	3	0.94	24	0.125
274	A	4	4	0.90	24	0.167
275	A	2	2	1.00	32	0.062
276	A	4	3	1.00	30	0.100
277	A	4	3	1.00	34	0.088
278	A	4	3	1.00	19	0.158
279	A	4	3	1.00	15	0.200
280	A	4	3	1.00	19	0.158
281	A	3	2	1.00	17	0.118
282	A	4	4	1.00	25	0.160
283	A	2	2	1.00	39	0.051
284	A	4	4	1.00	20	0.200
285	A	7	7	1.00	20	0.350
286	A	6	6	1.00	18	0.333
287	A	5	5	1.00	17	0.294
288	A	3	3	1.00	20	0.150
289	A	4	4	1.00	20	0.200
290	A	5	4	1.00	20	0.200
291	A	5	4	1.00	20	0.200
292	A	4	4	1.00	20	0.200
293	A	7	7	1.00	20	0.350
294	A	6	6	1.00	18	0.333
295	A	3	3	1.00	17	0.176
296	A	4	4	1.00	20	0.200
297	A	5	5	1.00	20	0.250
298	A	5	5	1.00	20	0.250

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^2(A+Bx^2)\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=33

$$\frac{1}{5}Abx^5+\frac{1}{7}(bB+Ac)x^7+\frac{1}{9}Bcx^9$$

```
[Out] 1/5*A*b*x^5+1/7*(A*c+B*b)*x^7+1/9*B*c*x^9
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {1598, 459}

$$\frac{1}{7}x^7(Ac+bB)+\frac{1}{5}Abx^5+\frac{1}{9}Bcx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^5)/5 + ((b*B + A*c)*x^7)/7 + (B*c*x^9)/9
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^2\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx &= \int x^4\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int\left(Abx^4+(bB+Ac)x^6+Bcx^8\right)\,dx\\ &= \frac{1}{5}Abx^5+\frac{1}{7}(bB+Ac)x^7+\frac{1}{9}Bcx^9\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{5}Abx^5+\frac{1}{7}(bB+Ac)x^7+\frac{1}{9}Bcx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x^2)*(b*x^2 + c*x^4),x]
[Out] (A*b*x^5)/5 + ((b*B + A*c)*x^7)/7 + (B*c*x^9)/9
```

Maple [A]
time = 0.16, size = 28, normalized size = 0.85

method	result	size
default	$\frac{A b\,x^{5}}{5}+\frac{(A c+B b) x^{7}}{7}+\frac{B c\,x^{9}}{9}$	28
norman	$\frac{B c\,x^{9}}{9}+\left(\frac{A c}{7}+\frac{B b}{7}\right) x^{7}+\frac{A b\,x^{5}}{5}$	29
risch	$\frac{1}{5}A b\,x^{5}+\frac{1}{7}x^{7} A c+\frac{1}{7}b B\,x^{7}+\frac{1}{9}B c\,x^{9}$	30
gosper	$\frac{x^{5}\left(35 B c\,x^{4}+45 A c\,x^{2}+45 b B\,x^{2}+63 A b\right)}{315}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 1/5*A*b*x^5+1/7*(A*c+B*b)*x^7+1/9*B*c*x^9
```

Maxima [A]
time = 0.28, size = 27, normalized size = 0.82

$$\frac{1}{9}\,Bcx^{9}+\frac{1}{7}\,(Bb+Ac)x^{7}+\frac{1}{5}\,Abx^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/9*B*c*x^9 + 1/7*(B*b + A*c)*x^7 + 1/5*A*b*x^5
```

Fricas [A]
time = 2.91, size = 27, normalized size = 0.82

$$\frac{1}{9}\,Bcx^{9}+\frac{1}{7}\,(Bb+Ac)x^{7}+\frac{1}{5}\,Abx^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/9*B*c*x^9 + 1/7*(B*b + A*c)*x^7 + 1/5*A*b*x^5
```

Sympy [A]
time = 0.01, size = 29, normalized size = 0.88

$$\frac{Abx^{5}}{5}+\frac{Bcx^{9}}{9}+x^{7}\left(\frac{Ac}{7}+\frac{Bb}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] A*b*x**5/5 + B*c*x**9/9 + x**7*(A*c/7 + B*b/7)
```

Giac [A]
time = 1.46, size = 29, normalized size = 0.88

$$\frac{1}{9}\,Bcx^9+\frac{1}{7}\,Bbx^7+\frac{1}{7}\,Acx^7+\frac{1}{5}\,Abx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/9*B*c*x^9 + 1/7*B*b*x^7 + 1/7*A*c*x^7 + 1/5*A*b*x^5
```

Mupad [B]
time = 0.18, size = 28, normalized size = 0.85

$$\frac{B\,c\,x^9}{9}+\left(\frac{A\,c}{7}+\frac{B\,b}{7}\right)\,x^7+\frac{A\,b\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

```
[Out] x^7*((A*c)/7 + (B*b)/7) + (A*b*x^5)/5 + (B*c*x^9)/9
```

3.2 $\int x(A+Bx^2)\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=33

$$\frac{1}{4}Abx^4+\frac{1}{6}(bB+Ac)x^6+\frac{1}{8}Bcx^8$$

```
[Out] 1/4*A*b*x^4+1/6*(A*c+B*b)*x^6+1/8*B*c*x^8
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {1598, 457, 77}

$$\frac{1}{6}x^6(Ac+bB)+\frac{1}{4}Abx^4+\frac{1}{8}Bcx^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^4)/4 + ((b*B + A*c)*x^6)/6 + (B*c*x^8)/8
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int x(A+Bx^2)(bx^2+cx^4)\,dx=\int x^3(A+Bx^2)(b+cx^2)\,dx$$
$$=\frac{1}{2}\text{Subst}\left(\int x(A+Bx)(b+cx)\,dx,x,x^2\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int \left(Abx+(bB+Ac)x^2+Bcx^3\right)\,dx,x,x^2\right)$$
$$=\frac{1}{4}Abx^4+\frac{1}{6}(bB+Ac)x^6+\frac{1}{8}Bcx^8$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{4}Abx^4+\frac{1}{6}(bB+Ac)x^6+\frac{1}{8}Bcx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x^2)*(b*x^2 + c*x^4),x]
[Out] (A*b*x^4)/4 + ((b*B + A*c)*x^6)/6 + (B*c*x^8)/8
```

Maple [A]
time = 0.21, size = 28, normalized size = 0.85

method	result	size
default	$\frac{Ab\,x^4}{4}+\frac{(Ac+Bb)x^6}{6}+\frac{Bc\,x^8}{8}$	28
norman	$\frac{Bc\,x^8}{8}+\left(\frac{Ac}{6}+\frac{Bb}{6}\right)x^6+\frac{Ab\,x^4}{4}$	29
risch	$\frac{1}{4}Ab\,x^4+\frac{1}{6}x^6Ac+\frac{1}{6}bB\,x^6+\frac{1}{8}Bc\,x^8$	30
gosper	$\frac{x^4\left(3Bc\,x^4+4Ac\,x^2+4bB\,x^2+6Ab\right)}{24}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 1/4*A*b*x^4+1/6*(A*c+B*b)*x^6+1/8*B*c*x^8
```

Maxima [A]
time = 0.29, size = 27, normalized size = 0.82

$$\frac{1}{8}\,Bcx^8+\frac{1}{6}\,(Bb+Ac)x^6+\frac{1}{4}\,Abx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/8*B*c*x^8 + 1/6*(B*b + A*c)*x^6 + 1/4*A*b*x^4
```

Fricas [A]
time = 3.16, size = 27, normalized size = 0.82

$$\frac{1}{8}\,Bcx^8 + \frac{1}{6}\,(Bb + Ac)x^6 + \frac{1}{4}\,Abx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/8*B*c*x^8 + 1/6*(B*b + A*c)*x^6 + 1/4*A*b*x^4
```

Sympy [A]
time = 0.01, size = 29, normalized size = 0.88

$$\frac{Abx^4}{4} + \frac{Bcx^8}{8} + x^6\left(\frac{Ac}{6} + \frac{Bb}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] A*b*x**4/4 + B*c*x**8/8 + x**6*(A*c/6 + B*b/6)
```

Giac [A]
time = 1.66, size = 29, normalized size = 0.88

$$\frac{1}{8}\,Bcx^8 + \frac{1}{6}\,Bbx^6 + \frac{1}{6}\,Acx^6 + \frac{1}{4}\,Abx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/6*A*c*x^6 + 1/4*A*b*x^4
```

Mupad [B]
time = 0.06, size = 28, normalized size = 0.85

$$\frac{B\,c\,x^8}{8} + \left(\frac{A\,c}{6} + \frac{B\,b}{6}\right)\,x^6 + \frac{A\,b\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

```
[Out] x^6*((A*c)/6 + (B*b)/6) + (A*b*x^4)/4 + (B*c*x^8)/8
```

3.3 $\int (A+Bx^2)(bx^2+cx^4)\,dx$

Optimal. Leaf size=33

$$\frac{1}{3}Abx^3+\frac{1}{5}(bB+Ac)x^5+\frac{1}{7}Bcx^7$$

```
[Out] 1/3*A*b*x^3+1/5*(A*c+B*b)*x^5+1/7*B*c*x^7
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1607, 459}

$$\frac{1}{5}x^5(Ac+bB)+\frac{1}{3}Abx^3+\frac{1}{7}Bcx^7$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^3)/3 + ((b*B + A*c)*x^5)/5 + (B*c*x^7)/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx &= \int x^2\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Abx^2+(bB+Ac)x^4+Bcx^6\right)\,dx\\ &= \frac{1}{3}Abx^3+\frac{1}{5}(bB+Ac)x^5+\frac{1}{7}Bcx^7\end{aligned}$$

Mathematica [A]
time = 0.00, size = 33, normalized size = 1.00

$$\frac{1}{3}Abx^3+\frac{1}{5}(bB+Ac)x^5+\frac{1}{7}Bcx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^3)/3 + ((b*B + A*c)*x^5)/5 + (B*c*x^7)/7
```

Maple [A]
time = 0.13, size = 28, normalized size = 0.85

method	result	size
default	$\frac{A b \,x^{3}}{3}+\frac{\left(A c +B b \right) x^{5}}{5}+\frac{B c \,x^{7}}{7}$	28
norman	$\frac{B c \,x^{7}}{7}+\left(\frac{A c}{5}+\frac{B b}{5}\right) x^{5}+\frac{A b \,x^{3}}{3}$	29
risch	$\frac{1}{3} A b \,x^{3}+\frac{1}{5} x^{5} A c +\frac{1}{5} b B \,x^{5}+\frac{1}{7} B c \,x^{7}$	30
gosper	$\frac{x^{3} \left(15 B c \,x^{4}+21 A c \,x^{2}+21 b B \,x^{2}+35 A b \right)}{105}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*A*b*x^3+1/5*(A*c+B*b)*x^5+1/7*B*c*x^7
```

Maxima [A]
time = 0.30, size = 27, normalized size = 0.82

$$\frac{1}{7} \, B c x^{7} + \frac{1}{5} \, {\left(B b + A c\right)} x^{5} + \frac{1}{3} \, A b x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/7*B*c*x^7 + 1/5*(B*b + A*c)*x^5 + 1/3*A*b*x^3
```

Fricas [A]
time = 2.93, size = 27, normalized size = 0.82

$$\frac{1}{7} \, B c x^{7} + \frac{1}{5} \, {\left(B b + A c\right)} x^{5} + \frac{1}{3} \, A b x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/7*B*c*x^7 + 1/5*(B*b + A*c)*x^5 + 1/3*A*b*x^3
```

Sympy [A]
time = 0.01, size = 29, normalized size = 0.88

$$\frac{A b x^{3}}{3} + \frac{B c x^{7}}{7} + x^{5} \left(\frac{A c}{5} + \frac{B b}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] A*b*x**3/3 + B*c*x**7/7 + x**5*(A*c/5 + B*b/5)
```

Giac [A]
time = 1.15, size = 29, normalized size = 0.88

$$\frac{1}{7}Bcx^7 + \frac{1}{5}Bbx^5 + \frac{1}{5}Acx^5 + \frac{1}{3}Abx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/7*B*c*x^7 + 1/5*B*b*x^5 + 1/5*A*c*x^5 + 1/3*A*b*x^3
```

Mupad [B]
time = 0.04, size = 28, normalized size = 0.85

$$\frac{B\,c\,x^7}{7} + \left(\frac{A\,c}{5} + \frac{B\,b}{5}\right)\,x^5 + \frac{A\,b\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(b*x^2 + c*x^4),x)
```

```
[Out] x^5*((A*c)/5 + (B*b)/5) + (A*b*x^3)/3 + (B*c*x^7)/7
```

3.4 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x}\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}Abx^2 + \frac{1}{4}(bB + Ac)x^4 + \frac{1}{6}Bcx^6$$

```
[Out] 1/2*A*b*x^2+1/4*(A*c+B*b)*x^4+1/6*B*c*x^6
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1598, 455, 45}

$$\frac{1}{4}x^4(Ac + bB) + \frac{1}{2}Abx^2 + \frac{1}{6}Bcx^6$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x,x]
```

```
[Out] (A*b*x^2)/2 + ((b*B + A*c)*x^4)/4 + (B*c*x^6)/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x}\,dx = \int x(A+Bx^2)\left(b+cx^2\right)\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int (A+Bx)(b+cx)\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(Ab+(bB+Ac)x+Bcx^2\right)\,dx, x, x^2\right)$$
$$= \frac{1}{2}Abx^2+\frac{1}{4}(bB+Ac)x^4+\frac{1}{6}Bcx^6$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{2}Abx^2+\frac{1}{4}(bB+Ac)x^4+\frac{1}{6}Bcx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x,x]
[Out] (A*b*x^2)/2 + ((b*B + A*c)*x^4)/4 + (B*c*x^6)/6
```

Maple [A]
time = 0.12, size = 28, normalized size = 0.85

method	result	size
default	$\frac{Ab\,x^{2}}{2}+\frac{(Ac+Bb)x^{4}}{4}+\frac{Bc\,x^{6}}{6}$	28
norman	$\frac{Bc\,x^{6}}{6}+\left(\frac{Ac}{4}+\frac{Bb}{4}\right)x^{4}+\frac{Ab\,x^{2}}{2}$	29
risch	$\frac{1}{2}Ab\,x^{2}+\frac{1}{4}x^{4}Ac+\frac{1}{4}bB\,x^{4}+\frac{1}{6}Bc\,x^{6}$	30
gosper	$\frac{x^{2}\left(2Bc\,x^{4}+3Ac\,x^{2}+3bB\,x^{2}+6Ab\right)}{12}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x,x,method=_RETURNVERBOSE)
[Out] 1/2*A*b*x^2+1/4*(A*c+B*b)*x^4+1/6*B*c*x^6
```

Maxima [A]
time = 0.64, size = 27, normalized size = 0.82

$$\frac{1}{6}\,Bcx^6+\frac{1}{4}\,(Bb+Ac)x^4+\frac{1}{2}\,Abx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*B*c*x^6 + 1/4*(B*b + A*c)*x^4 + 1/2*A*b*x^2
```

Fricas [A]
time = 2.43, size = 27, normalized size = 0.82

$$\frac{1}{6}Bcx^6 + \frac{1}{4}(Bb + Ac)x^4 + \frac{1}{2}Abx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*B*c*x^6 + 1/4*(B*b + A*c)*x^4 + 1/2*A*b*x^2
```

Sympy [A]
time = 0.01, size = 29, normalized size = 0.88

$$\frac{Abx^2}{2} + \frac{Bcx^6}{6} + x^4\left(\frac{Ac}{4} + \frac{Bb}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x,x)
```

```
[Out] A*b*x**2/2 + B*c*x**6/6 + x**4*(A*c/4 + B*b/4)
```

Giac [A]
time = 0.70, size = 29, normalized size = 0.88

$$\frac{1}{6}Bcx^6 + \frac{1}{4}Bbx^4 + \frac{1}{4}Acx^4 + \frac{1}{2}Abx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x,x, algorithm="giac")
```

```
[Out] 1/6*B*c*x^6 + 1/4*B*b*x^4 + 1/4*A*c*x^4 + 1/2*A*b*x^2
```

Mupad [B]
time = 0.06, size = 28, normalized size = 0.85

$$\frac{B\,c\,x^6}{6} + \left(\frac{A\,c}{4} + \frac{B\,b}{4}\right)\,x^4 + \frac{A\,b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x,x)
```

```
[Out] x^4*((A*c)/4 + (B*b)/4) + (A*b*x^2)/2 + (B*c*x^6)/6
```

3.5 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx$

Optimal. Leaf size=28

$$Abx+\frac{1}{3}(bB+Ac)x^3+\frac{1}{5}Bcx^5$$

[Out] A*b*x+1/3*(A*c+B*b)*x^3+1/5*B*c*x^5

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1598, 380}

$$\frac{1}{3}x^3(Ac+bB)+Abx+\frac{1}{5}Bcx^5$$

Antiderivative was successfully verified.

[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^2,x]

[Out] A*b*x + ((b*B + A*c)*x^3)/3 + (B*c*x^5)/5

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx &= \int (A+Bx^2)(b+cx^2)\,dx\\ &= \int \left(Ab+(bB+Ac)x^2+Bcx^4\right)dx\\ &= Abx+\frac{1}{3}(bB+Ac)x^3+\frac{1}{5}Bcx^5\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$Abx+\frac{1}{3}(bB+Ac)x^3+\frac{1}{5}Bcx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^2,x]
[Out] A*b*x + ((b*B + A*c)*x^3)/3 + (B*c*x^5)/5
```

Maple [A]
time = 0.17, size = 25, normalized size = 0.89

method	result	size
default	$A b x +\frac{\left(A c +B b \right) x^{3}}{3}+\frac{B c \,x^{5}}{5}$	25
risch	$A b x +\frac{1}{3} x^{3} A c +\frac{1}{3} b B \,x^{3}+\frac{1}{5} B c \,x^{5}$	27
gosper	$\frac{x \left(3 B c \,x^{4}+5 A c \,x^{2}+5 b B \,x^{2}+15 A b \right)}{15}$	30
norman	$\frac{\left(\frac{A c}{3}+\frac{B b}{3}\right) x^{4}+A b \,x^{2}+\frac{B c \,x^{6}}{5}}{x}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^2,x,method=_RETURNVERBOSE)
[Out] A*b*x+1/3*(A*c+B*b)*x^3+1/5*B*c*x^5
```

Maxima [A]
time = 0.31, size = 24, normalized size = 0.86

$$\frac{1}{5}\,Bcx^{5} + \frac{1}{3}\,(Bb + Ac)x^{3} + Abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="maxima")
[Out] 1/5*B*c*x^5 + 1/3*(B*b + A*c)*x^3 + A*b*x
```

Fricas [A]
time = 1.43, size = 24, normalized size = 0.86

$$\frac{1}{5}\,Bcx^{5} + \frac{1}{3}\,(Bb + Ac)x^{3} + Abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="fricas")
[Out] 1/5*B*c*x^5 + 1/3*(B*b + A*c)*x^3 + A*b*x
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.93

$$Abx + \frac{Bcx^{5}}{5} + x^{3}\left(\frac{Ac}{3} + \frac{Bb}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**2,x)
```

```
[Out] A*b*x + B*c*x**5/5 + x**3*(A*c/3 + B*b/3)
```

Giac [A]
time = 0.77, size = 26, normalized size = 0.93

$$\frac{1}{5}Bcx^5 + \frac{1}{3}Bbx^3 + \frac{1}{3}Acx^3 + Abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/3*A*c*x^3 + A*b*x
```

Mupad [B]
time = 0.06, size = 25, normalized size = 0.89

$$\frac{B\,c\,x^5}{5} + \left(\frac{A\,c}{3} + \frac{B\,b}{3}\right)\,x^3 + A\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^2,x)
```

```
[Out] x^3*((A*c)/3 + (B*b)/3) + A*b*x + (B*c*x^5)/5
```

3.6 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}\,dx$

Optimal. Leaf size=29

$$\frac{1}{2}(bB+Ac)x^2+\frac{1}{4}Bcx^4+Ab\log(x)$$

```
[Out] 1/2*(A*c+B*b)*x^2+1/4*B*c*x^4+A*b*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1598, 457, 77}

$$\frac{1}{2}x^2(Ac+bB)+Ab\log(x)+\frac{1}{4}Bcx^4$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^3,x]
```

```
[Out] ((b*B + A*c)*x^2)/2 + (B*c*x^4)/4 + A*b*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}\,dx = \int \frac{(A+Bx^2)(b+cx^2)}{x}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(bB+Ac+\frac{Ab}{x}+Bcx\right)\,dx, x, x^2\right)$$
$$= \frac{1}{2}(bB+Ac)x^2+\frac{1}{4}Bcx^4+Ab\log(x)$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$\frac{1}{2}(bB+Ac)x^2+\frac{1}{4}Bcx^4+Ab\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^3,x]
```

```
[Out] ((b*B + A*c)*x^2)/2 + (B*c*x^4)/4 + A*b*Log[x]
```

Maple [A]
time = 0.09, size = 28, normalized size = 0.97

method	result	size
default	$\frac{B c \,x^{4}}{4}+\frac{A c \,x^{2}}{2}+\frac{b B \,x^{2}}{2}+A b \ln \left(x \right)$	28
norman	$\frac{\left(\frac{A c}{2}+\frac{B b}{2}\right) x^{4}+\frac{B c \,x^{6}}{4}}{x^{2}}+A b \ln \left(x \right)$	32
risch	$\frac{B c \,x^{4}}{4}+\frac{A c \,x^{2}}{2}+\frac{b B \,x^{2}}{2}+\frac{c \,A^{2}}{4 B}+\frac{A b}{2}+\frac{B \,b^{2}}{4 c}+A b \ln \left(x \right)$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*B*c*x^4+1/2*A*c*x^2+1/2*b*B*x^2+A*b*ln(x)
```

Maxima [A]
time = 0.27, size = 28, normalized size = 0.97

$$\frac{1}{4}\,Bcx^4+\frac{1}{2}\,(Bb+Ac)x^2+\frac{1}{2}\,Ab\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*B*c*x^4 + 1/2*(B*b + A*c)*x^2 + 1/2*A*b*log(x^2)
```

Fricas [A]
time = 1.60, size = 25, normalized size = 0.86

$$\frac{1}{4}Bcx^4 + \frac{1}{2}(Bb + Ac)x^2 + Ab\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^3,x, algorithm="fricas")
[Out] 1/4*B*c*x^4 + 1/2*(B*b + A*c)*x^2 + A*b*log(x)
```

Sympy [A]
time = 0.03, size = 27, normalized size = 0.93

$$Ab\log(x) + \frac{Bcx^4}{4} + x^2\left(\frac{Ac}{2} + \frac{Bb}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**3,x)
[Out] A*b*log(x) + B*c*x**4/4 + x**2*(A*c/2 + B*b/2)
```

Giac [A]
time = 0.65, size = 30, normalized size = 1.03

$$\frac{1}{4}Bcx^4 + \frac{1}{2}Bbx^2 + \frac{1}{2}Acx^2 + \frac{1}{2}Ab\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^3,x, algorithm="giac")
[Out] 1/4*B*c*x^4 + 1/2*B*b*x^2 + 1/2*A*c*x^2 + 1/2*A*b*log(x^2)
```

Mupad [B]
time = 0.06, size = 26, normalized size = 0.90

$$x^2\left(\frac{A\,c}{2} + \frac{B\,b}{2}\right) + \frac{B\,c\,x^4}{4} + A\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^3,x)
[Out] x^2*((A*c)/2 + (B*b)/2) + (B*c*x^4)/4 + A*b*log(x)
```

3.7 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4} dx$

Optimal. Leaf size=26

$$-\frac{Ab}{x}+(bB+Ac)x+\frac{1}{3}Bcx^3$$

```
[Out] -A*b/x+(A*c+B*b)*x+1/3*B*c*x^3
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1598, 459}

$$x(Ac+bB)-\frac{Ab}{x}+\frac{1}{3}Bcx^3$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^4,x]
```

```
[Out] -((A*b)/x) + (b*B + A*c)*x + (B*c*x^3)/3
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4} dx &= \int \frac{(A+Bx^2)(b+cx^2)}{x^2} dx\\ &= \int \left(bB\left(1+\frac{Ac}{bB}\right)+\frac{Ab}{x^2}+Bcx^2\right) dx\\ &= -\frac{Ab}{x}+(bB+Ac)x+\frac{1}{3}Bcx^3\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$-\frac{Ab}{x}+(bB+Ac)x+\frac{1}{3}Bcx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^4,x]
[Out] -((A*b)/x) + (b*B + A*c)*x + (B*c*x^3)/3
```

Maple [A]
time = 0.03, size = 24, normalized size = 0.92

method	result	size
default	$\frac{Bc\,x^3}{3}+Acx+bBx-\frac{Ab}{x}$	24
risch	$\frac{Bc\,x^3}{3}+Acx+bBx-\frac{Ab}{x}$	24
norman	$\frac{(Ac+Bb)x^4-Ab\,x^2+\frac{Bc\,x^6}{3}}{x^3}$	31
gosper	$-\frac{-Bc\,x^4-3Ac\,x^2-3bB\,x^2+3Ab}{3x}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/3*B*c*x^3+A*c*x+b*B*x-A*b/x
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.92

$$\frac{1}{3}\,Bcx^3+(Bb+Ac)x-\frac{Ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^4,x, algorithm="maxima")
[Out] 1/3*B*c*x^3 + (B*b + A*c)*x - A*b/x
```

Fricas [A]
time = 1.14, size = 28, normalized size = 1.08

$$\frac{Bcx^4+3\,(Bb+Ac)x^2-3\,Ab}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(B*c*x^4 + 3*(B*b + A*c)*x^2 - 3*A*b)/x
```

Sympy [**A**]
time = 0.03, size = 20, normalized size = 0.77

$$-\frac{Ab}{x}+\frac{Bcx^3}{3}+x(Ac+Bb)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**4,x)
[Out] -A*b/x + B*c*x**3/3 + x*(A*c + B*b)
```

Giac [**A**]
time = 0.74, size = 23, normalized size = 0.88

$$\frac{1}{3}Bcx^3+Bbx+Acx-\frac{Ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^4,x, algorithm="giac")
[Out] 1/3*B*c*x^3 + B*b*x + A*c*x - A*b/x
```

Mupad [**B**]
time = 0.07, size = 24, normalized size = 0.92

$$x\,\left(A\,c+B\,b\right)-\frac{A\,b}{x}+\frac{B\,c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^4,x)
[Out] x*(A*c + B*b) - (A*b)/x + (B*c*x^3)/3
```

3.8 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5}\,dx$

Optimal. Leaf size=29

$$-\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2+(bB+Ac)\log(x)$$

```
[Out] -1/2*A*b/x^2+1/2*B*c*x^2+(A*c+B*b)*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1598, 457, 77}

$$\log(x)(Ac+bB)-\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^5,x]
```

```
[Out] -1/2*(A*b)/x^2 + (B*c*x^2)/2 + (b*B + A*c)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5}\,dx = \int \frac{(A+Bx^2)(b+cx^2)}{x^3}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(Bc+\frac{Ab}{x^2}+\frac{bB+Ac}{x}\right)\,dx, x, x^2\right)$$
$$= -\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2+(bB+Ac)\log(x)$$

Mathematica [**A**]
time = 0.01, size = 29, normalized size = 1.00

$$-\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2+(bB+Ac)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^5,x]
```

```
[Out] -1/2*(A*b)/x^2 + (B*c*x^2)/2 + (b*B + A*c)*Log[x]
```

Maple [**A**]
time = 0.03, size = 26, normalized size = 0.90

method	result	size
default	$-\frac{Ab}{2x^2}+\frac{Bc\,x^2}{2}+(Ac+Bb)\ln(x)$	26
risch	$-\frac{Ab}{2x^2}+\frac{Bc\,x^2}{2}+A\ln(x)\,c+bB\ln(x)$	26
norman	$\frac{-\frac{1}{2}Ab\,x^2+\frac{1}{2}Bc\,x^6}{x^4}+(Ac+Bb)\ln(x)$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A*b/x^2+1/2*B*c*x^2+(A*c+B*b)*ln(x)
```

Maxima [**A**]
time = 0.27, size = 28, normalized size = 0.97

$$\frac{1}{2}\,Bcx^2+\frac{1}{2}\,(Bb+Ac)\log\left(x^2\right)-\frac{Ab}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*B*c*x^2 + 1/2*(B*b + A*c)*log(x^2) - 1/2*A*b/x^2
```

Fricas [A]
time = 1.33, size = 30, normalized size = 1.03

$$\frac{Bcx^4 + 2\,(Bb + Ac)x^2\log{(x)} - Ab}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/2*(B*c*x^4 + 2*(B*b + A*c)*x^2*log(x) - A*b)/x^2
```

Sympy [A]
time = 0.08, size = 26, normalized size = 0.90

$$-\frac{Ab}{2x^2} + \frac{Bcx^2}{2} + (Ac + Bb)\log{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**5,x)
```

```
[Out] -A*b/(2*x**2) + B*c*x**2/2 + (A*c + B*b)*log(x)
```

Giac [A]
time = 0.63, size = 42, normalized size = 1.45

$$\frac{1}{2}\,Bcx^2 + \frac{1}{2}\,(Bb + Ac)\log\left(x^2\right) - \frac{Bbx^2 + Acx^2 + Ab}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*B*c*x^2 + 1/2*(B*b + A*c)*log(x^2) - 1/2*(B*b*x^2 + A*c*x^2 + A*b)/x^2
```

Mupad [B]
time = 0.04, size = 25, normalized size = 0.86

$$\ln{(x)}\,\left(A\,c + B\,b\right) - \frac{A\,b}{2\,x^2} + \frac{B\,c\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^5,x)
```

```
[Out] log(x)*(A*c + B*b) - (A*b)/(2*x^2) + (B*c*x^2)/2
```

3.9 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6} \, dx$

Optimal. Leaf size=26

$$-\frac{Ab}{3x^3}-\frac{bB+Ac}{x}+Bcx$$

[Out] -1/3*A*b/x^3+(-A*c-B*b)/x+B*c*x

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1598, 459}

$$-\frac{Ac+bB}{x}-\frac{Ab}{3x^3}+Bcx$$

Antiderivative was successfully verified.

[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^6,x]

[Out] -1/3*(A*b)/x^3 - (b*B + A*c)/x + B*c*x

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6} \, dx &= \int \frac{(A+Bx^2)(b+cx^2)}{x^4} \, dx \\
&= \int \left(Bc+\frac{Ab}{x^4}+\frac{bB+Ac}{x^2}\right) dx \\
&= -\frac{Ab}{3x^3}-\frac{bB+Ac}{x}+Bcx
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.04

$$-\frac{Ab}{3x^3} + \frac{-bB - Ac}{x} + Bcx$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^6,x]
[Out] -1/3*(A*b)/x^3 + (-(b*B) - A*c)/x + B*c*x
```

Maple [A]
time = 0.03, size = 25, normalized size = 0.96

method	result	size
default	$Bcx - \frac{Ab}{3x^3} - \frac{Ac+Bb}{x}$	25
risch	$Bcx + \frac{(-Ac-Bb)x^2 - \frac{Ab}{3}}{x^3}$	28
gosper	$-\frac{-3Bc\,x^4+3Ac\,x^2+3bB\,x^2+Ab}{3x^3}$	31
norman	$\frac{(-Ac-Bb)x^4+Bc\,x^6-\frac{Ab\,x^2}{3}}{x^5}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^6,x,method=_RETURNVERBOSE)
[Out] B*c*x-1/3*A*b/x^3-(A*c+B*b)/x
```

Maxima [A]
time = 0.30, size = 26, normalized size = 1.00

$$Bcx - \frac{3\,(Bb + Ac)x^2 + Ab}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^6,x, algorithm="maxima")
[Out] B*c*x - 1/3*(3*(B*b + A*c)*x^2 + A*b)/x^3
```

Fricas [A]
time = 1.55, size = 29, normalized size = 1.12

$$\frac{3\,Bcx^4 - 3\,(Bb + Ac)x^2 - Ab}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/3*(3*B*c*x^4 - 3*(B*b + A*c)*x^2 - A*b)/x^3
```

Sympy [A]
time = 0.09, size = 27, normalized size = 1.04

$$Bcx + \frac{-Ab + x^2(-3Ac - 3Bb)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**6,x)
[Out] B*c*x + (-A*b + x**2*(-3*A*c - 3*B*b))/(3*x**3)
```

Giac [A]
time = 0.62, size = 28, normalized size = 1.08

$$Bcx - \frac{3\,Bbx^2 + 3\,Acx^2 + Ab}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^6,x, algorithm="giac")
[Out] B*c*x - 1/3*(3*B*b*x^2 + 3*A*c*x^2 + A*b)/x^3
```

Mupad [B]
time = 0.06, size = 26, normalized size = 1.00

$$B\,c\,x - \frac{\left(A\,c + B\,b\right)\,x^2 + \frac{A\,b}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^6,x)
[Out] B*c*x - ((A*b)/3 + x^2*(A*c + B*b))/x^3
```

3.10 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7}\,dx$

Optimal. Leaf size=29

$$-\frac{Ab}{4x^4}-\frac{bB+Ac}{2x^2}+Bc\log(x)$$

```
[Out] -1/4*A*b/x^4+1/2*(-A*c-B*b)/x^2+B*c*ln(x)
```

Rubi [A]

time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {1598, 457, 77}

$$-\frac{Ac+bB}{2x^2}-\frac{Ab}{4x^4}+Bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^7,x]
```

```
[Out] -1/4*(A*b)/x^4 - (b*B + A*c)/(2*x^2) + B*c*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)}{x^5}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)}{x^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{Ab}{x^3}+\frac{bB+Ac}{x^2}+\frac{Bc}{x}\right)\,dx, x, x^2\right) \\
&= -\frac{Ab}{4x^4}-\frac{bB+Ac}{2x^2}+Bc\log(x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.07

$$-\frac{Ab}{4x^4}+\frac{-bB-Ac}{2x^2}+Bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^7,x]
```

```
[Out] -1/4*(A*b)/x^4 + (-(b*B) - A*c)/(2*x^2) + B*c*Log[x]
```

Maple [A]
time = 0.03, size = 26, normalized size = 0.90

method	result	size
default	$-\frac{Ab}{4x^4}-\frac{Ac+Bb}{2x^2}+Bc\ln(x)$	26
risch	$\frac{\left(-\frac{Ac}{2}-\frac{Bb}{2}\right)x^2-\frac{Ab}{4}}{x^4}+Bc\ln(x)$	29
norman	$\frac{\left(-\frac{Ac}{2}-\frac{Bb}{2}\right)x^4-\frac{Ab\,x^2}{4}}{x^6}+Bc\ln(x)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*A*b/x^4-1/2*(A*c+B*b)/x^2+B*c*ln(x)
```

Maxima [A]
time = 0.27, size = 30, normalized size = 1.03

$$\frac{1}{2}\,Bc\log\left(x^2\right)-\frac{2\,(Bb+Ac)x^2+Ab}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^7,x, algorithm="maxima")
[Out] 1/2*B*c*log(x^2) - 1/4*(2*(B*b + A*c)*x^2 + A*b)/x^4
```

Fricas [A]
time = 2.14, size = 31, normalized size = 1.07

$$\frac{4\,Bcx^4\log\left(x\right)-2\,(Bb+Ac)x^2-Ab}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^7,x, algorithm="fricas")
[Out] 1/4*(4*B*c*x^4*log(x) - 2*(B*b + A*c)*x^2 - A*b)/x^4
```

Sympy [A]
time = 0.18, size = 29, normalized size = 1.00

$$Bc\log\left(x\right)+\frac{-Ab+x^2(-2Ac-2Bb)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**7,x)
[Out] B*c*log(x) + (-A*b + x**2*(-2*A*c - 2*B*b))/(4*x**4)
```

Giac [A]
time = 0.68, size = 39, normalized size = 1.34

$$\frac{1}{2}\,Bc\log\left(x^2\right)-\frac{3\,Bcx^4+2\,Bbx^2+2\,Acx^2+Ab}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^7,x, algorithm="giac")
[Out] 1/2*B*c*log(x^2) - 1/4*(3*B*c*x^4 + 2*B*b*x^2 + 2*A*c*x^2 + A*b)/x^4
```

Mupad [B]
time = 0.07, size = 29, normalized size = 1.00

$$B\,c\,\ln\left(x\right)-\frac{\left(\frac{A\,c}{2}+\frac{B\,b}{2}\right)\,x^2+\frac{A\,b}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^7,x)
[Out] B*c*log(x) - ((A*b)/4 + x^2*((A*c)/2 + (B*b)/2))/x^4
```

3.11 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8} dx$

Optimal. Leaf size=31

$$-\frac{Ab}{5x^5}-\frac{bB+Ac}{3x^3}-\frac{Bc}{x}$$

```
[Out] -1/5*A*b/x^5+1/3*(-A*c-B*b)/x^3-B*c/x
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1598, 459}

$$-\frac{Ac+bB}{3x^3}-\frac{Ab}{5x^5}-\frac{Bc}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^8,x]
```

```
[Out] -1/5*(A*b)/x^5 - (b*B + A*c)/(3*x^3) - (B*c)/x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)}{x^6}\,dx\\
&= \int \left(\frac{Ab}{x^6}+\frac{bB+Ac}{x^4}+\frac{Bc}{x^2}\right)dx\\
&= -\frac{Ab}{5x^5}-\frac{bB+Ac}{3x^3}-\frac{Bc}{x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.06

$$-\frac{Ab}{5x^5}+\frac{-bB-Ac}{3x^3}-\frac{Bc}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^8,x]
```

```
[Out] -1/5*(A*b)/x^5 + (-(b*B) - A*c)/(3*x^3) - (B*c)/x
```

Maple [A]
time = 0.02, size = 28, normalized size = 0.90

method	result	size
default	$-\frac{Ab}{5x^5}-\frac{Ac+Bb}{3x^3}-\frac{Bc}{x}$	28
risch	$\frac{-Bc\,x^4+\left(-\frac{Ac}{3}-\frac{Bb}{3}\right)x^2-\frac{Ab}{5}}{x^5}$	30
gosper	$-\frac{15Bc\,x^4+5Ac\,x^2+5bB\,x^2+3Ab}{15x^5}$	32
norman	$\frac{\left(-\frac{Ac}{3}-\frac{Bb}{3}\right)x^4-\frac{Ab\,x^2}{5}-Bc\,x^6}{x^7}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*A*b/x^5-1/3*(A*c+B*b)/x^3-B*c/x
```

Maxima [A]
time = 0.28, size = 29, normalized size = 0.94

$$-\frac{15\,Bcx^4+5\,(Bb+Ac)x^2+3\,Ab}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/15*(15*B*c*x^4 + 5*(B*b + A*c)*x^2 + 3*A*b)/x^5
```

Fricas [A]
time = 3.69, size = 29, normalized size = 0.94

$$-\frac{15\,Bcx^4+5\,(Bb+Ac)x^2+3\,Ab}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/15*(15*B*c*x^4 + 5*(B*b + A*c)*x^2 + 3*A*b)/x^5
```

Sympy [A]
time = 0.18, size = 32, normalized size = 1.03

$$\frac{-3Ab - 15Bcx^4 + x^2(-5Ac - 5Bb)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**8,x)
[Out] (-3*A*b - 15*B*c*x**4 + x**2*(-5*A*c - 5*B*b))/(15*x**5)
```

Giac [A]
time = 0.59, size = 31, normalized size = 1.00

$$-\frac{15\,Bcx^4 + 5\,Bbx^2 + 5\,Acx^2 + 3\,Ab}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^8,x, algorithm="giac")
[Out] -1/15*(15*B*c*x^4 + 5*B*b*x^2 + 5*A*c*x^2 + 3*A*b)/x^5
```

Mupad [B]
time = 0.04, size = 29, normalized size = 0.94

$$-\frac{B\,c\,x^4 + \left(\frac{A\,c}{3} + \frac{B\,b}{3}\right)\,x^2 + \frac{A\,b}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^8,x)
[Out] -((A*b)/5 + x^2*((A*c)/3 + (B*b)/3) + B*c*x^4)/x^5
```

3.12 $\int (A+Bx^2)(bx^2+cx^4)^2\,dx$

Optimal. Leaf size=55

$$\frac{1}{5}Ab^2x^5+\frac{1}{7}b(bB+2Ac)x^7+\frac{1}{9}c(2bB+Ac)x^9+\frac{1}{11}Bc^2x^{11}$$

```
[Out] 1/5*A*b^2*x^5+1/7*b*(2*A*c+B*b)*x^7+1/9*c*(A*c+2*B*b)*x^9+1/11*B*c^2*x^11
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1607, 459}

$$\frac{1}{5}Ab^2x^5+\frac{1}{9}cx^9(Ac+2bB)+\frac{1}{7}bx^7(2Ac+bB)+\frac{1}{11}Bc^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (A*b^2*x^5)/5 + (b*(b*B + 2*A*c)*x^7)/7 + (c*(2*b*B + A*c)*x^9)/9 + (B*c^2*
x^11)/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int (A+Bx^2)(bx^2+cx^4)^2\,dx &= \int x^4(A+Bx^2)(b+cx^2)^2\,dx\\ &= \int \left(Ab^2x^4+b(bB+2Ac)x^6+c(2bB+Ac)x^8+Bc^2x^{10}\right)dx\\ &= \frac{1}{5}Ab^2x^5+\frac{1}{7}b(bB+2Ac)x^7+\frac{1}{9}c(2bB+Ac)x^9+\frac{1}{11}Bc^2x^{11}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{5}Ab^2x^5+\frac{1}{7}b(bB+2Ac)x^7+\frac{1}{9}c(2bB+Ac)x^9+\frac{1}{11}Bc^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (A*b^2*x^5)/5 + (b*(b*B + 2*A*c)*x^7)/7 + (c*(2*b*B + A*c)*x^9)/9 + (B*c^2*
x^11)/11
```

Maple [A]
time = 0.43, size = 52, normalized size = 0.95

method	result	size
default	$\frac{B\,c^2x^{11}}{11}+\frac{(A\,c^2+2bBc)x^9}{9}+\frac{(2Abc+b^2B)x^7}{7}+\frac{A\,b^2x^5}{5}$	52
norman	$\frac{B\,c^2x^{11}}{11}+\left(\frac{1}{9}A\,c^2+\frac{2}{9}bBc\right)x^9+\left(\frac{2}{7}Abc+\frac{1}{7}b^2B\right)x^7+\frac{A\,b^2x^5}{5}$	52
risch	$\frac{1}{11}B\,c^2x^{11}+\frac{1}{9}x^9A\,c^2+\frac{2}{9}x^9bBc+\frac{2}{7}x^7Abc+\frac{1}{7}b^2B\,x^7+\frac{1}{5}A\,b^2x^5$	54
gosper	$\frac{x^5(315B\,c^2x^6+385A\,c^2x^4+770x^4bBc+990Abc\,x^2+495b^2B\,x^2+693b^2A)}{3465}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*B*c^2*x^11+1/9*(A*c^2+2*B*b*c)*x^9+1/7*(2*A*b*c+B*b^2)*x^7+1/5*A*b^2*x
^5
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.93

$$\frac{1}{11}\,Bc^2x^{11}+\frac{1}{9}\,(2\,Bbc+Ac^2)x^9+\frac{1}{5}\,Ab^2x^5+\frac{1}{7}\,(Bb^2+2\,Abc)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/11*B*c^2*x^11 + 1/9*(2*B*b*c + A*c^2)*x^9 + 1/5*A*b^2*x^5 + 1/7*(B*b^2 +
2*A*b*c)*x^7
```

Fricas [A]
time = 2.60, size = 51, normalized size = 0.93

$$\frac{1}{11}\,Bc^2x^{11}+\frac{1}{9}\,(2\,Bbc+Ac^2)x^9+\frac{1}{5}\,Ab^2x^5+\frac{1}{7}\,(Bb^2+2\,Abc)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/11*B*c^2*x^11 + 1/9*(2*B*b*c + A*c^2)*x^9 + 1/5*A*b^2*x^5 + 1/7*(B*b^2 +
2*A*b*c)*x^7
```

Sympy [A]
time = 0.01, size = 56, normalized size = 1.02

$$\frac{Ab^2x^5}{5} + \frac{Bc^2x^{11}}{11} + x^9\left(\frac{Ac^2}{9} + \frac{2Bbc}{9}\right) + x^7 \cdot \left(\frac{2Abc}{7} + \frac{Bb^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] A*b**2*x**5/5 + B*c**2*x**11/11 + x**9*(A*c**2/9 + 2*B*b*c/9) + x**7*(2*A*b
*c/7 + B*b**2/7)
```

Giac [A]
time = 0.62, size = 53, normalized size = 0.96

$$\frac{1}{11}\,Bc^2x^{11} + \frac{2}{9}\,Bbcx^9 + \frac{1}{9}\,Ac^2x^9 + \frac{1}{7}\,Bb^2x^7 + \frac{2}{7}\,Abcx^7 + \frac{1}{5}\,Ab^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/11*B*c^2*x^11 + 2/9*B*b*c*x^9 + 1/9*A*c^2*x^9 + 1/7*B*b^2*x^7 + 2/7*A*b*c
*x^7 + 1/5*A*b^2*x^5
```

Mupad [B]
time = 0.09, size = 51, normalized size = 0.93

$$x^7\left(\frac{B\,b^2}{7} + \frac{2\,A\,c\,b}{7}\right) + x^9\left(\frac{A\,c^2}{9} + \frac{2\,B\,b\,c}{9}\right) + \frac{A\,b^2\,x^5}{5} + \frac{B\,c^2\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^7*((B*b^2)/7 + (2*A*b*c)/7) + x^9*((A*c^2)/9 + (2*B*b*c)/9) + (A*b^2*x^5)
/5 + (B*c^2*x^11)/11
```

3.13 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x} dx$

Optimal. Leaf size=55

$$\frac{1}{4}Ab^2x^4 + \frac{1}{6}b(bB + 2Ac)x^6 + \frac{1}{8}c(2bB + Ac)x^8 + \frac{1}{10}Bc^2x^{10}$$

```
[Out] 1/4*A*b^2*x^4+1/6*b*(2*A*c+B*b)*x^6+1/8*c*(A*c+2*B*b)*x^8+1/10*B*c^2*x^10
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 77}

$$\frac{1}{4}Ab^2x^4 + \frac{1}{8}cx^8(Ac + 2bB) + \frac{1}{6}bx^6(2Ac + bB) + \frac{1}{10}Bc^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] (A*b^2*x^4)/4 + (b*(b*B + 2*A*c)*x^6)/6 + (c*(2*b*B + A*c)*x^8)/8 + (B*c^2*
x^10)/10
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x}\,dx = \int x^3(A+Bx^2)\left(b+cx^2\right)^2\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int x(A+Bx)(b+cx)^2\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(Ab^2x+b(bB+2Ac)x^2+c(2bB+Ac)x^3+Bc^2x^4\right)\,dx, x, x^2\right)$$
$$= \frac{1}{4}Ab^2x^4+\frac{1}{6}b(bB+2Ac)x^6+\frac{1}{8}c(2bB+Ac)x^8+\frac{1}{10}Bc^2x^{10}$$

Mathematica [**A**]
time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{4}Ab^2x^4+\frac{1}{6}b(bB+2Ac)x^6+\frac{1}{8}c(2bB+Ac)x^8+\frac{1}{10}Bc^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] (A*b^2*x^4)/4 + (b*(b*B + 2*A*c)*x^6)/6 + (c*(2*b*B + A*c)*x^8)/8 + (B*c^2*
x^10)/10
```

Maple [**A**]
time = 0.40, size = 52, normalized size = 0.95

method	result	size
default	$\frac{B\,c^2x^{10}}{10}+\frac{(A\,c^2+2bBc)x^8}{8}+\frac{(2Abc+b^2B)x^6}{6}+\frac{A\,b^2x^4}{4}$	52
norman	$\frac{B\,c^2x^{10}}{10}+\left(\frac{1}{8}A\,c^2+\frac{1}{4}bBc\right)x^8+\left(\frac{1}{3}Abc+\frac{1}{6}b^2B\right)x^6+\frac{A\,b^2x^4}{4}$	52
risch	$\frac{1}{10}B\,c^2x^{10}+\frac{1}{8}x^8A\,c^2+\frac{1}{4}x^8bBc+\frac{1}{3}x^6Abc+\frac{1}{6}b^2B\,x^6+\frac{1}{4}A\,b^2x^4$	54
gosper	$\frac{x^4\left(12B\,c^2x^6+15A\,c^2x^4+30x^4bBc+40Abc\,x^2+20b^2B\,x^2+30b^2A\right)}{120}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*B*c^2*x^10+1/8*(A*c^2+2*B*b*c)*x^8+1/6*(2*A*b*c+B*b^2)*x^6+1/4*A*b^2*x
^4
```

Maxima [**A**]
time = 0.28, size = 51, normalized size = 0.93

$$\frac{1}{10}\,Bc^2x^{10}+\frac{1}{8}\left(2\,Bbc+Ac^2\right)x^8+\frac{1}{4}\,Ab^2x^4+\frac{1}{6}\left(Bb^2+2\,Abc\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/10*B*c^2*x^10 + 1/8*(2*B*b*c + A*c^2)*x^8 + 1/4*A*b^2*x^4 + 1/6*(B*b^2 +
2*A*b*c)*x^6
```

Fricas [A]
time = 1.81, size = 51, normalized size = 0.93

$$\frac{1}{10}Bc^2x^{10} + \frac{1}{8}\left(2\,Bbc + Ac^2\right)x^8 + \frac{1}{4}Ab^2x^4 + \frac{1}{6}\left(Bb^2 + 2\,Abc\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/10*B*c^2*x^10 + 1/8*(2*B*b*c + A*c^2)*x^8 + 1/4*A*b^2*x^4 + 1/6*(B*b^2 +
2*A*b*c)*x^6
```

Sympy [A]
time = 0.01, size = 53, normalized size = 0.96

$$\frac{Ab^2x^4}{4} + \frac{Bc^2x^{10}}{10} + x^8\left(\frac{Ac^2}{8} + \frac{Bbc}{4}\right) + x^6\left(\frac{Abc}{3} + \frac{Bb^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x,x)
```

```
[Out] A*b**2*x**4/4 + B*c**2*x**10/10 + x**8*(A*c**2/8 + B*b*c/4) + x**6*(A*b*c/3
 + B*b**2/6)
```

Giac [A]
time = 0.65, size = 53, normalized size = 0.96

$$\frac{1}{10}Bc^2x^{10} + \frac{1}{4}Bbcx^8 + \frac{1}{8}Ac^2x^8 + \frac{1}{6}Bb^2x^6 + \frac{1}{3}Abcx^6 + \frac{1}{4}Ab^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/10*B*c^2*x^10 + 1/4*B*b*c*x^8 + 1/8*A*c^2*x^8 + 1/6*B*b^2*x^6 + 1/3*A*b*c
*x^6 + 1/4*A*b^2*x^4
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.93

$$x^6\left(\frac{B\,b^2}{6} + \frac{A\,c\,b}{3}\right) + x^8\left(\frac{A\,c^2}{8} + \frac{B\,b\,c}{4}\right) + \frac{A\,b^2\,x^4}{4} + \frac{B\,c^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x,x)
```

```
[Out] x^6*((B*b^2)/6 + (A*b*c)/3) + x^8*((A*c^2)/8 + (B*b*c)/4) + (A*b^2*x^4)/4 +
 (B*c^2*x^10)/10
```

3.14 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2}\,dx$

Optimal. Leaf size=55

$$\frac{1}{3}Ab^2x^3 + \frac{1}{5}b(bB+2Ac)x^5 + \frac{1}{7}c(2bB+Ac)x^7 + \frac{1}{9}Bc^2x^9$$

```
[Out] 1/3*A*b^2*x^3+1/5*b*(2*A*c+B*b)*x^5+1/7*c*(A*c+2*B*b)*x^7+1/9*B*c^2*x^9
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1598, 459}

$$\frac{1}{3}Ab^2x^3 + \frac{1}{7}cx^7(Ac+2bB) + \frac{1}{5}bx^5(2Ac+bB) + \frac{1}{9}Bc^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] (A*b^2*x^3)/3 + (b*(b*B + 2*A*c)*x^5)/5 + (c*(2*b*B + A*c)*x^7)/7 + (B*c^2*
x^9)/9
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2}\,dx &= \int x^2(A+Bx^2)\left(b+cx^2\right)^2\,dx \\
&= \int \left(Ab^2x^2 + b(bB+2Ac)x^4 + c(2bB+Ac)x^6 + Bc^2x^8\right)dx \\
&= \frac{1}{3}Ab^2x^3 + \frac{1}{5}b(bB+2Ac)x^5 + \frac{1}{7}c(2bB+Ac)x^7 + \frac{1}{9}Bc^2x^9
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{3}Ab^2x^3+\frac{1}{5}b(bB+2Ac)x^5+\frac{1}{7}c(2bB+Ac)x^7+\frac{1}{9}Bc^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] (A*b^2*x^3)/3 + (b*(b*B + 2*A*c)*x^5)/5 + (c*(2*b*B + A*c)*x^7)/7 + (B*c^2*
x^9)/9
```

Maple [A]
time = 0.42, size = 52, normalized size = 0.95

method	result	size
default	$\frac{B\,c^2x^9}{9}+\frac{(A\,c^2+2bBc)x^7}{7}+\frac{(2Abc+b^2B)x^5}{5}+\frac{A\,b^2x^3}{3}$	52
risch	$\frac{1}{9}B\,c^2x^9+\frac{1}{7}x^7A\,c^2+\frac{2}{7}x^7bBc+\frac{2}{5}x^5Abc+\frac{1}{5}b^2B\,x^5+\frac{1}{3}A\,b^2x^3$	54
gosper	$\frac{x^3(35B\,c^2x^6+45A\,c^2x^4+90x^4bBc+126Abc\,x^2+63b^2B\,x^2+105b^2A)}{315}$	56
norman	$\frac{(\frac{1}{7}A\,c^2+\frac{2}{7}bBc)x^8+(\frac{2}{5}Abc+\frac{1}{5}b^2B)x^6+\frac{A\,b^2x^4}{3}+\frac{B\,c^2x^{10}}{9}}{x}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*B*c^2*x^9+1/7*(A*c^2+2*B*b*c)*x^7+1/5*(2*A*b*c+B*b^2)*x^5+1/3*A*b^2*x^3
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.93

$$\frac{1}{9}\,Bc^2x^9+\frac{1}{7}\,(2\,Bbc+Ac^2)x^7+\frac{1}{3}\,Ab^2x^3+\frac{1}{5}\,(Bb^2+2\,Abc)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*B*c^2*x^9 + 1/7*(2*B*b*c + A*c^2)*x^7 + 1/3*A*b^2*x^3 + 1/5*(B*b^2 + 2*
A*b*c)*x^5
```

Fricas [A]
time = 1.25, size = 51, normalized size = 0.93

$$\frac{1}{9}\,Bc^2x^9+\frac{1}{7}\,(2\,Bbc+Ac^2)x^7+\frac{1}{3}\,Ab^2x^3+\frac{1}{5}\,(Bb^2+2\,Abc)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*B*c^2*x^9 + 1/7*(2*B*b*c + A*c^2)*x^7 + 1/3*A*b^2*x^3 + 1/5*(B*b^2 + 2*
A*b*c)*x^5
```

Sympy [A]
time = 0.01, size = 56, normalized size = 1.02

$$\frac{Ab^2x^3}{3} + \frac{Bc^2x^9}{9} + x^7\left(\frac{Ac^2}{7} + \frac{2Bbc}{7}\right) + x^5 \cdot \left(\frac{2Abc}{5} + \frac{Bb^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**2,x)
```

```
[Out] A*b**2*x**3/3 + B*c**2*x**9/9 + x**7*(A*c**2/7 + 2*B*b*c/7) + x**5*(2*A*b*c
/5 + B*b**2/5)
```

Giac [A]
time = 1.14, size = 53, normalized size = 0.96

$$\frac{1}{9}\,Bc^2x^9 + \frac{2}{7}\,Bbcx^7 + \frac{1}{7}\,Ac^2x^7 + \frac{1}{5}\,Bb^2x^5 + \frac{2}{5}\,Abcx^5 + \frac{1}{3}\,Ab^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/7*A*c^2*x^7 + 1/5*B*b^2*x^5 + 2/5*A*b*c*x
^5 + 1/3*A*b^2*x^3
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.93

$$x^5\,\left(\frac{B\,b^2}{5}+\frac{2\,A\,c\,b}{5}\right)+x^7\,\left(\frac{A\,c^2}{7}+\frac{2\,B\,b\,c}{7}\right)+\frac{A\,b^2\,x^3}{3}+\frac{B\,c^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^2,x)
```

```
[Out] x^5*((B*b^2)/5 + (2*A*b*c)/5) + x^7*((A*c^2)/7 + (2*B*b*c)/7) + (A*b^2*x^3)
/3 + (B*c^2*x^9)/9
```

3.15 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3} dx$

Optimal. Leaf size=42

$$-\frac{(bB-Ac)(b+cx^2)^3}{6c^2}+\frac{B(b+cx^2)^4}{8c^2}$$

```
[Out] -1/6*(-A*c+B*b)*(c*x^2+b)^3/c^2+1/8*B*(c*x^2+b)^4/c^2
```

Rubi [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 455, 45}

$$\frac{B(b+cx^2)^4}{8c^2}-\frac{(b+cx^2)^3(bB-Ac)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] -1/6*((b*B - A*c)*(b + c*x^2)^3)/c^2 + (B*(b + c*x^2)^4)/(8*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^3}\,dx = \int x\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int (A+Bx)(b+cx)^2\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bB+Ac)(b+cx)^2}{c}+\frac{B(b+cx)^3}{c}\right)\,dx, x, x^2\right)$$
$$= -\frac{(bB-Ac)\,(b+cx^2)^3}{6c^2}+\frac{B(b+cx^2)^4}{8c^2}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.21

$$\frac{1}{24}x^2\left(12Ab^2+6b(bB+2Ac)x^2+4c(2bB+Ac)x^4+3Bc^2x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] (x^2*(12*A*b^2 + 6*b*(b*B + 2*A*c)*x^2 + 4*c*(2*b*B + A*c)*x^4 + 3*B*c^2*x^
6))/24
```

Maple [A]
time = 0.52, size = 52, normalized size = 1.24

method	result	size
default	$\frac{B\,c^2x^8}{8}+\frac{(A\,c^2+2bBc)x^6}{6}+\frac{(2Abc+b^2B)x^4}{4}+\frac{b^2A\,x^2}{2}$	52
risch	$\frac{1}{8}B\,c^2x^8+\frac{1}{6}x^6A\,c^2+\frac{1}{3}x^6bBc+\frac{1}{2}x^4Abc+\frac{1}{4}b^2B\,x^4+\frac{1}{2}b^2A\,x^2$	54
gosper	$\frac{x^2(3B\,c^2x^6+4A\,c^2x^4+8x^4bBc+12Abc\,x^2+6b^2B\,x^2+12b^2A)}{24}$	56
norman	$\frac{(\frac{1}{6}A\,c^2+\frac{1}{3}bBc)x^8+(\frac{1}{2}Abc+\frac{1}{4}b^2B)x^6+\frac{A\,b^2x^4}{2}+\frac{B\,c^2x^{10}}{8}}{x^2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*B*c^2*x^8+1/6*(A*c^2+2*B*b*c)*x^6+1/4*(2*A*b*c+B*b^2)*x^4+1/2*b^2*A*x^2
```

Maxima [A]
time = 0.29, size = 51, normalized size = 1.21

$$\frac{1}{8}\,Bc^2x^8+\frac{1}{6}\,\left(2\,Bbc+Ac^2\right)x^6+\frac{1}{2}\,Ab^2x^2+\frac{1}{4}\,\left(Bb^2+2\,Abc\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*B*c^2*x^8 + 1/6*(2*B*b*c + A*c^2)*x^6 + 1/2*A*b^2*x^2 + 1/4*(B*b^2 + 2*
A*b*c)*x^4
```

Fricas [A]
time = 1.91, size = 51, normalized size = 1.21

$$\frac{1}{8}\,Bc^2x^8+\frac{1}{6}\left(2\,Bbc+Ac^2\right)x^6+\frac{1}{2}\,Ab^2x^2+\frac{1}{4}\left(Bb^2+2\,Abc\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*B*c^2*x^8 + 1/6*(2*B*b*c + A*c^2)*x^6 + 1/2*A*b^2*x^2 + 1/4*(B*b^2 + 2*
A*b*c)*x^4
```

Sympy [A]
time = 0.01, size = 53, normalized size = 1.26

$$\frac{Ab^2x^2}{2}+\frac{Bc^2x^8}{8}+x^6\left(\frac{Ac^2}{6}+\frac{Bbc}{3}\right)+x^4\left(\frac{Abc}{2}+\frac{Bb^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**3,x)
```

```
[Out] A*b**2*x**2/2 + B*c**2*x**8/8 + x**6*(A*c**2/6 + B*b*c/3) + x**4*(A*b*c/2 +
 B*b**2/4)
```

Giac [A]
time = 1.84, size = 53, normalized size = 1.26

$$\frac{1}{8}\,Bc^2x^8+\frac{1}{3}\,Bbcx^6+\frac{1}{6}\,Ac^2x^6+\frac{1}{4}\,Bb^2x^4+\frac{1}{2}\,Abcx^4+\frac{1}{2}\,Ab^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/8*B*c^2*x^8 + 1/3*B*b*c*x^6 + 1/6*A*c^2*x^6 + 1/4*B*b^2*x^4 + 1/2*A*b*c*x
^4 + 1/2*A*b^2*x^2
```

Mupad [B]
time = 0.05, size = 51, normalized size = 1.21

$$x^4\left(\frac{B\,b^2}{4}+\frac{A\,c\,b}{2}\right)+x^6\left(\frac{A\,c^2}{6}+\frac{B\,b\,c}{3}\right)+\frac{A\,b^2\,x^2}{2}+\frac{B\,c^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^3,x)
```

```
[Out] x^4*((B*b^2)/4 + (A*b*c)/2) + x^6*((A*c^2)/6 + (B*b*c)/3) + (A*b^2*x^2)/2 +
 (B*c^2*x^8)/8
```

3.16 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4} \, dx$

Optimal. Leaf size=50

$$Ab^2x + \frac{1}{3}b(bB + 2Ac)x^3 + \frac{1}{5}c(2bB + Ac)x^5 + \frac{1}{7}Bc^2x^7$$

```
[Out] A*b^2*x+1/3*b*(2*A*c+B*b)*x^3+1/5*c*(A*c+2*B*b)*x^5+1/7*B*c^2*x^7
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1598, 380}

$$Ab^2x + \frac{1}{5}cx^5(Ac + 2bB) + \frac{1}{3}bx^3(2Ac + bB) + \frac{1}{7}Bc^2x^7$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] A*b^2*x + (b*(b*B + 2*A*c)*x^3)/3 + (c*(2*b*B + A*c)*x^5)/5 + (B*c^2*x^7)/7
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4} \, dx &= \int (A+Bx^2)(b+cx^2)^2 \, dx \\
&= \int \left(Ab^2 + b(bB+2Ac)x^2 + c(2bB+Ac)x^4 + Bc^2x^6\right) dx \\
&= Ab^2x + \frac{1}{3}b(bB+2Ac)x^3 + \frac{1}{5}c(2bB+Ac)x^5 + \frac{1}{7}Bc^2x^7
\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 50, normalized size = 1.00

$$Ab^2x + \frac{1}{3}b(bB + 2Ac)x^3 + \frac{1}{5}c(2bB + Ac)x^5 + \frac{1}{7}Bc^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] A*b^2*x + (b*(b*B + 2*A*c)*x^3)/3 + (c*(2*b*B + A*c)*x^5)/5 + (B*c^2*x^7)/7
```

Maple [**A**]
time = 0.50, size = 49, normalized size = 0.98

method	result	size
default	$\frac{B\,c^2x^7}{7}+\frac{(A\,c^2+2bBc)x^5}{5}+\frac{(2Abc+b^2B)x^3}{3}+A\,b^2x$	49
risch	$\frac{1}{7}B\,c^2x^7+\frac{1}{5}x^5A\,c^2+\frac{2}{5}x^5bBc+\frac{2}{3}x^3Abc+\frac{1}{3}b^2B\,x^3+A\,b^2x$	51
gosper	$\frac{x(15B\,c^2x^6+21A\,c^2x^4+42x^4bBc+70Abc\,x^2+35b^2B\,x^2+105b^2A)}{105}$	54
norman	$\frac{(\frac{1}{5}A\,c^2+\frac{2}{5}bBc)x^8+(\frac{2}{3}Abc+\frac{1}{3}b^2B)x^6+A\,b^2x^4+\frac{B\,c^2x^{10}}{7}}{x^3}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*B*c^2*x^7+1/5*(A*c^2+2*B*b*c)*x^5+1/3*(2*A*b*c+B*b^2)*x^3+A*b^2*x
```

Maxima [**A**]
time = 0.28, size = 48, normalized size = 0.96

$$\frac{1}{7}\,Bc^2x^7 + \frac{1}{5}\,\left(2\,Bbc + Ac^2\right)x^5 + Ab^2x + \frac{1}{3}\,\left(Bb^2 + 2\,Abc\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*B*c^2*x^7 + 1/5*(2*B*b*c + A*c^2)*x^5 + A*b^2*x + 1/3*(B*b^2 + 2*A*b*c)
*x^3
```

Fricas [**A**]
time = 2.50, size = 48, normalized size = 0.96

$$\frac{1}{7}\,Bc^2x^7 + \frac{1}{5}\,\left(2\,Bbc + Ac^2\right)x^5 + Ab^2x + \frac{1}{3}\,\left(Bb^2 + 2\,Abc\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/7*B*c^2*x^7 + 1/5*(2*B*b*c + A*c^2)*x^5 + A*b^2*x + 1/3*(B*b^2 + 2*A*b*c)
*x^3
```

Sympy [A]
time = 0.01, size = 53, normalized size = 1.06

$$Ab^2x+\frac{Bc^2x^7}{7}+x^5\left(\frac{Ac^2}{5}+\frac{2Bbc}{5}\right)+x^3\cdot\left(\frac{2Abc}{3}+\frac{Bb^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**4,x)
```

```
[Out] A*b**2*x + B*c**2*x**7/7 + x**5*(A*c**2/5 + 2*B*b*c/5) + x**3*(2*A*b*c/3 +
B*b**2/3)
```

Giac [A]
time = 0.88, size = 50, normalized size = 1.00

$$\frac{1}{7}Bc^2x^7+\frac{2}{5}Bbcx^5+\frac{1}{5}Ac^2x^5+\frac{1}{3}Bb^2x^3+\frac{2}{3}Abcx^3+Ab^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/7*B*c^2*x^7 + 2/5*B*b*c*x^5 + 1/5*A*c^2*x^5 + 1/3*B*b^2*x^3 + 2/3*A*b*c*x
^3 + A*b^2*x
```

Mupad [B]
time = 0.05, size = 48, normalized size = 0.96

$$x^3\left(\frac{B\,b^2}{3}+\frac{2\,A\,c\,b}{3}\right)+x^5\left(\frac{A\,c^2}{5}+\frac{2\,B\,b\,c}{5}\right)+\frac{B\,c^2\,x^7}{7}+A\,b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^4,x)
```

```
[Out] x^3*((B*b^2)/3 + (2*A*b*c)/3) + x^5*((A*c^2)/5 + (2*B*b*c)/5) + (B*c^2*x^7)
/7 + A*b^2*x
```

3.17 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5}\,dx$

Optimal. Leaf size=43

$$Abcx^2+\frac{1}{4}Ac^2x^4+\frac{B(b+cx^2)^3}{6c}+Ab^2\log(x)$$

[Out] A*b*c*x^2+1/4*A*c^2*x^4+1/6*B*(c*x^2+b)^3/c+A*b^2*ln(x)

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 457, 81, 45}

$$Ab^2\log(x)+Abcx^2+\frac{1}{4}Ac^2x^4+\frac{B(b+cx^2)^3}{6c}$$

Antiderivative was successfully verified.

[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^5,x]

[Out] A*b*c*x^2 + (A*c^2*x^4)/4 + (B*(b + c*x^2)^3)/(6*c) + A*b^2*Log[x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^5}\,dx &= \int \frac{(A+Bx^2)\,(b+cx^2)^2}{x}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^2}{x}\,dx,x,x^2\right)\\ &= \frac{B(b+cx^2)^3}{6c}+\frac{1}{2}A\text{Subst}\left(\int \frac{(b+cx)^2}{x}\,dx,x,x^2\right)\\ &= \frac{B(b+cx^2)^3}{6c}+\frac{1}{2}A\text{Subst}\left(\int \left(2bc+\frac{b^2}{x}+c^2x\right)\,dx,x,x^2\right)\\ &= Abcx^2+\frac{1}{4}Ac^2x^4+\frac{B(b+cx^2)^3}{6c}+Ab^2\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.19

$$\frac{1}{2}b(bB+2Ac)x^2+\frac{1}{4}c(2bB+Ac)x^4+\frac{1}{6}Bc^2x^6+Ab^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^5,x]
[Out] (b*(b*B + 2*A*c)*x^2)/2 + (c*(2*b*B + A*c)*x^4)/4 + (B*c^2*x^6)/6 + A*b^2*L
og[x]
```

Maple [A]
time = 0.36, size = 51, normalized size = 1.19

method	result	size
default	$\frac{B\,c^2x^6}{6}+\frac{A\,c^2x^4}{4}+\frac{x^4bBc}{2}+Abc\,x^2+\frac{b^2B\,x^2}{2}+A\,b^2\ln(x)$	51
risch	$\frac{B\,c^2x^6}{6}+\frac{A\,c^2x^4}{4}+\frac{x^4bBc}{2}+Abc\,x^2+\frac{b^2B\,x^2}{2}+A\,b^2\ln(x)$	51
norman	$\frac{(\frac{1}{4}A\,c^2+\frac{1}{2}bBc)x^8+(Abc+\frac{1}{2}b^2B)x^6+\frac{B\,c^2x^{10}}{6}}{x^4}+A\,b^2\ln(x)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*B*c^2*x^6+1/4*A*c^2*x^4+1/2*x^4*b*B*c+A*b*c*x^2+1/2*b^2*B*x^2+A*b^2*ln(
x)
```

Maxima [A]
time = 0.28, size = 52, normalized size = 1.21

$$\frac{1}{6}\,Bc^2x^6+\frac{1}{4}\,(2\,Bbc+Ac^2)x^4+\frac{1}{2}\,Ab^2\log\left(x^2\right)+\frac{1}{2}\,(Bb^2+2\,Abc)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*B*c^2*x^6 + 1/4*(2*B*b*c + A*c^2)*x^4 + 1/2*A*b^2*log(x^2) + 1/2*(B*b^2
 + 2*A*b*c)*x^2
```

Fricas [A]
time = 2.06, size = 49, normalized size = 1.14

$$\frac{1}{6}\,Bc^2x^6+\frac{1}{4}\,(2\,Bbc+Ac^2)x^4+Ab^2\log\left(x\right)+\frac{1}{2}\,(Bb^2+2\,Abc)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/6*B*c^2*x^6 + 1/4*(2*B*b*c + A*c^2)*x^4 + A*b^2*log(x) + 1/2*(B*b^2 + 2*A
*b*c)*x^2
```

Sympy [A]
time = 0.05, size = 49, normalized size = 1.14

$$Ab^2\log\left(x\right)+\frac{Bc^2x^6}{6}+x^4\left(\frac{Ac^2}{4}+\frac{Bbc}{2}\right)+x^2\left(Abc+\frac{Bb^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**5,x)
```

```
[Out] A*b**2*log(x) + B*c**2*x**6/6 + x**4*(A*c**2/4 + B*b*c/2) + x**2*(A*b*c + B
*b**2/2)
```

Giac [A]
time = 0.93, size = 53, normalized size = 1.23

$$\frac{1}{6}\,Bc^2x^6+\frac{1}{2}\,Bbcx^4+\frac{1}{4}\,Ac^2x^4+\frac{1}{2}\,Bb^2x^2+Abcx^2+\frac{1}{2}\,Ab^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/6*B*c^2*x^6 + 1/2*B*b*c*x^4 + 1/4*A*c^2*x^4 + 1/2*B*b^2*x^2 + A*b*c*x^2 +
 1/2*A*b^2*log(x^2)
```

Mupad [B]
time = 0.04, size = 48, normalized size = 1.12

$$x^2\left(\frac{B\,b^2}{2}+A\,c\,b\right)+x^4\left(\frac{A\,c^2}{4}+\frac{B\,b\,c}{2}\right)+\frac{B\,c^2\,x^6}{6}+A\,b^2\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^5,x)
```

```
[Out] x^2*((B*b^2)/2 + A*b*c) + x^4*((A*c^2)/4 + (B*b*c)/2) + (B*c^2*x^6)/6 + A*b
^2*log(x)
```

3.18 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6} dx$

Optimal. Leaf size=48

$$-\frac{Ab^2}{x}+b(bB+2Ac)x+\frac{1}{3}c(2bB+Ac)x^3+\frac{1}{5}Bc^2x^5$$

```
[Out] -A*b^2/x+b*(2*A*c+B*b)*x+1/3*c*(A*c+2*B*b)*x^3+1/5*B*c^2*x^5
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$-\frac{Ab^2}{x}+\frac{1}{3}cx^3(Ac+2bB)+bx(2Ac+bB)+\frac{1}{5}Bc^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^6,x]
```

```
[Out] -((A*b^2)/x) + b*(b*B + 2*A*c)*x + (c*(2*b*B + A*c)*x^3)/3 + (B*c^2*x^5)/5
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6} dx &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^2} dx\\ &= \int \left(b(bB+2Ac)+\frac{Ab^2}{x^2}+c(2bB+Ac)x^2+Bc^2x^4\right) dx\\ &= -\frac{Ab^2}{x}+b(bB+2Ac)x+\frac{1}{3}c(2bB+Ac)x^3+\frac{1}{5}Bc^2x^5\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 1.00

$$-\frac{Ab^2}{x}+b(bB+2Ac)x+\frac{1}{3}c(2bB+Ac)x^3+\frac{1}{5}Bc^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^6,x]
[Out] -((A*b^2)/x) + b*(b*B + 2*A*c)*x + (c*(2*b*B + A*c)*x^3)/3 + (B*c^2*x^5)/5
```

Maple [A]
time = 0.35, size = 49, normalized size = 1.02

method	result	size
default	$\frac{B\,c^2x^5}{5}+\frac{A\,c^2x^3}{3}+\frac{2Bbc\,x^3}{3}+2Abcx+b^2Bx-\frac{A\,b^2}{x}$	49
risch	$\frac{B\,c^2x^5}{5}+\frac{A\,c^2x^3}{3}+\frac{2Bbc\,x^3}{3}+2Abcx+b^2Bx-\frac{A\,b^2}{x}$	49
norman	$\frac{\left(\frac{1}{3}A\,c^2+\frac{2}{3}bBc\right)x^8+\left(2Abc+b^2B\right)x^6-A\,b^2x^4+\frac{B\,c^2x^{10}}{5}}{x^5}$	55
gosper	$-\frac{-3B\,c^2x^6-5A\,c^2x^4-10x^4bBc-30Abc\,x^2-15b^2B\,x^2+15b^2A}{15x}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x,method=_RETURNVERBOSE)
[Out] 1/5*B*c^2*x^5+1/3*A*c^2*x^3+2/3*B*b*c*x^3+2*A*b*c*x+b^2*B*x-A*b^2/x
```

Maxima [A]
time = 0.28, size = 48, normalized size = 1.00

$$\frac{1}{5}\,Bc^2x^5+\frac{1}{3}\,\left(2\,Bbc+Ac^2\right)x^3-\frac{Ab^2}{x}+\left(Bb^2+2\,Abc\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x, algorithm="maxima")
[Out] 1/5*B*c^2*x^5 + 1/3*(2*B*b*c + A*c^2)*x^3 - A*b^2/x + (B*b^2 + 2*A*b*c)*x
```

Fricas [A]
time = 2.10, size = 53, normalized size = 1.10

$$\frac{3\,Bc^2x^6+5\,(2\,Bbc+Ac^2)x^4-15\,Ab^2+15\,(Bb^2+2\,Abc)x^2}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*c^2*x^6 + 5*(2*B*b*c + A*c^2)*x^4 - 15*A*b^2 + 15*(B*b^2 + 2*A*b*
c)*x^2)/x
```

Sympy [A]
time = 0.04, size = 48, normalized size = 1.00

$$-\frac{Ab^2}{x}+\frac{Bc^2x^5}{5}+x^3\left(\frac{Ac^2}{3}+\frac{2Bbc}{3}\right)+x\left(2Abc+Bb^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**6,x)
```

```
[Out] -A*b**2/x + B*c**2*x**5/5 + x**3*(A*c**2/3 + 2*B*b*c/3) + x*(2*A*b*c + B*b*
*2)
```

Giac [A]
time = 0.80, size = 48, normalized size = 1.00

$$\frac{1}{5}\,Bc^2x^5+\frac{2}{3}\,Bbcx^3+\frac{1}{3}\,Ac^2x^3+Bb^2x+2\,Abcx-\frac{Ab^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/5*B*c^2*x^5 + 2/3*B*b*c*x^3 + 1/3*A*c^2*x^3 + B*b^2*x + 2*A*b*c*x - A*b^2
/x
```

Mupad [B]
time = 0.05, size = 48, normalized size = 1.00

$$x^3\,\left(\frac{A\,c^2}{3}+\frac{2\,B\,b\,c}{3}\right)+x\,\left(B\,b^2+2\,A\,c\,b\right)-\frac{A\,b^2}{x}+\frac{B\,c^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^6,x)
```

```
[Out] x^3*((A*c^2)/3 + (2*B*b*c)/3) + x*(B*b^2 + 2*A*b*c) - (A*b^2)/x + (B*c^2*x^
5)/5
```

3.19 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7} dx$

Optimal. Leaf size=51

$$-\frac{Ab^2}{2x^2} + \frac{1}{2}c(2bB + Ac)x^2 + \frac{1}{4}Bc^2x^4 + b(bB + 2Ac)\log(x)$$

```
[Out] -1/2*A*b^2/x^2+1/2*c*(A*c+2*B*b)*x^2+1/4*B*c^2*x^4+b*(2*A*c+B*b)*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 77}

$$-\frac{Ab^2}{2x^2} + \frac{1}{2}cx^2(Ac + 2bB) + b\log(x)(2Ac + bB) + \frac{1}{4}Bc^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] -1/2*(A*b^2)/x^2 + (c*(2*b*B + A*c)*x^2)/2 + (B*c^2*x^4)/4 + b*(b*B + 2*A*c
)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx = \int \frac{(A+Bx^2)(b+cx^2)^2}{x^3}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^2}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(c(2bB+Ac)+\frac{Ab^2}{x^2}+\frac{b(bB+2Ac)}{x}+Bc^2x\right)\,dx, x, x^2\right)$$
$$= -\frac{Ab^2}{2x^2}+\frac{1}{2}c(2bB+Ac)x^2+\frac{1}{4}Bc^2x^4+b(bB+2Ac)\log(x)$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2Ab^2}{x^2}+2c(2bB+Ac)x^2+Bc^2x^4+4b(bB+2Ac)\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] ((-2*A*b^2)/x^2 + 2*c*(2*b*B + A*c)*x^2 + B*c^2*x^4 + 4*b*(b*B + 2*A*c)*Log
[x])/4
```

Maple [A]
time = 0.40, size = 48, normalized size = 0.94

method	result	size
default	$\frac{B\,c^2x^4}{4}+\frac{A\,c^2x^2}{2}+Bbc\,x^2-\frac{A\,b^2}{2x^2}+b(2Ac+Bb)\ln(x)$	48
norman	$\frac{\left(\frac{1}{2}A\,c^2+bBc\right)x^8-\frac{A\,b^2x^4}{2}+\frac{B\,c^2x^{10}}{4}}{x^6}+\left(2Abc+b^2B\right)\ln(x)$	54
risch	$\frac{B\,c^2x^4}{4}+\frac{A\,c^2x^2}{2}+Bbc\,x^2+\frac{A^2c^2}{4B}+Abc+b^2B-\frac{A\,b^2}{2x^2}+2A\ln(x)\,bc+b^2B\ln(x)$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*B*c^2*x^4+1/2*A*c^2*x^2+B*b*c*x^2-1/2*A*b^2/x^2+b*(2*A*c+B*b)*ln(x)
```

Maxima [A]
time = 0.29, size = 52, normalized size = 1.02

$$\frac{1}{4}\,Bc^2x^4+\frac{1}{2}\,(2\,Bbc+Ac^2)x^2+\frac{1}{2}\,(Bb^2+2\,Abc)\log\left(x^2\right)-\frac{Ab^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*B*c^2*x^4 + 1/2*(2*B*b*c + A*c^2)*x^2 + 1/2*(B*b^2 + 2*A*b*c)*log(x^2)
- 1/2*A*b^2/x^2
```

Fricas [A]
time = 1.04, size = 54, normalized size = 1.06

$$\frac{Bc^2x^6 + 2\,(2\,Bbc + Ac^2)x^4 + 4\,(Bb^2 + 2\,Abc)x^2\log\left(x\right) - 2\,Ab^2}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^2*x^6 + 2*(2*B*b*c + A*c^2)*x^4 + 4*(B*b^2 + 2*A*b*c)*x^2*log(x) -
 2*A*b^2)/x^2
```

Sympy [A]
time = 0.10, size = 48, normalized size = 0.94

$$-\frac{Ab^2}{2x^2} + \frac{Bc^2x^4}{4} + b(2Ac + Bb)\log\left(x\right) + x^2\left(\frac{Ac^2}{2} + Bbc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**7,x)
```

```
[Out] -A*b**2/(2*x**2) + B*c**2*x**4/4 + b*(2*A*c + B*b)*log(x) + x**2*(A*c**2/2
+ B*b*c)
```

Giac [A]
time = 0.72, size = 70, normalized size = 1.37

$$\frac{1}{4}\,Bc^2x^4 + Bbcx^2 + \frac{1}{2}\,Ac^2x^2 + \frac{1}{2}\,\left(Bb^2 + 2\,Abc\right)\log\left(x^2\right) - \frac{Bb^2x^2 + 2\,Abcx^2 + Ab^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/4*B*c^2*x^4 + B*b*c*x^2 + 1/2*A*c^2*x^2 + 1/2*(B*b^2 + 2*A*b*c)*log(x^2)
- 1/2*(B*b^2*x^2 + 2*A*b*c*x^2 + A*b^2)/x^2
```

Mupad [B]
time = 0.05, size = 48, normalized size = 0.94

$$x^2\left(\frac{A\,c^2}{2} + B\,b\,c\right) + \ln\left(x\right)\,\left(B\,b^2 + 2\,A\,c\,b\right) - \frac{A\,b^2}{2\,x^2} + \frac{B\,c^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^7,x)
```

```
[Out] x^2*((A*c^2)/2 + B*b*c) + log(x)*(B*b^2 + 2*A*b*c) - (A*b^2)/(2*x^2) + (B*c
^2*x^4)/4
```

3.20 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8} \, dx$

Optimal. Leaf size=48

$$-\frac{Ab^2}{3x^3}-\frac{b(bB+2Ac)}{x}+c(2bB+Ac)x+\frac{1}{3}Bc^2x^3$$

```
[Out] -1/3*A*b^2/x^3-b*(2*A*c+B*b)/x+c*(A*c+2*B*b)*x+1/3*B*c^2*x^3
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$-\frac{Ab^2}{3x^3}+cx(Ac+2bB)-\frac{b(2Ac+bB)}{x}+\frac{1}{3}Bc^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^8,x]
```

```
[Out] -1/3*(A*b^2)/x^3 - (b*(b*B + 2*A*c))/x + c*(2*b*B + A*c)*x + (B*c^2*x^3)/3
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8} \, dx &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^4} \, dx\\ &= \int \left(c(2bB+Ac)+\frac{Ab^2}{x^4}+\frac{b(bB+2Ac)}{x^2}+Bc^2x^2\right) dx\\ &= -\frac{Ab^2}{3x^3}-\frac{b(bB+2Ac)}{x}+c(2bB+Ac)x+\frac{1}{3}Bc^2x^3\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.04

$$-\frac{Ab^2}{3x^3}+\frac{-b^2B-2Abc}{x}+c(2bB+Ac)x+\frac{1}{3}Bc^2x^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^8,x]
```

```
[Out] -1/3*(A*b^2)/x^3 + (-(b^2*B) - 2*A*b*c)/x + c*(2*b*B + A*c)*x + (B*c^2*x^3)
/3
```

Maple [A]
time = 0.36, size = 46, normalized size = 0.96

method	result	size
default	$\frac{B\,c^{2}x^{3}}{3}+A\,c^{2}x+2bBcx-\frac{A\,b^{2}}{3x^{3}}-\frac{b(2Ac+Bb)}{x}$	46
risch	$\frac{B\,c^{2}x^{3}}{3}+A\,c^{2}x+2bBcx+\frac{(-2Abc-b^{2}B)x^{2}-\frac{b^{2}A}{3}}{x^{3}}$	50
gosper	$-\frac{-B\,c^{2}x^{6}-3A\,c^{2}x^{4}-6x^{4}bBc+6Abc\,x^{2}+3b^{2}B\,x^{2}+b^{2}A}{3x^{3}}$	55
norman	$\frac{(A\,c^{2}+2bBc)x^{8}+(-2Abc-b^{2}B)x^{6}-\frac{A\,b^{2}x^{4}}{3}+\frac{B\,c^{2}x^{10}}{3}}{x^{7}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*B*c^2*x^3+A*c^2*x+2*b*B*c*x-1/3*A*b^2/x^3-b*(2*A*c+B*b)/x
```

Maxima [A]
time = 0.28, size = 50, normalized size = 1.04

$$\frac{1}{3}\,Bc^2x^3+\left(2\,Bbc+Ac^2\right)x-\frac{Ab^2+3\,(Bb^2+2\,Abc)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x, algorithm="maxima")
```

```
[Out] 1/3*B*c^2*x^3 + (2*B*b*c + A*c^2)*x - 1/3*(A*b^2 + 3*(B*b^2 + 2*A*b*c)*x^2)
/x^3
```

Fricas [A]
time = 1.71, size = 52, normalized size = 1.08

$$\frac{Bc^2x^6+3\,(2\,Bbc+Ac^2)x^4-Ab^2-3\,(Bb^2+2\,Abc)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x, algorithm="fricas")
```

```
[Out] 1/3*(B*c^2*x^6 + 3*(2*B*b*c + A*c^2)*x^4 - A*b^2 - 3*(B*b^2 + 2*A*b*c)*x^2)
/x^3
```

Sympy [A]
time = 0.11, size = 51, normalized size = 1.06

$$\frac{Bc^2x^3}{3} + x\left(Ac^2 + 2Bbc\right) + \frac{-Ab^2 + x^2(-6Abc - 3Bb^2)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**8,x)
```

```
[Out] B*c**2*x**3/3 + x*(A*c**2 + 2*B*b*c) + (-A*b**2 + x**2*(-6*A*b*c - 3*B*b**2
))/(3*x**3)
```

Giac [A]
time = 0.60, size = 50, normalized size = 1.04

$$\frac{1}{3}\,Bc^2x^3 + 2\,Bbcx + Ac^2x - \frac{3\,Bb^2x^2 + 6\,Abcx^2 + Ab^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x, algorithm="giac")
```

```
[Out] 1/3*B*c^2*x^3 + 2*B*b*c*x + A*c^2*x - 1/3*(3*B*b^2*x^2 + 6*A*b*c*x^2 + A*b^
2)/x^3
```

Mupad [B]
time = 0.05, size = 50, normalized size = 1.04

$$x\,\left(A\,c^2+2\,B\,b\,c\right)-\frac{x^2\,\left(B\,b^2+2\,A\,c\,b\right)+\frac{A\,b^2}{3}}{x^3}+\frac{B\,c^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^8,x)
```

```
[Out] x*(A*c^2 + 2*B*b*c) - (x^2*(B*b^2 + 2*A*b*c) + (A*b^2)/3)/x^3 + (B*c^2*x^3)
/3
```

3.21 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9} \, dx$

Optimal. Leaf size=51

$$-\frac{Ab^2}{4x^4}-\frac{b(bB+2Ac)}{2x^2}+\frac{1}{2}Bc^2x^2+c(2bB+Ac)\log(x)$$

```
[Out] -1/4*A*b^2/x^4-1/2*b*(2*A*c+B*b)/x^2+1/2*B*c^2*x^2+c*(A*c+2*B*b)*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 77}

$$-\frac{Ab^2}{4x^4}-\frac{b(2Ac+bB)}{2x^2}+c\log(x)(Ac+2bB)+\frac{1}{2}Bc^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^9,x]
```

```
[Out] -1/4*(A*b^2)/x^4 - (b*(b*B + 2*A*c))/(2*x^2) + (B*c^2*x^2)/2 + c*(2*b*B + A
*c)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9}\,dx = \int \frac{(A+Bx^2)(b+cx^2)^2}{x^5}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^2}{x^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(Bc^2+\frac{Ab^2}{x^3}+\frac{b(bB+2Ac)}{x^2}+\frac{c(2bB+Ac)}{x}\right)\,dx, x, x^2\right)$$
$$= -\frac{Ab^2}{4x^4}-\frac{b(bB+2Ac)}{2x^2}+\frac{1}{2}Bc^2x^2+c(2bB+Ac)\log(x)$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.98

$$-\frac{Ab(b+4cx^2)+2Bx^2(b^2-c^2x^4)}{4x^4}+c(2bB+Ac)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^9,x]
[Out] -1/4*(A*b*(b + 4*c*x^2) + 2*B*x^2*(b^2 - c^2*x^4))/x^4 + c*(2*b*B + A*c)*Lo
g[x]
```

Maple [A]
time = 0.36, size = 46, normalized size = 0.90

method	result	size
default	$-\frac{A\,b^2}{4x^4}-\frac{b(2Ac+Bb)}{2x^2}+\frac{B\,c^2x^2}{2}+c(Ac+2Bb)\ln(x)$	46
risch	$\frac{B\,c^2x^2}{2}+\frac{(-Abc-\frac{1}{2}b^2B)x^2-\frac{b^2A}{4}}{x^4}+A\ln(x)\,c^2+2B\ln(x)\,bc$	52
norman	$\frac{(-Abc-\frac{1}{2}b^2B)x^6-\frac{A\,b^2x^4}{4}+\frac{B\,c^2x^{10}}{2}}{x^8}+(A\,c^2+2bBc)\ln(x)$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x,method=_RETURNVERBOSE)
[Out] -1/4*A*b^2/x^4-1/2*b*(2*A*c+B*b)/x^2+1/2*B*c^2*x^2+c*(A*c+2*B*b)*ln(x)
```

Maxima [A]
time = 0.28, size = 54, normalized size = 1.06

$$\frac{1}{2}\,Bc^2x^2+\frac{1}{2}\left(2\,Bbc+Ac^2\right)\log\left(x^2\right)-\frac{Ab^2+2\,(Bb^2+2\,Abc)x^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*B*c^2*x^2 + 1/2*(2*B*b*c + A*c^2)*log(x^2) - 1/4*(A*b^2 + 2*(B*b^2 + 2*
A*b*c)*x^2)/x^4
```

Fricas [A]
time = 1.57, size = 55, normalized size = 1.08

$$\frac{2\,Bc^{2}x^{6}+4\,(2\,Bbc+Ac^{2})x^{4}\log{(x)}-Ab^{2}-2\,(Bb^{2}+2\,Abc)x^{2}}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*c^2*x^6 + 4*(2*B*b*c + A*c^2)*x^4*log(x) - A*b^2 - 2*(B*b^2 + 2*A*
b*c)*x^2)/x^4
```

Sympy [A]
time = 0.27, size = 51, normalized size = 1.00

$$\frac{Bc^{2}x^{2}}{2}+c(Ac+2Bb)\log{(x)}+\frac{-Ab^{2}+x^{2}(-4Abc-2Bb^{2})}{4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**9,x)
```

```
[Out] B*c**2*x**2/2 + c*(A*c + 2*B*b)*log(x) + (-A*b**2 + x**2*(-4*A*b*c - 2*B*b*
*2))/(4*x**4)
```

Giac [A]
time = 0.61, size = 72, normalized size = 1.41

$$\frac{1}{2}\,Bc^{2}x^{2}+\frac{1}{2}\,\left(2\,Bbc+Ac^{2}\right)\log\left(x^{2}\right)-\frac{6\,Bbcx^{4}+3\,Ac^{2}x^{4}+2\,Bb^{2}x^{2}+4\,Abcx^{2}+Ab^{2}}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*B*c^2*x^2 + 1/2*(2*B*b*c + A*c^2)*log(x^2) - 1/4*(6*B*b*c*x^4 + 3*A*c^2
*x^4 + 2*B*b^2*x^2 + 4*A*b*c*x^2 + A*b^2)/x^4
```

Mupad [B]
time = 0.08, size = 51, normalized size = 1.00

$$\ln{(x)}\,\left(A\,c^{2}+2\,B\,b\,c\right)-\frac{x^{2}\,\left(\frac{B\,b^{2}}{2}+A\,c\,b\right)+\frac{A\,b^{2}}{4}}{x^{4}}+\frac{B\,c^{2}\,x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^9,x)
```

```
[Out] log(x)*(A*c^2 + 2*B*b*c) - (x^2*((B*b^2)/2 + A*b*c) + (A*b^2)/4)/x^4 + (B*c
^2*x^2)/2
```

3.22 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}}\,dx$

Optimal. Leaf size=48

$$-\frac{Ab^2}{5x^5}-\frac{b(bB+2Ac)}{3x^3}-\frac{c(2bB+Ac)}{x}+Bc^2x$$

```
[Out] -1/5*A*b^2/x^5-1/3*b*(2*A*c+B*b)/x^3-c*(A*c+2*B*b)/x+B*c^2*x
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1598, 459}

$$-\frac{Ab^2}{5x^5}-\frac{b(2Ac+bB)}{3x^3}-\frac{c(Ac+2bB)}{x}+Bc^2x$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^10,x]
```

```
[Out] -1/5*(A*b^2)/x^5 - (b*(b*B + 2*A*c))/(3*x^3) - (c*(2*b*B + A*c))/x + B*c^2*
x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^6}\,dx \\
&= \int \left(Bc^2+\frac{Ab^2}{x^6}+\frac{b(bB+2Ac)}{x^4}+\frac{c(2bB+Ac)}{x^2}\right)dx \\
&= -\frac{Ab^2}{5x^5}-\frac{b(bB+2Ac)}{3x^3}-\frac{c(2bB+Ac)}{x}+Bc^2x
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 1.00

$$-\frac{Ab^2}{5x^5}-\frac{b(bB+2Ac)}{3x^3}-\frac{c(2bB+Ac)}{x}+Bc^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^10,x]
```

```
[Out] -1/5*(A*b^2)/x^5 - (b*(b*B + 2*A*c))/(3*x^3) - (c*(2*b*B + A*c))/x + B*c^2*
x
```

Maple [A]
time = 0.36, size = 45, normalized size = 0.94

method	result	size
default	$-\frac{A\,b^2}{5x^5}-\frac{b(2Ac+Bb)}{3x^3}-\frac{c(Ac+2Bb)}{x}+B\,c^2x$	45
risch	$B\,c^2x+\frac{(-A\,c^2-2bBc)x^4+(-\frac{2}{3}Abc-\frac{1}{3}b^2B)x^2-\frac{b^2A}{5}}{x^5}$	51
norman	$\frac{(-\frac{2}{3}Abc-\frac{1}{3}b^2B)x^6+(-A\,c^2-2bBc)x^8+B\,c^2x^{10}-\frac{A\,b^2x^4}{5}}{x^9}$	55
gosper	$-\frac{-15B\,c^2x^6+15A\,c^2x^4+30x^4bBc+10Abc\,x^2+5b^2B\,x^2+3b^2A}{15x^5}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*A*b^2/x^5-1/3*b*(2*A*c+B*b)/x^3-c*(A*c+2*B*b)/x+B*c^2*x
```

Maxima [A]
time = 0.28, size = 51, normalized size = 1.06

$$Bc^2x-\frac{15\,(2\,Bbc+Ac^2)x^4+3\,Ab^2+5\,(Bb^2+2\,Abc)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x, algorithm="maxima")
```

```
[Out] B*c^2*x - 1/15*(15*(2*B*b*c + A*c^2)*x^4 + 3*A*b^2 + 5*(B*b^2 + 2*A*b*c)*x^
2)/x^5
```

Fricas [A]
time = 1.32, size = 53, normalized size = 1.10

$$\frac{15\,Bc^2x^6-15\,(2\,Bbc+Ac^2)x^4-3\,Ab^2-5\,(Bb^2+2\,Abc)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x, algorithm="fricas")
```

```
[Out] 1/15*(15*B*c^2*x^6 - 15*(2*B*b*c + A*c^2)*x^4 - 3*A*b^2 - 5*(B*b^2 + 2*A*b*
c)*x^2)/x^5
```

Sympy [A]
time = 0.31, size = 54, normalized size = 1.12

$$Bc^2x + \frac{-3Ab^2 + x^4(-15Ac^2 - 30Bbc) + x^2(-10Abc - 5Bb^2)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**10,x)
```

```
[Out] B*c**2*x + (-3*A*b**2 + x**4*(-15*A*c**2 - 30*B*b*c) + x**2*(-10*A*b*c - 5*
B*b**2))/(15*x**5)
```

Giac [A]
time = 1.12, size = 53, normalized size = 1.10

$$Bc^2x - \frac{30\,Bbcx^4 + 15\,Ac^2x^4 + 5\,Bb^2x^2 + 10\,Abcx^2 + 3\,Ab^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x, algorithm="giac")
```

```
[Out] B*c^2*x - 1/15*(30*B*b*c*x^4 + 15*A*c^2*x^4 + 5*B*b^2*x^2 + 10*A*b*c*x^2 +
3*A*b^2)/x^5
```

Mupad [B]
time = 0.07, size = 50, normalized size = 1.04

$$B\,c^2\,x - \frac{x^2\,\left(\frac{B\,b^2}{3} + \frac{2\,A\,c\,b}{3}\right) + x^4\,\left(A\,c^2 + 2\,B\,b\,c\right) + \frac{A\,b^2}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^10,x)
```

```
[Out] B*c^2*x - (x^2*((B*b^2)/3 + (2*A*b*c)/3) + x^4*(A*c^2 + 2*B*b*c) + (A*b^2)/
5)/x^5
```

3.23 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}}\,dx$

Optimal. Leaf size=51

$$-\frac{Ab^2}{6x^6}-\frac{b(bB+2Ac)}{4x^4}-\frac{c(2bB+Ac)}{2x^2}+Bc^2\log(x)$$

```
[Out] -1/6*A*b^2/x^6-1/4*b*(2*A*c+B*b)/x^4-1/2*c*(A*c+2*B*b)/x^2+B*c^2*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 77}

$$-\frac{Ab^2}{6x^6}-\frac{b(2Ac+bB)}{4x^4}-\frac{c(Ac+2bB)}{2x^2}+Bc^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^11,x]
```

```
[Out] -1/6*(A*b^2)/x^6 - (b*(b*B + 2*A*c))/(4*x^4) - (c*(2*b*B + A*c))/(2*x^2) +
B*c^2*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}}\,dx = \int \frac{(A+Bx^2)(b+cx^2)^2}{x^7}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^2}{x^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{Ab^2}{x^4}+\frac{b(bB+2Ac)}{x^3}+\frac{c(2bB+Ac)}{x^2}+\frac{Bc^2}{x}\right)dx, x, x^2\right)$$
$$= -\frac{Ab^2}{6x^6}-\frac{b(bB+2Ac)}{4x^4}-\frac{c(2bB+Ac)}{2x^2}+Bc^2\log(x)$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 1.04

$$-\frac{3bBx^2(b+4cx^2)+2A(b^2+3bcx^2+3c^2x^4)}{12x^6}+Bc^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^11,x]
```

```
[Out] -1/12*(3*b*B*x^2*(b + 4*c*x^2) + 2*A*(b^2 + 3*b*c*x^2 + 3*c^2*x^4))/x^6 + B
*c^2*Log[x]
```

Maple [A]
time = 0.36, size = 46, normalized size = 0.90

method	result	size
default	$-\frac{A\,b^2}{6x^6}-\frac{b(2Ac+Bb)}{4x^4}-\frac{c(Ac+2Bb)}{2x^2}+B\,c^2\ln(x)$	46
risch	$\frac{(-\frac{1}{2}A\,c^2-bBc)x^4+(-\frac{1}{2}Abc-\frac{1}{4}b^2B)x^2-\frac{b^2A}{6}}{x^6}+B\,c^2\ln(x)$	52
norman	$\frac{(-\frac{1}{2}A\,c^2-bBc)x^8+(-\frac{1}{2}Abc-\frac{1}{4}b^2B)x^6-\frac{A\,b^2x^4}{6}}{x^{10}}+B\,c^2\ln(x)$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*A*b^2/x^6-1/4*b*(2*A*c+B*b)/x^4-1/2*c*(A*c+2*B*b)/x^2+B*c^2*ln(x)
```

Maxima [A]
time = 0.28, size = 55, normalized size = 1.08

$$\frac{1}{2}\,Bc^2\log\left(x^2\right)-\frac{6\,(2\,Bbc+Ac^2)x^4+2\,Ab^2+3\,(Bb^2+2\,Abc)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*B*c^2*log(x^2) - 1/12*(6*(2*B*b*c + A*c^2)*x^4 + 2*A*b^2 + 3*(B*b^2 + 2
*A*b*c)*x^2)/x^6
```

Fricas [A]
time = 1.64, size = 55, normalized size = 1.08

$$\frac{12\,Bc^2x^6\log{(x)} - 6\,(2\,Bbc + Ac^2)x^4 - 2\,Ab^2 - 3\,(Bb^2 + 2\,Abc)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x, algorithm="fricas")
```

```
[Out] 1/12*(12*B*c^2*x^6*log(x) - 6*(2*B*b*c + A*c^2)*x^4 - 2*A*b^2 - 3*(B*b^2 +
2*A*b*c)*x^2)/x^6
```

Sympy [A]
time = 0.53, size = 56, normalized size = 1.10

$$Bc^2\log{(x)} + \frac{-2Ab^2 + x^4(-6Ac^2 - 12Bbc) + x^2(-6Abc - 3Bb^2)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**11,x)
```

```
[Out] B*c**2*log(x) + (-2*A*b**2 + x**4*(-6*A*c**2 - 12*B*b*c) + x**2*(-6*A*b*c -
 3*B*b**2))/(12*x**6)
```

Giac [A]
time = 1.79, size = 66, normalized size = 1.29

$$\frac{1}{2}\,Bc^2\log\left(x^2\right) - \frac{11\,Bc^2x^6 + 12\,Bbcx^4 + 6\,Ac^2x^4 + 3\,Bb^2x^2 + 6\,Abcx^2 + 2\,Ab^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x, algorithm="giac")
```

```
[Out] 1/2*B*c^2*log(x^2) - 1/12*(11*B*c^2*x^6 + 12*B*b*c*x^4 + 6*A*c^2*x^4 + 3*B*
b^2*x^2 + 6*A*b*c*x^2 + 2*A*b^2)/x^6
```

Mupad [B]
time = 0.09, size = 51, normalized size = 1.00

$$B\,c^2\,\ln{(x)} - \frac{x^2\,\left(\frac{B\,b^2}{4} + \frac{A\,c\,b}{2}\right) + x^4\,\left(\frac{A\,c^2}{2} + B\,b\,c\right) + \frac{A\,b^2}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^11,x)
```

```
[Out] B*c^2*log(x) - (x^2*((B*b^2)/4 + (A*b*c)/2) + x^4*((A*c^2)/2 + B*b*c) + (A*
b^2)/6)/x^6
```

3.24 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}} \, dx$

Optimal. Leaf size=53

$$-\frac{Ab^2}{7x^7}-\frac{b(bB+2Ac)}{5x^5}-\frac{c(2bB+Ac)}{3x^3}-\frac{Bc^2}{x}$$

```
[Out] -1/7*A*b^2/x^7-1/5*b*(2*A*c+B*b)/x^5-1/3*c*(A*c+2*B*b)/x^3-B*c^2/x
```

Rubi [A]

time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1598, 459}

$$-\frac{Ab^2}{7x^7}-\frac{b(2Ac+bB)}{5x^5}-\frac{c(Ac+2bB)}{3x^3}-\frac{Bc^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^12,x]
```

```
[Out] -1/7*(A*b^2)/x^7 - (b*(b*B + 2*A*c))/(5*x^5) - (c*(2*b*B + A*c))/(3*x^3) -
(B*c^2)/x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}} \, dx &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^8} \, dx \\
&= \int \left(\frac{Ab^2}{x^8}+\frac{b(bB+2Ac)}{x^6}+\frac{c(2bB+Ac)}{x^4}+\frac{Bc^2}{x^2}\right) dx \\
&= -\frac{Ab^2}{7x^7}-\frac{b(bB+2Ac)}{5x^5}-\frac{c(2bB+Ac)}{3x^3}-\frac{Bc^2}{x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 1.11

$$-\frac{Ab^2}{7x^7}+\frac{-b^2B-2Abc}{5x^5}+\frac{-2bBc-Ac^2}{3x^3}-\frac{Bc^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^12,x]
```

```
[Out] -1/7*(A*b^2)/x^7 + (-(b^2*B) - 2*A*b*c)/(5*x^5) + (-2*b*B*c - A*c^2)/(3*x^3
) - (B*c^2)/x
```

Maple [A]
time = 0.36, size = 48, normalized size = 0.91

method	result	size
default	$-\frac{A\,b^{2}}{7x^{7}}-\frac{b(2Ac+Bb)}{5x^{5}}-\frac{c(Ac+2Bb)}{3x^{3}}-\frac{B\,c^{2}}{x}$	48
risch	$\frac{-B\,c^{2}x^{6}+(-\frac{1}{3}A\,c^{2}-\frac{2}{3}bBc)x^{4}+(-\frac{2}{5}Abc-\frac{1}{5}b^{2}B)x^{2}-\frac{b^{2}A}{7}}{x^{7}}$	53
gosper	$-\frac{105B\,c^{2}x^{6}+35A\,c^{2}x^{4}+70x^{4}bBc+42Abc\,x^{2}+21b^{2}B\,x^{2}+15b^{2}A}{105x^{7}}$	56
norman	$\frac{(-\frac{1}{3}A\,c^{2}-\frac{2}{3}bBc)x^{8}+(-\frac{2}{5}Abc-\frac{1}{5}b^{2}B)x^{6}-\frac{A\,b^{2}x^{4}}{7}-B\,c^{2}x^{10}}{x^{11}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*A*b^2/x^7-1/5*b*(2*A*c+B*b)/x^5-1/3*c*(A*c+2*B*b)/x^3-B*c^2/x
```

Maxima [A]
time = 0.28, size = 53, normalized size = 1.00

$$-\frac{105\,Bc^2x^6+35\,(2\,Bbc+Ac^2)x^4+15\,Ab^2+21\,(Bb^2+2\,Abc)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x, algorithm="maxima")
```

```
[Out] -1/105*(105*B*c^2*x^6 + 35*(2*B*b*c + A*c^2)*x^4 + 15*A*b^2 + 21*(B*b^2 + 2
*A*b*c)*x^2)/x^7
```

Fricas [A]
time = 1.69, size = 53, normalized size = 1.00

$$-\frac{105\,Bc^2x^6+35\,(2\,Bbc+Ac^2)x^4+15\,Ab^2+21\,(Bb^2+2\,Abc)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x, algorithm="fricas")
```

```
[Out] -1/105*(105*B*c^2*x^6 + 35*(2*B*b*c + A*c^2)*x^4 + 15*A*b^2 + 21*(B*b^2 + 2
*A*b*c)*x^2)/x^7
```

Sympy [A]
time = 0.57, size = 58, normalized size = 1.09

$$\frac{-15Ab^2 - 105Bc^2x^6 + x^4(-35Ac^2 - 70Bbc) + x^2(-42Abc - 21Bb^2)}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**12,x)
```

```
[Out] (-15*A*b**2 - 105*B*c**2*x**6 + x**4*(-35*A*c**2 - 70*B*b*c) + x**2*(-42*A*
b*c - 21*B*b**2))/(105*x**7)
```

Giac [A]
time = 1.60, size = 55, normalized size = 1.04

$$-\frac{105\,Bc^2x^6 + 70\,Bbcx^4 + 35\,Ac^2x^4 + 21\,Bb^2x^2 + 42\,Abcx^2 + 15\,Ab^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/105*(105*B*c^2*x^6 + 70*B*b*c*x^4 + 35*A*c^2*x^4 + 21*B*b^2*x^2 + 42*A*b
*c*x^2 + 15*A*b^2)/x^7
```

Mupad [B]
time = 0.04, size = 52, normalized size = 0.98

$$-\frac{x^2\left(\frac{B\,b^2}{5}+\frac{2\,A\,c\,b}{5}\right)+x^4\left(\frac{A\,c^2}{3}+\frac{2\,B\,b\,c}{3}\right)+\frac{A\,b^2}{7}+B\,c^2\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^12,x)
```

```
[Out] -(x^2*((B*b^2)/5 + (2*A*b*c)/5) + x^4*((A*c^2)/3 + (2*B*b*c)/3) + (A*b^2)/7
 + B*c^2*x^6)/x^7
```

3.25 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2} \, dx$

Optimal. Leaf size=75

$$\frac{1}{5}Ab^3x^5+\frac{1}{7}b^2(bB+3Ac)x^7+\frac{1}{3}bc(bB+Ac)x^9+\frac{1}{11}c^2(3bB+Ac)x^{11}+\frac{1}{13}Bc^3x^{13}$$

```
[Out] 1/5*A*b^3*x^5+1/7*b^2*(3*A*c+B*b)*x^7+1/3*b*c*(A*c+B*b)*x^9+1/11*c^2*(A*c+3
*B*b)*x^11+1/13*B*c^3*x^13
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1598, 459}

$$\frac{1}{5}Ab^3x^5+\frac{1}{7}b^2x^7(3Ac+bB)+\frac{1}{11}c^2x^{11}(Ac+3bB)+\frac{1}{3}bcx^9(Ac+bB)+\frac{1}{13}Bc^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] (A*b^3*x^5)/5 + (b^2*(b*B + 3*A*c)*x^7)/7 + (b*c*(b*B + A*c)*x^9)/3 + (c^2*
(3*b*B + A*c)*x^11)/11 + (B*c^3*x^13)/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2} \, dx &= \int x^4(A+Bx^2)\left(b+cx^2\right)^3 \, dx \\
&= \int \left(Ab^3x^4+b^2(bB+3Ac)x^6+3bc(bB+Ac)x^8+c^2(3bB+Ac)x^{10}+Bc^3x^1 \\
&= \frac{1}{5}Ab^3x^5+\frac{1}{7}b^2(bB+3Ac)x^7+\frac{1}{3}bc(bB+Ac)x^9+\frac{1}{11}c^2(3bB+Ac)x^{11}+\frac{1}{13}
\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 75, normalized size = 1.00

$$\frac{1}{5}Ab^3x^5+\frac{1}{7}b^2(bB+3Ac)x^7+\frac{1}{3}bc(bB+Ac)x^9+\frac{1}{11}c^2(3bB+Ac)x^{11}+\frac{1}{13}Bc^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] (A*b^3*x^5)/5 + (b^2*(b*B + 3*A*c)*x^7)/7 + (b*c*(b*B + A*c)*x^9)/3 + (c^2*
(3*b*B + A*c)*x^11)/11 + (B*c^3*x^13)/13
```

Maple [**A**]
time = 0.57, size = 76, normalized size = 1.01

method	result	s
default	$\frac{Bc^3x^{13}}{13}+\frac{\left(Ac^3+3Bbc^2\right)x^{11}}{11}+\frac{\left(3Abc^2+3Bb^2c\right)x^9}{9}+\frac{\left(3Ab^2c+Bb^3\right)x^7}{7}+\frac{Ab^3x^5}{5}$	7
risch	$\frac{1}{13}Bc^3x^{13}+\frac{1}{11}x^{11}Ac^3+\frac{3}{11}x^{11}Bbc^2+\frac{1}{3}x^9Abc^2+\frac{1}{3}x^9Bb^2c+\frac{3}{7}x^7Ab^2c+\frac{1}{7}Bb^3x^7+\frac{1}{5}Ab^3x^5$	7
norman	$\frac{\left(\frac{1}{11}Ac^3+\frac{3}{11}Bbc^2\right)x^{12}+\left(\frac{1}{3}Abc^2+\frac{1}{3}Bb^2c\right)x^{10}+\left(\frac{3}{7}Ab^2c+\frac{1}{7}Bb^3\right)x^8+\frac{Ab^3x^6}{5}+\frac{Bc^3x^{14}}{13}}{x}$	7
gosper	$\frac{x^5\left(1155Bc^3x^8+1365Ac^3x^6+4095x^6Bbc^2+5005Abc^2x^4+5005x^4Bb^2c+6435Ab^2cx^2+2145x^2Bb^3+3003Ab^3\right)}{15015}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*B*c^3*x^13+1/11*(A*c^3+3*B*b*c^2)*x^11+1/9*(3*A*b*c^2+3*B*b^2*c)*x^9+1
/7*(3*A*b^2*c+B*b^3)*x^7+1/5*A*b^3*x^5
```

Maxima [**A**]
time = 0.27, size = 73, normalized size = 0.97

$$\frac{1}{13}\,Bc^3x^{13}+\frac{1}{11}\left(3\,Bbc^2+Ac^3\right)x^{11}+\frac{1}{3}\left(Bb^2c+Abc^2\right)x^9+\frac{1}{5}\,Ab^3x^5+\frac{1}{7}\left(Bb^3+3\,Ab^2c\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c + A*b*c^2)*x
^9 + 1/5*A*b^3*x^5 + 1/7*(B*b^3 + 3*A*b^2*c)*x^7
```

Fricas [**A**]
time = 1.54, size = 73, normalized size = 0.97

$$\frac{1}{13}\,Bc^3x^{13}+\frac{1}{11}\left(3\,Bbc^2+Ac^3\right)x^{11}+\frac{1}{3}\left(Bb^2c+Abc^2\right)x^9+\frac{1}{5}\,Ab^3x^5+\frac{1}{7}\left(Bb^3+3\,Ab^2c\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c + A*b*c^2)*x
^9 + 1/5*A*b^3*x^5 + 1/7*(B*b^3 + 3*A*b^2*c)*x^7
```

Sympy [A]
time = 0.01, size = 80, normalized size = 1.07

$$\frac{Ab^3x^5}{5} + \frac{Bc^3x^{13}}{13} + x^{11}\left(\frac{Ac^3}{11} + \frac{3Bbc^2}{11}\right) + x^9\left(\frac{Abc^2}{3} + \frac{Bb^2c}{3}\right) + x^7\cdot\left(\frac{3Ab^2c}{7} + \frac{Bb^3}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**2,x)
```

```
[Out] A*b**3*x**5/5 + B*c**3*x**13/13 + x**11*(A*c**3/11 + 3*B*b*c**2/11) + x**9*
(A*b*c**2/3 + B*b**2*c/3) + x**7*(3*A*b**2*c/7 + B*b**3/7)
```

Giac [A]
time = 0.94, size = 77, normalized size = 1.03

$$\frac{1}{13}\,Bc^3x^{13} + \frac{3}{11}\,Bbc^2x^{11} + \frac{1}{11}\,Ac^3x^{11} + \frac{1}{3}\,Bb^2cx^9 + \frac{1}{3}\,Abc^2x^9 + \frac{1}{7}\,Bb^3x^7 + \frac{3}{7}\,Ab^2cx^7 + \frac{1}{5}\,Ab^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/13*B*c^3*x^13 + 3/11*B*b*c^2*x^11 + 1/11*A*c^3*x^11 + 1/3*B*b^2*c*x^9 + 1
/3*A*b*c^2*x^9 + 1/7*B*b^3*x^7 + 3/7*A*b^2*c*x^7 + 1/5*A*b^3*x^5
```

Mupad [B]
time = 0.06, size = 69, normalized size = 0.92

$$x^7\left(\frac{B\,b^3}{7} + \frac{3\,A\,c\,b^2}{7}\right) + x^{11}\left(\frac{A\,c^3}{11} + \frac{3\,B\,b\,c^2}{11}\right) + \frac{A\,b^3\,x^5}{5} + \frac{B\,c^3\,x^{13}}{13} + \frac{b\,c\,x^9\,(A\,c + B\,b)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^2,x)
```

```
[Out] x^7*((B*b^3)/7 + (3*A*b^2*c)/7) + x^11*((A*c^3)/11 + (3*B*b*c^2)/11) + (A*b
^3*x^5)/5 + (B*c^3*x^13)/13 + (b*c*x^9*(A*c + B*b))/3
```

3.26 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3} dx$

Optimal. Leaf size=68

$$\frac{b(bB-Ac)(b+cx^2)^4}{8c^3}-\frac{(2bB-Ac)(b+cx^2)^5}{10c^3}+\frac{B(b+cx^2)^6}{12c^3}$$

```
[Out] 1/8*b*(-A*c+B*b)*(c*x^2+b)^4/c^3-1/10*(-A*c+2*B*b)*(c*x^2+b)^5/c^3+1/12*B*(
c*x^2+b)^6/c^3
```

Rubi [A]
time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 77}

$$-\frac{(b+cx^2)^5(2bB-Ac)}{10c^3}+\frac{b(b+cx^2)^4(bB-Ac)}{8c^3}+\frac{B(b+cx^2)^6}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] (b*(b*B - A*c)*(b + c*x^2)^4)/(8*c^3) - ((2*b*B - A*c)*(b + c*x^2)^5)/(10*c
^3) + (B*(b + c*x^2)^6)/(12*c^3)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^3}\,dx &= \int x^3\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int x(A+Bx)(b+cx)^3\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b(bB-Ac)(b+cx)^3}{c^2}+\frac{(-2bB+Ac)(b+cx)^4}{c^2}+\frac{B(b+cx)^5}{c^2}\right.\right.\\
&= \frac{b(bB-Ac)\left(b+cx^2\right)^4}{8c^3}-\frac{(2bB-Ac)\left(b+cx^2\right)^5}{10c^3}+\frac{B\left(b+cx^2\right)^6}{12c^3}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 69, normalized size = 1.01

$$\frac{1}{120}x^4\left(30Ab^3+20b^2(bB+3Ac)x^2+45bc(bB+Ac)x^4+12c^2(3bB+Ac)x^6+10Bc^3x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] (x^4*(30*A*b^3 + 20*b^2*(b*B + 3*A*c)*x^2 + 45*b*c*(b*B + A*c)*x^4 + 12*c^2
*(3*b*B + A*c)*x^6 + 10*B*c^3*x^8))/120
```

Maple [A]
time = 0.40, size = 76, normalized size = 1.12

method	result	siz
default	$\frac{B\,c^3x^{12}}{12}+\frac{\left(A\,c^3+3Bb\,c^2\right)x^{10}}{10}+\frac{\left(3Ab\,c^2+3B\,b^2c\right)x^8}{8}+\frac{\left(3A\,b^2c+B\,b^3\right)x^6}{6}+\frac{A\,b^3x^4}{4}$	76
risch	$\frac{1}{12}B\,c^3x^{12}+\frac{1}{10}x^{10}A\,c^3+\frac{3}{10}x^{10}Bb\,c^2+\frac{3}{8}x^8Ab\,c^2+\frac{3}{8}x^8B\,b^2c+\frac{1}{2}x^6A\,b^2c+\frac{1}{6}b^3B\,x^6+\frac{1}{4}A\,b^3x^4$	78
norman	$\frac{\left(\frac{1}{10}A\,c^3+\frac{3}{10}Bb\,c^2\right)x^{12}+\left(\frac{3}{8}Ab\,c^2+\frac{3}{8}B\,b^2c\right)x^{10}+\left(\frac{1}{2}A\,b^2c+\frac{1}{6}B\,b^3\right)x^8+\frac{A\,b^3x^6}{4}+\frac{B\,c^3x^{14}}{12}}{x^2}$	79
gosper	$\frac{x^4\left(10B\,c^3x^8+12A\,c^3x^6+36x^6Bb\,c^2+45Ab\,c^2x^4+45x^4B\,b^2c+60A\,b^2c\,x^2+20x^2B\,b^3+30A\,b^3\right)}{120}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*B*c^3*x^12+1/10*(A*c^3+3*B*b*c^2)*x^10+1/8*(3*A*b*c^2+3*B*b^2*c)*x^8+1
/6*(3*A*b^2*c+B*b^3)*x^6+1/4*A*b^3*x^4
```

Maxima [A]
time = 0.27, size = 73, normalized size = 1.07

$$\frac{1}{12}\,Bc^3x^{12}+\frac{1}{10}\,\left(3\,Bbc^2+Ac^3\right)x^{10}+\frac{3}{8}\,\left(Bb^2c+Abc^2\right)x^8+\frac{1}{4}\,Ab^3x^4+\frac{1}{6}\,\left(Bb^3+3\,Ab^2c\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c + A*b*c^2)*x
^8 + 1/4*A*b^3*x^4 + 1/6*(B*b^3 + 3*A*b^2*c)*x^6
```

Fricas [A]
time = 1.84, size = 73, normalized size = 1.07

$$\frac{1}{12}\,Bc^3x^{12}+\frac{1}{10}\,\left(3\,Bbc^2+Ac^3\right)x^{10}+\frac{3}{8}\,\left(Bb^2c+Abc^2\right)x^8+\frac{1}{4}\,Ab^3x^4+\frac{1}{6}\,\left(Bb^3+3\,Ab^2c\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c + A*b*c^2)*x
^8 + 1/4*A*b^3*x^4 + 1/6*(B*b^3 + 3*A*b^2*c)*x^6
```

Sympy [A]
time = 0.02, size = 82, normalized size = 1.21

$$\frac{Ab^3x^4}{4}+\frac{Bc^3x^{12}}{12}+x^{10}\left(\frac{Ac^3}{10}+\frac{3Bbc^2}{10}\right)+x^8\cdot\left(\frac{3Abc^2}{8}+\frac{3Bb^2c}{8}\right)+x^6\left(\frac{Ab^2c}{2}+\frac{Bb^3}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**3,x)
```

```
[Out] A*b**3*x**4/4 + B*c**3*x**12/12 + x**10*(A*c**3/10 + 3*B*b*c**2/10) + x**8*
(3*A*b*c**2/8 + 3*B*b**2*c/8) + x**6*(A*b**2*c/2 + B*b**3/6)
```

Giac [A]
time = 0.93, size = 77, normalized size = 1.13

$$\frac{1}{12}\,Bc^3x^{12}+\frac{3}{10}\,Bbc^2x^{10}+\frac{1}{10}\,Ac^3x^{10}+\frac{3}{8}\,Bb^2cx^8+\frac{3}{8}\,Abc^2x^8+\frac{1}{6}\,Bb^3x^6+\frac{1}{2}\,Ab^2cx^6+\frac{1}{4}\,Ab^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/12*B*c^3*x^12 + 3/10*B*b*c^2*x^10 + 1/10*A*c^3*x^10 + 3/8*B*b^2*c*x^8 + 3
/8*A*b*c^2*x^8 + 1/6*B*b^3*x^6 + 1/2*A*b^2*c*x^6 + 1/4*A*b^3*x^4
```

Mupad [B]
time = 0.03, size = 69, normalized size = 1.01

$$x^6\,\left(\frac{B\,b^3}{6}+\frac{A\,c\,b^2}{2}\right)+x^{10}\,\left(\frac{A\,c^3}{10}+\frac{3\,B\,b\,c^2}{10}\right)+\frac{A\,b^3\,x^4}{4}+\frac{B\,c^3\,x^{12}}{12}+\frac{3\,b\,c\,x^8\,\left(A\,c+B\,b\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^3,x)
```

```
[Out] x^6*((B*b^3)/6 + (A*b^2*c)/2) + x^10*((A*c^3)/10 + (3*B*b*c^2)/10) + (A*b^3
*x^4)/4 + (B*c^3*x^12)/12 + (3*b*c*x^8*(A*c + B*b))/8
```

3.27 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}\,dx$

Optimal. Leaf size=75

$$\frac{1}{3}Ab^3x^3+\frac{1}{5}b^2(bB+3Ac)x^5+\frac{3}{7}bc(bB+Ac)x^7+\frac{1}{9}c^2(3bB+Ac)x^9+\frac{1}{11}Bc^3x^{11}$$

```
[Out] 1/3*A*b^3*x^3+1/5*b^2*(3*A*c+B*b)*x^5+3/7*b*c*(A*c+B*b)*x^7+1/9*c^2*(A*c+3*
B*b)*x^9+1/11*B*c^3*x^11
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {1598, 459}

$$\frac{1}{3}Ab^3x^3+\frac{1}{5}b^2x^5(3Ac+bB)+\frac{1}{9}c^2x^9(Ac+3bB)+\frac{3}{7}bcx^7(Ac+bB)+\frac{1}{11}Bc^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^4,x]
```

```
[Out] (A*b^3*x^3)/3 + (b^2*(b*B + 3*A*c)*x^5)/5 + (3*b*c*(b*B + A*c)*x^7)/7 + (c^
2*(3*b*B + A*c)*x^9)/9 + (B*c^3*x^11)/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}\,dx &= \int x^2(A+Bx^2)\left(b+cx^2\right)^3\,dx\\ &= \int \left(Ab^3x^2+b^2(bB+3Ac)x^4+3bc(bB+Ac)x^6+c^2(3bB+Ac)x^8+Bc^3x^{10}\right)dx\\ &= \frac{1}{3}Ab^3x^3+\frac{1}{5}b^2(bB+3Ac)x^5+\frac{3}{7}bc(bB+Ac)x^7+\frac{1}{9}c^2(3bB+Ac)x^9+\frac{1}{11}Bc^3x^{11}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 75, normalized size = 1.00

$$\frac{1}{3}Ab^3x^3+\frac{1}{5}b^2(bB+3Ac)x^5+\frac{3}{7}bc(bB+Ac)x^7+\frac{1}{9}c^2(3bB+Ac)x^9+\frac{1}{11}Bc^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^4,x]
```

```
[Out] (A*b^3*x^3)/3 + (b^2*(b*B + 3*A*c)*x^5)/5 + (3*b*c*(b*B + A*c)*x^7)/7 + (c^
2*(3*b*B + A*c)*x^9)/9 + (B*c^3*x^11)/11
```

Maple [A]
time = 0.41, size = 76, normalized size = 1.01

method	result	size
default	$\frac{B\,c^3x^{11}}{11}+\frac{(A\,c^3+3Bb\,c^2)x^9}{9}+\frac{(3Ab\,c^2+3B\,b^2c)x^7}{7}+\frac{(3A\,b^2c+B\,b^3)x^5}{5}+\frac{A\,b^3x^3}{3}$	76
risch	$\frac{1}{11}B\,c^3x^{11}+\frac{1}{9}x^9A\,c^3+\frac{1}{3}x^9Bb\,c^2+\frac{3}{7}x^7Ab\,c^2+\frac{3}{7}x^7B\,b^2c+\frac{3}{5}x^5A\,b^2c+\frac{1}{5}x^5B\,b^3+\frac{1}{3}A\,b^3x^3$	78
norman	$\frac{(\frac{1}{9}A\,c^3+\frac{1}{3}Bb\,c^2)x^{12}+(\frac{3}{7}Ab\,c^2+\frac{3}{7}B\,b^2c)x^{10}+(\frac{3}{5}A\,b^2c+\frac{1}{5}B\,b^3)x^8+\frac{A\,b^3x^6}{3}+\frac{B\,c^3x^{14}}{11}}{x^3}$	79
gosper	$\frac{x^3(315B\,c^3x^8+385A\,c^3x^6+1155x^6Bb\,c^2+1485Ab\,c^2x^4+1485x^4B\,b^2c+2079A\,b^2c\,x^2+693x^2B\,b^3+1155A\,b^3)}{3465}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*B*c^3*x^11+1/9*(A*c^3+3*B*b*c^2)*x^9+1/7*(3*A*b*c^2+3*B*b^2*c)*x^7+1/5
*(3*A*b^2*c+B*b^3)*x^5+1/3*A*b^3*x^3
```

Maxima [A]
time = 0.29, size = 73, normalized size = 0.97

$$\frac{1}{11}\,Bc^3x^{11}+\frac{1}{9}\,(3\,Bbc^2+Ac^3)x^9+\frac{3}{7}\,(Bb^2c+Abc^2)x^7+\frac{1}{3}\,Ab^3x^3+\frac{1}{5}\,(Bb^3+3\,Ab^2c)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/11*B*c^3*x^11 + 1/9*(3*B*b*c^2 + A*c^3)*x^9 + 3/7*(B*b^2*c + A*b*c^2)*x^7
 + 1/3*A*b^3*x^3 + 1/5*(B*b^3 + 3*A*b^2*c)*x^5
```

Fricas [A]
time = 1.43, size = 73, normalized size = 0.97

$$\frac{1}{11}\,Bc^3x^{11}+\frac{1}{9}\,(3\,Bbc^2+Ac^3)x^9+\frac{3}{7}\,(Bb^2c+Abc^2)x^7+\frac{1}{3}\,Ab^3x^3+\frac{1}{5}\,(Bb^3+3\,Ab^2c)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/11*B*c^3*x^11 + 1/9*(3*B*b*c^2 + A*c^3)*x^9 + 3/7*(B*b^2*c + A*b*c^2)*x^7
 + 1/3*A*b^3*x^3 + 1/5*(B*b^3 + 3*A*b^2*c)*x^5
```

Sympy [A]
time = 0.01, size = 82, normalized size = 1.09

$$\frac{Ab^3x^3}{3} + \frac{Bc^3x^{11}}{11} + x^9\left(\frac{Ac^3}{9} + \frac{Bbc^2}{3}\right) + x^7\cdot\left(\frac{3Abc^2}{7} + \frac{3Bb^2c}{7}\right) + x^5\cdot\left(\frac{3Ab^2c}{5} + \frac{Bb^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**4,x)
```

```
[Out] A*b**3*x**3/3 + B*c**3*x**11/11 + x**9*(A*c**3/9 + B*b*c**2/3) + x**7*(3*A*
b*c**2/7 + 3*B*b**2*c/7) + x**5*(3*A*b**2*c/5 + B*b**3/5)
```

Giac [A]
time = 1.44, size = 77, normalized size = 1.03

$$\frac{1}{11}Bc^3x^{11} + \frac{1}{3}Bbc^2x^9 + \frac{1}{9}Ac^3x^9 + \frac{3}{7}Bb^2cx^7 + \frac{3}{7}Abc^2x^7 + \frac{1}{5}Bb^3x^5 + \frac{3}{5}Ab^2cx^5 + \frac{1}{3}Ab^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/11*B*c^3*x^11 + 1/3*B*b*c^2*x^9 + 1/9*A*c^3*x^9 + 3/7*B*b^2*c*x^7 + 3/7*A
*b*c^2*x^7 + 1/5*B*b^3*x^5 + 3/5*A*b^2*c*x^5 + 1/3*A*b^3*x^3
```

Mupad [B]
time = 0.03, size = 69, normalized size = 0.92

$$x^5\left(\frac{B\,b^3}{5} + \frac{3\,A\,c\,b^2}{5}\right) + x^9\left(\frac{A\,c^3}{9} + \frac{B\,b\,c^2}{3}\right) + \frac{A\,b^3\,x^3}{3} + \frac{B\,c^3\,x^{11}}{11} + \frac{3\,b\,c\,x^7\,(A\,c + B\,b)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^4,x)
```

```
[Out] x^5*((B*b^3)/5 + (3*A*b^2*c)/5) + x^9*((A*c^3)/9 + (B*b*c^2)/3) + (A*b^3*x^
3)/3 + (B*c^3*x^11)/11 + (3*b*c*x^7*(A*c + B*b))/7
```

3.28 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5} dx$

Optimal. Leaf size=42

$$-\frac{(bB-Ac)(b+cx^2)^4}{8c^2}+\frac{B(b+cx^2)^5}{10c^2}$$

```
[Out] -1/8*(-A*c+B*b)*(c*x^2+b)^4/c^2+1/10*B*(c*x^2+b)^5/c^2
```

Rubi [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 455, 45}

$$\frac{B(b+cx^2)^5}{10c^2}-\frac{(b+cx^2)^4(bB-Ac)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^5,x]
```

```
[Out] -1/8*((b*B - A*c)*(b + c*x^2)^4)/c^2 + (B*(b + c*x^2)^5)/(10*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5}\,dx = \int x(A+Bx^2)\left(b+cx^2\right)^3\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int (A+Bx)(b+cx)^3\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bB+Ac)(b+cx)^3}{c} + \frac{B(b+cx)^4}{c}\right)\,dx, x, x^2\right)$$
$$= -\frac{(bB-Ac)\left(b+cx^2\right)^4}{8c^2} + \frac{B\left(b+cx^2\right)^5}{10c^2}$$

Mathematica [A]
time = 0.01, size = 69, normalized size = 1.64

$$\frac{1}{40}x^2\left(20Ab^3+10b^2(bB+3Ac)x^2+20bc(bB+Ac)x^4+5c^2(3bB+Ac)x^6+4Bc^3x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^5,x]
```

```
[Out] (x^2*(20*A*b^3 + 10*b^2*(b*B + 3*A*c)*x^2 + 20*b*c*(b*B + A*c)*x^4 + 5*c^2*
(3*b*B + A*c)*x^6 + 4*B*c^3*x^8))/40
```

Maple [A]
time = 0.39, size = 76, normalized size = 1.81

method	result	size
default	$\frac{B\,c^3x^{10}}{10}+\frac{(A\,c^3+3Bb\,c^2)x^8}{8}+\frac{(3Ab\,c^2+3B\,b^2c)x^6}{6}+\frac{(3A\,b^2c+B\,b^3)x^4}{4}+\frac{A\,b^3x^2}{2}$	76
risch	$\frac{1}{10}B\,c^3x^{10}+\frac{1}{8}x^8A\,c^3+\frac{3}{8}x^8Bb\,c^2+\frac{1}{2}x^6Ab\,c^2+\frac{1}{2}x^6B\,b^2c+\frac{3}{4}x^4A\,b^2c+\frac{1}{4}x^4B\,b^3+\frac{1}{2}A\,b^3x^2$	78
norman	$\frac{(\frac{1}{8}A\,c^3+\frac{3}{8}Bb\,c^2)x^{12}+(\frac{1}{2}Ab\,c^2+\frac{1}{2}B\,b^2c)x^{10}+(\frac{3}{4}A\,b^2c+\frac{1}{4}B\,b^3)x^8+\frac{A\,b^3x^6}{2}+\frac{B\,c^3x^{14}}{10}}{x^4}$	79
gosper	$\frac{x^2(4B\,c^3x^8+5A\,c^3x^6+15x^6Bb\,c^2+20Ab\,c^2x^4+20x^4B\,b^2c+30A\,b^2c\,x^2+10x^2B\,b^3+20A\,b^3)}{40}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*B*c^3*x^10+1/8*(A*c^3+3*B*b*c^2)*x^8+1/6*(3*A*b*c^2+3*B*b^2*c)*x^6+1/4
*(3*A*b^2*c+B*b^3)*x^4+1/2*A*b^3*x^2
```

Maxima [A]
time = 0.29, size = 73, normalized size = 1.74

$$\frac{1}{10}\,Bc^3x^{10}+\frac{1}{8}\left(3\,Bbc^2+Ac^3\right)x^8+\frac{1}{2}\left(Bb^2c+Abc^2\right)x^6+\frac{1}{2}\,Ab^3x^2+\frac{1}{4}\left(Bb^3+3\,Ab^2c\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/10*B*c^3*x^10 + 1/8*(3*B*b*c^2 + A*c^3)*x^8 + 1/2*(B*b^2*c + A*b*c^2)*x^6
 + 1/2*A*b^3*x^2 + 1/4*(B*b^3 + 3*A*b^2*c)*x^4
```

Fricas [A]
time = 1.40, size = 73, normalized size = 1.74

$$\frac{1}{10}\,Bc^3x^{10}+\frac{1}{8}\left(3\,Bbc^2+Ac^3\right)x^8+\frac{1}{2}\left(Bb^2c+Abc^2\right)x^6+\frac{1}{2}\,Ab^3x^2+\frac{1}{4}\left(Bb^3+3\,Ab^2c\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/10*B*c^3*x^10 + 1/8*(3*B*b*c^2 + A*c^3)*x^8 + 1/2*(B*b^2*c + A*b*c^2)*x^6
 + 1/2*A*b^3*x^2 + 1/4*(B*b^3 + 3*A*b^2*c)*x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(36) = 72.
time = 0.02, size = 80, normalized size = 1.90

$$\frac{Ab^3x^2}{2}+\frac{Bc^3x^{10}}{10}+x^8\left(\frac{Ac^3}{8}+\frac{3Bbc^2}{8}\right)+x^6\left(\frac{Abc^2}{2}+\frac{Bb^2c}{2}\right)+x^4\cdot\left(\frac{3Ab^2c}{4}+\frac{Bb^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**5,x)
```

```
[Out] A*b**3*x**2/2 + B*c**3*x**10/10 + x**8*(A*c**3/8 + 3*B*b*c**2/8) + x**6*(A*
b*c**2/2 + B*b**2*c/2) + x**4*(3*A*b**2*c/4 + B*b**3/4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(38) = 76.
time = 1.38, size = 77, normalized size = 1.83

$$\frac{1}{10}\,Bc^3x^{10}+\frac{3}{8}\,Bbc^2x^8+\frac{1}{8}\,Ac^3x^8+\frac{1}{2}\,Bb^2cx^6+\frac{1}{2}\,Abc^2x^6+\frac{1}{4}\,Bb^3x^4+\frac{3}{4}\,Ab^2cx^4+\frac{1}{2}\,Ab^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/10*B*c^3*x^10 + 3/8*B*b*c^2*x^8 + 1/8*A*c^3*x^8 + 1/2*B*b^2*c*x^6 + 1/2*A
*b*c^2*x^6 + 1/4*B*b^3*x^4 + 3/4*A*b^2*c*x^4 + 1/2*A*b^3*x^2
```

Mupad [B]
time = 0.03, size = 69, normalized size = 1.64

$$x^4\left(\frac{B\,b^3}{4}+\frac{3\,A\,c\,b^2}{4}\right)+x^8\left(\frac{A\,c^3}{8}+\frac{3\,B\,b\,c^2}{8}\right)+\frac{A\,b^3\,x^2}{2}+\frac{B\,c^3\,x^{10}}{10}+\frac{b\,c\,x^6\,\left(A\,c+B\,b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^5,x)
```

```
[Out] x^4*((B*b^3)/4 + (3*A*b^2*c)/4) + x^8*((A*c^3)/8 + (3*B*b*c^2)/8) + (A*b^3*
x^2)/2 + (B*c^3*x^10)/10 + (b*c*x^6*(A*c + B*b))/2
```

3.29 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6}\,dx$

Optimal. Leaf size=70

$$Ab^3x+\frac{1}{3}b^2(bB+3Ac)x^3+\frac{3}{5}bc(bB+Ac)x^5+\frac{1}{7}c^2(3bB+Ac)x^7+\frac{1}{9}Bc^3x^9$$

```
[Out] A*b^3*x+1/3*b^2*(3*A*c+B*b)*x^3+3/5*b*c*(A*c+B*b)*x^5+1/7*c^2*(A*c+3*B*b)*x
^7+1/9*B*c^3*x^9
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1598, 380}

$$Ab^3x+\frac{1}{3}b^2x^3(3Ac+bB)+\frac{1}{7}c^2x^7(Ac+3bB)+\frac{3}{5}bcx^5(Ac+bB)+\frac{1}{9}Bc^3x^9$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^6,x]
```

```
[Out] A*b^3*x + (b^2*(b*B + 3*A*c)*x^3)/3 + (3*b*c*(b*B + A*c)*x^5)/5 + (c^2*(3*b
*B + A*c)*x^7)/7 + (B*c^3*x^9)/9
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6}\,dx &= \int (A+Bx^2)(b+cx^2)^3\,dx\\ &= \int \left(Ab^3+b^2(bB+3Ac)x^2+3bc(bB+Ac)x^4+c^2(3bB+Ac)x^6+Bc^3x^8\right)\\ &= Ab^3x+\frac{1}{3}b^2(bB+3Ac)x^3+\frac{3}{5}bc(bB+Ac)x^5+\frac{1}{7}c^2(3bB+Ac)x^7+\frac{1}{9}Bc^3x\end{aligned}$$

Mathematica [A]
time = 0.01, size = 70, normalized size = 1.00

$$Ab^3x+\frac{1}{3}b^2(bB+3Ac)x^3+\frac{3}{5}bc(bB+Ac)x^5+\frac{1}{7}c^2(3bB+Ac)x^7+\frac{1}{9}Bc^3x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^6,x]
```

```
[Out] A*b^3*x + (b^2*(b*B + 3*A*c)*x^3)/3 + (3*b*c*(b*B + A*c)*x^5)/5 + (c^2*(3*b
*B + A*c)*x^7)/7 + (B*c^3*x^9)/9
```

Maple [A]
time = 0.49, size = 73, normalized size = 1.04

method	result	size
default	$\frac{B\,c^3x^9}{9}+\frac{(A\,c^3+3Bb\,c^2)x^7}{7}+\frac{(3Ab\,c^2+3B\,b^2c)x^5}{5}+\frac{(3A\,b^2c+B\,b^3)x^3}{3}+A\,b^3x$	73
risch	$\frac{1}{9}B\,c^3x^9+\frac{1}{7}x^7A\,c^3+\frac{3}{7}x^7Bb\,c^2+\frac{3}{5}x^5Ab\,c^2+\frac{3}{5}x^5B\,b^2c+x^3A\,b^2c+\frac{1}{3}x^3B\,b^3+A\,b^3x$	74
norman	$\frac{\left(\frac{1}{7}A\,c^3+\frac{3}{7}Bb\,c^2\right)x^{12}+\left(\frac{3}{5}Ab\,c^2+\frac{3}{5}B\,b^2c\right)x^{10}+\left(A\,b^2c+\frac{1}{3}B\,b^3\right)x^8+A\,b^3x^6+\frac{B\,c^3x^{14}}{9}}{x^5}$	77
gosper	$\frac{x\left(35B\,c^3x^8+45A\,c^3x^6+135x^6Bb\,c^2+189Ab\,c^2x^4+189x^4B\,b^2c+315A\,b^2c\,x^2+105x^2B\,b^3+315A\,b^3\right)}{315}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*B*c^3*x^9+1/7*(A*c^3+3*B*b*c^2)*x^7+1/5*(3*A*b*c^2+3*B*b^2*c)*x^5+1/3*(
3*A*b^2*c+B*b^3)*x^3+A*b^3*x
```

Maxima [A]
time = 0.28, size = 70, normalized size = 1.00

$$\frac{1}{9}\,Bc^3x^9+\frac{1}{7}\left(3\,Bbc^2+Ac^3\right)x^7+\frac{3}{5}\left(Bb^2c+Abc^2\right)x^5+Ab^3x+\frac{1}{3}\left(Bb^3+3\,Ab^2c\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x, algorithm="maxima")
```

```
[Out] 1/9*B*c^3*x^9 + 1/7*(3*B*b*c^2 + A*c^3)*x^7 + 3/5*(B*b^2*c + A*b*c^2)*x^5 +
 A*b^3*x + 1/3*(B*b^3 + 3*A*b^2*c)*x^3
```

Fricas [A]
time = 1.39, size = 70, normalized size = 1.00

$$\frac{1}{9}\,Bc^3x^9+\frac{1}{7}\left(3\,Bbc^2+Ac^3\right)x^7+\frac{3}{5}\left(Bb^2c+Abc^2\right)x^5+Ab^3x+\frac{1}{3}\left(Bb^3+3\,Ab^2c\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x, algorithm="fricas")
```

```
[Out] 1/9*B*c^3*x^9 + 1/7*(3*B*b*c^2 + A*c^3)*x^7 + 3/5*(B*b^2*c + A*b*c^2)*x^5 +
 A*b^3*x + 1/3*(B*b^3 + 3*A*b^2*c)*x^3
```

Sympy [A]
time = 0.01, size = 76, normalized size = 1.09

$$Ab^3x + \frac{Bc^3x^9}{9} + x^7\left(\frac{Ac^3}{7} + \frac{3Bbc^2}{7}\right) + x^5\cdot\left(\frac{3Abc^2}{5} + \frac{3Bb^2c}{5}\right) + x^3\left(Ab^2c + \frac{Bb^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**6,x)
```

```
[Out] A*b**3*x + B*c**3*x**9/9 + x**7*(A*c**3/7 + 3*B*b*c**2/7) + x**5*(3*A*b*c**
2/5 + 3*B*b**2*c/5) + x**3*(A*b**2*c + B*b**3/3)
```

Giac [A]
time = 1.08, size = 73, normalized size = 1.04

$$\frac{1}{9}\,Bc^3x^9 + \frac{3}{7}\,Bbc^2x^7 + \frac{1}{7}\,Ac^3x^7 + \frac{3}{5}\,Bb^2cx^5 + \frac{3}{5}\,Abc^2x^5 + \frac{1}{3}\,Bb^3x^3 + Ab^2cx^3 + Ab^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x, algorithm="giac")
```

```
[Out] 1/9*B*c^3*x^9 + 3/7*B*b*c^2*x^7 + 1/7*A*c^3*x^7 + 3/5*B*b^2*c*x^5 + 3/5*A*b
*c^2*x^5 + 1/3*B*b^3*x^3 + A*b^2*c*x^3 + A*b^3*x
```

Mupad [B]
time = 0.03, size = 65, normalized size = 0.93

$$x^3\left(\frac{B\,b^3}{3} + A\,c\,b^2\right) + x^7\left(\frac{A\,c^3}{7} + \frac{3\,B\,b\,c^2}{7}\right) + \frac{B\,c^3\,x^9}{9} + A\,b^3\,x + \frac{3\,b\,c\,x^5\,(A\,c + B\,b)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^6,x)
```

```
[Out] x^3*((B*b^3)/3 + A*b^2*c) + x^7*((A*c^3)/7 + (3*B*b*c^2)/7) + (B*c^3*x^9)/9
 + A*b^3*x + (3*b*c*x^5*(A*c + B*b))/5
```

3.30 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7} dx$

Optimal. Leaf size=60

$$\frac{3}{2}Ab^2cx^2 + \frac{3}{4}Abc^2x^4 + \frac{1}{6}Ac^3x^6 + \frac{B(b+cx^2)^4}{8c} + Ab^3\log(x)$$

```
[Out] 3/2*A*b^2*c*x^2+3/4*A*b*c^2*x^4+1/6*A*c^3*x^6+1/8*B*(c*x^2+b)^4/c+A*b^3*ln(
x)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1598, 457, 81, 45}

$$Ab^3\log(x) + \frac{3}{2}Ab^2cx^2 + \frac{3}{4}Abc^2x^4 + \frac{1}{6}Ac^3x^6 + \frac{B(b+cx^2)^4}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^7,x]
```

```
[Out] (3*A*b^2*c*x^2)/2 + (3*A*b*c^2*x^4)/4 + (A*c^3*x^6)/6 + (B*(b + c*x^2)^4)/(
8*c) + A*b^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x}\,dx,x,x^2\right)\\ &= \frac{B(b+cx^2)^4}{8c}+\frac{1}{2}A\text{Subst}\left(\int \frac{(b+cx)^3}{x}\,dx,x,x^2\right)\\ &= \frac{B(b+cx^2)^4}{8c}+\frac{1}{2}A\text{Subst}\left(\int \left(3b^2c+\frac{b^3}{x}+3bc^2x+c^3x^2\right)\,dx,x,x^2\right)\\ &= \frac{3}{2}Ab^2cx^2+\frac{3}{4}Abc^2x^4+\frac{1}{6}Ac^3x^6+\frac{B(b+cx^2)^4}{8c}+Ab^3\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 71, normalized size = 1.18

$$\frac{1}{2}b^2(bB+3Ac)x^2+\frac{3}{4}bc(bB+Ac)x^4+\frac{1}{6}c^2(3bB+Ac)x^6+\frac{1}{8}Bc^3x^8+Ab^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^7,x]
```

```
[Out] (b^2*(b*B + 3*A*c)*x^2)/2 + (3*b*c*(b*B + A*c)*x^4)/4 + (c^2*(3*b*B + A*c)*
x^6)/6 + (B*c^3*x^8)/8 + A*b^3*Log[x]
```

Maple [A]
time = 0.42, size = 76, normalized size = 1.27

method	result	size
default	$\frac{B\,c^{3}x^{8}}{8}+\frac{A\,c^{3}x^{6}}{6}+\frac{x^{6}Bb\,c^{2}}{2}+\frac{3Ab\,c^{2}x^{4}}{4}+\frac{3x^{4}B\,b^{2}c}{4}+\frac{3A\,b^{2}c\,x^{2}}{2}+\frac{x^{2}B\,b^{3}}{2}+A\,b^{3}\ln\left(x\right)$	76
risch	$\frac{B\,c^{3}x^{8}}{8}+\frac{A\,c^{3}x^{6}}{6}+\frac{x^{6}Bb\,c^{2}}{2}+\frac{3Ab\,c^{2}x^{4}}{4}+\frac{3x^{4}B\,b^{2}c}{4}+\frac{3A\,b^{2}c\,x^{2}}{2}+\frac{x^{2}B\,b^{3}}{2}+A\,b^{3}\ln\left(x\right)$	76
norman	$\frac{\left(\frac{1}{6}A\,c^{3}+\frac{1}{2}Bb\,c^{2}\right)x^{12}+\left(\frac{3}{4}Ab\,c^{2}+\frac{3}{4}B\,b^{2}c\right)x^{10}+\left(\frac{3}{2}A\,b^{2}c+\frac{1}{2}B\,b^{3}\right)x^{8}+\frac{B\,c^{3}x^{14}}{8}}{x^{6}}+A\,b^{3}\ln\left(x\right)$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*B*c^3*x^8+1/6*A*c^3*x^6+1/2*x^6*B*b*c^2+3/4*A*b*c^2*x^4+3/4*x^4*B*b^2*c
+3/2*A*b^2*c*x^2+1/2*x^2*B*b^3+A*b^3*ln(x)
```

Maxima [**A**]
time = 0.28, size = 74, normalized size = 1.23

$$\frac{1}{8}Bc^3x^8 + \frac{1}{6}\left(3Bbc^2 + Ac^3\right)x^6 + \frac{3}{4}\left(Bb^2c + Abc^2\right)x^4 + \frac{1}{2}Ab^3\log\left(x^2\right) + \frac{1}{2}\left(Bb^3 + 3Ab^2c\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/8*B*c^3*x^8 + 1/6*(3*B*b*c^2 + A*c^3)*x^6 + 3/4*(B*b^2*c + A*b*c^2)*x^4 +
 1/2*A*b^3*log(x^2) + 1/2*(B*b^3 + 3*A*b^2*c)*x^2
```

Fricas [**A**]
time = 1.57, size = 71, normalized size = 1.18

$$\frac{1}{8}Bc^3x^8 + \frac{1}{6}\left(3Bbc^2 + Ac^3\right)x^6 + \frac{3}{4}\left(Bb^2c + Abc^2\right)x^4 + Ab^3\log\left(x\right) + \frac{1}{2}\left(Bb^3 + 3Ab^2c\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/8*B*c^3*x^8 + 1/6*(3*B*b*c^2 + A*c^3)*x^6 + 3/4*(B*b^2*c + A*b*c^2)*x^4 +
 A*b^3*log(x) + 1/2*(B*b^3 + 3*A*b^2*c)*x^2
```

Sympy [**A**]
time = 0.06, size = 80, normalized size = 1.33

$$Ab^3\log\left(x\right) + \frac{Bc^3x^8}{8} + x^6\left(\frac{Ac^3}{6} + \frac{Bbc^2}{2}\right) + x^4\cdot\left(\frac{3Abc^2}{4} + \frac{3Bb^2c}{4}\right) + x^2\cdot\left(\frac{3Ab^2c}{2} + \frac{Bb^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**7,x)
```

```
[Out] A*b**3*log(x) + B*c**3*x**8/8 + x**6*(A*c**3/6 + B*b*c**2/2) + x**4*(3*A*b*
c**2/4 + 3*B*b**2*c/4) + x**2*(3*A*b**2*c/2 + B*b**3/2)
```

Giac [**A**]
time = 0.71, size = 78, normalized size = 1.30

$$\frac{1}{8}Bc^3x^8 + \frac{1}{2}Bbc^2x^6 + \frac{1}{6}Ac^3x^6 + \frac{3}{4}Bb^2cx^4 + \frac{3}{4}Abc^2x^4 + \frac{1}{2}Bb^3x^2 + \frac{3}{2}Ab^2cx^2 + \frac{1}{2}Ab^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/8*B*c^3*x^8 + 1/2*B*b*c^2*x^6 + 1/6*A*c^3*x^6 + 3/4*B*b^2*c*x^4 + 3/4*A*b
*c^2*x^4 + 1/2*B*b^3*x^2 + 3/2*A*b^2*c*x^2 + 1/2*A*b^3*log(x^2)
```

Mupad [B]
time = 0.03, size = 67, normalized size = 1.12

$$x^2\left(\frac{B\,b^3}{2}+\frac{3\,A\,c\,b^2}{2}\right)+x^6\left(\frac{A\,c^3}{6}+\frac{B\,b\,c^2}{2}\right)+\frac{B\,c^3\,x^8}{8}+A\,b^3\,\ln\left(x\right)+\frac{3\,b\,c\,x^4\,\left(A\,c+B\,b\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^7,x)
```

```
[Out] x^2*((B*b^3)/2 + (3*A*b^2*c)/2) + x^6*((A*c^3)/6 + (B*b*c^2)/2) + (B*c^3*x^
8)/8 + A*b^3*log(x) + (3*b*c*x^4*(A*c + B*b))/4
```

3.31 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8} \, dx$

Optimal. Leaf size=65

$$-\frac{Ab^3}{x}+b^2(bB+3Ac)x+bc(bB+Ac)x^3+\frac{1}{5}c^2(3bB+Ac)x^5+\frac{1}{7}Bc^3x^7$$

```
[Out] -A*b^3/x+b^2*(3*A*c+B*b)*x+b*c*(A*c+B*b)*x^3+1/5*c^2*(A*c+3*B*b)*x^5+1/7*B*
c^3*x^7
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1598, 459}

$$-\frac{Ab^3}{x}+b^2x(3Ac+bB)+\frac{1}{5}c^2x^5(Ac+3bB)+bcx^3(Ac+bB)+\frac{1}{7}Bc^3x^7$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^8,x]
```

```
[Out] -((A*b^3)/x) + b^2*(b*B + 3*A*c)*x + b*c*(b*B + A*c)*x^3 + (c^2*(3*b*B + A*
c)*x^5)/5 + (B*c^3*x^7)/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8} \, dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^2} \, dx \\
&= \int \left(b^2(bB+3Ac)+\frac{Ab^3}{x^2}+3bc(bB+Ac)x^2+c^2(3bB+Ac)x^4+Bc^3x^6\right) dx \\
&= -\frac{Ab^3}{x}+b^2(bB+3Ac)x+bc(bB+Ac)x^3+\frac{1}{5}c^2(3bB+Ac)x^5+\frac{1}{7}Bc^3x^7
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 1.00

$$-\frac{Ab^3}{x}+b^2(bB+3Ac)x+bc(bB+Ac)x^3+\frac{1}{5}c^2(3bB+Ac)x^5+\frac{1}{7}Bc^3x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^8,x]
```

```
[Out] -((A*b^3)/x) + b^2*(b*B + 3*A*c)*x + b*c*(b*B + A*c)*x^3 + (c^2*(3*b*B + A*
c)*x^5)/5 + (B*c^3*x^7)/7
```

Maple [A]
time = 0.37, size = 71, normalized size = 1.09

method	result	size
default	$\frac{B\,c^3x^7}{7}+\frac{A\,c^3x^5}{5}+\frac{3Bb\,c^2x^5}{5}+Ab\,c^2x^3+B\,b^2c\,x^3+3A\,b^2cx+B\,b^3x-\frac{A\,b^3}{x}$	71
risch	$\frac{B\,c^3x^7}{7}+\frac{A\,c^3x^5}{5}+\frac{3Bb\,c^2x^5}{5}+Ab\,c^2x^3+B\,b^2c\,x^3+3A\,b^2cx+B\,b^3x-\frac{A\,b^3}{x}$	71
norman	$\frac{\left(\frac{1}{5}A\,c^3+\frac{3}{5}Bb\,c^2\right)x^{12}+\left(Ab\,c^2+B\,b^2c\right)x^{10}+\left(3A\,b^2c+B\,b^3\right)x^8-A\,b^3x^6+\frac{B\,c^3x^{14}}{7}}{x^7}$	76
gosper	$-\frac{-5B\,c^3x^8-7A\,c^3x^6-21x^6Bb\,c^2-35Ab\,c^2x^4-35x^4B\,b^2c-105A\,b^2c\,x^2-35x^2B\,b^3+35A\,b^3}{35x}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*B*c^3*x^7+1/5*A*c^3*x^5+3/5*B*b*c^2*x^5+A*b*c^2*x^3+B*b^2*c*x^3+3*A*b^2
*c*x+B*b^3*x-A*b^3/x
```

Maxima [A]
time = 0.28, size = 69, normalized size = 1.06

$$\frac{1}{7}\,Bc^3x^7+\frac{1}{5}\,\left(3\,Bbc^2+Ac^3\right)x^5+\left(Bb^2c+Abc^2\right)x^3-\frac{Ab^3}{x}+\left(Bb^3+3\,Ab^2c\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x, algorithm="maxima")
```

```
[Out] 1/7*B*c^3*x^7 + 1/5*(3*B*b*c^2 + A*c^3)*x^5 + (B*b^2*c + A*b*c^2)*x^3 - A*b
^3/x + (B*b^3 + 3*A*b^2*c)*x
```

Fricas [A]
time = 1.48, size = 75, normalized size = 1.15

$$\frac{5\,Bc^3x^8+7\,(3\,Bbc^2+Ac^3)x^6+35\,(Bb^2c+Abc^2)x^4-35\,Ab^3+35\,(Bb^3+3\,Ab^2c)x^2}{35\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x, algorithm="fricas")
```

```
[Out] 1/35*(5*B*c^3*x^8 + 7*(3*B*b*c^2 + A*c^3)*x^6 + 35*(B*b^2*c + A*b*c^2)*x^4
- 35*A*b^3 + 35*(B*b^3 + 3*A*b^2*c)*x^2)/x
```

Sympy [A]
time = 0.06, size = 68, normalized size = 1.05

$$-\frac{Ab^3}{x} + \frac{Bc^3x^7}{7} + x^5\left(\frac{Ac^3}{5} + \frac{3Bbc^2}{5}\right) + x^3\left(Abc^2 + Bb^2c\right) + x\left(3Ab^2c + Bb^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**8,x)
```

```
[Out] -A*b**3/x + B*c**3*x**7/7 + x**5*(A*c**3/5 + 3*B*b*c**2/5) + x**3*(A*b*c**2
 + B*b**2*c) + x*(3*A*b**2*c + B*b**3)
```

Giac [A]
time = 0.77, size = 70, normalized size = 1.08

$$\frac{1}{7}Bc^3x^7 + \frac{3}{5}Bbc^2x^5 + \frac{1}{5}Ac^3x^5 + Bb^2cx^3 + Abc^2x^3 + Bb^3x + 3Ab^2cx - \frac{Ab^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x, algorithm="giac")
```

```
[Out] 1/7*B*c^3*x^7 + 3/5*B*b*c^2*x^5 + 1/5*A*c^3*x^5 + B*b^2*c*x^3 + A*b*c^2*x^3
 + B*b^3*x + 3*A*b^2*c*x - A*b^3/x
```

Mupad [B]
time = 0.03, size = 65, normalized size = 1.00

$$x\left(B\,b^3 + 3\,A\,c\,b^2\right) + x^5\left(\frac{A\,c^3}{5} + \frac{3\,B\,b\,c^2}{5}\right) - \frac{A\,b^3}{x} + \frac{B\,c^3\,x^7}{7} + b\,c\,x^3\left(A\,c + B\,b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^8,x)
```

```
[Out] x*(B*b^3 + 3*A*b^2*c) + x^5*((A*c^3)/5 + (3*B*b*c^2)/5) - (A*b^3)/x + (B*c^
3*x^7)/7 + b*c*x^3*(A*c + B*b)
```

3.32 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9}\,dx$

Optimal. Leaf size=71

$$-\frac{Ab^3}{2x^2}+\frac{3}{2}bc(bB+Ac)x^2+\frac{1}{4}c^2(3bB+Ac)x^4+\frac{1}{6}Bc^3x^6+b^2(bB+3Ac)\log(x)$$

```
[Out] -1/2*A*b^3/x^2+3/2*b*c*(A*c+B*b)*x^2+1/4*c^2*(A*c+3*B*b)*x^4+1/6*B*c^3*x^6+
b^2*(3*A*c+B*b)*ln(x)
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 77}

$$-\frac{Ab^3}{2x^2}+b^2\log(x)(3Ac+bB)+\frac{1}{4}c^2x^4(Ac+3bB)+\frac{3}{2}bcx^2(Ac+bB)+\frac{1}{6}Bc^3x^6$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^9,x]
```

```
[Out] -1/2*(A*b^3)/x^2 + (3*b*c*(b*B + A*c)*x^2)/2 + (c^2*(3*b*B + A*c)*x^4)/4 +
(B*c^3*x^6)/6 + b^2*(b*B + 3*A*c)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9}\,dx = \int \frac{(A+Bx^2)(b+cx^2)^3}{x^3}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(3bc(bB+Ac) + \frac{Ab^3}{x^2} + \frac{b^2(bB+3Ac)}{x} + c^2(3bB+Ac)x + Bc^3\right.\right.$$
$$= -\frac{Ab^3}{2x^2} + \frac{3}{2}bc(bB+Ac)x^2 + \frac{1}{4}c^2(3bB+Ac)x^4 + \frac{1}{6}Bc^3x^6 + b^2(bB+3Ac)\log$$

Mathematica [A]
time = 0.02, size = 73, normalized size = 1.03

$$-\frac{Ab^3}{2x^2} + \frac{3}{2}bc(bB+Ac)x^2 + \frac{1}{4}c^2(3bB+Ac)x^4 + \frac{1}{6}Bc^3x^6 + \left(b^3B+3Ab^2c\right)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^9,x]
```

```
[Out] -1/2*(A*b^3)/x^2 + (3*b*c*(b*B + A*c)*x^2)/2 + (c^2*(3*b*B + A*c)*x^4)/4 +
(B*c^3*x^6)/6 + (b^3*B + 3*A*b^2*c)*Log[x]
```

Maple [A]
time = 0.37, size = 73, normalized size = 1.03

method	result	size
default	$\frac{B\,c^3x^6}{6}+\frac{A\,c^3x^4}{4}+\frac{3Bb\,c^2x^4}{4}+\frac{3Ab\,c^2x^2}{2}+\frac{3B\,b^2c\,x^2}{2}-\frac{A\,b^3}{2x^2}+b^2(3Ac+Bb)\ln(x)$	73
risch	$\frac{B\,c^3x^6}{6}+\frac{A\,c^3x^4}{4}+\frac{3Bb\,c^2x^4}{4}+\frac{3Ab\,c^2x^2}{2}+\frac{3B\,b^2c\,x^2}{2}-\frac{A\,b^3}{2x^2}+3A\ln(x)\,b^2c+B\ln(x)\,b^3$	75
norman	$\frac{\left(\frac{1}{4}A\,c^3+\frac{3}{4}Bb\,c^2\right)x^{12}+\left(\frac{3}{2}Ab\,c^2+\frac{3}{2}B\,b^2c\right)x^{10}-\frac{A\,b^3x^6}{2}+\frac{B\,c^3x^{14}}{6}}{x^8}+\left(3A\,b^2c+B\,b^3\right)\ln(x)$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*B*c^3*x^6+1/4*A*c^3*x^4+3/4*B*b*c^2*x^4+3/2*A*b*c^2*x^2+3/2*B*b^2*c*x^2
-1/2*A*b^3/x^2+b^2*(3*A*c+B*b)*ln(x)
```

Maxima [A]
time = 0.29, size = 74, normalized size = 1.04

$$\frac{1}{6}\,Bc^3x^6 + \frac{1}{4}\left(3\,Bbc^2 + Ac^3\right)x^4 + \frac{3}{2}\left(Bb^2c + Abc^2\right)x^2 - \frac{Ab^3}{2\,x^2} + \frac{1}{2}\left(Bb^3 + 3\,Ab^2c\right)\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x, algorithm="maxima")
```

```
[Out] 1/6*B*c^3*x^6 + 1/4*(3*B*b*c^2 + A*c^3)*x^4 + 3/2*(B*b^2*c + A*b*c^2)*x^2 -
 1/2*A*b^3/x^2 + 1/2*(B*b^3 + 3*A*b^2*c)*log(x^2)
```

Fricas [A]
time = 1.70, size = 77, normalized size = 1.08

$$\frac{2\,Bc^3x^8+3\,(3\,Bbc^2+Ac^3)x^6+18\,(Bb^2c+Abc^2)x^4-6\,Ab^3+12\,(Bb^3+3\,Ab^2c)x^2\log\left(x\right)}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*c^3*x^8 + 3*(3*B*b*c^2 + A*c^3)*x^6 + 18*(B*b^2*c + A*b*c^2)*x^4
- 6*A*b^3 + 12*(B*b^3 + 3*A*b^2*c)*x^2*log(x))/x^2
```

Sympy [A]
time = 0.12, size = 78, normalized size = 1.10

$$-\frac{Ab^3}{2x^2}+\frac{Bc^3x^6}{6}+b^2\cdot(3Ac+Bb)\log\left(x\right)+x^4\left(\frac{Ac^3}{4}+\frac{3Bbc^2}{4}\right)+x^2\cdot\left(\frac{3Abc^2}{2}+\frac{3Bb^2c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**9,x)
```

```
[Out] -A*b**3/(2*x**2) + B*c**3*x**6/6 + b**2*(3*A*c + B*b)*log(x) + x**4*(A*c**3
/4 + 3*B*b*c**2/4) + x**2*(3*A*b*c**2/2 + 3*B*b**2*c/2)
```

Giac [A]
time = 0.68, size = 97, normalized size = 1.37

$$\frac{1}{6}\,Bc^3x^6+\frac{3}{4}\,Bbc^2x^4+\frac{1}{4}\,Ac^3x^4+\frac{3}{2}\,Bb^2cx^2+\frac{3}{2}\,Abc^2x^2+\frac{1}{2}\,(Bb^3+3\,Ab^2c)\log\left(x^2\right)-\frac{Bb^3x^2+3\,Ab^2cx^2+Ab^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x, algorithm="giac")
```

```
[Out] 1/6*B*c^3*x^6 + 3/4*B*b*c^2*x^4 + 1/4*A*c^3*x^4 + 3/2*B*b^2*c*x^2 + 3/2*A*b
*c^2*x^2 + 1/2*(B*b^3 + 3*A*b^2*c)*log(x^2) - 1/2*(B*b^3*x^2 + 3*A*b^2*c*x^
2 + A*b^3)/x^2
```

Mupad [B]
time = 0.04, size = 67, normalized size = 0.94

$$x^4\left(\frac{A\,c^3}{4}+\frac{3\,B\,b\,c^2}{4}\right)+\ln\left(x\right)\,\left(B\,b^3+3\,A\,c\,b^2\right)-\frac{A\,b^3}{2\,x^2}+\frac{B\,c^3\,x^6}{6}+\frac{3\,b\,c\,x^2\,\left(A\,c+B\,b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^9,x)
```

```
[Out] x^4*((A*c^3)/4 + (3*B*b*c^2)/4) + log(x)*(B*b^3 + 3*A*b^2*c) - (A*b^3)/(2*x
^2) + (B*c^3*x^6)/6 + (3*b*c*x^2*(A*c + B*b))/2
```

3.33 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx$

Optimal. Leaf size=69

$$-\frac{Ab^3}{3x^3}-\frac{b^2(bB+3Ac)}{x}+3bc(bB+Ac)x+\frac{1}{3}c^2(3bB+Ac)x^3+\frac{1}{5}Bc^3x^5$$

```
[Out] -1/3*A*b^3/x^3-b^2*(3*A*c+B*b)/x+3*b*c*(A*c+B*b)*x+1/3*c^2*(A*c+3*B*b)*x^3+
1/5*B*c^3*x^5
```

Rubi [A]

time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$-\frac{Ab^3}{3x^3}-\frac{b^2(3Ac+bB)}{x}+\frac{1}{3}c^2x^3(Ac+3bB)+3bcx(Ac+bB)+\frac{1}{5}Bc^3x^5$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^10,x]
```

```
[Out] -1/3*(A*b^3)/x^3 - (b^2*(b*B + 3*A*c))/x + 3*b*c*(b*B + A*c)*x + (c^2*(3*b*
B + A*c)*x^3)/3 + (B*c^3*x^5)/5
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^4}\,dx\\ &= \int \left(3bc(bB+Ac)+\frac{Ab^3}{x^4}+\frac{b^2(bB+3Ac)}{x^2}+c^2(3bB+Ac)x^2+Bc^3x^4\right)dx\\ &= -\frac{Ab^3}{3x^3}-\frac{b^2(bB+3Ac)}{x}+3bc(bB+Ac)x+\frac{1}{3}c^2(3bB+Ac)x^3+\frac{1}{5}Bc^3x^5\end{aligned}$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 1.03

$$-\frac{Ab^3}{3x^3}+\frac{-b^3B-3Ab^2c}{x}+3bc(bB+Ac)x+\frac{1}{3}c^2(3bB+Ac)x^3+\frac{1}{5}Bc^3x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^10,x]
```

```
[Out] -1/3*(A*b^3)/x^3 + (-(b^3*B) - 3*A*b^2*c)/x + 3*b*c*(b*B + A*c)*x + (c^2*(3
*b*B + A*c)*x^3)/3 + (B*c^3*x^5)/5
```

Maple [A]
time = 0.38, size = 70, normalized size = 1.01

method	result	size
default	$\frac{B\,c^3x^5}{5}+\frac{A\,c^3x^3}{3}+Bb\,c^2x^3+3Ab\,c^2x+3B\,b^2cx-\frac{A\,b^3}{3x^3}-\frac{b^2(3Ac+Bb)}{x}$	70
risch	$\frac{B\,c^3x^5}{5}+\frac{A\,c^3x^3}{3}+Bb\,c^2x^3+3Ab\,c^2x+3B\,b^2cx+\frac{(-3A\,b^2c-B\,b^3)x^2-\frac{A\,b^3}{3}}{x^3}$	74
norman	$\frac{(\frac{1}{3}A\,c^3+Bb\,c^2)x^{12}+(3Ab\,c^2+3B\,b^2c)x^{10}+(-3A\,b^2c-B\,b^3)x^8-\frac{A\,b^3x^6}{3}+\frac{B\,c^3x^{14}}{5}}{x^9}$	78
gosper	$-\frac{-3B\,c^3x^8-5A\,c^3x^6-15x^6Bb\,c^2-45Ab\,c^2x^4-45x^4B\,b^2c+45A\,b^2c\,x^2+15x^2B\,b^3+5A\,b^3}{15x^3}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*B*c^3*x^5+1/3*A*c^3*x^3+B*b*c^2*x^3+3*A*b*c^2*x+3*B*b^2*c*x-1/3*A*b^3/x
^3-b^2*(3*A*c+B*b)/x
```

Maxima [A]
time = 0.27, size = 73, normalized size = 1.06

$$\frac{1}{5}\,Bc^3x^5+\frac{1}{3}\left(3\,Bbc^2+Ac^3\right)x^3+3\left(Bb^2c+Abc^2\right)x-\frac{Ab^3+3\,(Bb^3+3\,Ab^2c)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x, algorithm="maxima")
```

```
[Out] 1/5*B*c^3*x^5 + 1/3*(3*B*b*c^2 + A*c^3)*x^3 + 3*(B*b^2*c + A*b*c^2)*x - 1/3
*(A*b^3 + 3*(B*b^3 + 3*A*b^2*c)*x^2)/x^3
```

Fricas [A]
time = 1.68, size = 75, normalized size = 1.09

$$\frac{3\,Bc^3x^8+5\,(3\,Bbc^2+Ac^3)x^6+45\,(Bb^2c+Abc^2)x^4-5\,Ab^3-15\,(Bb^3+3\,Ab^2c)x^2}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*c^3*x^8 + 5*(3*B*b*c^2 + A*c^3)*x^6 + 45*(B*b^2*c + A*b*c^2)*x^4
- 5*A*b^3 - 15*(B*b^3 + 3*A*b^2*c)*x^2)/x^3
```

Sympy [A]
time = 0.13, size = 75, normalized size = 1.09

$$\frac{Bc^3x^5}{5} + x^3\left(\frac{Ac^3}{3} + Bbc^2\right) + x\left(3Abc^2 + 3Bb^2c\right) + \frac{-Ab^3 + x^2(-9Ab^2c - 3Bb^3)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**10,x)
```

```
[Out] B*c**3*x**5/5 + x**3*(A*c**3/3 + B*b*c**2) + x*(3*A*b*c**2 + 3*B*b**2*c) +
(-A*b**3 + x**2*(-9*A*b**2*c - 3*B*b**3))/(3*x**3)
```

Giac [A]
time = 0.71, size = 74, normalized size = 1.07

$$\frac{1}{5}\,Bc^3x^5 + Bbc^2x^3 + \frac{1}{3}\,Ac^3x^3 + 3\,Bb^2cx + 3\,Abc^2x - \frac{3\,Bb^3x^2 + 9\,Ab^2cx^2 + Ab^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x, algorithm="giac")
```

```
[Out] 1/5*B*c^3*x^5 + B*b*c^2*x^3 + 1/3*A*c^3*x^3 + 3*B*b^2*c*x + 3*A*b*c^2*x - 1
/3*(3*B*b^3*x^2 + 9*A*b^2*c*x^2 + A*b^3)/x^3
```

Mupad [B]
time = 0.06, size = 68, normalized size = 0.99

$$x^3\left(\frac{A\,c^3}{3} + B\,b\,c^2\right) - \frac{\frac{A\,b^3}{3} + x^2\,\left(B\,b^3 + 3\,A\,c\,b^2\right)}{x^3} + \frac{B\,c^3\,x^5}{5} + 3\,b\,c\,x\,\left(A\,c + B\,b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^10,x)
```

```
[Out] x^3*((A*c^3)/3 + B*b*c^2) - ((A*b^3)/3 + x^2*(B*b^3 + 3*A*b^2*c))/x^3 + (B*
c^3*x^5)/5 + 3*b*c*x*(A*c + B*b)
```

3.34 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}} dx$

Optimal. Leaf size=72

$$-\frac{Ab^3}{4x^4} - \frac{b^2(bB+3Ac)}{2x^2} + \frac{1}{2}c^2(3bB+Ac)x^2 + \frac{1}{4}Bc^3x^4 + 3bc(bB+Ac)\log(x)$$

```
[Out] -1/4*A*b^3/x^4-1/2*b^2*(3*A*c+B*b)/x^2+1/2*c^2*(A*c+3*B*b)*x^2+1/4*B*c^3*x^
4+3*b*c*(A*c+B*b)*ln(x)
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 77}

$$-\frac{Ab^3}{4x^4} - \frac{b^2(3Ac+bB)}{2x^2} + \frac{1}{2}c^2x^2(Ac+3bB) + 3bc\log(x)(Ac+bB) + \frac{1}{4}Bc^3x^4$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^11,x]
```

```
[Out] -1/4*(A*b^3)/x^4 - (b^2*(b*B + 3*A*c))/(2*x^2) + (c^2*(3*b*B + A*c)*x^2)/2
+ (B*c^3*x^4)/4 + 3*b*c*(b*B + A*c)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}}\,dx = \int \frac{(A+Bx^2)(b+cx^2)^3}{x^5}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(c^2(3bB+Ac)+\frac{Ab^3}{x^3}+\frac{b^2(bB+3Ac)}{x^2}+\frac{3bc(bB+Ac)}{x}+Bc\right.\right.$$
$$= -\frac{Ab^3}{4x^4}-\frac{b^2(bB+3Ac)}{2x^2}+\frac{1}{2}c^2(3bB+Ac)x^2+\frac{1}{4}Bc^3x^4+3bc(bB+Ac)\log$$

Mathematica [A]
time = 0.02, size = 73, normalized size = 1.01

$$\frac{-A(b^3+6b^2cx^2-2c^3x^6)+Bx^2(-2b^3+6bc^2x^4+c^3x^6)}{4x^4}+3bc(bB+Ac)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^11,x]
```

```
[Out] (-(A*(b^3 + 6*b^2*c*x^2 - 2*c^3*x^6)) + B*x^2*(-2*b^3 + 6*b*c^2*x^4 + c^3*x
^6))/(4*x^4) + 3*b*c*(b*B + A*c)*Log[x]
```

Maple [A]
time = 0.46, size = 68, normalized size = 0.94

method	result
default	$\frac{B\,c^3x^4}{4}+\frac{A\,c^3x^2}{2}+\frac{3Bb\,c^2x^2}{2}-\frac{A\,b^3}{4x^4}-\frac{b^2(3Ac+Bb)}{2x^2}+3bc(Ac+Bb)\ln(x)$
norman	$\frac{\left(\frac{1}{2}A\,c^3+\frac{3}{2}Bb\,c^2\right)x^{12}+\left(-\frac{3}{2}A\,b^2c-\frac{1}{2}B\,b^3\right)x^8-\frac{A\,b^3x^6}{4}+\frac{B\,c^3x^{14}}{4}}{x^{10}}+(3Ab\,c^2+3B\,b^2c)\ln(x)$
risch	$\frac{B\,c^3x^4}{4}+\frac{A\,c^3x^2}{2}+\frac{3Bb\,c^2x^2}{2}+\frac{c^3A^2}{4B}+\frac{3Ab\,c^2}{2}+\frac{9B\,b^2c}{4}+\frac{\left(-\frac{3}{2}A\,b^2c-\frac{1}{2}B\,b^3\right)x^2-\frac{A\,b^3}{4}}{x^4}+3A\ln(x)\,b\,c^2+3B\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*B*c^3*x^4+1/2*A*c^3*x^2+3/2*B*b*c^2*x^2-1/4*A*b^3/x^4-1/2*b^2*(3*A*c+B*
b)/x^2+3*b*c*(A*c+B*b)*ln(x)
```

Maxima [A]
time = 0.28, size = 76, normalized size = 1.06

$$\frac{1}{4}Bc^3x^4+\frac{1}{2}\left(3\,Bbc^2+Ac^3\right)x^2+\frac{3}{2}\left(Bb^2c+Abc^2\right)\log\left(x^2\right)-\frac{Ab^3+2\,(Bb^3+3\,Ab^2c)x^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x, algorithm="maxima")
```

```
[Out] 1/4*B*c^3*x^4 + 1/2*(3*B*b*c^2 + A*c^3)*x^2 + 3/2*(B*b^2*c + A*b*c^2)*log(x
^2) - 1/4*(A*b^3 + 2*(B*b^3 + 3*A*b^2*c)*x^2)/x^4
```

Fricas [A]
time = 1.55, size = 76, normalized size = 1.06

$$\frac{Bc^3x^8 + 2\,(3\,Bbc^2 + Ac^3)x^6 + 12\,(Bb^2c + Abc^2)x^4\log\left(x\right) - Ab^3 - 2\,(Bb^3 + 3\,Ab^2c)x^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^3*x^8 + 2*(3*B*b*c^2 + A*c^3)*x^6 + 12*(B*b^2*c + A*b*c^2)*x^4*log
(x) - A*b^3 - 2*(B*b^3 + 3*A*b^2*c)*x^2)/x^4
```

Sympy [A]
time = 0.30, size = 75, normalized size = 1.04

$$\frac{Bc^3x^4}{4} + 3bc(Ac + Bb)\log\left(x\right) + x^2\left(\frac{Ac^3}{2} + \frac{3Bbc^2}{2}\right) + \frac{-Ab^3 + x^2(-6Ab^2c - 2Bb^3)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**11,x)
```

```
[Out] B*c**3*x**4/4 + 3*b*c*(A*c + B*b)*log(x) + x**2*(A*c**3/2 + 3*B*b*c**2/2) +
 (-A*b**3 + x**2*(-6*A*b**2*c - 2*B*b**3))/(4*x**4)
```

Giac [A]
time = 0.82, size = 98, normalized size = 1.36

$$\frac{1}{4}\,Bc^3x^4 + \frac{3}{2}\,Bbc^2x^2 + \frac{1}{2}\,Ac^3x^2 + \frac{3}{2}\,(Bb^2c + Abc^2)\log\left(x^2\right) - \frac{9\,Bb^2cx^4 + 9\,Abc^2x^4 + 2\,Bb^3x^2 + 6\,Ab^2cx^2 + Ab^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x, algorithm="giac")
```

```
[Out] 1/4*B*c^3*x^4 + 3/2*B*b*c^2*x^2 + 1/2*A*c^3*x^2 + 3/2*(B*b^2*c + A*b*c^2)*l
og(x^2) - 1/4*(9*B*b^2*c*x^4 + 9*A*b*c^2*x^4 + 2*B*b^3*x^2 + 6*A*b^2*c*x^2
+ A*b^3)/x^4
```

Mupad [B]
time = 0.07, size = 76, normalized size = 1.06

$$\ln\left(x\right)\,\left(3\,B\,b^2\,c + 3\,A\,b\,c^2\right) - \frac{\frac{A\,b^3}{4} + x^2\,\left(\frac{B\,b^3}{2} + \frac{3\,A\,c\,b^2}{2}\right)}{x^4} + x^2\,\left(\frac{A\,c^3}{2} + \frac{3\,B\,b\,c^2}{2}\right) + \frac{B\,c^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^11,x)
```

```
[Out] log(x)*(3*A*b*c^2 + 3*B*b^2*c) - ((A*b^3)/4 + x^2*((B*b^3)/2 + (3*A*b^2*c)/
2))/x^4 + x^2*((A*c^3)/2 + (3*B*b*c^2)/2) + (B*c^3*x^4)/4
```

3.35 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}}\,dx$

Optimal. Leaf size=68

$$-\frac{Ab^3}{5x^5}-\frac{b^2(bB+3Ac)}{3x^3}-\frac{3bc(bB+Ac)}{x}+c^2(3bB+Ac)x+\frac{1}{3}Bc^3x^3$$

```
[Out] -1/5*A*b^3/x^5-1/3*b^2*(3*A*c+B*b)/x^3-3*b*c*(A*c+B*b)/x+c^2*(A*c+3*B*b)*x+
1/3*B*c^3*x^3
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1598, 459}

$$-\frac{Ab^3}{5x^5}-\frac{b^2(3Ac+bB)}{3x^3}+c^2x(Ac+3bB)-\frac{3bc(Ac+bB)}{x}+\frac{1}{3}Bc^3x^3$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^12,x]
```

```
[Out] -1/5*(A*b^3)/x^5 - (b^2*(b*B + 3*A*c))/(3*x^3) - (3*b*c*(b*B + A*c))/x + c^
2*(3*b*B + A*c)*x + (B*c^3*x^3)/3
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^6}\,dx\\
&= \int \left(c^2(3bB+Ac)+\frac{Ab^3}{x^6}+\frac{b^2(bB+3Ac)}{x^4}+\frac{3bc(bB+Ac)}{x^2}+Bc^3x^2\right)dx\\
&= -\frac{Ab^3}{5x^5}-\frac{b^2(bB+3Ac)}{3x^3}-\frac{3bc(bB+Ac)}{x}+c^2(3bB+Ac)x+\frac{1}{3}Bc^3x^3
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 68, normalized size = 1.00

$$-\frac{Ab^3}{5x^5}-\frac{b^2(bB+3Ac)}{3x^3}-\frac{3bc(bB+Ac)}{x}+c^2(3bB+Ac)x+\frac{1}{3}Bc^3x^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^12,x]
```

```
[Out] -1/5*(A*b^3)/x^5 - (b^2*(b*B + 3*A*c))/(3*x^3) - (3*b*c*(b*B + A*c))/x + c^
2*(3*b*B + A*c)*x + (B*c^3*x^3)/3
```

Maple [A]
time = 0.38, size = 64, normalized size = 0.94

method	result	size
default	$\frac{B\,c^3x^3}{3}+A\,c^3x+3Bb\,c^2x-\frac{A\,b^3}{5x^5}-\frac{b^2(3Ac+Bb)}{3x^3}-\frac{3bc(Ac+Bb)}{x}$	64
risch	$\frac{B\,c^3x^3}{3}+A\,c^3x+3Bb\,c^2x+\frac{(-3Ab\,c^2-3B\,b^2c)x^4+(-A\,b^2c-\frac{1}{3}B\,b^3)x^2-\frac{A\,b^3}{5}}{x^5}$	73
norman	$\frac{(-A\,b^2c-\frac{1}{3}B\,b^3)x^8+(A\,c^3+3Bb\,c^2)x^{12}+(-3Ab\,c^2-3B\,b^2c)x^{10}-\frac{A\,b^3x^6}{5}+\frac{B\,c^3x^{14}}{3}}{x^{11}}$	78
gosper	$-\frac{-5B\,c^3x^8-15A\,c^3x^6-45x^6Bb\,c^2+45Ab\,c^2x^4+45x^4B\,b^2c+15A\,b^2c\,x^2+5x^2B\,b^3+3A\,b^3}{15x^5}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*B*c^3*x^3+A*c^3*x+3*B*b*c^2*x-1/5*A*b^3/x^5-1/3*b^2*(3*A*c+B*b)/x^3-3*b
*c*(A*c+B*b)/x
```

Maxima [A]
time = 0.28, size = 73, normalized size = 1.07

$$\frac{1}{3}\,Bc^3x^3+\left(3\,Bbc^2+Ac^3\right)x-\frac{45\,(Bb^2c+Abc^2)x^4+3\,Ab^3+5\,(Bb^3+3\,Ab^2c)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x, algorithm="maxima")
```

```
[Out] 1/3*B*c^3*x^3 + (3*B*b*c^2 + A*c^3)*x - 1/15*(45*(B*b^2*c + A*b*c^2)*x^4 +
3*A*b^3 + 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^5
```

Fricas [A]
time = 1.42, size = 75, normalized size = 1.10

$$\frac{5\,Bc^3x^8+15\,(3\,Bbc^2+Ac^3)x^6-45\,(Bb^2c+Abc^2)x^4-3\,Ab^3-5\,(Bb^3+3\,Ab^2c)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x, algorithm="fricas")
```

```
[Out] 1/15*(5*B*c^3*x^8 + 15*(3*B*b*c^2 + A*c^3)*x^6 - 45*(B*b^2*c + A*b*c^2)*x^4
 - 3*A*b^3 - 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^5
```

Sympy [A]
time = 0.34, size = 78, normalized size = 1.15

$$\frac{Bc^3x^3}{3} + x\left(Ac^3 + 3Bbc^2\right) + \frac{-3Ab^3 + x^4(-45Abc^2 - 45Bb^2c) + x^2(-15Ab^2c - 5Bb^3)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**12,x)
```

```
[Out] B*c**3*x**3/3 + x*(A*c**3 + 3*B*b*c**2) + (-3*A*b**3 + x**4*(-45*A*b*c**2 -
 45*B*b**2*c) + x**2*(-15*A*b**2*c - 5*B*b**3))/(15*x**5)
```

Giac [A]
time = 0.75, size = 75, normalized size = 1.10

$$\frac{1}{3}\,Bc^3x^3 + 3\,Bbc^2x + Ac^3x - \frac{45\,Bb^2cx^4 + 45\,Abc^2x^4 + 5\,Bb^3x^2 + 15\,Ab^2cx^2 + 3\,Ab^3}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x, algorithm="giac")
```

```
[Out] 1/3*B*c^3*x^3 + 3*B*b*c^2*x + A*c^3*x - 1/15*(45*B*b^2*c*x^4 + 45*A*b*c^2*x
^4 + 5*B*b^3*x^2 + 15*A*b^2*c*x^2 + 3*A*b^3)/x^5
```

Mupad [B]
time = 0.06, size = 73, normalized size = 1.07

$$x\left(A\,c^3 + 3\,B\,b\,c^2\right) - \frac{x^4\,(3\,B\,b^2\,c + 3\,A\,b\,c^2) + \frac{A\,b^3}{5} + x^2\,\left(\frac{B\,b^3}{3} + A\,c\,b^2\right)}{x^5} + \frac{B\,c^3\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^12,x)
```

```
[Out] x*(A*c^3 + 3*B*b*c^2) - (x^4*(3*A*b*c^2 + 3*B*b^2*c) + (A*b^3)/5 + x^2*((B*
b^3)/3 + A*b^2*c))/x^5 + (B*c^3*x^3)/3
```

3.36 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}}\,dx$

Optimal. Leaf size=71

$$-\frac{Ab^3}{6x^6}-\frac{b^2(bB+3Ac)}{4x^4}-\frac{3bc(bB+Ac)}{2x^2}+\frac{1}{2}Bc^3x^2+c^2(3bB+Ac)\log(x)$$

```
[Out] -1/6*A*b^3/x^6-1/4*b^2*(3*A*c+B*b)/x^4-3/2*b*c*(A*c+B*b)/x^2+1/2*B*c^3*x^2+
c^2*(A*c+3*B*b)*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 77}

$$-\frac{Ab^3}{6x^6}-\frac{b^2(3Ac+bB)}{4x^4}+c^2\log(x)(Ac+3bB)-\frac{3bc(Ac+bB)}{2x^2}+\frac{1}{2}Bc^3x^2$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^13,x]
```

```
[Out] -1/6*(A*b^3)/x^6 - (b^2*(b*B + 3*A*c))/(4*x^4) - (3*b*c*(b*B + A*c))/(2*x^2
) + (B*c^3*x^2)/2 + c^2*(3*b*B + A*c)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}}\,dx = \int \frac{(A+Bx^2)(b+cx^2)^3}{x^7}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(Bc^3 + \frac{Ab^3}{x^4} + \frac{b^2(bB+3Ac)}{x^3} + \frac{3bc(bB+Ac)}{x^2} + \frac{c^2(3bB+Ac)}{x}\right.\right.$$
$$= -\frac{Ab^3}{6x^6} - \frac{b^2(bB+3Ac)}{4x^4} - \frac{3bc(bB+Ac)}{2x^2} + \frac{1}{2}Bc^3x^2 + c^2(3bB+Ac)\log(x)$$

Mathematica [A]
time = 0.03, size = 71, normalized size = 1.00

$$-\frac{Ab^3}{6x^6} - \frac{b^2(bB+3Ac)}{4x^4} - \frac{3bc(bB+Ac)}{2x^2} + \frac{1}{2}Bc^3x^2 + c^2(3bB+Ac)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^13,x]
```

```
[Out] -1/6*(A*b^3)/x^6 - (b^2*(b*B + 3*A*c))/(4*x^4) - (3*b*c*(b*B + A*c))/(2*x^2
) + (B*c^3*x^2)/2 + c^2*(3*b*B + A*c)*Log[x]
```

Maple [A]
time = 0.38, size = 64, normalized size = 0.90

method	result	size
default	$-\frac{A\,b^3}{6x^6} - \frac{b^2(3Ac+Bb)}{4x^4} - \frac{3bc(Ac+Bb)}{2x^2} + \frac{B\,c^3x^2}{2} + c^2(Ac+3Bb)\ln(x)$	64
risch	$\frac{B\,c^3x^2}{2} + \frac{(-\frac{3}{2}Ab\,c^2 - \frac{3}{2}B\,b^2c)x^4 + (-\frac{3}{4}A\,b^2c - \frac{1}{4}B\,b^3)x^2 - \frac{A\,b^3}{6}}{x^6} + A\ln(x)\,c^3 + 3B\ln(x)\,b\,c^2$	75
norman	$\frac{(-\frac{3}{2}Ab\,c^2 - \frac{3}{2}B\,b^2c)x^{10} + (-\frac{3}{4}A\,b^2c - \frac{1}{4}B\,b^3)x^8 - \frac{A\,b^3x^6}{6} + \frac{B\,c^3x^{14}}{2}}{x^{12}} + (A\,c^3 + 3Bb\,c^2)\ln(x)$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*A*b^3/x^6-1/4*b^2*(3*A*c+B*b)/x^4-3/2*b*c*(A*c+B*b)/x^2+1/2*B*c^3*x^2+
c^2*(A*c+3*B*b)*ln(x)
```

Maxima [A]
time = 0.28, size = 77, normalized size = 1.08

$$\frac{1}{2}\,Bc^3x^2 + \frac{1}{2}\left(3\,Bbc^2 + Ac^3\right)\log\left(x^2\right) - \frac{18\,(Bb^2c + Abc^2)x^4 + 2\,Ab^3 + 3\,(Bb^3 + 3\,Ab^2c)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x, algorithm="maxima")
```

```
[Out] 1/2*B*c^3*x^2 + 1/2*(3*B*b*c^2 + A*c^3)*log(x^2) - 1/12*(18*(B*b^2*c + A*b*
c^2)*x^4 + 2*A*b^3 + 3*(B*b^3 + 3*A*b^2*c)*x^2)/x^6
```

Fricas [A]
time = 1.70, size = 77, normalized size = 1.08

$$\frac{6\,Bc^3x^8+12\,(3\,Bbc^2+Ac^3)x^6\log{(x)}-18\,(Bb^2c+Abc^2)x^4-2\,Ab^3-3\,(Bb^3+3\,Ab^2c)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x, algorithm="fricas")
```

```
[Out] 1/12*(6*B*c^3*x^8 + 12*(3*B*b*c^2 + A*c^3)*x^6*log(x) - 18*(B*b^2*c + A*b*c
^2)*x^4 - 2*A*b^3 - 3*(B*b^3 + 3*A*b^2*c)*x^2)/x^6
```

Sympy [A]
time = 0.68, size = 78, normalized size = 1.10

$$\frac{Bc^3x^2}{2}+c^2(Ac+3Bb)\log{(x)}+\frac{-2Ab^3+x^4(-18Abc^2-18Bb^2c)+x^2(-9Ab^2c-3Bb^3)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**13,x)
```

```
[Out] B*c**3*x**2/2 + c**2*(A*c + 3*B*b)*log(x) + (-2*A*b**3 + x**4*(-18*A*b*c**2
 - 18*B*b**2*c) + x**2*(-9*A*b**2*c - 3*B*b**3))/(12*x**6)
```

Giac [A]
time = 0.69, size = 99, normalized size = 1.39

$$\frac{1}{2}\,Bc^3x^2+\frac{1}{2}\,\left(3\,Bbc^2+Ac^3\right)\log\left(x^2\right)-\frac{33\,Bbc^2x^6+11\,Ac^3x^6+18\,Bb^2cx^4+18\,Abc^2x^4+3\,Bb^3x^2+9\,Ab^2cx^2+2\,Ab^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x, algorithm="giac")
```

```
[Out] 1/2*B*c^3*x^2 + 1/2*(3*B*b*c^2 + A*c^3)*log(x^2) - 1/12*(33*B*b*c^2*x^6 + 1
1*A*c^3*x^6 + 18*B*b^2*c*x^4 + 18*A*b*c^2*x^4 + 3*B*b^3*x^2 + 9*A*b^2*c*x^2
 + 2*A*b^3)/x^6
```

Mupad [B]
time = 0.09, size = 75, normalized size = 1.06

$$\ln\left(x\right)\,\left(A\,c^3+3\,B\,b\,c^2\right)-\frac{x^4\,\left(\frac{3\,B\,b^2\,c}{2}+\frac{3\,A\,b\,c^2}{2}\right)+\frac{A\,b^3}{6}+x^2\,\left(\frac{B\,b^3}{4}+\frac{3\,A\,c\,b^2}{4}\right)}{x^6}+\frac{B\,c^3\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^13,x)
```

```
[Out] log(x)*(A*c^3 + 3*B*b*c^2) - (x^4*((3*A*b*c^2)/2 + (3*B*b^2*c)/2) + (A*b^3)
/6 + x^2*((B*b^3)/4 + (3*A*b^2*c)/4))/x^6 + (B*c^3*x^2)/2
```

3.37 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}} dx$

Optimal. Leaf size=66

$$-\frac{Ab^3}{7x^7}-\frac{b^2(bB+3Ac)}{5x^5}-\frac{bc(bB+Ac)}{x^3}-\frac{c^2(3bB+Ac)}{x}+Bc^3x$$

```
[Out] -1/7*A*b^3/x^7-1/5*b^2*(3*A*c+B*b)/x^5-b*c*(A*c+B*b)/x^3-c^2*(A*c+3*B*b)/x+
B*c^3*x
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1598, 459}

$$-\frac{Ab^3}{7x^7}-\frac{b^2(3Ac+bB)}{5x^5}-\frac{c^2(Ac+3bB)}{x}-\frac{bc(Ac+bB)}{x^3}+Bc^3x$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^14,x]
```

```
[Out] -1/7*(A*b^3)/x^7 - (b^2*(b*B + 3*A*c))/(5*x^5) - (b*c*(b*B + A*c))/x^3 - (c
^2*(3*b*B + A*c))/x + B*c^3*x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^8}\,dx\\
&= \int \left(Bc^3+\frac{Ab^3}{x^8}+\frac{b^2(bB+3Ac)}{x^6}+\frac{3bc(bB+Ac)}{x^4}+\frac{c^2(3bB+Ac)}{x^2}\right)dx\\
&= -\frac{Ab^3}{7x^7}-\frac{b^2(bB+3Ac)}{5x^5}-\frac{bc(bB+Ac)}{x^3}-\frac{c^2(3bB+Ac)}{x}+Bc^3x
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 66, normalized size = 1.00

$$-\frac{Ab^3}{7x^7}-\frac{b^2(bB+3Ac)}{5x^5}-\frac{bc(bB+Ac)}{x^3}-\frac{c^2(3bB+Ac)}{x}+Bc^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^14,x]
```

```
[Out] -1/7*(A*b^3)/x^7 - (b^2*(b*B + 3*A*c))/(5*x^5) - (b*c*(b*B + A*c))/x^3 - (c
^2*(3*b*B + A*c))/x + B*c^3*x
```

Maple [A]
time = 0.36, size = 63, normalized size = 0.95

method	result	size
default	$-\frac{A\,b^{3}}{7x^{7}}-\frac{b^{2}(3Ac+Bb)}{5x^{5}}-\frac{bc(Ac+Bb)}{x^{3}}-\frac{c^{2}(Ac+3Bb)}{x}+B\,c^{3}x$	63
risch	$B\,c^{3}x+\frac{(-A\,c^{3}-3Bb\,c^{2})x^{6}+(-Ab\,c^{2}-B\,b^{2}c)x^{4}+(-\frac{3}{5}A\,b^{2}c-\frac{1}{5}B\,b^{3})x^{2}-\frac{A\,b^{3}}{7}}{x^{7}}$	74
norman	$\frac{(-\frac{3}{5}A\,b^{2}c-\frac{1}{5}B\,b^{3})x^{8}+(-A\,c^{3}-3Bb\,c^{2})x^{12}+(-Ab\,c^{2}-B\,b^{2}c)x^{10}+B\,c^{3}x^{14}-\frac{A\,b^{3}x^{6}}{7}}{x^{13}}$	78
gosper	$-\frac{-35B\,c^{3}x^{8}+35A\,c^{3}x^{6}+105x^{6}Bb\,c^{2}+35Ab\,c^{2}x^{4}+35x^{4}B\,b^{2}c+21A\,b^{2}c\,x^{2}+7x^{2}B\,b^{3}+5A\,b^{3}}{35x^{7}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*A*b^3/x^7-1/5*b^2*(3*A*c+B*b)/x^5-b*c*(A*c+B*b)/x^3-c^2*(A*c+3*B*b)/x+
B*c^3*x
```

Maxima [A]
time = 0.27, size = 73, normalized size = 1.11

$$Bc^3x-\frac{35\,(3\,Bbc^2+Ac^3)x^6+35\,(Bb^2c+Abc^2)x^4+5\,Ab^3+7\,(Bb^3+3\,Ab^2c)x^2}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x, algorithm="maxima")
```

```
[Out] B*c^3*x - 1/35*(35*(3*B*b*c^2 + A*c^3)*x^6 + 35*(B*b^2*c + A*b*c^2)*x^4 + 5
*A*b^3 + 7*(B*b^3 + 3*A*b^2*c)*x^2)/x^7
```

Fricas [A]
time = 1.37, size = 75, normalized size = 1.14

$$\frac{35\,Bc^3x^8-35\,(3\,Bbc^2+Ac^3)x^6-35\,(Bb^2c+Abc^2)x^4-5\,Ab^3-7\,(Bb^3+3\,Ab^2c)x^2}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x, algorithm="fricas")
```

```
[Out] 1/35*(35*B*c^3*x^8 - 35*(3*B*b*c^2 + A*c^3)*x^6 - 35*(B*b^2*c + A*b*c^2)*x^
4 - 5*A*b^3 - 7*(B*b^3 + 3*A*b^2*c)*x^2)/x^7
```

Sympy [A]
time = 0.80, size = 80, normalized size = 1.21

$$Bc^3x + \frac{-5Ab^3 + x^6(-35Ac^3 - 105Bbc^2) + x^4(-35Abc^2 - 35Bb^2c) + x^2(-21Ab^2c - 7Bb^3)}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**14,x)
```

```
[Out] B*c**3*x + (-5*A*b**3 + x**6*(-35*A*c**3 - 105*B*b*c**2) + x**4*(-35*A*b*c*
*2 - 35*B*b**2*c) + x**2*(-21*A*b**2*c - 7*B*b**3))/(35*x**7)
```

Giac [A]
time = 0.62, size = 77, normalized size = 1.17

$$Bc^3x - \frac{105\,Bbc^2x^6 + 35\,Ac^3x^6 + 35\,Bb^2cx^4 + 35\,Abc^2x^4 + 7\,Bb^3x^2 + 21\,Ab^2cx^2 + 5\,Ab^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x, algorithm="giac")
```

```
[Out] B*c^3*x - 1/35*(105*B*b*c^2*x^6 + 35*A*c^3*x^6 + 35*B*b^2*c*x^4 + 35*A*b*c^
2*x^4 + 7*B*b^3*x^2 + 21*A*b^2*c*x^2 + 5*A*b^3)/x^7
```

Mupad [B]
time = 0.08, size = 71, normalized size = 1.08

$$B\,c^3\,x - \frac{x^4\,(B\,b^2\,c + A\,b\,c^2) + \frac{A\,b^3}{7} + x^2\,\left(\frac{B\,b^3}{5} + \frac{3\,A\,c\,b^2}{5}\right) + x^6\,(A\,c^3 + 3\,B\,b\,c^2)}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^14,x)
```

```
[Out] B*c^3*x - (x^4*(A*b*c^2 + B*b^2*c) + (A*b^3)/7 + x^2*((B*b^3)/5 + (3*A*b^2*
c)/5) + x^6*(A*c^3 + 3*B*b*c^2))/x^7
```

3.38 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}}\,dx$

Optimal. Leaf size=63

$$-\frac{b^3B}{6x^6}-\frac{3b^2Bc}{4x^4}-\frac{3bBc^2}{2x^2}-\frac{A(b+cx^2)^4}{8bx^8}+Bc^3\log(x)$$

```
[Out] -1/6*b^3*B/x^6-3/4*b^2*B*c/x^4-3/2*b*B*c^2/x^2-1/8*A*(c*x^2+b)^4/b/x^8+B*c^
3*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 457, 79, 45}

$$-\frac{A(b+cx^2)^4}{8bx^8}-\frac{b^3B}{6x^6}-\frac{3b^2Bc}{4x^4}-\frac{3bBc^2}{2x^2}+Bc^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^15,x]
```

```
[Out] -1/6*(b^3*B)/x^6 - (3*b^2*B*c)/(4*x^4) - (3*b*B*c^2)/(2*x^2) - (A*(b + c*x^
2)^4)/(8*b*x^8) + B*c^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^{15}}\,dx &= \int \frac{(A+Bx^2)\left(b+cx^2\right)^3}{x^9}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^5}\,dx, x, x^2\right)\\
&= -\frac{A(b+cx^2)^4}{8bx^8}+\frac{1}{2}B\text{Subst}\left(\int \frac{(b+cx)^3}{x^4}\,dx, x, x^2\right)\\
&= -\frac{A(b+cx^2)^4}{8bx^8}+\frac{1}{2}B\text{Subst}\left(\int \left(\frac{b^3}{x^4}+\frac{3b^2c}{x^3}+\frac{3bc^2}{x^2}+\frac{c^3}{x}\right)dx, x, x^2\right)\\
&= -\frac{b^3B}{6x^6}-\frac{3b^2Bc}{4x^4}-\frac{3bBc^2}{2x^2}-\frac{A(b+cx^2)^4}{8bx^8}+Bc^3\log(x)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 77, normalized size = 1.22

$$
-\frac{2bBx^2(2b^2+9bcx^2+18c^2x^4)+3A(b^3+4b^2cx^2+6bc^2x^4+4c^3x^6)}{24x^8}+Bc^3\log(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^15,x]
```

```
[Out] -1/24*(2*b*B*x^2*(2*b^2 + 9*b*c*x^2 + 18*c^2*x^4) + 3*A*(b^3 + 4*b^2*c*x^2
+ 6*b*c^2*x^4 + 4*c^3*x^6))/x^8 + B*c^3*Log[x]
```

Maple [A]
time = 0.37, size = 64, normalized size = 1.02

method	result	size
default	$-\frac{3bc(Ac+Bb)}{4x^4}-\frac{b^2(3Ac+Bb)}{6x^6}-\frac{c^2(Ac+3Bb)}{2x^2}-\frac{A\,b^3}{8x^8}+B\,c^3\ln(x)$	64
risch	$\frac{\left(-\frac{1}{2}A\,c^3-\frac{3}{2}Bb\,c^2\right)x^6+\left(-\frac{3}{4}Ab\,c^2-\frac{3}{4}B\,b^2c\right)x^4+\left(-\frac{1}{2}A\,b^2c-\frac{1}{6}B\,b^3\right)x^2-\frac{A\,b^3}{8}}{x^8}+B\,c^3\ln(x)$	75

norman	$\frac{\left(-\frac{1}{2}A\,c^3-\frac{3}{2}Bb\,c^2\right)x^{12}+\left(-\frac{3}{4}Ab\,c^2-\frac{3}{4}B\,b^2c\right)x^{10}+\left(-\frac{1}{2}A\,b^2c-\frac{1}{6}B\,b^3\right)x^8-\frac{A\,b^3x^6}{8}}{x^{14}}+B\,c^3\ln\left(x\right)$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*b*c*(A*c+B*b)/x^4-1/6*b^2*(3*A*c+B*b)/x^6-1/2*c^2*(A*c+3*B*b)/x^2-1/8*
A*b^3/x^8+B*c^3*ln(x)
```

Maxima [A]
time = 0.30, size = 77, normalized size = 1.22

$$\frac{1}{2}\,Bc^3\log\left(x^2\right)-\frac{12\,(3\,Bbc^2+Ac^3)x^6+18\,(Bb^2c+Abc^2)x^4+3\,Ab^3+4\,(Bb^3+3\,Ab^2c)x^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x, algorithm="maxima")
```

```
[Out] 1/2*B*c^3*log(x^2) - 1/24*(12*(3*B*b*c^2 + A*c^3)*x^6 + 18*(B*b^2*c + A*b*c
^2)*x^4 + 3*A*b^3 + 4*(B*b^3 + 3*A*b^2*c)*x^2)/x^8
```

Fricas [A]
time = 1.77, size = 77, normalized size = 1.22

$$\frac{24\,Bc^3x^8\log\left(x\right)-12\,(3\,Bbc^2+Ac^3)x^6-18\,(Bb^2c+Abc^2)x^4-3\,Ab^3-4\,(Bb^3+3\,Ab^2c)x^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x, algorithm="fricas")
```

```
[Out] 1/24*(24*B*c^3*x^8*log(x) - 12*(3*B*b*c^2 + A*c^3)*x^6 - 18*(B*b^2*c + A*b*
c^2)*x^4 - 3*A*b^3 - 4*(B*b^3 + 3*A*b^2*c)*x^2)/x^8
```

Sympy [A]
time = 1.33, size = 82, normalized size = 1.30

$$Bc^3\log\left(x\right)+\frac{-3Ab^3+x^6(-12Ac^3-36Bbc^2)+x^4(-18Abc^2-18Bb^2c)+x^2(-12Ab^2c-4Bb^3)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**15,x)
```

```
[Out] B*c**3*log(x) + (-3*A*b**3 + x**6*(-12*A*c**3 - 36*B*b*c**2) + x**4*(-18*A*
b*c**2 - 18*B*b**2*c) + x**2*(-12*A*b**2*c - 4*B*b**3))/(24*x**8)
```

Giac [A]
time = 0.57, size = 90, normalized size = 1.43

$$\frac{1}{2}\,Bc^3\log\left(x^2\right)-\frac{25\,Bc^3x^8+36\,Bbc^2x^6+12\,Ac^3x^6+18\,Bb^2cx^4+18\,Abc^2x^4+4\,Bb^3x^2+12\,Ab^2cx^2+3\,Ab^3}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x, algorithm="giac")
```

```
[Out] 1/2*B*c^3*log(x^2) - 1/24*(25*B*c^3*x^8 + 36*B*b*c^2*x^6 + 12*A*c^3*x^6 + 1
8*B*b^2*c*x^4 + 18*A*b*c^2*x^4 + 4*B*b^3*x^2 + 12*A*b^2*c*x^2 + 3*A*b^3)/x^
8
```

Mupad [B]
time = 0.10, size = 75, normalized size = 1.19

$$B\,c^3\,\ln\left(x\right)-\frac{x^4\,\left(\frac{3\,B\,b^2\,c}{4}+\frac{3\,A\,b\,c^2}{4}\right)+\frac{A\,b^3}{8}+x^2\,\left(\frac{B\,b^3}{6}+\frac{A\,c\,b^2}{2}\right)+x^6\,\left(\frac{A\,c^3}{2}+\frac{3\,B\,b\,c^2}{2}\right)}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^15,x)
```

```
[Out] B*c^3*log(x) - (x^4*((3*A*b*c^2)/4 + (3*B*b^2*c)/4) + (A*b^3)/8 + x^2*((B*b
^3)/6 + (A*b^2*c)/2) + x^6*((A*c^3)/2 + (3*B*b*c^2)/2))/x^8
```

3.39 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}} \, dx$

Optimal. Leaf size=73

$$-\frac{Ab^3}{9x^9}-\frac{b^2(bB+3Ac)}{7x^7}-\frac{3bc(bB+Ac)}{5x^5}-\frac{c^2(3bB+Ac)}{3x^3}-\frac{Bc^3}{x}$$

```
[Out] -1/9*A*b^3/x^9-1/7*b^2*(3*A*c+B*b)/x^7-3/5*b*c*(A*c+B*b)/x^5-1/3*c^2*(A*c+3
*B*b)/x^3-B*c^3/x
```

Rubi [A]

time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1598, 459}

$$-\frac{Ab^3}{9x^9}-\frac{b^2(3Ac+bB)}{7x^7}-\frac{c^2(Ac+3bB)}{3x^3}-\frac{3bc(Ac+bB)}{5x^5}-\frac{Bc^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^16,x]
```

```
[Out] -1/9*(A*b^3)/x^9 - (b^2*(b*B + 3*A*c))/(7*x^7) - (3*b*c*(b*B + A*c))/(5*x^5
) - (c^2*(3*b*B + A*c))/(3*x^3) - (B*c^3)/x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}} \, dx &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^{10}} \, dx \\
&= \int \left(\frac{Ab^3}{x^{10}}+\frac{b^2(bB+3Ac)}{x^8}+\frac{3bc(bB+Ac)}{x^6}+\frac{c^2(3bB+Ac)}{x^4}+\frac{Bc^3}{x^2}\right) dx \\
&= -\frac{Ab^3}{9x^9}-\frac{b^2(bB+3Ac)}{7x^7}-\frac{3bc(bB+Ac)}{5x^5}-\frac{c^2(3bB+Ac)}{3x^3}-\frac{Bc^3}{x}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 73, normalized size = 1.00

$$-\frac{Ab^3}{9x^9}-\frac{b^2(bB+3Ac)}{7x^7}-\frac{3bc(bB+Ac)}{5x^5}-\frac{c^2(3bB+Ac)}{3x^3}-\frac{Bc^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^16,x]
```

```
[Out] -1/9*(A*b^3)/x^9 - (b^2*(b*B + 3*A*c))/(7*x^7) - (3*b*c*(b*B + A*c))/(5*x^5
) - (c^2*(3*b*B + A*c))/(3*x^3) - (B*c^3)/x
```

Maple [A]
time = 0.46, size = 66, normalized size = 0.90

method	result	size
default	$-\frac{A\,b^3}{9x^9}-\frac{b^2(3Ac+Bb)}{7x^7}-\frac{3bc(Ac+Bb)}{5x^5}-\frac{c^2(Ac+3Bb)}{3x^3}-\frac{B\,c^3}{x}$	66
risch	$\frac{-B\,c^3x^8+(-\frac{1}{3}A\,c^3-Bb\,c^2)x^6+(-\frac{3}{5}Ab\,c^2-\frac{3}{5}B\,b^2c)x^4+(-\frac{3}{7}A\,b^2c-\frac{1}{7}B\,b^3)x^2-\frac{A\,b^3}{9}}{x^9}$	76
norman	$\frac{(-\frac{1}{3}A\,c^3-Bb\,c^2)x^{12}+(-\frac{3}{5}Ab\,c^2-\frac{3}{5}B\,b^2c)x^{10}+(-\frac{3}{7}A\,b^2c-\frac{1}{7}B\,b^3)x^8-\frac{A\,b^3x^6}{9}-B\,c^3x^{14}}{x^{15}}$	79
gosper	$-\frac{315B\,c^3x^8+105A\,c^3x^6+315x^6Bb\,c^2+189Ab\,c^2x^4+189x^4B\,b^2c+135A\,b^2c\,x^2+45x^2B\,b^3+35A\,b^3}{315x^9}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*A*b^3/x^9-1/7*b^2*(3*A*c+B*b)/x^7-3/5*b*c*(A*c+B*b)/x^5-1/3*c^2*(A*c+3
*B*b)/x^3-B*c^3/x
```

Maxima [A]
time = 0.28, size = 75, normalized size = 1.03

$$-\frac{315\,Bc^3x^8+105\,(3\,Bbc^2+Ac^3)x^6+189\,(Bb^2c+Abc^2)x^4+35\,Ab^3+45\,(Bb^3+3\,Ab^2c)x^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x, algorithm="maxima")
```

```
[Out] -1/315*(315*B*c^3*x^8 + 105*(3*B*b*c^2 + A*c^3)*x^6 + 189*(B*b^2*c + A*b*c^
2)*x^4 + 35*A*b^3 + 45*(B*b^3 + 3*A*b^2*c)*x^2)/x^9
```

Fricas [A]
time = 1.44, size = 75, normalized size = 1.03

$$-\frac{315\,Bc^3x^8+105\,(3\,Bbc^2+Ac^3)x^6+189\,(Bb^2c+Abc^2)x^4+35\,Ab^3+45\,(Bb^3+3\,Ab^2c)x^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x, algorithm="fricas")
```

```
[Out] -1/315*(315*B*c^3*x^8 + 105*(3*B*b*c^2 + A*c^3)*x^6 + 189*(B*b^2*c + A*b*c^
2)*x^4 + 35*A*b^3 + 45*(B*b^3 + 3*A*b^2*c)*x^2)/x^9
```

Sympy [A]
time = 2.46, size = 83, normalized size = 1.14

$$\frac{-35Ab^3 - 315Bc^3x^8 + x^6(-105Ac^3 - 315Bbc^2) + x^4(-189Abc^2 - 189Bb^2c) + x^2(-135Ab^2c - 45Bb^3)}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**16,x)
```

```
[Out] (-35*A*b**3 - 315*B*c**3*x**8 + x**6*(-105*A*c**3 - 315*B*b*c**2) + x**4*(-
189*A*b*c**2 - 189*B*b**2*c) + x**2*(-135*A*b**2*c - 45*B*b**3))/(315*x**9)
```

Giac [A]
time = 0.67, size = 79, normalized size = 1.08

$$-\frac{315\,Bc^3x^8 + 315\,Bbc^2x^6 + 105\,Ac^3x^6 + 189\,Bb^2cx^4 + 189\,Abc^2x^4 + 45\,Bb^3x^2 + 135\,Ab^2cx^2 + 35\,Ab^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x, algorithm="giac")
```

```
[Out] -1/315*(315*B*c^3*x^8 + 315*B*b*c^2*x^6 + 105*A*c^3*x^6 + 189*B*b^2*c*x^4 +
 189*A*b*c^2*x^4 + 45*B*b^3*x^2 + 135*A*b^2*c*x^2 + 35*A*b^3)/x^9
```

Mupad [B]
time = 0.07, size = 74, normalized size = 1.01

$$-\frac{x^4\left(\frac{3\,B\,b^2\,c}{5}+\frac{3\,A\,b\,c^2}{5}\right)+\frac{A\,b^3}{9}+x^2\left(\frac{B\,b^3}{7}+\frac{3\,A\,c\,b^2}{7}\right)+x^6\left(\frac{A\,c^3}{3}+B\,b\,c^2\right)+B\,c^3\,x^8}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^16,x)
```

```
[Out] -(x^4*((3*A*b*c^2)/5 + (3*B*b^2*c)/5) + (A*b^3)/9 + x^2*((B*b^3)/7 + (3*A*b
^2*c)/7) + x^6*((A*c^3)/3 + B*b*c^2) + B*c^3*x^8)/x^9
```

3.40 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}\,dx$

Optimal. Leaf size=49

$$-\frac{A(b+cx^2)^4}{10bx^{10}}-\frac{(5bB-Ac)\,(b+cx^2)^4}{40b^2x^8}$$

```
[Out] -1/10*A*(c*x^2+b)^4/b/x^10-1/40*(-A*c+5*B*b)*(c*x^2+b)^4/b^2/x^8
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1598, 457, 79, 37}

$$-\frac{(b+cx^2)^4\,(5bB-Ac)}{40b^2x^8}-\frac{A(b+cx^2)^4}{10bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^17,x]
```

```
[Out] -1/10*(A*(b + c*x^2)^4)/(b*x^10) - ((5*b*B - A*c)*(b + c*x^2)^4)/(40*b^2*x^
8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^{17}}\,dx &= \int \frac{(A+Bx^2)\left(b+cx^2\right)^3}{x^{11}}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^6}\,dx,x,x^2\right)\\ &= -\frac{A\left(b+cx^2\right)^4}{10bx^{10}}+\frac{(5bB-Ac)\text{Subst}\left(\int \frac{(b+cx)^3}{x^5}\,dx,x,x^2\right)}{10b}\\ &= -\frac{A\left(b+cx^2\right)^4}{10bx^{10}}-\frac{(5bB-Ac)\left(b+cx^2\right)^4}{40b^2x^8}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 78, normalized size = 1.59

$$-\frac{5Bx^2(b^3+4b^2cx^2+6bc^2x^4+4c^3x^6)+A(4b^3+15b^2cx^2+20bc^2x^4+10c^3x^6)}{40x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^17,x]
```

```
[Out] -1/40*(5*B*x^2*(b^3 + 4*b^2*c*x^2 + 6*b*c^2*x^4 + 4*c^3*x^6) + A*(4*b^3 + 1
5*b^2*c*x^2 + 20*b*c^2*x^4 + 10*c^3*x^6))/x^10
```

Maple [A]
time = 0.41, size = 66, normalized size = 1.35

method	result	size
default	$-\frac{c^2(Ac+3Bb)}{4x^4}-\frac{bc(Ac+Bb)}{2x^6}-\frac{B\,c^3}{2x^2}-\frac{b^2(3Ac+Bb)}{8x^8}-\frac{A\,b^3}{10x^{10}}$	66
risch	$\frac{-\frac{B\,c^3x^8}{2}+(-\frac{1}{4}A\,c^3-\frac{3}{4}Bb\,c^2)x^6+(-\frac{1}{2}Ab\,c^2-\frac{1}{2}B\,b^2c)x^4+(-\frac{3}{8}A\,b^2c-\frac{1}{8}B\,b^3)x^2-\frac{A\,b^3}{10}}{x^{10}}$	76
norman	$\frac{(-\frac{1}{4}A\,c^3-\frac{3}{4}Bb\,c^2)x^{12}+(-\frac{1}{2}Ab\,c^2-\frac{1}{2}B\,b^2c)x^{10}+(-\frac{3}{8}A\,b^2c-\frac{1}{8}B\,b^3)x^8-\frac{A\,b^3x^6}{10}-\frac{B\,c^3x^{14}}{2}}{x^{16}}$	79
gosper	$-\frac{20B\,c^3x^8+10A\,c^3x^6+30x^6Bb\,c^2+20Ab\,c^2x^4+20x^4B\,b^2c+15A\,b^2c\,x^2+5x^2B\,b^3+4A\,b^3}{40x^{10}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*c^2*(A*c+3*B*b)/x^4-1/2*b*c*(A*c+B*b)/x^6-1/2*B*c^3/x^2-1/8*b^2*(3*A*c
+B*b)/x^8-1/10*A*b^3/x^10
```

Maxima [A]
time = 0.28, size = 75, normalized size = 1.53

$$-\frac{20\,Bc^3x^8+10\,(3\,Bbc^2+Ac^3)x^6+20\,(Bb^2c+Abc^2)x^4+4\,Ab^3+5\,(Bb^3+3\,Ab^2c)x^2}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x, algorithm="maxima")
```

```
[Out] -1/40*(20*B*c^3*x^8 + 10*(3*B*b*c^2 + A*c^3)*x^6 + 20*(B*b^2*c + A*b*c^2)*x
^4 + 4*A*b^3 + 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^10
```

Fricas [A]
time = 1.46, size = 75, normalized size = 1.53

$$-\frac{20\,Bc^3x^8+10\,(3\,Bbc^2+Ac^3)x^6+20\,(Bb^2c+Abc^2)x^4+4\,Ab^3+5\,(Bb^3+3\,Ab^2c)x^2}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x, algorithm="fricas")
```

```
[Out] -1/40*(20*B*c^3*x^8 + 10*(3*B*b*c^2 + A*c^3)*x^6 + 20*(B*b^2*c + A*b*c^2)*x
^4 + 4*A*b^3 + 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^10
```

Sympy [A]
time = 3.55, size = 83, normalized size = 1.69

$$\frac{-4Ab^3-20Bc^3x^8+x^6(-10Ac^3-30Bbc^2)+x^4(-20Abc^2-20Bb^2c)+x^2(-15Ab^2c-5Bb^3)}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**17,x)
```

```
[Out] (-4*A*b**3 - 20*B*c**3*x**8 + x**6*(-10*A*c**3 - 30*B*b*c**2) + x**4*(-20*A
*b*c**2 - 20*B*b**2*c) + x**2*(-15*A*b**2*c - 5*B*b**3))/(40*x**10)
```

Giac [A]
time = 0.64, size = 79, normalized size = 1.61

$$-\frac{20\,Bc^3x^8+30\,Bbc^2x^6+10\,Ac^3x^6+20\,Bb^2cx^4+20\,Abc^2x^4+5\,Bb^3x^2+15\,Ab^2cx^2+4\,Ab^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x, algorithm="giac")
```

```
[Out] -1/40*(20*B*c^3*x^8 + 30*B*b*c^2*x^6 + 10*A*c^3*x^6 + 20*B*b^2*c*x^4 + 20*A
*b*c^2*x^4 + 5*B*b^3*x^2 + 15*A*b^2*c*x^2 + 4*A*b^3)/x^10
```

Mupad [B]
time = 0.07, size = 76, normalized size = 1.55

$$-\frac{x^4\left(\frac{B\,b^2\,c}{2}+\frac{A\,b\,c^2}{2}\right)+\frac{A\,b^3}{10}+x^2\left(\frac{B\,b^3}{8}+\frac{3\,A\,c\,b^2}{8}\right)+x^6\left(\frac{A\,c^3}{4}+\frac{3\,B\,b\,c^2}{4}\right)+\frac{B\,c^3\,x^8}{2}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^17,x)
```

```
[Out] -(x^4*((A*b*c^2)/2 + (B*b^2*c)/2) + (A*b^3)/10 + x^2*((B*b^3)/8 + (3*A*b^2*
c)/8) + x^6*((A*c^3)/4 + (3*B*b*c^2)/4) + (B*c^3*x^8)/2)/x^10
```

3.41 $\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=119

$$\frac{b^3(bB-Ac)x}{c^5}-\frac{b^2(bB-Ac)x^3}{3c^4}+\frac{b(bB-Ac)x^5}{5c^3}-\frac{(bB-Ac)x^7}{7c^2}+\frac{Bx^9}{9c}-\frac{b^{7/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{11/2}}$$

```
[Out] b^3*(-A*c+B*b)*x/c^5-1/3*b^2*(-A*c+B*b)*x^3/c^4+1/5*b*(-A*c+B*b)*x^5/c^3-1/
7*(-A*c+B*b)*x^7/c^2+1/9*B*x^9/c-b^(7/2)*(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2
))/c^(11/2)
```

Rubi [A]
time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1598, 470, 308, 211}

$$-\frac{b^{7/2}(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{11/2}}+\frac{b^3x(bB-Ac)}{c^5}-\frac{b^2x^3(bB-Ac)}{3c^4}+\frac{bx^5(bB-Ac)}{5c^3}-\frac{x^7(bB-Ac)}{7c^2}+\frac{Bx^9}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b^3*(b*B - A*c)*x)/c^5 - (b^2*(b*B - A*c)*x^3)/(3*c^4) + (b*(b*B - A*c)*x^
5)/(5*c^3) - ((b*B - A*c)*x^7)/(7*c^2) + (B*x^9)/(9*c) - (b^(7/2)*(b*B - A*
c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(11/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx &= \int \frac{x^8(A+Bx^2)}{b+cx^2}\,dx\\ &= \frac{Bx^9}{9c}-\frac{(9bB-9Ac)\int \frac{x^8}{b+cx^2}\,dx}{9c}\\ &= \frac{Bx^9}{9c}-\frac{(9bB-9Ac)\int \left(-\frac{b^3}{c^4}+\frac{b^2x^2}{c^3}-\frac{bx^4}{c^2}+\frac{x^6}{c}+\frac{b^4}{c^4(b+cx^2)}\right)dx}{9c}\\ &= \frac{b^3(bB-Ac)x}{c^5}-\frac{b^2(bB-Ac)x^3}{3c^4}+\frac{b(bB-Ac)x^5}{5c^3}-\frac{(bB-Ac)x^7}{7c^2}+\frac{Bx^9}{9c}-\frac{(b^4(bB-\\ &= \frac{b^3(bB-Ac)x}{c^5}-\frac{b^2(bB-Ac)x^3}{3c^4}+\frac{b(bB-Ac)x^5}{5c^3}-\frac{(bB-Ac)x^7}{7c^2}+\frac{Bx^9}{9c}-\frac{b^{7/2}(bB-}{}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 119, normalized size = 1.00

$$\frac{b^3(bB-Ac)x}{c^5}-\frac{b^2(bB-Ac)x^3}{3c^4}+\frac{b(bB-Ac)x^5}{5c^3}+\frac{(-bB+Ac)x^7}{7c^2}+\frac{Bx^9}{9c}-\frac{b^{7/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] (b^3*(b*B - A*c)*x)/c^5 - (b^2*(b*B - A*c)*x^3)/(3*c^4) + (b*(b*B - A*c)*x^
5)/(5*c^3) + ((-(b*B) + A*c)*x^7)/(7*c^2) + (B*x^9)/(9*c) - (b^(7/2)*(b*B -
 A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(11/2)
```

Maple [A]
time = 0.46, size = 123, normalized size = 1.03

method	result
default	$-\frac{-\frac{1}{9}Bx^9c^4-\frac{1}{7}Ac^4x^7+\frac{1}{7}Bbc^3x^7+\frac{1}{5}Abc^3x^5-\frac{1}{5}Bb^2c^2x^5-\frac{1}{3}Ab^2c^2x^3+\frac{1}{3}Bb^3cx^3+Ab^3cx-Bb^4x}{c^5}+\frac{b^4(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^5\sqrt{bc}}$

risch	$\frac{B\,x^{9}}{9c}+\frac{A\,x^{7}}{7c}-\frac{Bb\,x^{7}}{7c^{2}}-\frac{Ab\,x^{5}}{5c^{2}}+\frac{B\,b^{2}x^{5}}{5c^{3}}+\frac{A\,b^{2}x^{3}}{3c^{3}}-\frac{B\,b^{3}x^{3}}{3c^{4}}-\frac{A\,b^{3}x}{c^{4}}+\frac{B\,b^{4}x}{c^{5}}+\frac{\sqrt{-bc}\,b^{3}\ln\left(-\sqrt{-bc}\,x+b\right)A}{2c^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^5*(-1/9*B*x^9*c^4-1/7*A*c^4*x^7+1/7*B*b*c^3*x^7+1/5*A*b*c^3*x^5-1/5*B*
b^2*c^2*x^5-1/3*A*b^2*c^2*x^3+1/3*B*b^3*c*x^3+A*b^3*c*x-B*b^4*x)+b^4*(A*c-B
*b)/c^5/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [A]
time = 0.49, size = 124, normalized size = 1.04

$$-\frac{(Bb^5-Ab^4c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^5}+\frac{35\,Bc^4x^9-45\,(Bbc^3-Ac^4)x^7+63\,(Bb^2c^2-Abc^3)x^5-105\,(Bb^3c-Ab^2c^2)x^3+315\,(Bb^4-Ab^3c)x}{315\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -(B*b^5 - A*b^4*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/315*(35*B*c^4*
x^9 - 45*(B*b*c^3 - A*c^4)*x^7 + 63*(B*b^2*c^2 - A*b*c^3)*x^5 - 105*(B*b^3*
c - A*b^2*c^2)*x^3 + 315*(B*b^4 - A*b^3*c)*x)/c^5
```

Fricas [A]
time = 1.45, size = 274, normalized size = 2.30

$$\left[\frac{70\,Bc^4x^9-90\,(Bbc^3-Ac^4)x^7+126\,(Bb^2c^2-Abc^3)x^5-210\,(Bb^3c-Ab^2c^2)x^3-315\,(Bb^4-Ab^3c)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+630\,(Bb^4-Ab^3c)x}{630\,c^5},\ \frac{35\,Bc^4x^9-45\,(Bbc^3-Ac^4)x^7+63\,(Bb^2c^2-Abc^3)x^5-105\,(Bb^3c-Ab^2c^2)x^3-315\,(Bb^4-Ab^3c)\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)+315\,(Bb^4-Ab^3c)x}{315\,c^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/630*(70*B*c^4*x^9 - 90*(B*b*c^3 - A*c^4)*x^7 + 126*(B*b^2*c^2 - A*b*c^3)
*x^5 - 210*(B*b^3*c - A*b^2*c^2)*x^3 - 315*(B*b^4 - A*b^3*c)*sqrt(-b/c)*log
((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 630*(B*b^4 - A*b^3*c)*x)/c^5
, 1/315*(35*B*c^4*x^9 - 45*(B*b*c^3 - A*c^4)*x^7 + 63*(B*b^2*c^2 - A*b*c^3)
*x^5 - 105*(B*b^3*c - A*b^2*c^2)*x^3 - 315*(B*b^4 - A*b^3*c)*sqrt(b/c)*arct
an(c*x*sqrt(b/c)/b) + 315*(B*b^4 - A*b^3*c)*x)/c^5]
```

Sympy [A]
time = 0.22, size = 204, normalized size = 1.71

$$\frac{Bx^9}{9c}+x^7\left(\frac{A}{7c}-\frac{Bb}{7c^2}\right)+x^5\left(-\frac{Ab}{5c^2}+\frac{Bb^2}{5c^3}\right)+x^3\left(\frac{Ab^2}{3c^3}-\frac{Bb^3}{3c^4}\right)+x\left(-\frac{Ab^3}{c^4}+\frac{Bb^4}{c^5}\right)+\frac{\sqrt{-\frac{b^7}{c^{11}}}\,(-Ac+Bb)\log\left(-\frac{c^5\sqrt{-\frac{b^7}{c^{11}}}\,(-Ac+Bb)}{-Ab^3c+Bb^4}+x\right)}{2}-\frac{\sqrt{-\frac{b^7}{c^{11}}}\,(-Ac+Bb)\log\left(\frac{c^5\sqrt{-\frac{b^7}{c^{11}}}\,(-Ac+Bb)}{-Ab^3c+Bb^4}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**9/(9*c) + x**7*(A/(7*c) - B*b/(7*c**2)) + x**5*(-A*b/(5*c**2) + B*b**2
/(5*c**3)) + x**3*(A*b**2/(3*c**3) - B*b**3/(3*c**4)) + x*(-A*b**3/c**4 + B
*b**4/c**5) + sqrt(-b**7/c**11)*(-A*c + B*b)*log(-c**5*sqrt(-b**7/c**11)*(-
A*c + B*b)/(-A*b**3*c + B*b**4) + x)/2 - sqrt(-b**7/c**11)*(-A*c + B*b)*log
(c**5*sqrt(-b**7/c**11)*(-A*c + B*b)/(-A*b**3*c + B*b**4) + x)/2
```

Giac [A]
time = 0.61, size = 133, normalized size = 1.12

$$-\frac{(Bb^5 - Ab^4c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^5} + \frac{35\,Bc^8x^9 - 45\,Bbc^7x^7 + 45\,Ac^8x^7 + 63\,Bb^2c^6x^5 - 63\,Abc^7x^5 - 105\,Bb^3c^5x^3 + 105\,Ab^2c^6x^3 + 315\,Bb^4c^4x - 315\,Ab^3c^5x}{315\,c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -(B*b^5 - A*b^4*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/315*(35*B*c^8*
x^9 - 45*B*b*c^7*x^7 + 45*A*c^8*x^7 + 63*B*b^2*c^6*x^5 - 63*A*b*c^7*x^5 - 1
05*B*b^3*c^5*x^3 + 105*A*b^2*c^6*x^3 + 315*B*b^4*c^4*x - 315*A*b^3*c^5*x)/c
^9
```

Mupad [B]
time = 0.17, size = 144, normalized size = 1.21

$$x^7\left(\frac{A}{7\,c}-\frac{B\,b}{7\,c^2}\right)+\frac{B\,x^9}{9\,c}+\frac{b^2\,x^3\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{3\,c^2}-\frac{b^{7/2}\,\mathrm{atan}\left(\frac{b^{7/2}\,\sqrt{c}\,x\,(A\,c-B\,b)}{B\,b^5-A\,b^4\,c}\right)\,(A\,c-B\,b)}{c^{11/2}}-\frac{b\,x^5\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{5\,c}-\frac{b^3\,x\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^7*(A/(7*c) - (B*b)/(7*c^2)) + (B*x^9)/(9*c) + (b^2*x^3*(A/c - (B*b)/c^2))
/(3*c^2) - (b^(7/2)*atan((b^(7/2)*c^(1/2)*x*(A*c - B*b))/(B*b^5 - A*b^4*c))
*(A*c - B*b))/c^(11/2) - (b*x^5*(A/c - (B*b)/c^2))/(5*c) - (b^3*x*(A/c - (B
*b)/c^2))/c^3
```

3.42 $\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=96

$$-\frac{b^2(bB-Ac)x^2}{2c^4}+\frac{b(bB-Ac)x^4}{4c^3}-\frac{(bB-Ac)x^6}{6c^2}+\frac{Bx^8}{8c}+\frac{b^3(bB-Ac)\log(b+cx^2)}{2c^5}$$

```
[Out] -1/2*b^2*(-A*c+B*b)*x^2/c^4+1/4*b*(-A*c+B*b)*x^4/c^3-1/6*(-A*c+B*b)*x^6/c^2
+1/8*B*x^8/c+1/2*b^3*(-A*c+B*b)*ln(c*x^2+b)/c^5
```

Rubi [A]
time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 78}

$$\frac{b^3(bB-Ac)\log(b+cx^2)}{2c^5}-\frac{b^2x^2(bB-Ac)}{2c^4}+\frac{bx^4(bB-Ac)}{4c^3}-\frac{x^6(bB-Ac)}{6c^2}+\frac{Bx^8}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -1/2*(b^2*(b*B - A*c)*x^2)/c^4 + (b*(b*B - A*c)*x^4)/(4*c^3) - ((b*B - A*c)
*x^6)/(6*c^2) + (B*x^8)/(8*c) + (b^3*(b*B - A*c)*Log[b + c*x^2])/(2*c^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx &= \int \frac{x^7(A+Bx^2)}{b+cx^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{b+cx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b^2(bB-Ac)}{c^4}+\frac{b(bB-Ac)x}{c^3}+\frac{(-bB+Ac)x^2}{c^2}+\frac{Bx^3}{c}+\frac{b^3(bB-Ac)}{c^4(b+cx)}\right.\right.\\ &= -\frac{b^2(bB-Ac)x^2}{2c^4}+\frac{b(bB-Ac)x^4}{4c^3}-\frac{(bB-Ac)x^6}{6c^2}+\frac{Bx^8}{8c}+\frac{b^3(bB-Ac)\log(b+cx^2)}{2c^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 92, normalized size = 0.96

$$\frac{cx^2(-12b^3B+6b^2c(2A+Bx^2)-2bc^2x^2(3A+2Bx^2)+c^3x^4(4A+3Bx^2))+12b^3(bB-Ac)\log(b+cx^2)}{24c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (c*x^2*(-12*b^3*B + 6*b^2*c*(2*A + B*x^2) - 2*b*c^2*x^2*(3*A + 2*B*x^2) + c
^3*x^4*(4*A + 3*B*x^2)) + 12*b^3*(b*B - A*c)*Log[b + c*x^2])/(24*c^5)
```

Maple [A]
time = 0.38, size = 98, normalized size = 1.02

method	result	size
norman	$\frac{\frac{B x^9}{8c}+\frac{(Ac-Bb)x^7}{6c^2}-\frac{b(Ac-Bb)x^5}{4c^3}+\frac{b^2(Ac-Bb)x^3}{2c^4}}{x}-\frac{b^3(Ac-Bb)\ln(c x^2+b)}{2c^5}$	92
default	$\frac{\frac{1}{4}B c^3x^8+\frac{1}{3}A c^3x^6-\frac{1}{3}x^6Bb c^2-\frac{1}{2}Ab c^2x^4+\frac{1}{2}x^4B b^2c+A b^2c x^2-x^2B b^3}{2c^4}-\frac{b^3(Ac-Bb)\ln(c x^2+b)}{2c^5}$	98
risch	$\frac{B x^8}{8c}+\frac{A x^6}{6c}-\frac{x^6Bb}{6c^2}-\frac{Ab x^4}{4c^2}+\frac{x^4B b^2}{4c^3}+\frac{A b^2x^2}{2c^3}-\frac{x^2B b^3}{2c^4}-\frac{b^3\ln(c x^2+b)A}{2c^4}+\frac{b^4\ln(c x^2+b)B}{2c^5}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^4*(1/4*B*c^3*x^8+1/3*A*c^3*x^6-1/3*x^6*B*b*c^2-1/2*A*b*c^2*x^4+1/2*x^
4*B*b^2*c+A*b^2*c*x^2-x^2*B*b^3)-1/2*b^3*(A*c-B*b)/c^5*ln(c*x^2+b)
```

Maxima [A]
time = 0.28, size = 97, normalized size = 1.01

$$\frac{3\,Bc^3x^8-4\,(Bbc^2-Ac^3)x^6+6\,(Bb^2c-Abc^2)x^4-12\,(Bb^3-Ab^2c)x^2}{24\,c^4}+\frac{(Bb^4-Ab^3c)\log(cx^2+b)}{2\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/24*(3*B*c^3*x^8 - 4*(B*b*c^2 - A*c^3)*x^6 + 6*(B*b^2*c - A*b*c^2)*x^4 - 1
2*(B*b^3 - A*b^2*c)*x^2)/c^4 + 1/2*(B*b^4 - A*b^3*c)*log(c*x^2 + b)/c^5
```

Fricas [A]
time = 1.51, size = 98, normalized size = 1.02

$$\frac{3\,Bc^4x^8-4\,(Bbc^3-Ac^4)x^6+6\,(Bb^2c^2-Abc^3)x^4-12\,(Bb^3c-Ab^2c^2)x^2+12\,(Bb^4-Ab^3c)\log{(cx^2+b)}}{24\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*B*c^4*x^8 - 4*(B*b*c^3 - A*c^4)*x^6 + 6*(B*b^2*c^2 - A*b*c^3)*x^4 -
 12*(B*b^3*c - A*b^2*c^2)*x^2 + 12*(B*b^4 - A*b^3*c)*log(c*x^2 + b))/c^5
```

Sympy [A]
time = 0.18, size = 94, normalized size = 0.98

$$\frac{Bx^8}{8c}+\frac{b^3(-Ac+Bb)\log{(b+cx^2)}}{2c^5}+x^6\left(\frac{A}{6c}-\frac{Bb}{6c^2}\right)+x^4\left(-\frac{Ab}{4c^2}+\frac{Bb^2}{4c^3}\right)+x^2\left(\frac{Ab^2}{2c^3}-\frac{Bb^3}{2c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**8/(8*c) + b**3*(-A*c + B*b)*log(b + c*x**2)/(2*c**5) + x**6*(A/(6*c) -
 B*b/(6*c**2)) + x**4*(-A*b/(4*c**2) + B*b**2/(4*c**3)) + x**2*(A*b**2/(2*c
**3) - B*b**3/(2*c**4))
```

Giac [A]
time = 0.63, size = 101, normalized size = 1.05

$$\frac{3\,Bc^3x^8-4\,Bbc^2x^6+4\,Ac^3x^6+6\,Bb^2cx^4-6\,Abc^2x^4-12\,Bb^3x^2+12\,Ab^2cx^2}{24\,c^4}+\frac{(Bb^4-Ab^3c)\log{(|cx^2+b|)}}{2\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/24*(3*B*c^3*x^8 - 4*B*b*c^2*x^6 + 4*A*c^3*x^6 + 6*B*b^2*c*x^4 - 6*A*b*c^2
*x^4 - 12*B*b^3*x^2 + 12*A*b^2*c*x^2)/c^4 + 1/2*(B*b^4 - A*b^3*c)*log(abs(c
*x^2 + b))/c^5
```

Mupad [B]
time = 0.06, size = 100, normalized size = 1.04

$$x^6\left(\frac{A}{6\,c}-\frac{B\,b}{6\,c^2}\right)+\frac{B\,x^8}{8\,c}+\frac{\ln\left(c\,x^2+b\right)\,\left(B\,b^4-A\,b^3\,c\right)}{2\,c^5}+\frac{b^2\,x^2\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{2\,c^2}-\frac{b\,x^4\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^6*(A/(6*c) - (B*b)/(6*c^2)) + (B*x^8)/(8*c) + (log(b + c*x^2)*(B*b^4 - A*
b^3*c))/(2*c^5) + (b^2*x^2*(A/c - (B*b)/c^2))/(2*c^2) - (b*x^4*(A/c - (B*b)
/c^2))/(4*c)
```

3.43 $\int \frac{x^8(A+Bx^2)}{bx^2+cx^4} \, dx$

Optimal. Leaf size=98

$$-\frac{b^2(bB-Ac)x}{c^4}+\frac{b(bB-Ac)x^3}{3c^3}-\frac{(bB-Ac)x^5}{5c^2}+\frac{Bx^7}{7c}+\frac{b^{5/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}$$

```
[Out] -b^2*(-A*c+B*b)*x/c^4+1/3*b*(-A*c+B*b)*x^3/c^3-1/5*(-A*c+B*b)*x^5/c^2+1/7*B
*x^7/c+b^(5/2)*(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(9/2)
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1598, 470, 308, 211}

$$\frac{b^{5/2}(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{b^2x(bB-Ac)}{c^4}+\frac{bx^3(bB-Ac)}{3c^3}-\frac{x^5(bB-Ac)}{5c^2}+\frac{Bx^7}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -((b^2*(b*B - A*c)*x)/c^4) + (b*(b*B - A*c)*x^3)/(3*c^3) - ((b*B - A*c)*x^5
)/(5*c^2) + (B*x^7)/(7*c) + (b^(5/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]
])/c^(9/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx &= \int \frac{x^6(A+Bx^2)}{b+cx^2}\,dx\\ &= \frac{Bx^7}{7c} - \frac{(7bB-7Ac)\int \frac{x^6}{b+cx^2}\,dx}{7c}\\ &= \frac{Bx^7}{7c} - \frac{(7bB-7Ac)\int \left(\frac{b^2}{c^3} - \frac{bx^2}{c^2} + \frac{x^4}{c} - \frac{b^3}{c^3(b+cx^2)}\right)dx}{7c}\\ &= -\frac{b^2(bB-Ac)x}{c^4} + \frac{b(bB-Ac)x^3}{3c^3} - \frac{(bB-Ac)x^5}{5c^2} + \frac{Bx^7}{7c} + \frac{(b^3(bB-Ac))\int \frac{1}{b+cx^2}\,dx}{c^4}\\ &= -\frac{b^2(bB-Ac)x}{c^4} + \frac{b(bB-Ac)x^3}{3c^3} - \frac{(bB-Ac)x^5}{5c^2} + \frac{Bx^7}{7c} + \frac{b^{5/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{b}}\right.}{c^{9/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 98, normalized size = 1.00

$$-\frac{b^2(bB-Ac)x}{c^4} + \frac{b(bB-Ac)x^3}{3c^3} + \frac{(-bB+Ac)x^5}{5c^2} + \frac{Bx^7}{7c} + \frac{b^{5/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] -((b^2*(b*B - A*c)*x)/c^4) + (b*(b*B - A*c)*x^3)/(3*c^3) + ((-(b*B) + A*c)*
x^5)/(5*c^2) + (B*x^7)/(7*c) + (b^(5/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt
[b]])/c^(9/2)
```

Maple [A]
time = 0.41, size = 99, normalized size = 1.01

method	result
default	$\frac{\frac{1}{7}B\,c^3x^7+\frac{1}{5}A\,c^3x^5-\frac{1}{5}Bb\,c^2x^5-\frac{1}{3}Ab\,c^2x^3+\frac{1}{3}B\,b^2c\,x^3+A\,b^2cx-B\,b^3x}{c^4}-\frac{b^3(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^4\sqrt{bc}}$

risch	$\frac{B\,x^7}{7c}+\frac{A\,x^5}{5c}-\frac{Bb\,x^5}{5c^2}-\frac{Ab\,x^3}{3c^2}+\frac{B\,b^2x^3}{3c^3}+\frac{A\,b^2x}{c^3}-\frac{B\,b^3x}{c^4}+\frac{\sqrt{-bc}\;b^2\ln\left(-\sqrt{-bc}\;x-b\right)A}{2c^4}-\frac{\sqrt{-bc}\;b^3\ln\left(-\sqrt{\right.}\right.}{2c^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/7*B*c^3*x^7+1/5*A*c^3*x^5-1/5*B*b*c^2*x^5-1/3*A*b*c^2*x^3+1/3*B*b^
2*c*x^3+A*b^2*c*x-B*b^3*x)-b^3*(A*c-B*b)/c^4/(b*c)^(1/2)*arctan(c*x/(b*c)^(
1/2))
```

Maxima [A]
time = 0.49, size = 100, normalized size = 1.02

$$\frac{(Bb^4-Ab^3c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\;c^4}+\frac{15\,Bc^3x^7-21\,(Bbc^2-Ac^3)x^5+35\,(Bb^2c-Abc^2)x^3-105\,(Bb^3-Ab^2c)x}{105\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] (B*b^4 - A*b^3*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*B*c^3*x
^7 - 21*(B*b*c^2 - A*c^3)*x^5 + 35*(B*b^2*c - A*b*c^2)*x^3 - 105*(B*b^3 - A
*b^2*c)*x)/c^4
```

Fricas [A]
time = 1.32, size = 228, normalized size = 2.33

$$\left[\frac{30\,Bc^3x^7-42\,(Bbc^2-Ac^3)x^5+70\,(Bb^2c-Abc^2)x^3-105\,(Bb^3-Ab^2c)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-210\,(Bb^3-Ab^2c)x}{210\,c^4},\ \frac{15\,Bc^3x^7-21\,(Bbc^2-Ac^3)x^5+35\,(Bb^2c-Abc^2)x^3+105\,(Bb^3-Ab^2c)\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)-105\,(Bb^3-Ab^2c)x}{105\,c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/210*(30*B*c^3*x^7 - 42*(B*b*c^2 - A*c^3)*x^5 + 70*(B*b^2*c - A*b*c^2)*x^
3 - 105*(B*b^3 - A*b^2*c)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*
x^2 + b)) - 210*(B*b^3 - A*b^2*c)*x)/c^4, 1/105*(15*B*c^3*x^7 - 21*(B*b*c^2
 - A*c^3)*x^5 + 35*(B*b^2*c - A*b*c^2)*x^3 + 105*(B*b^3 - A*b^2*c)*sqrt(b/c
)*arctan(c*x*sqrt(b/c)/b) - 105*(B*b^3 - A*b^2*c)*x)/c^4]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(87) = 174$.
time = 0.20, size = 180, normalized size = 1.84

$$\frac{Bx^7}{7c}+x^5\left(\frac{A}{5c}-\frac{Bb}{5c^2}\right)+x^3\left(-\frac{Ab}{3c^2}+\frac{Bb^2}{3c^3}\right)+x\left(\frac{Ab^2}{c^3}-\frac{Bb^3}{c^4}\right)-\frac{\sqrt{-\frac{b^5}{c^9}}\,(-Ac+Bb)\log\left(-\frac{c^4\sqrt{-\frac{b^5}{c^9}}\,(-Ac+Bb)}{-Ab^2c+Bb^3}+x\right)}{2}+\frac{\sqrt{-\frac{b^5}{c^9}}\,(-Ac+Bb)\log\left(\frac{c^4\sqrt{-\frac{b^5}{c^9}}\,(-Ac+Bb)}{-Ab^2c+Bb^3}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**7/(7*c) + x**5*(A/(5*c) - B*b/(5*c**2)) + x**3*(-A*b/(3*c**2) + B*b**2
/(3*c**3)) + x*(A*b**2/c**3 - B*b**3/c**4) - sqrt(-b**5/c**9)*(-A*c + B*b)*
log(-c**4*sqrt(-b**5/c**9)*(-A*c + B*b)/(-A*b**2*c + B*b**3) + x)/2 + sqrt(
-b**5/c**9)*(-A*c + B*b)*log(c**4*sqrt(-b**5/c**9)*(-A*c + B*b)/(-A*b**2*c
+ B*b**3) + x)/2
```

Giac [A]
time = 0.60, size = 108, normalized size = 1.10

$$\frac{(Bb^4 - Ab^3c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^4} + \frac{15\,Bc^6x^7 - 21\,Bbc^5x^5 + 21\,Ac^6x^5 + 35\,Bb^2c^4x^3 - 35\,Abc^5x^3 - 105\,Bb^3c^3x + 105\,Ab^2c^4x}{105\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*b^4 - A*b^3*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*B*c^6*x
^7 - 21*B*b*c^5*x^5 + 21*A*c^6*x^5 + 35*B*b^2*c^4*x^3 - 35*A*b*c^5*x^3 - 10
5*B*b^3*c^3*x + 105*A*b^2*c^4*x)/c^7
```

Mupad [B]
time = 0.04, size = 118, normalized size = 1.20

$$x^5\left(\frac{A}{5\,c} - \frac{B\,b}{5\,c^2}\right) + \frac{B\,x^7}{7\,c} + \frac{b^{5/2}\,\mathrm{atan}\left(\frac{b^{5/2}\,\sqrt{c}\,x\,(A\,c-B\,b)}{B\,b^4-A\,b^3\,c}\right)\,(A\,c - B\,b)}{c^{9/2}} - \frac{b\,x^3\,\left(\frac{A}{c} - \frac{B\,b}{c^2}\right)}{3\,c} + \frac{b^2\,x\,\left(\frac{A}{c} - \frac{B\,b}{c^2}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^5*(A/(5*c) - (B*b)/(5*c^2)) + (B*x^7)/(7*c) + (b^(5/2)*atan((b^(5/2)*c^(1
/2)*x*(A*c - B*b))/(B*b^4 - A*b^3*c))*(A*c - B*b))/c^(9/2) - (b*x^3*(A/c -
(B*b)/c^2))/(3*c) + (b^2*x*(A/c - (B*b)/c^2))/c^2
```

3.44 $\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=75

$$\frac{b(bB-Ac)x^2}{2c^3}-\frac{(bB-Ac)x^4}{4c^2}+\frac{Bx^6}{6c}-\frac{b^2(bB-Ac)\log\left(b+cx^2\right)}{2c^4}$$

```
[Out] 1/2*b*(-A*c+B*b)*x^2/c^3-1/4*(-A*c+B*b)*x^4/c^2+1/6*B*x^6/c-1/2*b^2*(-A*c+B
*b)*ln(c*x^2+b)/c^4
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$-\frac{b^2(bB-Ac)\log\left(b+cx^2\right)}{2c^4}+\frac{bx^2(bB-Ac)}{2c^3}-\frac{x^4(bB-Ac)}{4c^2}+\frac{Bx^6}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b*(b*B - A*c)*x^2)/(2*c^3) - ((b*B - A*c)*x^4)/(4*c^2) + (B*x^6)/(6*c) - (
b^2*(b*B - A*c)*Log[b + c*x^2])/(2*c^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx &= \int \frac{x^5(A+Bx^2)}{b+cx^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{b+cx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b(bB-Ac)}{c^3}+\frac{(-bB+Ac)x}{c^2}+\frac{Bx^2}{c}-\frac{b^2(bB-Ac)}{c^3(b+cx)}\right)\,dx, x, x^2\right)\\ &= \frac{b(bB-Ac)x^2}{2c^3}-\frac{(bB-Ac)x^4}{4c^2}+\frac{Bx^6}{6c}-\frac{b^2(bB-Ac)\log\left(b+cx^2\right)}{2c^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 0.95

$$\frac{cx^2(6b^2B-3bc(2A+Bx^2)+c^2x^2(3A+2Bx^2))+6b^2(-bB+Ac)\log\left(b+cx^2\right)}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (c*x^2*(6*b^2*B - 3*b*c*(2*A + B*x^2) + c^2*x^2*(3*A + 2*B*x^2)) + 6*b^2*(-
(b*B) + A*c)*Log[b + c*x^2])/(12*c^4)
```

Maple [A]
time = 0.38, size = 74, normalized size = 0.99

method	result	size
norman	$\frac{\frac{B\,x^7}{6c}+\frac{(Ac-Bb)x^5}{4c^2}-\frac{b(Ac-Bb)x^3}{2c^3}}{x}+\frac{b^2(Ac-Bb)\ln\left(c\,x^2+b\right)}{2c^4}$	73
default	$-\frac{-\frac{1}{3}B\,c^2x^6-\frac{1}{2}A\,c^2x^4+\frac{1}{2}x^4bBc+Abc\,x^2-b^2B\,x^2}{2c^3}+\frac{b^2(Ac-Bb)\ln\left(c\,x^2+b\right)}{2c^4}$	74
risch	$\frac{B\,x^6}{6c}+\frac{A\,x^4}{4c}-\frac{x^4bB}{4c^2}-\frac{Ab\,x^2}{2c^2}+\frac{b^2B\,x^2}{2c^3}+\frac{b^2\ln\left(c\,x^2+b\right)A}{2c^3}-\frac{b^3\ln\left(c\,x^2+b\right)B}{2c^4}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c^3*(-1/3*B*c^2*x^6-1/2*A*c^2*x^4+1/2*x^4*b*B*c+A*b*c*x^2-b^2*B*x^2)+1
/2*b^2*(A*c-B*b)/c^4*ln(c*x^2+b)
```

Maxima [A]
time = 0.29, size = 74, normalized size = 0.99

$$\frac{2\,Bc^2x^6-3\,(Bbc-Ac^2)x^4+6\,(Bb^2-Abc)x^2}{12\,c^3}-\frac{(Bb^3-Ab^2c)\log\left(cx^2+b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/12*(2*B*c^2*x^6 - 3*(B*b*c - A*c^2)*x^4 + 6*(B*b^2 - A*b*c)*x^2)/c^3 - 1/
2*(B*b^3 - A*b^2*c)*log(c*x^2 + b)/c^4
```

Fricas [A]
time = 1.61, size = 75, normalized size = 1.00

$$\frac{2\,Bc^3x^6-3\,(Bbc^2-Ac^3)x^4+6\,(Bb^2c-Abc^2)x^2-6\,(Bb^3-Ab^2c)\log\left(cx^2+b\right)}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*c^3*x^6 - 3*(B*b*c^2 - A*c^3)*x^4 + 6*(B*b^2*c - A*b*c^2)*x^2 - 6
*(B*b^3 - A*b^2*c)*log(c*x^2 + b))/c^4
```

Sympy [A]
time = 0.16, size = 70, normalized size = 0.93

$$\frac{Bx^6}{6c}-\frac{b^2(-Ac+Bb)\log\left(b+cx^2\right)}{2c^4}+x^4\left(\frac{A}{4c}-\frac{Bb}{4c^2}\right)+x^2\left(-\frac{Ab}{2c^2}+\frac{Bb^2}{2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**6/(6*c) - b**2*(-A*c + B*b)*log(b + c*x**2)/(2*c**4) + x**4*(A/(4*c) -
 B*b/(4*c**2)) + x**2*(-A*b/(2*c**2) + B*b**2/(2*c**3))
```

Giac [A]
time = 0.64, size = 77, normalized size = 1.03

$$\frac{2\,Bc^2x^6-3\,Bbcx^4+3\,Ac^2x^4+6\,Bb^2x^2-6\,Abcx^2}{12\,c^3}-\frac{(Bb^3-Ab^2c)\log\left(|cx^2+b|\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/12*(2*B*c^2*x^6 - 3*B*b*c*x^4 + 3*A*c^2*x^4 + 6*B*b^2*x^2 - 6*A*b*c*x^2)/
c^3 - 1/2*(B*b^3 - A*b^2*c)*log(abs(c*x^2 + b))/c^4
```

Mupad [B]
time = 0.09, size = 76, normalized size = 1.01

$$x^4\left(\frac{A}{4\,c}-\frac{B\,b}{4\,c^2}\right)+\frac{B\,x^6}{6\,c}-\frac{\ln\left(c\,x^2+b\right)\,\left(B\,b^3-A\,b^2\,c\right)}{2\,c^4}-\frac{b\,x^2\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^4*(A/(4*c) - (B*b)/(4*c^2)) + (B*x^6)/(6*c) - (log(b + c*x^2)*(B*b^3 - A*
b^2*c))/(2*c^4) - (b*x^2*(A/c - (B*b)/c^2))/(2*c)
```

3.45 $\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=77

$$\frac{b(bB-Ac)x}{c^3}-\frac{(bB-Ac)x^3}{3c^2}+\frac{Bx^5}{5c}-\frac{b^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}$$

```
[Out] b*(-A*c+B*b)*x/c^3-1/3*(-A*c+B*b)*x^3/c^2+1/5*B*x^5/c-b^(3/2)*(-A*c+B*b)*ar
ctan(x*c^(1/2)/b^(1/2))/c^(7/2)
```

Rubi [A]

time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 470, 308, 211}

$$-\frac{b^{3/2}(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}+\frac{bx(bB-Ac)}{c^3}-\frac{x^3(bB-Ac)}{3c^2}+\frac{Bx^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b*(b*B - A*c)*x)/c^3 - ((b*B - A*c)*x^3)/(3*c^2) + (B*x^5)/(5*c) - (b^(3/2
)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx &= \int \frac{x^4(A+Bx^2)}{b+cx^2}\,dx\\ &= \frac{Bx^5}{5c}-\frac{(5bB-5Ac)\int \frac{x^4}{b+cx^2}\,dx}{5c}\\ &= \frac{Bx^5}{5c}-\frac{(5bB-5Ac)\int \left(-\frac{b}{c^2}+\frac{x^2}{c}+\frac{b^2}{c^2(b+cx^2)}\right)dx}{5c}\\ &= \frac{b(bB-Ac)x}{c^3}-\frac{(bB-Ac)x^3}{3c^2}+\frac{Bx^5}{5c}-\frac{(b^2(bB-Ac))\int \frac{1}{b+cx^2}\,dx}{c^3}\\ &= \frac{b(bB-Ac)x}{c^3}-\frac{(bB-Ac)x^3}{3c^2}+\frac{Bx^5}{5c}-\frac{b^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 1.00

$$\frac{b(bB-Ac)x}{c^3}+\frac{(-bB+Ac)x^3}{3c^2}+\frac{Bx^5}{5c}-\frac{b^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] (b*(b*B - A*c)*x)/c^3 + ((-(b*B) + A*c)*x^3)/(3*c^2) + (B*x^5)/(5*c) - (b^(
3/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(7/2)
```

Maple [A]
time = 0.40, size = 75, normalized size = 0.97

method	result
default	$-\frac{-\frac{1}{5}Bc^2x^5-\frac{1}{3}Ac^2x^3+\frac{1}{3}Bbcx^3+Abcx-b^2Bx}{c^3}+\frac{b^2(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^3\sqrt{bc}}$
risch	$\frac{Bx^5}{5c}+\frac{Ax^3}{3c}-\frac{Bbx^3}{3c^2}-\frac{Abx}{c^2}+\frac{b^2Bx}{c^3}+\frac{\sqrt{-bc}\,b\ln\left(-\sqrt{-bc}\,x+b\right)A}{2c^3}-\frac{\sqrt{-bc}\,b^2\ln\left(-\sqrt{-bc}\,x+b\right)B}{2c^4}-\sqrt{-b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3*(-1/5*B*c^2*x^5-1/3*A*c^2*x^3+1/3*B*b*c*x^3+A*b*c*x-b^2*B*x)+b^2*(A*
c-B*b)/c^3/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [A]
time = 0.49, size = 78, normalized size = 1.01

$$-\frac{(Bb^3 - Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^3} + \frac{3\,Bc^2x^5 - 5\,(Bbc - Ac^2)x^3 + 15\,(Bb^2 - Abc)x}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -(B*b^3 - A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*B*c^2*x^
5 - 5*(B*b*c - A*c^2)*x^3 + 15*(B*b^2 - A*b*c)*x)/c^3
```

Fricas [A]
time = 1.48, size = 178, normalized size = 2.31

$$\left[\frac{6\,Bc^2x^5 - 10\,(Bbc - Ac^2)x^3 - 15\,(Bb^2 - Abc)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2 + 2\,cx\sqrt{-\frac{b}{c}} - b}{cx^2 + b}\right) + 30\,(Bb^2 - Abc)x}{30\,c^3}, \frac{3\,Bc^2x^5 - 5\,(Bbc - Ac^2)x^3 - 15\,(Bb^2 - Abc)\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right) + 15\,(Bb^2 - Abc)x}{15\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/30*(6*B*c^2*x^5 - 10*(B*b*c - A*c^2)*x^3 - 15*(B*b^2 - A*b*c)*sqrt(-b/c)
*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 30*(B*b^2 - A*b*c)*x)/c^
3, 1/15*(3*B*c^2*x^5 - 5*(B*b*c - A*c^2)*x^3 - 15*(B*b^2 - A*b*c)*sqrt(b/c)
*arctan(c*x*sqrt(b/c)/b) + 15*(B*b^2 - A*b*c)*x)/c^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(68) = 136$.
time = 0.18, size = 153, normalized size = 1.99

$$\frac{Bx^5}{5c} + x^3\left(\frac{A}{3c} - \frac{Bb}{3c^2}\right) + x\left(-\frac{Ab}{c^2} + \frac{Bb^2}{c^3}\right) + \frac{\sqrt{-\frac{b^3}{c^7}}\,(-Ac + Bb)\log\left(-\frac{c^3\sqrt{-\frac{b^3}{c^7}}\,(-Ac+Bb)}{-Abc+Bb^2} + x\right)}{2} - \frac{\sqrt{-\frac{b^3}{c^7}}\,(-Ac + Bb)\log\left(\frac{c^3\sqrt{-\frac{b^3}{c^7}}\,(-Ac+Bb)}{-Abc+Bb^2} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**5/(5*c) + x**3*(A/(3*c) - B*b/(3*c**2)) + x*(-A*b/c**2 + B*b**2/c**3)
+ sqrt(-b**3/c**7)*(-A*c + B*b)*log(-c**3*sqrt(-b**3/c**7)*(-A*c + B*b)/(-A
*b*c + B*b**2) + x)/2 - sqrt(-b**3/c**7)*(-A*c + B*b)*log(c**3*sqrt(-b**3/c
**7)*(-A*c + B*b)/(-A*b*c + B*b**2) + x)/2
```

Giac [A]
time = 0.62, size = 85, normalized size = 1.10

$$-\frac{(Bb^3 - Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^3} + \frac{3\,Bc^4x^5 - 5\,Bbc^3x^3 + 5\,Ac^4x^3 + 15\,Bb^2c^2x - 15\,Abc^3x}{15\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -(B*b^3 - A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*B*c^4*x^
5 - 5*B*b*c^3*x^3 + 5*A*c^4*x^3 + 15*B*b^2*c^2*x - 15*A*b*c^3*x)/c^5
```

Mupad [B]
time = 0.07, size = 96, normalized size = 1.25

$$x^3\left(\frac{A}{3\,c} - \frac{B\,b}{3\,c^2}\right) + \frac{B\,x^5}{5\,c} - \frac{b^{3/2}\,\mathrm{atan}\left(\frac{b^{3/2}\,\sqrt{c}\,x\,(A\,c-B\,b)}{B\,b^3-A\,b^2\,c}\right)\,(A\,c - B\,b)}{c^{7/2}} - \frac{b\,x\,\left(\frac{A}{c} - \frac{B\,b}{c^2}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^3*(A/(3*c) - (B*b)/(3*c^2)) + (B*x^5)/(5*c) - (b^(3/2)*atan((b^(3/2)*c^(1
/2)*x*(A*c - B*b))/(B*b^3 - A*b^2*c))*(A*c - B*b))/c^(7/2) - (b*x*(A/c - (B
*b)/c^2))/c
```

3.46 $\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=54

$$-\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{b(bB-Ac)\log(b+cx^2)}{2c^3}$$

```
[Out] -1/2*(-A*c+B*b)*x^2/c^2+1/4*B*x^4/c+1/2*b*(-A*c+B*b)*ln(c*x^2+b)/c^3
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 78}

$$\frac{b(bB-Ac)\log(b+cx^2)}{2c^3}-\frac{x^2(bB-Ac)}{2c^2}+\frac{Bx^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -1/2*((b*B - A*c)*x^2)/c^2 + (B*x^4)/(4*c) + (b*(b*B - A*c)*Log[b + c*x^2])
/(2*c^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx &= \int \frac{x^3(A+Bx^2)}{b+cx^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{b+cx}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{-bB+Ac}{c^2}+\frac{Bx}{c}+\frac{b(bB-Ac)}{c^2(b+cx)}\right)dx, x, x^2\right) \\
&= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{b(bB-Ac)\log(b+cx^2)}{2c^3}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 0.87

$$\frac{cx^2(-2bB+2Ac+Bcx^2)+2b(bB-Ac)\log(b+cx^2)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (c*x^2*(-2*b*B + 2*A*c + B*c*x^2) + 2*b*(b*B - A*c)*Log[b + c*x^2])/(4*c^3)
```

Maple [A]
time = 0.43, size = 50, normalized size = 0.93

method	result	size
default	$\frac{\frac{1}{2}Bc\,x^4+Ac\,x^2-bB\,x^2}{2c^2}-\frac{b(Ac-Bb)\ln(c\,x^2+b)}{2c^3}$	50
norman	$\frac{\frac{B\,x^5}{4c}+\frac{(Ac-Bb)x^3}{2c^2}}{x}-\frac{b(Ac-Bb)\ln(c\,x^2+b)}{2c^3}$	54
risch	$\frac{B\,x^4}{4c}+\frac{A\,x^2}{2c}-\frac{Bb\,x^2}{2c^2}+\frac{A^2}{4Bc}-\frac{Ab}{2c^2}+\frac{B\,b^2}{4c^3}-\frac{b\ln(c\,x^2+b)A}{2c^2}+\frac{b^2\ln(c\,x^2+b)B}{2c^3}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^2*(1/2*B*c*x^4+A*c*x^2-b*B*x^2)-1/2*b/c^3*(A*c-B*b)*ln(c*x^2+b)
```

Maxima [A]
time = 0.27, size = 50, normalized size = 0.93

$$\frac{Bcx^4-2\,(Bb-Ac)x^2}{4\,c^2}+\frac{(Bb^2-Abc)\log(cx^2+b)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(B*c*x^4 - 2*(B*b - A*c)*x^2)/c^2 + 1/2*(B*b^2 - A*b*c)*log(c*x^2 + b)/
c^3
```

Fricas [A]
time = 1.54, size = 51, normalized size = 0.94

$$\frac{Bc^2x^4 - 2\left(Bbc - Ac^2\right)x^2 + 2\left(Bb^2 - Abc\right)\log\left(cx^2 + b\right)}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^2*x^4 - 2*(B*b*c - A*c^2)*x^2 + 2*(B*b^2 - A*b*c)*log(c*x^2 + b))/
c^3
```

Sympy [A]
time = 0.14, size = 46, normalized size = 0.85

$$\frac{Bx^4}{4c} + \frac{b(-Ac + Bb)\log\left(b + cx^2\right)}{2c^3} + x^2\left(\frac{A}{2c} - \frac{Bb}{2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**4/(4*c) + b*(-A*c + B*b)*log(b + c*x**2)/(2*c**3) + x**2*(A/(2*c) - B*
b/(2*c**2))
```

Giac [A]
time = 0.66, size = 52, normalized size = 0.96

$$\frac{Bcx^4 - 2\,Bbx^2 + 2\,Acx^2}{4\,c^2} + \frac{\left(Bb^2 - Abc\right)\log\left(\left|cx^2 + b\right|\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/4*(B*c*x^4 - 2*B*b*x^2 + 2*A*c*x^2)/c^2 + 1/2*(B*b^2 - A*b*c)*log(abs(c*x
^2 + b))/c^3
```

Mupad [B]
time = 0.07, size = 52, normalized size = 0.96

$$x^2\left(\frac{A}{2\,c} - \frac{B\,b}{2\,c^2}\right) + \frac{\ln\left(c\,x^2 + b\right)\,\left(B\,b^2 - A\,b\,c\right)}{2\,c^3} + \frac{B\,x^4}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^2*(A/(2*c) - (B*b)/(2*c^2)) + (log(b + c*x^2)*(B*b^2 - A*b*c))/(2*c^3) +
(B*x^4)/(4*c)
```

3.47 $\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=58

$$-\frac{(bB-Ac)x}{c^2}+\frac{Bx^3}{3c}+\frac{\sqrt{b}\,(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}$$

```
[Out] -(-A*c+B*b)*x/c^2+1/3*B*x^3/c+(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))*b^(1/2)/
c^(5/2)
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1598, 470, 327, 211}

$$\frac{\sqrt{b}\,(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{x(bB-Ac)}{c^2}+\frac{Bx^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -(((b*B - A*c)*x)/c^2) + (B*x^3)/(3*c) + (Sqrt[b]*(b*B - A*c)*ArcTan[(Sqrt[
c]*x)/Sqrt[b]])/c^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx &= \int \frac{x^2(A+Bx^2)}{b+cx^2}\,dx\\ &= \frac{Bx^3}{3c} - \frac{(3bB-3Ac)\int \frac{x^2}{b+cx^2}\,dx}{3c}\\ &= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{(b(bB-Ac))\int \frac{1}{b+cx^2}\,dx}{c^2}\\ &= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\sqrt{b}\,(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.98

$$\frac{(-bB+Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\sqrt{b}\,(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] ((-(b*B) + A*c)*x)/c^2 + (B*x^3)/(3*c) + (Sqrt[b]*(b*B - A*c)*ArcTan[(Sqrt[
c]*x)/Sqrt[b]])/c^(5/2)
```

Maple [A]
time = 0.39, size = 51, normalized size = 0.88

method	result
default	$\frac{\frac{1}{3}Bc\,x^3+Acx-bBx}{c^2} - \frac{b(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^2\sqrt{bc}}$
risch	$\frac{B\,x^3}{3c} + \frac{Ax}{c} - \frac{bBx}{c^2} + \frac{\sqrt{-bc}\,\ln\left(-\sqrt{-bc}\,x-b\right)A}{2c^2} - \frac{\sqrt{-bc}\,\ln\left(-\sqrt{-bc}\,x-b\right)Bb}{2c^3} - \frac{\sqrt{-bc}\,\ln\left(\sqrt{-bc}\,x-b\right)}{2c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/3*B*c*x^3+A*c*x-b*B*x)-b*(A*c-B*b)/c^2/(b*c)^(1/2)*arctan(c*x/(b*c
)^(1/2))
```

Maxima [A]
time = 0.50, size = 53, normalized size = 0.91

$$\frac{(Bb^2 - Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^2} + \frac{Bcx^3 - 3\,(Bb - Ac)x}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] (B*b^2 - A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(B*c*x^3 - 3*(B
*b - A*c)*x)/c^2
```

Fricas [A]
time = 1.55, size = 129, normalized size = 2.22

$$\left[\frac{2\,Bcx^3 - 3\,(Bb - Ac)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2 - 2\,cx\sqrt{-\frac{b}{c}} - b}{cx^2 + b}\right) - 6\,(Bb - Ac)x}{6\,c^2}, \frac{Bcx^3 + 3\,(Bb - Ac)\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right) - 3\,(Bb - Ac)x}{3\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*B*c*x^3 - 3*(B*b - A*c)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) -
b)/(c*x^2 + b)) - 6*(B*b - A*c)*x)/c^2, 1/3*(B*c*x^3 + 3*(B*b - A*c)*sqrt(b
/c)*arctan(c*x*sqrt(b/c)/b) - 3*(B*b - A*c)*x)/c^2]
```

Sympy [A]
time = 0.17, size = 90, normalized size = 1.55

$$\frac{Bx^3}{3c} + x\left(\frac{A}{c} - \frac{Bb}{c^2}\right) - \frac{\sqrt{-\frac{b}{c^5}}\,(-Ac + Bb)\log\left(-c^2\sqrt{-\frac{b}{c^5}} + x\right)}{2} + \frac{\sqrt{-\frac{b}{c^5}}\,(-Ac + Bb)\log\left(c^2\sqrt{-\frac{b}{c^5}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**3/(3*c) + x*(A/c - B*b/c**2) - sqrt(-b/c**5)*(-A*c + B*b)*log(-c**2*sq
rt(-b/c**5) + x)/2 + sqrt(-b/c**5)*(-A*c + B*b)*log(c**2*sqrt(-b/c**5) + x)
/2
```

Giac [A]
time = 0.66, size = 57, normalized size = 0.98

$$\frac{(Bb^2 - Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^2} + \frac{Bc^2x^3 - 3\,Bbcx + 3\,Ac^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*b^2 - A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(B*c^2*x^3 - 3*
B*b*c*x + 3*A*c^2*x)/c^3
```

Mupad [B]
time = 0.11, size = 70, normalized size = 1.21

$$x\left(\frac{A}{c} - \frac{B\,b}{c^2}\right) + \frac{B\,x^3}{3\,c} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c}\,x\,(A\,c-B\,b)}{B\,b^2-A\,b\,c}\right)\,(A\,c - B\,b)}{c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x*(A/c - (B*b)/c^2) + (B*x^3)/(3*c) + (b^(1/2)*atan((b^(1/2)*c^(1/2)*x*(A*c
 - B*b))/(B*b^2 - A*b*c))*(A*c - B*b))/c^(5/2)
```

3.48 $\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=35

$$\frac{Bx^2}{2c}-\frac{(bB-Ac)\log\left(b+cx^2\right)}{2c^2}$$

[Out] 1/2*B*x^2/c-1/2*(-A*c+B*b)*ln(c*x^2+b)/c^2

Rubi [A]
time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 455, 45}

$$\frac{Bx^2}{2c}-\frac{(bB-Ac)\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4),x]

[Out] (B*x^2)/(2*c) - ((b*B - A*c)*Log[b + c*x^2])/(2*c^2)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx &= \int \frac{x(A+Bx^2)}{b+cx^2}\,dx \\ &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{b+cx}\,dx, x, x^2\right) \\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{B}{c}+\frac{-bB+Ac}{c(b+cx)}\right)\,dx, x, x^2\right) \\ &= \frac{Bx^2}{2c}-\frac{(bB-Ac)\log\left(b+cx^2\right)}{2c^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.89

$$\frac{Bcx^2+(-bB+Ac)\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] (B*c*x^2 + (-(b*B) + A*c)*Log[b + c*x^2])/(2*c^2)
```

Maple [A]
time = 0.37, size = 32, normalized size = 0.91

method	result	size
default	$\frac{B\,x^2}{2c}+\frac{(Ac-Bb)\ln\left(c\,x^2+b\right)}{2c^2}$	32
norman	$\frac{B\,x^2}{2c}+\frac{(Ac-Bb)\ln\left(c\,x^2+b\right)}{2c^2}$	32
risch	$\frac{B\,x^2}{2c}+\frac{\ln\left(c\,x^2+b\right)A}{2c}-\frac{\ln\left(c\,x^2+b\right)Bb}{2c^2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*B*x^2/c+1/2/c^2*(A*c-B*b)*ln(c*x^2+b)
```

Maxima [A]
time = 0.28, size = 31, normalized size = 0.89

$$\frac{Bx^2}{2\,c}-\frac{(Bb-Ac)\log\left(cx^2+b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*B*x^2/c - 1/2*(B*b - A*c)*log(c*x^2 + b)/c^2
```

Fricas [A]
time = 1.59, size = 30, normalized size = 0.86

$$\frac{Bcx^2 - (Bb - Ac)\log(cx^2 + b)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(B*c*x^2 - (B*b - A*c)*log(c*x^2 + b))/c^2
```

Sympy [A]
time = 0.12, size = 27, normalized size = 0.77

$$\frac{Bx^2}{2c} - \frac{(-Ac + Bb)\log(b + cx^2)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**2/(2*c) - (-A*c + B*b)*log(b + c*x**2)/(2*c**2)
```

Giac [A]
time = 0.89, size = 32, normalized size = 0.91

$$\frac{Bx^2}{2c} - \frac{(Bb - Ac)\log(|cx^2 + b|)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c - 1/2*(B*b - A*c)*log(abs(c*x^2 + b))/c^2
```

Mupad [B]
time = 0.06, size = 31, normalized size = 0.89

$$\frac{B\,x^2}{2\,c} + \frac{\ln(c\,x^2 + b)\,(A\,c - B\,b)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] (B*x^2)/(2*c) + (log(b + c*x^2)*(A*c - B*b))/(2*c^2)
```

3.49 $\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=40

$$\frac{Bx}{c}-\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,c^{3/2}}$$

```
[Out] B*x/c-(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(3/2)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 396, 211}

$$\frac{Bx}{c}-\frac{(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt[b]*c^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx = \int \frac{A+Bx^2}{b+cx^2}\,dx$$
$$= \frac{Bx}{c} - \frac{(bB-Ac)\int \frac{1}{b+cx^2}\,dx}{c}$$
$$= \frac{Bx}{c} - \frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,c^{3/2}}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 1.00

$$\frac{Bx}{c} - \frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt[b]*c^(3/2))
```

Maple [A]
time = 0.42, size = 34, normalized size = 0.85

method	result	size
default	$\frac{Bx}{c}+\frac{(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c\sqrt{bc}}$	34
risch	$\frac{Bx}{c}-\frac{\ln\left(cx+\sqrt{-bc}\right)A}{2\sqrt{-bc}}+\frac{\ln\left(cx+\sqrt{-bc}\right)Bb}{2c\sqrt{-bc}}+\frac{\ln\left(-cx+\sqrt{-bc}\right)A}{2\sqrt{-bc}}-\frac{\ln\left(-cx+\sqrt{-bc}\right)Bb}{2c\sqrt{-bc}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] B*x/c+(A*c-B*b)/c/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [A]
time = 0.49, size = 34, normalized size = 0.85

$$\frac{Bx}{c} - \frac{(Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] B*x/c - (B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c)
```

Fricas [A]
time = 1.84, size = 99, normalized size = 2.48

$$\left[\frac{2\,Bbcx + (Bb - Ac)\sqrt{-bc}\,\log\left(\frac{cx^2 - 2\,\sqrt{-bc}\,x - b}{cx^2 + b}\right)}{2\,bc^2}, \frac{Bbcx - (Bb - Ac)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{bc^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*b*c*x + (B*b - A*c)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(
c*x^2 + b)))/(b*c^2), (B*b*c*x - (B*b - A*c)*sqrt(b*c)*arctan(sqrt(b*c)*x/b
))/(b*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.
time = 0.14, size = 82, normalized size = 2.05

$$\frac{Bx}{c} + \frac{\sqrt{-\frac{1}{bc^3}}\,(-Ac + Bb)\log\left(-bc\sqrt{-\frac{1}{bc^3}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{bc^3}}\,(-Ac + Bb)\log\left(bc\sqrt{-\frac{1}{bc^3}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x/c + sqrt(-1/(b*c**3))*(-A*c + B*b)*log(-b*c*sqrt(-1/(b*c**3)) + x)/2 -
sqrt(-1/(b*c**3))*(-A*c + B*b)*log(b*c*sqrt(-1/(b*c**3)) + x)/2
```

Giac [A]
time = 0.68, size = 34, normalized size = 0.85

$$\frac{Bx}{c} - \frac{(Bb - Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] B*x/c - (B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c)
```

Mupad [B]
time = 0.05, size = 31, normalized size = 0.78

$$\frac{B\,x}{c}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,\left(A\,c-B\,b\right)}{\sqrt{b}\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4),x)
[Out] (B*x)/c + (atan((c^(1/2)*x)/b^(1/2))*(A*c - B*b))/(b^(1/2)*c^(3/2))
```

3.50 $\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=34

$$\frac{A\log(x)}{b}+\frac{(bB-Ac)\log(b+cx^2)}{2bc}$$

```
[Out] A*ln(x)/b+1/2*(-A*c+B*b)*ln(c*x^2+b)/b/c
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {1598, 457, 78}

$$\frac{(bB-Ac)\log(b+cx^2)}{2bc}+\frac{A\log(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (A*Log[x])/b + ((b*B - A*c)*Log[b + c*x^2])/(2*b*c)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx &= \int \frac{A+Bx^2}{x\,(b+cx^2)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x(b+cx)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{bx}+\frac{bB-Ac}{b(b+cx)}\right)dx, x, x^2\right) \\
&= \frac{A\log(x)}{b}+\frac{(bB-Ac)\log(b+cx^2)}{2bc}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.00

$$\frac{A\log(x)}{b}+\frac{(bB-Ac)\log(b+cx^2)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (A*Log[x])/b + ((b*B - A*c)*Log[b + c*x^2])/(2*b*c)
```

Maple [A]
time = 0.38, size = 33, normalized size = 0.97

method	result	size
default	$-\frac{(Ac-Bb)\ln(c\,x^2+b)}{2bc}+\frac{A\ln(x)}{b}$	33
norman	$-\frac{(Ac-Bb)\ln(c\,x^2+b)}{2bc}+\frac{A\ln(x)}{b}$	33
risch	$-\frac{\ln(c\,x^2+b)A}{2b}+\frac{\ln(c\,x^2+b)B}{2c}+\frac{A\ln(x)}{b}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(A*c-B*b)/b/c*ln(c*x^2+b)+A*ln(x)/b
```

Maxima [A]
time = 0.29, size = 35, normalized size = 1.03

$$\frac{A\log(x^2)}{2\,b}+\frac{(Bb-Ac)\log(cx^2+b)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*A*log(x^2)/b + 1/2*(B*b - A*c)*log(c*x^2 + b)/(b*c)
```

Fricas [**A**]
time = 2.19, size = 32, normalized size = 0.94

$$\frac{2\,Ac\log\left(x\right)+\left(Bb-Ac\right)\log\left(cx^{2}+b\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*A*c*log(x) + (B*b - A*c)*log(c*x^2 + b))/(b*c)
```

Sympy [**A**]
time = 0.38, size = 26, normalized size = 0.76

$$\frac{A\log\left(x\right)}{b}+\frac{\left(-Ac+Bb\right)\log\left(\frac{b}{c}+x^{2}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] A*log(x)/b + (-A*c + B*b)*log(b/c + x**2)/(2*b*c)
```

Giac [**A**]
time = 0.64, size = 34, normalized size = 1.00

$$\frac{A\log\left(|x|\right)}{b}+\frac{\left(Bb-Ac\right)\log\left(|cx^{2}+b|\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] A*log(abs(x))/b + 1/2*(B*b - A*c)*log(abs(c*x^2 + b))/(b*c)
```

Mupad [**B**]
time = 0.08, size = 32, normalized size = 0.94

$$\frac{A\,\ln\left(x\right)}{b}-\frac{\ln\left(c\,x^{2}+b\right)\,\left(A\,c-B\,b\right)}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] (A*log(x))/b - (log(b + c*x^2)*(A*c - B*b))/(2*b*c)
```

3.51 $\int \frac{A+Bx^2}{bx^2+cx^4} \, dx$

Optimal. Leaf size=42

$$-\frac{A}{bx}+\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}$$

```
[Out] -A/b/x+(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(3/2)/c^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1607, 464, 211}

$$\frac{(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{A}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(A/(b*x)) + ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{bx^2+cx^4}\,dx = \int \frac{A+Bx^2}{x^2(b+cx^2)}\,dx$$
$$= -\frac{A}{bx} - \frac{(-bB+Ac)\int \frac{1}{b+cx^2}\,dx}{b}$$
$$= -\frac{A}{bx} + \frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 1.00

$$-\frac{A}{bx} + \frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(A/(b*x)) + ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

Maple [A]
time = 0.41, size = 37, normalized size = 0.88

method	result	si
default	$\frac{(-Ac+Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{b\sqrt{bc}} - \frac{A}{bx}$	37
risch	$-\frac{A}{bx} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3c\_Z^2+A^2c^2-2ABbc+B^2b^2\right)} \_R\ln\left(\left(3\_R^2b^3c+2A^2c^2-4ABbc+2B^2b^2\right)x+\left(Ab^2c-Bb^3\right)\_R\right)\right)}{2}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (-A*c+B*b)/b/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))-A/b/x
```

Maxima [A]
time = 0.49, size = 36, normalized size = 0.86

$$\frac{(Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b} - \frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] (B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - A/(b*x)
```

Fricas [A]
time = 2.03, size = 105, normalized size = 2.50

$$\left[\frac{(Bb-Ac)\sqrt{-bc}\,x\log\left(\frac{cx^2+2\sqrt{-bc}\,x-b}{cx^2+b}\right)-2\,Abc}{2\,b^2cx},\ \frac{(Bb-Ac)\sqrt{bc}\,x\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)-Abc}{b^2cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b - A*c)*sqrt(-b*c)*x*log((c*x^2 + 2*sqrt(-b*c)*x - b)/(c*x^2 + b)
) - 2*A*b*c)/(b^2*c*x), ((B*b - A*c)*sqrt(b*c)*x*arctan(sqrt(b*c)*x/b) - A*
b*c)/(b^2*c*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.
time = 0.16, size = 82, normalized size = 1.95

$$-\frac{A}{bx}-\frac{\sqrt{-\frac{1}{b^3c}}\,(-Ac+Bb)\log\left(-b^2\sqrt{-\frac{1}{b^3c}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{b^3c}}\,(-Ac+Bb)\log\left(b^2\sqrt{-\frac{1}{b^3c}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] -A/(b*x) - sqrt(-1/(b**3*c))*(-A*c + B*b)*log(-b**2*sqrt(-1/(b**3*c)) + x)/
2 + sqrt(-1/(b**3*c))*(-A*c + B*b)*log(b**2*sqrt(-1/(b**3*c)) + x)/2
```

Giac [A]
time = 0.64, size = 36, normalized size = 0.86

$$\frac{(Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b}-\frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - A/(b*x)
```

Mupad [B]
time = 0.09, size = 35, normalized size = 0.83

$$-\frac{A}{b\,x}-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,\left(A\,c-B\,b\right)}{b^{3/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(b*x^2 + c*x^4),x)
[Out] - A/(b*x) - (atan((c^(1/2)*x)/b^(1/2))*(A*c - B*b))/(b^(3/2)*c^(1/2))
```

3.52 $\int \frac{A+Bx^2}{bx^2-cx^4} \, dx$

Optimal. Leaf size=41

$$-\frac{A}{bx}+\frac{(bB+Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}$$

```
[Out] -A/b/x+(A*c+B*b)*arctanh(x*c^(1/2)/b^(1/2))/b^(3/2)/c^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1607, 464, 214}

$$\frac{(Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{A}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(b*x^2 - c*x^4),x]
```

```
[Out] -(A/(b*x)) + ((b*B + A*c)*ArcTanh[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{bx^2-cx^4}\,dx = \int \frac{A+Bx^2}{x^2(b-cx^2)}\,dx$$
$$= -\frac{A}{bx} + \frac{(bB+Ac)\int \frac{1}{b-cx^2}\,dx}{b}$$
$$= -\frac{A}{bx} + \frac{(bB+Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}$$

Mathematica [**A**]
time = 0.02, size = 41, normalized size = 1.00

$$-\frac{A}{bx} + \frac{(bB+Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(b*x^2 - c*x^4),x]
```

```
[Out] -(A/(b*x)) + ((b*B + A*c)*ArcTanh[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

Maple [**A**]
time = 0.50, size = 39, normalized size = 0.95

method	result	size
default	$-\frac{A}{bx}-\frac{(-Ac-Bb)\,\mathrm{arctanh}\left(\frac{cx}{\sqrt{bc}}\right)}{b\sqrt{bc}}$	39
risch	$-\frac{A}{bx}+\frac{\ln\left(x\sqrt{bc}+b\right)Ac}{2\sqrt{bc}\,b}+\frac{\ln\left(x\sqrt{bc}+b\right)B}{2\sqrt{bc}}-\frac{\ln\left(x\sqrt{bc}-b\right)Ac}{2\sqrt{bc}\,b}-\frac{\ln\left(x\sqrt{bc}-b\right)B}{2\sqrt{bc}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(-c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -A/b/x-(-A*c-B*b)/b/(b*c)^(1/2)*arctanh(c*x/(b*c)^(1/2))
```

Maxima [**A**]
time = 0.51, size = 51, normalized size = 1.24

$$-\frac{(Bb+Ac)\log\left(\frac{cx-\sqrt{bc}}{cx+\sqrt{bc}}\right)}{2\sqrt{bc}\,b} - \frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(B*b + A*c)*log((c*x - sqrt(b*c))/(c*x + sqrt(b*c)))/(sqrt(b*c)*b) - A
/(b*x)
```

Fricas [A]
time = 1.62, size = 103, normalized size = 2.51

$$\left[\frac{(Bb+Ac)\sqrt{bc}\,x\log\left(\frac{cx^2+2\sqrt{bc}\,x+b}{cx^2-b}\right)-2\,Abc}{2\,b^2cx},-\frac{(Bb+Ac)\sqrt{-bc}\,x\arctan\left(\frac{\sqrt{-bc}\,x}{b}\right)+Abc}{b^2cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b + A*c)*sqrt(b*c)*x*log((c*x^2 + 2*sqrt(b*c)*x + b)/(c*x^2 - b))
- 2*A*b*c)/(b^2*c*x), -((B*b + A*c)*sqrt(-b*c)*x*arctan(sqrt(-b*c)*x/b) + A
*b*c)/(b^2*c*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.
time = 0.17, size = 75, normalized size = 1.83

$$-\frac{A}{bx}-\frac{\sqrt{\frac{1}{b^3c}}\,(Ac+Bb)\log\left(-b^2\sqrt{\frac{1}{b^3c}}+x\right)}{2}+\frac{\sqrt{\frac{1}{b^3c}}\,(Ac+Bb)\log\left(b^2\sqrt{\frac{1}{b^3c}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(-c*x**4+b*x**2),x)
```

```
[Out] -A/(b*x) - sqrt(1/(b**3*c))*(A*c + B*b)*log(-b**2*sqrt(1/(b**3*c)) + x)/2 +
 sqrt(1/(b**3*c))*(A*c + B*b)*log(b**2*sqrt(1/(b**3*c)) + x)/2
```

Giac [A]
time = 0.85, size = 38, normalized size = 0.93

$$-\frac{(Bb+Ac)\arctan\left(\frac{cx}{\sqrt{-bc}}\right)}{\sqrt{-bc}\,b}-\frac{A}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -(B*b + A*c)*arctan(c*x/sqrt(-b*c))/(sqrt(-b*c)*b) - A/(b*x)
```

Mupad [B]
time = 0.15, size = 33, normalized size = 0.80

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c+B\,b)}{b^{3/2}\,\sqrt{c}}-\frac{A}{b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(b*x^2 - c*x^4),x)
[Out] (atanh((c^(1/2)*x)/b^(1/2))*(A*c + B*b))/(b^(3/2)*c^(1/2)) - A/(b*x)
```

3.53 $\int \frac{A+Bx^2}{x(bx^2+cx^4)}\,dx$

Optimal. Leaf size=49

$$-\frac{A}{2bx^2}+\frac{(bB-Ac)\log(x)}{b^2}-\frac{(bB-Ac)\log\left(b+cx^2\right)}{2b^2}$$

```
[Out] -1/2*A/b/x^2+(-A*c+B*b)*ln(x)/b^2-1/2*(-A*c+B*b)*ln(c*x^2+b)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$-\frac{(bB-Ac)\log\left(b+cx^2\right)}{2b^2}+\frac{\log(x)(bB-Ac)}{b^2}-\frac{A}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*A/(b*x^2) + ((b*B - A*c)*Log[x])/b^2 - ((b*B - A*c)*Log[b + c*x^2])/(2
*b^2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)}\,dx &= \int \frac{A+Bx^2}{x^3\left(b+cx^2\right)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2(b+cx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{bx^2}+\frac{bB-Ac}{b^2x}-\frac{c(bB-Ac)}{b^2(b+cx)}\right)dx,x,x^2\right)\\
&= -\frac{A}{2bx^2}+\frac{(bB-Ac)\log(x)}{b^2}-\frac{(bB-Ac)\log\left(b+cx^2\right)}{2b^2}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 1.00

$$
-\frac{A}{2bx^2}+\frac{(bB-Ac)\log(x)}{b^2}+\frac{(-bB+Ac)\log\left(b+cx^2\right)}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*A/(b*x^2) + ((b*B - A*c)*Log[x])/b^2 + ((-(b*B) + A*c)*Log[b + c*x^2])
/(2*b^2)
```

Maple [A]
time = 0.44, size = 46, normalized size = 0.94

method	result	size
default	$\frac{(Ac-Bb)\ln\left(c\,x^2+b\right)}{2b^2}-\frac{A}{2b\,x^2}+\frac{(-Ac+Bb)\ln(x)}{b^2}$	46
norman	$-\frac{A}{2b\,x^2}-\frac{(Ac-Bb)\ln(x)}{b^2}+\frac{(Ac-Bb)\ln\left(c\,x^2+b\right)}{2b^2}$	47
risch	$-\frac{A}{2b\,x^2}-\frac{\ln(x)Ac}{b^2}+\frac{\ln(x)B}{b}+\frac{\ln\left(-c\,x^2-b\right)Ac}{2b^2}-\frac{\ln\left(-c\,x^2-b\right)B}{2b}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(A*c-B*b)/b^2*ln(c*x^2+b)-1/2*A/b/x^2+(-A*c+B*b)*ln(x)/b^2
```

Maxima [A]
time = 0.30, size = 48, normalized size = 0.98

$$
-\frac{(Bb-Ac)\log\left(cx^2+b\right)}{2\,b^2}+\frac{(Bb-Ac)\log\left(x^2\right)}{2\,b^2}-\frac{A}{2\,bx^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(B*b - A*c)*log(c*x^2 + b)/b^2 + 1/2*(B*b - A*c)*log(x^2)/b^2 - 1/2*A/
(b*x^2)
```

Fricas [A]
time = 0.81, size = 47, normalized size = 0.96

$$-\frac{(Bb-Ac)x^2\log\left(cx^2+b\right)-2\,(Bb-Ac)x^2\log\left(x\right)+Ab}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*((B*b - A*c)*x^2*log(c*x^2 + b) - 2*(B*b - A*c)*x^2*log(x) + A*b)/(b^2
*x^2)
```

Sympy [A]
time = 0.39, size = 41, normalized size = 0.84

$$-\frac{A}{2bx^2}+\frac{(-Ac+Bb)\log\left(x\right)}{b^2}-\frac{(-Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2),x)
```

```
[Out] -A/(2*b*x**2) + (-A*c + B*b)*log(x)/b**2 - (-A*c + B*b)*log(b/c + x**2)/(2*
b**2)
```

Giac [A]
time = 0.85, size = 71, normalized size = 1.45

$$\frac{(Bb-Ac)\log\left(x^2\right)}{2\,b^2}-\frac{(Bbc-Ac^2)\log\left(|cx^2+b|\right)}{2\,b^2c}-\frac{Bbx^2-Acx^2+Ab}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(B*b - A*c)*log(x^2)/b^2 - 1/2*(B*b*c - A*c^2)*log(abs(c*x^2 + b))/(b^2
*c) - 1/2*(B*b*x^2 - A*c*x^2 + A*b)/(b^2*x^2)
```

Mupad [B]
time = 0.14, size = 46, normalized size = 0.94

$$\frac{\ln\left(c\,x^2+b\right)\,\left(A\,c-B\,b\right)}{2\,b^2}-\frac{A}{2\,b\,x^2}-\frac{\ln\left(x\right)\,\left(A\,c-B\,b\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(b*x^2 + c*x^4)),x)
```

```
[Out] (log(b + c*x^2)*(A*c - B*b))/(2*b^2) - A/(2*b*x^2) - (log(x)*(A*c - B*b))/b
^2
```

3.54 $\int \frac{A+Bx^2}{x^2(bx^2+cx^4)}\,dx$

Optimal. Leaf size=61

$$-\frac{A}{3bx^3}-\frac{bB-Ac}{b^2x}-\frac{\sqrt{c}\,(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}$$

```
[Out] -1/3*A/b/x^3+(A*c-B*b)/b^2/x-(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))*c^(1/2)/b
^(5/2)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 464, 331, 211}

$$-\frac{\sqrt{c}\,(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}-\frac{bB-Ac}{b^2x}-\frac{A}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*A/(b*x^3) - (b*B - A*c)/(b^2*x) - (Sqrt[c]*(b*B - A*c)*ArcTan[(Sqrt[c]
*x)/Sqrt[b]])/b^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^2(bx^2+cx^4)}\,dx &= \int \frac{A+Bx^2}{x^4(b+cx^2)}\,dx\\ &= -\frac{A}{3bx^3}-\frac{(-3bB+3Ac)\int \frac{1}{x^2(b+cx^2)}\,dx}{3b}\\ &= -\frac{A}{3bx^3}-\frac{bB-Ac}{b^2x}-\frac{(c(bB-Ac))\int \frac{1}{b+cx^2}\,dx}{b^2}\\ &= -\frac{A}{3bx^3}-\frac{bB-Ac}{b^2x}-\frac{\sqrt{c}\,(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.98

$$-\frac{A}{3bx^3}+\frac{-bB+Ac}{b^2x}-\frac{\sqrt{c}\,(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)),x]
[Out] -1/3*A/(b*x^3) + (-(b*B) + A*c)/(b^2*x) - (Sqrt[c]*(b*B - A*c)*ArcTan[(Sqrt
[c]*x)/Sqrt[b]])/b^(5/2)
```

Maple [A]
time = 0.41, size = 54, normalized size = 0.89

method	result
default	$\frac{c(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{b^2\sqrt{bc}}-\frac{A}{3b\,x^3}-\frac{-Ac+Bb}{b^2x}$

risch	$\frac{\frac{(Ac-Bb)x^2}{b^2}-\frac{A}{3b}}{x^3}+\frac{\sqrt{-bc}\,\ln\left(-cx-\sqrt{-bc}\right)Ac}{2b^3}-\frac{\sqrt{-bc}\,\ln\left(-cx-\sqrt{-bc}\right)B}{2b^2}-\frac{\sqrt{-bc}\,\ln\left(-cx+\sqrt{-bc}\right)Ac}{2b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] c*(A*c-B*b)/b^2/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))-1/3*A/b/x^3-1/b^2*(-A*c
+B*b)/x
```

Maxima [A]
time = 0.50, size = 56, normalized size = 0.92

$$-\frac{(Bbc-Ac^2)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b^2}-\frac{3\,(Bb-Ac)x^2+Ab}{3\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -(B*b*c - A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) - 1/3*(3*(B*b - A*c)
*x^2 + A*b)/(b^2*x^3)
```

Fricas [A]
time = 1.77, size = 135, normalized size = 2.21

$$\left[-\frac{3\,(Bb-Ac)x^3\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)+6\,(Bb-Ac)x^2+2\,Ab}{6\,b^2x^3},-\frac{3\,(Bb-Ac)x^3\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)+3\,(Bb-Ac)x^2+Ab}{3\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(B*b - A*c)*x^3*sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c*x
^2 + b)) + 6*(B*b - A*c)*x^2 + 2*A*b)/(b^2*x^3), -1/3*(3*(B*b - A*c)*x^3*sq
rt(c/b)*arctan(x*sqrt(c/b)) + 3*(B*b - A*c)*x^2 + A*b)/(b^2*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(49) = 98$.
time = 0.20, size = 129, normalized size = 2.11

$$\frac{\sqrt{-\frac{c}{b^5}}\,(-Ac+Bb)\log\left(-\frac{b^3\sqrt{-\frac{c}{b^5}}\,(-Ac+Bb)}{-Ac^2+Bbc}+x\right)}{2}-\frac{\sqrt{-\frac{c}{b^5}}\,(-Ac+Bb)\log\left(\frac{b^3\sqrt{-\frac{c}{b^5}}\,(-Ac+Bb)}{-Ac^2+Bbc}+x\right)}{2}+\frac{-Ab+x^2\cdot(3Ac-3Bb)}{3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-c/b**5)*(-A*c + B*b)*log(-b**3*sqrt(-c/b**5)*(-A*c + B*b)/(-A*c**2 +
B*b*c) + x)/2 - sqrt(-c/b**5)*(-A*c + B*b)*log(b**3*sqrt(-c/b**5)*(-A*c + B
*b)/(-A*c**2 + B*b*c) + x)/2 + (-A*b + x**2*(3*A*c - 3*B*b))/(3*b**2*x**3)
```

Giac [A]
time = 0.69, size = 57, normalized size = 0.93

$$-\frac{(Bbc - Ac^2)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b^2} - \frac{3\,Bbx^2 - 3\,Acx^2 + Ab}{3\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -(B*b*c - A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) - 1/3*(3*B*b*x^2 - 3
*A*c*x^2 + A*b)/(b^2*x^3)
```

Mupad [B]
time = 0.11, size = 53, normalized size = 0.87

$$\frac{\sqrt{c}\,\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c - B\,b)}{b^{5/2}} - \frac{\frac{A}{3\,b} - \frac{x^2\,(A\,c - B\,b)}{b^2}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)),x)
```

```
[Out] (c^(1/2)*atan((c^(1/2)*x)/b^(1/2))*(A*c - B*b))/b^(5/2) - (A/(3*b) - (x^2*(
A*c - B*b))/b^2)/x^3
```

3.55 $\int \frac{A+Bx^2}{x^3(bx^2+cx^4)}\,dx$

Optimal. Leaf size=70

$$-\frac{A}{4bx^4}-\frac{bB-Ac}{2b^2x^2}-\frac{c(bB-Ac)\log(x)}{b^3}+\frac{c(bB-Ac)\log(b+cx^2)}{2b^3}$$

```
[Out] -1/4*A/b/x^4+1/2*(A*c-B*b)/b^2/x^2-c*(-A*c+B*b)*ln(x)/b^3+1/2*c*(-A*c+B*b)*
ln(c*x^2+b)/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {1598, 457, 78}

$$\frac{c(bB-Ac)\log(b+cx^2)}{2b^3}-\frac{c\log(x)(bB-Ac)}{b^3}-\frac{bB-Ac}{2b^2x^2}-\frac{A}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*A/(b*x^4) - (b*B - A*c)/(2*b^2*x^2) - (c*(b*B - A*c)*Log[x])/b^3 + (c*
(b*B - A*c)*Log[b + c*x^2])/(2*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)}\,dx &= \int \frac{A+Bx^2}{x^5\left(b+cx^2\right)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3(b+cx)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{bx^3}+\frac{bB-Ac}{b^2x^2}-\frac{c(bB-Ac)}{b^3x}+\frac{c^2(bB-Ac)}{b^3(b+cx)}\right)\,dx, x, x^2\right) \\
&= -\frac{A}{4bx^4}-\frac{bB-Ac}{2b^2x^2}-\frac{c(bB-Ac)\log(x)}{b^3}+\frac{c(bB-Ac)\log\left(b+cx^2\right)}{2b^3}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 1.00

$$\frac{-b(Ab+2bBx^2-2Acx^2)+4c(-bB+Ac)x^4\log(x)+2c(bB-Ac)x^4\log\left(b+cx^2\right)}{4b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] (-(b*(A*b + 2*b*B*x^2 - 2*A*c*x^2)) + 4*c*(-(b*B) + A*c)*x^4*Log[x] + 2*c*(
b*B - A*c)*x^4*Log[b + c*x^2])/(4*b^3*x^4)
```

Maple [A]
time = 0.38, size = 64, normalized size = 0.91

method	result	size
default	$-\frac{c(Ac-Bb)\ln(c\,x^2+b)}{2b^3}-\frac{A}{4b\,x^4}-\frac{-Ac+Bb}{2b^2x^2}+\frac{c(Ac-Bb)\ln(x)}{b^3}$	64
norman	$\frac{-\frac{A}{4b}+\frac{(Ac-Bb)x^2}{2b^2}}{x^4}+\frac{c(Ac-Bb)\ln(x)}{b^3}-\frac{c(Ac-Bb)\ln(c\,x^2+b)}{2b^3}$	66
risch	$\frac{-\frac{A}{4b}+\frac{(Ac-Bb)x^2}{2b^2}}{x^4}+\frac{c^2\ln(x)A}{b^3}-\frac{c\ln(x)B}{b^2}-\frac{c^2\ln(c\,x^2+b)A}{2b^3}+\frac{c\ln(c\,x^2+b)B}{2b^2}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c*(A*c-B*b)/b^3*ln(c*x^2+b)-1/4*A/b/x^4-1/2*(-A*c+B*b)/b^2/x^2+c*(A*c-
B*b)/b^3*ln(x)
```

Maxima [A]
time = 0.28, size = 70, normalized size = 1.00

$$\frac{(Bbc-Ac^2)\log\left(cx^2+b\right)}{2\,b^3}-\frac{(Bbc-Ac^2)\log\left(x^2\right)}{2\,b^3}-\frac{2\,(Bb-Ac)x^2+Ab}{4\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*c - A*c^2)*log(c*x^2 + b)/b^3 - 1/2*(B*b*c - A*c^2)*log(x^2)/b^3 -
 1/4*(2*(B*b - A*c)*x^2 + A*b)/(b^2*x^4)
```

Fricas [**A**]
time = 0.92, size = 73, normalized size = 1.04

$$\frac{2\left(Bbc-Ac^2\right)x^4\log\left(cx^2+b\right)-4\left(Bbc-Ac^2\right)x^4\log\left(x\right)-Ab^2-2\left(Bb^2-Abc\right)x^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*b*c - A*c^2)*x^4*log(c*x^2 + b) - 4*(B*b*c - A*c^2)*x^4*log(x) -
A*b^2 - 2*(B*b^2 - A*b*c)*x^2)/(b^3*x^4)
```

Sympy [**A**]
time = 0.44, size = 61, normalized size = 0.87

$$\frac{-Ab+x^2\cdot(2Ac-2Bb)}{4b^2x^4}-\frac{c(-Ac+Bb)\log\left(x\right)}{b^3}+\frac{c(-Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2),x)
```

```
[Out] (-A*b + x**2*(2*A*c - 2*B*b))/(4*b**2*x**4) - c*(-A*c + B*b)*log(x)/b**3 +
c*(-A*c + B*b)*log(b/c + x**2)/(2*b**3)
```

Giac [**A**]
time = 0.78, size = 100, normalized size = 1.43

$$-\frac{\left(Bbc-Ac^2\right)\log\left(x^2\right)}{2\,b^3}+\frac{\left(Bbc^2-Ac^3\right)\log\left(\left|cx^2+b\right|\right)}{2\,b^3c}+\frac{3\,Bbcx^4-3\,Ac^2x^4-2\,Bb^2x^2+2\,Abcx^2-Ab^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*(B*b*c - A*c^2)*log(x^2)/b^3 + 1/2*(B*b*c^2 - A*c^3)*log(abs(c*x^2 + b
))/(b^3*c) + 1/4*(3*B*b*c*x^4 - 3*A*c^2*x^4 - 2*B*b^2*x^2 + 2*A*b*c*x^2 - A
*b^2)/(b^3*x^4)
```

Mupad [**B**]
time = 0.14, size = 70, normalized size = 1.00

$$\frac{\ln\left(x\right)\,\left(A\,c^2-B\,b\,c\right)}{b^3}-\frac{\ln\left(c\,x^2+b\right)\,\left(A\,c^2-B\,b\,c\right)}{2\,b^3}-\frac{\frac{A}{4\,b}-\frac{x^2\,\left(A\,c-B\,b\right)}{2\,b^2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(b*x^2 + c*x^4)),x)
```

```
[Out] (log(x)*(A*c^2 - B*b*c))/b^3 - (log(b + c*x^2)*(A*c^2 - B*b*c))/(2*b^3) - (
A/(4*b) - (x^2*(A*c - B*b))/(2*b^2))/x^4
```

3.56 $\int \frac{A+Bx^2}{x^4(bx^2+cx^4)}\,dx$

Optimal. Leaf size=78

$$-\frac{A}{5bx^5}-\frac{bB-Ac}{3b^2x^3}+\frac{c(bB-Ac)}{b^3x}+\frac{c^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}$$

```
[Out] -1/5*A/b/x^5+1/3*(A*c-B*b)/b^2/x^3+c*(-A*c+B*b)/b^3/x+c^(3/2)*(-A*c+B*b)*ar
ctan(x*c^(1/2)/b^(1/2))/b^(7/2)
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1598, 464, 331, 211}

$$\frac{c^{3/2}(bB-Ac)\mathrm{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}+\frac{c(bB-Ac)}{b^3x}-\frac{bB-Ac}{3b^2x^3}-\frac{A}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/5*A/(b*x^5) - (b*B - A*c)/(3*b^2*x^3) + (c*(b*B - A*c))/(b^3*x) + (c^(3/
2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^4\left(bx^2+cx^4\right)}\,dx &= \int \frac{A+Bx^2}{x^6\left(b+cx^2\right)}\,dx \\
&= -\frac{A}{5bx^5}-\frac{(-5bB+5Ac)\int \frac{1}{x^4(b+cx^2)}\,dx}{5b} \\
&= -\frac{A}{5bx^5}-\frac{bB-Ac}{3b^2x^3}-\frac{(c(bB-Ac))\int \frac{1}{x^2(b+cx^2)}\,dx}{b^2} \\
&= -\frac{A}{5bx^5}-\frac{bB-Ac}{3b^2x^3}+\frac{c(bB-Ac)}{b^3x}+\frac{\left(c^2(bB-Ac)\right)\int \frac{1}{b+cx^2}\,dx}{b^3} \\
&= -\frac{A}{5bx^5}-\frac{bB-Ac}{3b^2x^3}+\frac{c(bB-Ac)}{b^3x}+\frac{c^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 78, normalized size = 1.00

$$
-\frac{A}{5bx^5}+\frac{-bB+Ac}{3b^2x^3}+\frac{c(bB-Ac)}{b^3x}+\frac{c^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(b*x^2 + c*x^4)),x]
[Out] -1/5*A/(b*x^5) + (-(b*B) + A*c)/(3*b^2*x^3) + (c*(b*B - A*c))/(b^3*x) + (c^
(3/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(7/2)
```

Maple [A]
time = 0.42, size = 74, normalized size = 0.95

method	result
default	$-\frac{c^2(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{b^3\sqrt{bc}}-\frac{A}{5b\,x^5}-\frac{-Ac+Bb}{3b^2x^3}-\frac{c(Ac-Bb)}{b^3x}$

risch	$\frac{-\frac{c(Ac-Bb)x^4}{b^3}+\frac{(Ac-Bb)x^2}{3b^2}-\frac{A}{5b}}{x^5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^7\_Z^2+A^2c^5-2ABbc^4+B^2b^2c^3\right)}\_R\ln\left(\left(3\_R^2b^7+2A^2c^5-4ABbc^4+2B^2b^2c^3\right)\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -c^2*(A*c-B*b)/b^3/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))-1/5*A/b/x^5-1/3*(-A*
c+B*b)/b^2/x^3-c*(A*c-B*b)/b^3/x
```

Maxima [A]
time = 0.50, size = 79, normalized size = 1.01

$$\frac{(Bbc^2-Ac^3)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b^3}+\frac{15\,(Bbc-Ac^2)x^4-3\,Ab^2-5\,(Bb^2-Abc)x^2}{15\,b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] (B*b*c^2 - A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) + 1/15*(15*(B*b*c -
 A*c^2)*x^4 - 3*A*b^2 - 5*(B*b^2 - A*b*c)*x^2)/(b^3*x^5)
```

Fricas [A]
time = 1.71, size = 184, normalized size = 2.36

$$\left[-\frac{15\,(Bbc-Ac^2)x^5\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-30\,(Bbc-Ac^2)x^4+6\,Ab^2+10\,(Bb^2-Abc)x^2}{30\,b^3x^5},\frac{15\,(Bbc-Ac^2)x^5\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)+15\,(Bbc-Ac^2)x^4-3\,Ab^2-5\,(Bb^2-Abc)x^2}{15\,b^3x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(B*b*c - A*c^2)*x^5*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b
)/(c*x^2 + b)) - 30*(B*b*c - A*c^2)*x^4 + 6*A*b^2 + 10*(B*b^2 - A*b*c)*x^2)
/(b^3*x^5), 1/15*(15*(B*b*c - A*c^2)*x^5*sqrt(c/b)*arctan(x*sqrt(c/b)) + 15
*(B*b*c - A*c^2)*x^4 - 3*A*b^2 - 5*(B*b^2 - A*b*c)*x^2)/(b^3*x^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(68) = 136$.
time = 0.25, size = 163, normalized size = 2.09

$$-\frac{\sqrt{-\frac{c^3}{b^7}}\,(-Ac+Bb)\log\left(-\frac{b^4\sqrt{-\frac{c^3}{b^7}}\,(-Ac+Bb)}{-Ac^3+Bbc^2}+x\right)}{2}+\frac{\sqrt{-\frac{c^3}{b^7}}\,(-Ac+Bb)\log\left(\frac{b^4\sqrt{-\frac{c^3}{b^7}}\,(-Ac+Bb)}{-Ac^3+Bbc^2}+x\right)}{2}+\frac{-3Ab^2+x^4(-15Ac^2+15Bbc)+x^2\cdot(5Abc-5Bb^2)}{15b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(-c**3/b**7)*(-A*c + B*b)*log(-b**4*sqrt(-c**3/b**7)*(-A*c + B*b)/(-A*
c**3 + B*b*c**2) + x)/2 + sqrt(-c**3/b**7)*(-A*c + B*b)*log(b**4*sqrt(-c**3
/b**7)*(-A*c + B*b)/(-A*c**3 + B*b*c**2) + x)/2 + (-3*A*b**2 + x**4*(-15*A*
c**2 + 15*B*b*c) + x**2*(5*A*b*c - 5*B*b**2))/(15*b**3*x**5)
```

Giac [A]
time = 0.85, size = 81, normalized size = 1.04

$$\frac{(Bbc^2 - Ac^3)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b^3} + \frac{15\,Bbcx^4 - 15\,Ac^2x^4 - 5\,Bb^2x^2 + 5\,Abcx^2 - 3\,Ab^2}{15\,b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*b*c^2 - A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) + 1/15*(15*B*b*c*x^
4 - 15*A*c^2*x^4 - 5*B*b^2*x^2 + 5*A*b*c*x^2 - 3*A*b^2)/(b^3*x^5)
```

Mupad [B]
time = 0.11, size = 70, normalized size = 0.90

$$-\frac{\frac{A}{5\,b}-\frac{x^2\,(A\,c-B\,b)}{3\,b^2}+\frac{c\,x^4\,(A\,c-B\,b)}{b^3}}{x^5}-\frac{c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c-B\,b)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(b*x^2 + c*x^4)),x)
```

```
[Out] - (A/(5*b) - (x^2*(A*c - B*b))/(3*b^2) + (c*x^4*(A*c - B*b))/b^3)/x^5 - (c^
(3/2)*atan((c^(1/2)*x)/b^(1/2))*(A*c - B*b))/b^(7/2)
```

3.57 $\int \frac{A+Bx^2}{x^5(bx^2+cx^4)}\,dx$

Optimal. Leaf size=92

$$-\frac{A}{6bx^6}-\frac{bB-Ac}{4b^2x^4}+\frac{c(bB-Ac)}{2b^3x^2}+\frac{c^2(bB-Ac)\log(x)}{b^4}-\frac{c^2(bB-Ac)\log(b+cx^2)}{2b^4}$$

```
[Out] -1/6*A/b/x^6+1/4*(A*c-B*b)/b^2/x^4+1/2*c*(-A*c+B*b)/b^3/x^2+c^2*(-A*c+B*b)*
ln(x)/b^4-1/2*c^2*(-A*c+B*b)*ln(c*x^2+b)/b^4
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$-\frac{c^2(bB-Ac)\log(b+cx^2)}{2b^4}+\frac{c^2\log(x)(bB-Ac)}{b^4}+\frac{c(bB-Ac)}{2b^3x^2}-\frac{bB-Ac}{4b^2x^4}-\frac{A}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/6*A/(b*x^6) - (b*B - A*c)/(4*b^2*x^4) + (c*(b*B - A*c))/(2*b^3*x^2) + (c
^2*(b*B - A*c)*Log[x])/b^4 - (c^2*(b*B - A*c)*Log[b + c*x^2])/(2*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)}\,dx=\int \frac{A+Bx^2}{x^7\left(b+cx^2\right)}\,dx$$
$$=\frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4(b+cx)}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{bx^4}+\frac{bB-Ac}{b^2x^3}-\frac{c(bB-Ac)}{b^3x^2}+\frac{c^2(bB-Ac)}{b^4x}-\frac{c^3(bB-Ac)}{b^4(b+cx)}\right)\,dx,x,x$$
$$=-\frac{A}{6bx^6}-\frac{bB-Ac}{4b^2x^4}+\frac{c(bB-Ac)}{2b^3x^2}+\frac{c^2(bB-Ac)\log(x)}{b^4}-\frac{c^2(bB-Ac)\log\left(b+cx^2\right)}{2b^4}$$

Mathematica [A]
time = 0.03, size = 96, normalized size = 1.04

$$-\frac{A}{6bx^6}+\frac{-bB+Ac}{4b^2x^4}+\frac{c(bB-Ac)}{2b^3x^2}+\frac{\left(bBc^2-Ac^3\right)\log(x)}{b^4}+\frac{\left(-bBc^2+Ac^3\right)\log\left(b+cx^2\right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/6*A/(b*x^6) + (-(b*B) + A*c)/(4*b^2*x^4) + (c*(b*B - A*c))/(2*b^3*x^2) +
 ((b*B*c^2 - A*c^3)*Log[x])/b^4 + ((-(b*B*c^2) + A*c^3)*Log[b + c*x^2])/(2*
b^4)
```

Maple [A]
time = 0.38, size = 86, normalized size = 0.93

method	result	size
default	$\frac{(Ac-Bb)c^2\ln(cx^2+b)}{2b^4}-\frac{A}{6bx^6}-\frac{-Ac+Bb}{4b^2x^4}-\frac{c(Ac-Bb)}{2b^3x^2}-\frac{c^2(Ac-Bb)\ln(x)}{b^4}$	86
norman	$\frac{-\frac{A}{6b}+\frac{(Ac-Bb)x^2}{4b^2}-\frac{c(Ac-Bb)x^4}{2b^3}}{x^6}-\frac{c^2(Ac-Bb)\ln(x)}{b^4}+\frac{(Ac-Bb)c^2\ln(cx^2+b)}{2b^4}$	88
risch	$\frac{-\frac{A}{6b}+\frac{(Ac-Bb)x^2}{4b^2}-\frac{c(Ac-Bb)x^4}{2b^3}}{x^6}-\frac{c^3\ln(x)A}{b^4}+\frac{c^2\ln(x)B}{b^3}+\frac{c^3\ln(-cx^2-b)A}{2b^4}-\frac{c^2\ln(-cx^2-b)B}{2b^3}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(A*c-B*b)*c^2/b^4*ln(c*x^2+b)-1/6*A/b/x^6-1/4*(-A*c+B*b)/b^2/x^4-1/2*c*
(A*c-B*b)/b^3/x^2-c^2*(A*c-B*b)/b^4*ln(x)
```

Maxima [A]
time = 0.27, size = 96, normalized size = 1.04

$$-\frac{\left(Bbc^2-Ac^3\right)\log\left(cx^2+b\right)}{2\,b^4}+\frac{\left(Bbc^2-Ac^3\right)\log\left(x^2\right)}{2\,b^4}+\frac{6\,\left(Bbc-Ac^2\right)x^4-2\,Ab^2-3\,\left(Bb^2-Abc\right)x^2}{12\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(B*b*c^2 - A*c^3)*log(c*x^2 + b)/b^4 + 1/2*(B*b*c^2 - A*c^3)*log(x^2)/
b^4 + 1/12*(6*(B*b*c - A*c^2)*x^4 - 2*A*b^2 - 3*(B*b^2 - A*b*c)*x^2)/(b^3*x
^6)
```

Fricas [A]
time = 0.96, size = 98, normalized size = 1.07

$$-\frac{6\,(Bbc^2 - Ac^3)x^6\log\left(cx^2 + b\right) - 12\,(Bbc^2 - Ac^3)x^6\log\left(x\right) - 6\,(Bb^2c - Abc^2)x^4 + 2\,Ab^3 + 3\,(Bb^3 - Ab^2c)x^2}{12\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/12*(6*(B*b*c^2 - A*c^3)*x^6*log(c*x^2 + b) - 12*(B*b*c^2 - A*c^3)*x^6*lo
g(x) - 6*(B*b^2*c - A*b*c^2)*x^4 + 2*A*b^3 + 3*(B*b^3 - A*b^2*c)*x^2)/(b^4*
x^6)
```

Sympy [A]
time = 0.48, size = 88, normalized size = 0.96

$$\frac{-2Ab^2 + x^4(-6Ac^2 + 6Bbc) + x^2\cdot(3Abc - 3Bb^2)}{12b^3x^6} + \frac{c^2(-Ac + Bb)\log\left(x\right)}{b^4} - \frac{c^2(-Ac + Bb)\log\left(\frac{b}{c} + x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2),x)
```

```
[Out] (-2*A*b**2 + x**4*(-6*A*c**2 + 6*B*b*c) + x**2*(3*A*b*c - 3*B*b**2))/(12*b*
*3*x**6) + c**2*(-A*c + B*b)*log(x)/b**4 - c**2*(-A*c + B*b)*log(b/c + x**2
)/(2*b**4)
```

Giac [A]
time = 0.68, size = 126, normalized size = 1.37

$$\frac{(Bbc^2 - Ac^3)\log\left(x^2\right)}{2\,b^4} - \frac{(Bbc^3 - Ac^4)\log\left(|cx^2 + b|\right)}{2\,b^4c} - \frac{11\,Bbc^2x^6 - 11\,Ac^3x^6 - 6\,Bb^2cx^4 + 6\,Abc^2x^4 + 3\,Bb^3x^2 - 3\,Ab^2cx^2 + 2\,Ab^3}{12\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(B*b*c^2 - A*c^3)*log(x^2)/b^4 - 1/2*(B*b*c^3 - A*c^4)*log(abs(c*x^2 +
b))/(b^4*c) - 1/12*(11*B*b*c^2*x^6 - 11*A*c^3*x^6 - 6*B*b^2*c*x^4 + 6*A*b*c
^2*x^4 + 3*B*b^3*x^2 - 3*A*b^2*c*x^2 + 2*A*b^3)/(b^4*x^6)
```

Mupad [B]
time = 0.15, size = 92, normalized size = 1.00

$$\frac{\ln\left(c\,x^2+b\right)\,\left(A\,c^3-B\,b\,c^2\right)}{2\,b^4}-\frac{\frac{A}{6\,b}-\frac{x^2\,\left(A\,c-B\,b\right)}{4\,b^2}+\frac{c\,x^4\,\left(A\,c-B\,b\right)}{2\,b^3}}{x^6}-\frac{\ln\left(x\right)\,\left(A\,c^3-B\,b\,c^2\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(b*x^2 + c*x^4)),x)
```

```
[Out] (log(b + c*x^2)*(A*c^3 - B*b*c^2))/(2*b^4) - (A/(6*b) - (x^2*(A*c - B*b))/(
4*b^2) + (c*x^4*(A*c - B*b))/(2*b^3))/x^6 - (log(x)*(A*c^3 - B*b*c^2))/b^4
```

3.58 $\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=133

$$-\frac{b^2(4bB-3Ac)x}{c^5}+\frac{b(3bB-2Ac)x^3}{3c^4}-\frac{(2bB-Ac)x^5}{5c^3}+\frac{Bx^7}{7c^2}-\frac{b^3(bB-Ac)x}{2c^5\,(b+cx^2)}+\frac{b^{5/2}(9bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{11/2}}$$

```
[Out] -b^2*(-3*A*c+4*B*b)*x/c^5+1/3*b*(-2*A*c+3*B*b)*x^3/c^4-1/5*(-A*c+2*B*b)*x^5
/c^3+1/7*B*x^7/c^2-1/2*b^3*(-A*c+B*b)*x/c^5/(c*x^2+b)+1/2*b^(5/2)*(-7*A*c+9
*B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(11/2)
```

Rubi [A]
time = 0.11, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1598, 466, 1824, 211}

$$\frac{b^{5/2}(9bB-7Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{11/2}}-\frac{b^3x(bB-Ac)}{2c^5\,(b+cx^2)}-\frac{b^2x(4bB-3Ac)}{c^5}+\frac{bx^3(3bB-2Ac)}{3c^4}-\frac{x^5(2bB-Ac)}{5c^3}+\frac{Bx^7}{7c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b^2*(4*b*B - 3*A*c)*x)/c^5) + (b*(3*b*B - 2*A*c)*x^3)/(3*c^4) - ((2*b*B
- A*c)*x^5)/(5*c^3) + (B*x^7)/(7*c^2) - (b^3*(b*B - A*c)*x)/(2*c^5*(b + c*x
^2)) + (b^(5/2)*(9*b*B - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{x^8(A+Bx^2)}{(b+cx^2)^2}\,dx$$
$$= -\frac{b^3(bB-Ac)x}{2c^5(b+cx^2)} - \frac{\int \frac{-b^3(bB-Ac)+2b^2c(bB-Ac)x^2-2bc^2(bB-Ac)x^4+2c^3(bB-Ac)x^6-2Bc^4x^8}{b+cx^2}\,dx}{2c^5}$$
$$= -\frac{b^3(bB-Ac)x}{2c^5(b+cx^2)} - \frac{\int \Big(2b^2(4bB-3Ac) - 2bc(3bB-2Ac)x^2 + 2c^2(2bB-Ac)x^4 - 2B \ldots}{2c^5}$$
$$= -\frac{b^2(4bB-3Ac)x}{c^5} + \frac{b(3bB-2Ac)x^3}{3c^4} - \frac{(2bB-Ac)x^5}{5c^3} + \frac{Bx^7}{7c^2} - \frac{b^3(bB-Ac)x}{2c^5(b+cx^2)} + \frac{(b \ldots}{\ldots}$$
$$= -\frac{b^2(4bB-3Ac)x}{c^5} + \frac{b(3bB-2Ac)x^3}{3c^4} - \frac{(2bB-Ac)x^5}{5c^3} + \frac{Bx^7}{7c^2} - \frac{b^3(bB-Ac)x}{2c^5(b+cx^2)} + \frac{b^{5/\ldots}}{\ldots}$$

Mathematica [A]
time = 0.07, size = 134, normalized size = 1.01

$$-\frac{b^2(4bB-3Ac)x}{c^5} + \frac{b(3bB-2Ac)x^3}{3c^4} + \frac{(-2bB+Ac)x^5}{5c^3} + \frac{Bx^7}{7c^2} + \frac{(-b^4B+Ab^3c)\,x}{2c^5(b+cx^2)} + \frac{b^{5/2}(9bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b^2*(4*b*B - 3*A*c)*x)/c^5) + (b*(3*b*B - 2*A*c)*x^3)/(3*c^4) + ((-2*b*B
 + A*c)*x^5)/(5*c^3) + (B*x^7)/(7*c^2) + ((-(b^4*B) + A*b^3*c)*x)/(2*c^5*(b
 + c*x^2)) + (b^(5/2)*(9*b*B - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(11
/2))
```

Maple [A]
time = 0.42, size = 123, normalized size = 0.92

method	result

default	$\frac{\frac{1}{7}B\,c^{3}x^{7}+\frac{1}{5}A\,c^{3}x^{5}-\frac{2}{5}Bb\,c^{2}x^{5}-\frac{2}{3}Ab\,c^{2}x^{3}+B\,b^{2}c\,x^{3}+3A\,b^{2}cx-4B\,b^{3}x}{c^{5}}-\frac{b^{3}\left(\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)x}{c\,x^{2}+b}+\frac{(7Ac-9Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^{5}}$
risch	$\frac{B\,x^{7}}{7c^{2}}+\frac{A\,x^{5}}{5c^{2}}-\frac{2Bb\,x^{5}}{5c^{3}}-\frac{2Ab\,x^{3}}{3c^{3}}+\frac{B\,b^{2}x^{3}}{c^{4}}+\frac{3A\,b^{2}x}{c^{4}}-\frac{4B\,b^{3}x}{c^{5}}+\frac{\left(\frac{1}{2}A\,b^{3}c-\frac{1}{2}B\,b^{4}\right)x}{c^{5}(c\,x^{2}+b)}+\frac{7\sqrt{-bc}\;b^{2}\ln\left(-\sqrt{-bc}\;x-\right.}{4c^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(1/7*B*c^3*x^7+1/5*A*c^3*x^5-2/5*B*b*c^2*x^5-2/3*A*b*c^2*x^3+B*b^2*c*
x^3+3*A*b^2*c*x-4*B*b^3*x)-b^3/c^5*((-1/2*A*c+1/2*B*b)*x/(c*x^2+b)+1/2*(7*A
*c-9*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Maxima [A]
time = 0.50, size = 136, normalized size = 1.02

$$-\frac{(Bb^{4}-Ab^{3}c)x}{2\,(c^{6}x^{2}+bc^{5})}+\frac{(9\,Bb^{4}-7\,Ab^{3}c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^{5}}+\frac{15\,Bc^{3}x^{7}-21\,(2\,Bbc^{2}-Ac^{3})x^{5}+35\,(3\,Bb^{2}c-2\,Abc^{2})x^{3}-105\,(4\,Bb^{3}-3\,Ab^{2}c)x}{105\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b^4 - A*b^3*c)*x/(c^6*x^2 + b*c^5) + 1/2*(9*B*b^4 - 7*A*b^3*c)*arct
an(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/105*(15*B*c^3*x^7 - 21*(2*B*b*c^2 - A
*c^3)*x^5 + 35*(3*B*b^2*c - 2*A*b*c^2)*x^3 - 105*(4*B*b^3 - 3*A*b^2*c)*x)/c
^5
```

Fricas [A]
time = 2.08, size = 350, normalized size = 2.63

$$\left[\frac{60\,Bc^{4}x^{9}-12\,(9\,Bbc^{3}-7\,Ac^{4})x^{7}+28\,(9\,Bb^{2}c^{2}-7\,Abc^{3})x^{5}-140\,(9\,Bb^{3}c-7\,Ab^{2}c^{2})x^{3}-105\,(9\,Bb^{4}-7\,Ab^{3}c+(9\,Bb^{3}c-7\,Ab^{2}c^{2})x^{2})\sqrt{-\frac{b}{c}}\log\left(\frac{cx^{2}-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^{2}+b}\right)-210\,(9\,Bb^{4}-7\,Ab^{3}c)x}{420\,(c^{6}x^{2}+bc^{5})},\ \frac{30\,Bc^{4}x^{9}-6\,(9\,Bbc^{3}-7\,Ac^{4})x^{7}+14\,(9\,Bb^{2}c^{2}-7\,Abc^{3})x^{5}-70\,(9\,Bb^{3}c-7\,Ab^{2}c^{2})x^{3}+105\,(9\,Bb^{4}-7\,Ab^{3}c+(9\,Bb^{3}c-7\,Ab^{2}c^{2})x^{2})\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)-105\,(9\,Bb^{4}-7\,Ab^{3}c)x}{210\,(c^{6}x^{2}+bc^{5})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*B*c^4*x^9 - 12*(9*B*b*c^3 - 7*A*c^4)*x^7 + 28*(9*B*b^2*c^2 - 7*A
*b*c^3)*x^5 - 140*(9*B*b^3*c - 7*A*b^2*c^2)*x^3 - 105*(9*B*b^4 - 7*A*b^3*c
+ (9*B*b^3*c - 7*A*b^2*c^2)*x^2)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) -
 b)/(c*x^2 + b)) - 210*(9*B*b^4 - 7*A*b^3*c)*x)/(c^6*x^2 + b*c^5), 1/210*(3
0*B*c^4*x^9 - 6*(9*B*b*c^3 - 7*A*c^4)*x^7 + 14*(9*B*b^2*c^2 - 7*A*b*c^3)*x^
5 - 70*(9*B*b^3*c - 7*A*b^2*c^2)*x^3 + 105*(9*B*b^4 - 7*A*b^3*c + (9*B*b^3*
```

```
c - 7*A*b^2*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - 105*(9*B*b^4 - 7*
A*b^3*c)*x)/(c^6*x^2 + b*c^5)]
```

Sympy [A]
time = 0.41, size = 238, normalized size = 1.79

$$\frac{Bx^7}{7c^2}+x^5\left(\frac{A}{5c^2}-\frac{2Bb}{5c^3}\right)+x^3\left(-\frac{2Ab}{3c^3}+\frac{Bb^2}{c^4}\right)+x\left(\frac{3Ab^2}{c^4}-\frac{4Bb^3}{c^5}\right)+\frac{x(Ab^3c-Bb^4)}{2bc^5+2c^6x^2}-\frac{\sqrt{-\frac{b^5}{c^{11}}}(-7Ac+9Bb)\log\left(-\frac{c^5\sqrt{-\frac{b^5}{c^{11}}}(-7Ac+9Bb)}{-7Ab^2c+9Bb^3}+x\right)}{4}+\frac{\sqrt{-\frac{b^5}{c^{11}}}(-7Ac+9Bb)\log\left(\frac{c^5\sqrt{-\frac{b^5}{c^{11}}}(-7Ac+9Bb)}{-7Ab^2c+9Bb^3}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**7/(7*c**2) + x**5*(A/(5*c**2) - 2*B*b/(5*c**3)) + x**3*(-2*A*b/(3*c**3
) + B*b**2/c**4) + x*(3*A*b**2/c**4 - 4*B*b**3/c**5) + x*(A*b**3*c - B*b**4
)/(2*b*c**5 + 2*c**6*x**2) - sqrt(-b**5/c**11)*(-7*A*c + 9*B*b)*log(-c**5*s
qrt(-b**5/c**11)*(-7*A*c + 9*B*b)/(-7*A*b**2*c + 9*B*b**3) + x)/4 + sqrt(-b
**5/c**11)*(-7*A*c + 9*B*b)*log(c**5*sqrt(-b**5/c**11)*(-7*A*c + 9*B*b)/(-7
*A*b**2*c + 9*B*b**3) + x)/4
```

Giac [A]
time = 0.63, size = 139, normalized size = 1.05

$$\frac{(9\,Bb^4-7\,Ab^3c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^5}-\frac{Bb^4x-Ab^3cx}{2\,(cx^2+b)c^5}+\frac{15\,Bc^{12}x^7-42\,Bbc^{11}x^5+21\,Ac^{12}x^5+105\,Bb^2c^{10}x^3-70\,Abc^{11}x^3-420\,Bb^3c^9x+315\,Ab^2c^{10}x}{105\,c^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*B*b^4 - 7*A*b^3*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) - 1/2*(B*b^
4*x - A*b^3*c*x)/((c*x^2 + b)*c^5) + 1/105*(15*B*c^12*x^7 - 42*B*b*c^11*x^5
 + 21*A*c^12*x^5 + 105*B*b^2*c^10*x^3 - 70*A*b*c^11*x^3 - 420*B*b^3*c^9*x +
 315*A*b^2*c^10*x)/c^14
```

Mupad [B]
time = 0.10, size = 203, normalized size = 1.53

$$x\left(\frac{2\,b\,\left(\frac{2\,b\,\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{c}+\frac{B\,b^2}{c^4}\right)}{c}-\frac{b^2\,\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{c^2}\right)+x^5\,\left(\frac{A}{5\,c^2}-\frac{2\,B\,b}{5\,c^3}\right)-x^3\,\left(\frac{2\,b\,\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{3\,c}+\frac{B\,b^2}{3\,c^4}\right)+\frac{B\,x^7}{7\,c^2}-\frac{x\,\left(\frac{B\,b^4}{2}-\frac{A\,b^3\,c}{2}\right)}{c^6\,x^2+b\,c^5}+\frac{b^{5/2}\,\mathrm{atan}\left(\frac{b^{5/2}\,\sqrt{c}\,x\,(7\,A\,c-9\,B\,b)}{9\,B\,b^4-7\,A\,b^3\,c}\right)\,(7\,A\,c-9\,B\,b)}{2\,c^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^12*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x*((2*b*((2*b*(A/c^2 - (2*B*b)/c^3))/c + (B*b^2)/c^4))/c - (b^2*(A/c^2 - (2
*B*b)/c^3))/c^2) + x^5*(A/(5*c^2) - (2*B*b)/(5*c^3)) - x^3*((2*b*(A/c^2 - (
2*B*b)/c^3))/(3*c) + (B*b^2)/(3*c^4)) + (B*x^7)/(7*c^2) - (x*((B*b^4)/2 - (
A*b^3*c)/2))/(b*c^5 + c^6*x^2) + (b^(5/2)*atan((b^(5/2)*c^(1/2)*x*(7*A*c -
9*B*b))/(9*B*b^4 - 7*A*b^3*c))*(7*A*c - 9*B*b))/(2*c^(11/2))
```

3.59 $\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=105

$$\frac{b(3bB-2Ac)x^2}{2c^4}-\frac{(2bB-Ac)x^4}{4c^3}+\frac{Bx^6}{6c^2}-\frac{b^3(bB-Ac)}{2c^5\,(b+cx^2)}-\frac{b^2(4bB-3Ac)\log\left(b+cx^2\right)}{2c^5}$$

```
[Out] 1/2*b*(-2*A*c+3*B*b)*x^2/c^4-1/4*(-A*c+2*B*b)*x^4/c^3+1/6*B*x^6/c^2-1/2*b^3
*(-A*c+B*b)/c^5/(c*x^2+b)-1/2*b^2*(-3*A*c+4*B*b)*ln(c*x^2+b)/c^5
```

Rubi [A]
time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 78}

$$-\frac{b^3(bB-Ac)}{2c^5\,(b+cx^2)}-\frac{b^2(4bB-3Ac)\log\left(b+cx^2\right)}{2c^5}+\frac{bx^2(3bB-2Ac)}{2c^4}-\frac{x^4(2bB-Ac)}{4c^3}+\frac{Bx^6}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*(3*b*B - 2*A*c)*x^2)/(2*c^4) - ((2*b*B - A*c)*x^4)/(4*c^3) + (B*x^6)/(6*
c^2) - (b^3*(b*B - A*c))/(2*c^5*(b + c*x^2)) - (b^2*(4*b*B - 3*A*c)*Log[b +
 c*x^2])/(2*c^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{x^7(A+Bx^2)}{(b+cx^2)^2}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(b+cx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b(3bB-2Ac)}{c^4} + \frac{(-2bB+Ac)x}{c^3} + \frac{Bx^2}{c^2} + \frac{b^3(bB-Ac)}{c^4(b+cx)^2} - \frac{b^2(4bB-3A}{c^4(b+cx)}\right.\right.$$
$$= \frac{b(3bB-2Ac)x^2}{2c^4} - \frac{(2bB-Ac)x^4}{4c^3} + \frac{Bx^6}{6c^2} - \frac{b^3(bB-Ac)}{2c^5(b+cx^2)} - \frac{b^2(4bB-3Ac)\log(b+c}{2c^5}$$

Mathematica [A]
time = 0.05, size = 93, normalized size = 0.89

$$\frac{6bc(3bB-2Ac)x^2 + 3c^2(-2bB+Ac)x^4 + 2Bc^3x^6 + \frac{6b^3(-bB+Ac)}{b+cx^2} + 6b^2(-4bB+3Ac)\log(b+cx^2)}{12c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (6*b*c*(3*b*B - 2*A*c)*x^2 + 3*c^2*(-2*b*B + A*c)*x^4 + 2*B*c^3*x^6 + (6*b^
3*(-(b*B) + A*c))/(b + c*x^2) + 6*b^2*(-4*b*B + 3*A*c)*Log[b + c*x^2])/(12*
c^5)
```

Maple [A]
time = 0.38, size = 103, normalized size = 0.98

method	result	size
default	$-\frac{-\frac{B c^2 x^6}{6}+\frac{\left(-A c^2+2bBc\right)x^4}{4}+\frac{\left(2Abc-3b^2B\right)x^2}{2}}{c^4}+\frac{b^2\left(\frac{(3Ac-4Bb)\ln\left(c x^2+b\right)}{c}+\frac{b(Ac-Bb)}{c\left(c x^2+b\right)}\right)}{2c^4}$	103
norman	$\frac{\frac{B x^{11}}{6c}+\frac{(3Ac-4Bb)x^9}{12c^2}-\frac{b(3Ac-4Bb)x^7}{4c^3}+\frac{b\left(3A b^2 c-4B b^3\right)x^3}{2c^5}}{x^3(c x^2+b)}+\frac{b^2(3Ac-4Bb)\ln(c x^2+b)}{2c^5}$	108
risch	$\frac{B x^6}{6c^2}+\frac{A x^4}{4c^2}-\frac{x^4 bB}{2c^3}-\frac{Ab x^2}{c^3}+\frac{3b^2 B x^2}{2c^4}+\frac{b^3 A}{2c^4(c x^2+b)}-\frac{b^4 B}{2c^5(c x^2+b)}+\frac{3b^2\ln(c x^2+b)A}{2c^4}-\frac{2b^3\ln(c x^2+b)B}{c^5}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^4*(-1/6*B*c^2*x^6+1/4*(-A*c^2+2*B*b*c)*x^4+1/2*(2*A*b*c-3*B*b^2)*x^2)+
1/2*b^2/c^4*((3*A*c-4*B*b)/c*ln(c*x^2+b)+b*(A*c-B*b)/c/(c*x^2+b))
```

Maxima [A]
time = 0.27, size = 107, normalized size = 1.02

$$-\frac{Bb^4 - Ab^3c}{2\,(c^6x^2 + bc^5)} + \frac{2\,Bc^2x^6 - 3\,(2\,Bbc - Ac^2)x^4 + 6\,(3\,Bb^2 - 2\,Abc)x^2}{12\,c^4} - \frac{(4\,Bb^3 - 3\,Ab^2c)\log\left(cx^2 + b\right)}{2\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b^4 - A*b^3*c)/(c^6*x^2 + b*c^5) + 1/12*(2*B*c^2*x^6 - 3*(2*B*b*c -
 A*c^2)*x^4 + 6*(3*B*b^2 - 2*A*b*c)*x^2)/c^4 - 1/2*(4*B*b^3 - 3*A*b^2*c)*lo
g(c*x^2 + b)/c^5
```

Fricas [A]
time = 1.09, size = 148, normalized size = 1.41

$$\frac{2\,Bc^4x^8 - (4\,Bbc^3 - 3\,Ac^4)x^6 - 6\,Bb^4 + 6\,Ab^3c + 3\,(4\,Bb^2c^2 - 3\,Abc^3)x^4 + 6\,(3\,Bb^3c - 2\,Ab^2c^2)x^2 - 6\,(4\,Bb^4 - 3\,Ab^3c + (4\,Bb^3c - 3\,Ab^2c^2)x^2)\log\left(cx^2 + b\right)}{12\,(c^6x^2 + bc^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*c^4*x^8 - (4*B*b*c^3 - 3*A*c^4)*x^6 - 6*B*b^4 + 6*A*b^3*c + 3*(4*
B*b^2*c^2 - 3*A*b*c^3)*x^4 + 6*(3*B*b^3*c - 2*A*b^2*c^2)*x^2 - 6*(4*B*b^4 -
 3*A*b^3*c + (4*B*b^3*c - 3*A*b^2*c^2)*x^2)*log(c*x^2 + b))/(c^6*x^2 + b*c^
5)
```

Sympy [A]
time = 0.39, size = 104, normalized size = 0.99

$$\frac{Bx^6}{6c^2} - \frac{b^2(-3Ac + 4Bb)\log\left(b + cx^2\right)}{2c^5} + x^4\left(\frac{A}{4c^2} - \frac{Bb}{2c^3}\right) + x^2\left(-\frac{Ab}{c^3} + \frac{3Bb^2}{2c^4}\right) + \frac{Ab^3c - Bb^4}{2bc^5 + 2c^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**6/(6*c**2) - b**2*(-3*A*c + 4*B*b)*log(b + c*x**2)/(2*c**5) + x**4*(A/
(4*c**2) - B*b/(2*c**3)) + x**2*(-A*b/c**3 + 3*B*b**2/(2*c**4)) + (A*b**3*c
 - B*b**4)/(2*b*c**5 + 2*c**6*x**2)
```

Giac [A]
time = 0.74, size = 135, normalized size = 1.29

$$-\frac{(4\,Bb^3 - 3\,Ab^2c)\log\left(|cx^2 + b|\right)}{2\,c^5} + \frac{2\,Bc^4x^6 - 6\,Bbc^3x^4 + 3\,Ac^4x^4 + 18\,Bb^2c^2x^2 - 12\,Abc^3x^2}{12\,c^6} + \frac{4\,Bb^3cx^2 - 3\,Ab^2c^2x^2 + 3\,Bb^4 - 2\,Ab^3c}{2\,(cx^2 + b)c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*B*b^3 - 3*A*b^2*c)*log(abs(c*x^2 + b))/c^5 + 1/12*(2*B*c^4*x^6 - 6*
B*b*c^3*x^4 + 3*A*c^4*x^4 + 18*B*b^2*c^2*x^2 - 12*A*b*c^3*x^2)/c^6 + 1/2*(4
*B*b^3*c*x^2 - 3*A*b^2*c^2*x^2 + 3*B*b^4 - 2*A*b^3*c)/((c*x^2 + b)*c^5)
```

Mupad [B]
time = 0.10, size = 121, normalized size = 1.15

$$x^4\left(\frac{A}{4\,c^2}-\frac{B\,b}{2\,c^3}\right)-x^2\left(\frac{b\,\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{c}+\frac{B\,b^2}{2\,c^4}\right)+\frac{B\,x^6}{6\,c^2}-\frac{\ln\left(c\,x^2+b\right)\,\left(4\,B\,b^3-3\,A\,b^2\,c\right)}{2\,c^5}-\frac{B\,b^4-A\,b^3\,c}{2\,c\,\left(c^5\,x^2+b\,c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^4*(A/(4*c^2) - (B*b)/(2*c^3)) - x^2*((b*(A/c^2 - (2*B*b)/c^3))/c + (B*b^2
)/(2*c^4)) + (B*x^6)/(6*c^2) - (log(b + c*x^2)*(4*B*b^3 - 3*A*b^2*c))/(2*c^
5) - (B*b^4 - A*b^3*c)/(2*c*(b*c^4 + c^5*x^2))
```

3.60 $\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=110

$$\frac{b(3bB-2Ac)x}{c^4}-\frac{(2bB-Ac)x^3}{3c^3}+\frac{Bx^5}{5c^2}+\frac{b^2(bB-Ac)x}{2c^4\,(b+cx^2)}-\frac{b^{3/2}(7bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}$$

```
[Out] b*(-2*A*c+3*B*b)*x/c^4-1/3*(-A*c+2*B*b)*x^3/c^3+1/5*B*x^5/c^2+1/2*b^2*(-A*c
+B*b)*x/c^4/(c*x^2+b)-1/2*b^(3/2)*(-5*A*c+7*B*b)*arctan(x*c^(1/2)/b^(1/2))/
c^(9/2)
```

Rubi [A]
time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1598, 466, 1824, 211}

$$-\frac{b^{3/2}(7bB-5Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}+\frac{b^2x(bB-Ac)}{2c^4\,(b+cx^2)}+\frac{bx(3bB-2Ac)}{c^4}-\frac{x^3(2bB-Ac)}{3c^3}+\frac{Bx^5}{5c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*(3*b*B - 2*A*c)*x)/c^4 - ((2*b*B - A*c)*x^3)/(3*c^3) + (B*x^5)/(5*c^2) +
 (b^2*(b*B - A*c)*x)/(2*c^4*(b + c*x^2)) - (b^(3/2)*(7*b*B - 5*A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{x^6(A+Bx^2)}{(b+cx^2)^2}\,dx$$
$$= \frac{b^2(bB-Ac)x}{2c^4(b+cx^2)} - \frac{\int \frac{b^2(bB-Ac)-2bc(bB-Ac)x^2+2c^2(bB-Ac)x^4-2Bc^3x^6}{b+cx^2}\,dx}{2c^4}$$
$$= \frac{b^2(bB-Ac)x}{2c^4(b+cx^2)} - \frac{\int \left(-2b(3bB-2Ac)+2c(2bB-Ac)x^2-2Bc^2x^4+\frac{7b^3B-5Ab^2c}{b+cx^2}\right)\,dx}{2c^4}$$
$$= \frac{b(3bB-2Ac)x}{c^4} - \frac{(2bB-Ac)x^3}{3c^3} + \frac{Bx^5}{5c^2} + \frac{b^2(bB-Ac)x}{2c^4(b+cx^2)} - \frac{(b^2(7bB-5Ac))\int \frac{1}{b+cx^2}}{2c^4}$$
$$= \frac{b(3bB-2Ac)x}{c^4} - \frac{(2bB-Ac)x^3}{3c^3} + \frac{Bx^5}{5c^2} + \frac{b^2(bB-Ac)x}{2c^4(b+cx^2)} - \frac{b^{3/2}(7bB-5Ac)\tan^{-1}\left(\right.}{2c^{9/2}}$$

Mathematica [A]
time = 0.06, size = 111, normalized size = 1.01

$$\frac{b(3bB-2Ac)x}{c^4} + \frac{(-2bB+Ac)x^3}{3c^3} + \frac{Bx^5}{5c^2} - \frac{(-b^3B+Ab^2c)\,x}{2c^4(b+cx^2)} - \frac{b^{3/2}(7bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*(3*b*B - 2*A*c)*x)/c^4 + ((-2*b*B + A*c)*x^3)/(3*c^3) + (B*x^5)/(5*c^2)
- ((-(b^3*B) + A*b^2*c)*x)/(2*c^4*(b + c*x^2)) - (b^(3/2)*(7*b*B - 5*A*c)*A
rcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Maple [A]
time = 0.39, size = 100, normalized size = 0.91

method	result
default	$-\frac{-\frac{1}{5}B\,c^{2}x^{5}-\frac{1}{3}A\,c^{2}x^{3}+\frac{2}{3}Bbc\,x^{3}+2Abcx-3b^{2}Bx}{c^{4}}+\frac{b^{2}\left(\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)x}{c\,x^{2}+b}+\frac{(5Ac-7Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^{4}}$
risch	$\frac{B\,x^{5}}{5c^{2}}+\frac{A\,x^{3}}{3c^{2}}-\frac{2Bb\,x^{3}}{3c^{3}}-\frac{2Abx}{c^{3}}+\frac{3b^{2}Bx}{c^{4}}+\frac{\left(-\frac{1}{2}A\,b^{2}c+\frac{1}{2}B\,b^{3}\right)x}{c^{4}(c\,x^{2}+b)}+\frac{5\sqrt{-bc}\;b\ln\left(-\sqrt{-bc}\;x+b\right)A}{4c^{4}}-\frac{7\sqrt{-bc}\;b^{2}\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^4*(-1/5*B*c^2*x^5-1/3*A*c^2*x^3+2/3*B*b*c*x^3+2*A*b*c*x-3*b^2*B*x)+b^2
/c^4*((-1/2*A*c+1/2*B*b)*x/(c*x^2+b)+1/2*(5*A*c-7*B*b)/(b*c)^(1/2)*arctan(c
*x/(b*c)^(1/2)))
```

Maxima [A]
time = 0.51, size = 112, normalized size = 1.02

$$\frac{(Bb^3-Ab^2c)x}{2\,(c^5x^2+bc^4)}-\frac{(7\,Bb^3-5\,Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^4}+\frac{3\,Bc^2x^5-5\,(2\,Bbc-Ac^2)x^3+15\,(3\,Bb^2-2\,Abc)x}{15\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b^3 - A*b^2*c)*x/(c^5*x^2 + b*c^4) - 1/2*(7*B*b^3 - 5*A*b^2*c)*arcta
n(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/15*(3*B*c^2*x^5 - 5*(2*B*b*c - A*c^2)*
x^3 + 15*(3*B*b^2 - 2*A*b*c)*x)/c^4
```

Fricas [A]
time = 1.84, size = 298, normalized size = 2.71

$$\left[\frac{12\,Bc^3x^7-4\,(7\,Bbc^2-5\,Ac^3)x^5+20\,(7\,Bb^2c-5\,Abc^2)x^3-15\,(7\,Bb^3-5\,Ab^2c+(7\,Bb^2c-5\,Abc^2)x^2)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+30\,(7\,Bb^3-5\,Ab^2c)x}{60\,(c^5x^2+bc^4)},\ \frac{6\,Bc^3x^7-2\,(7\,Bbc^2-5\,Ac^3)x^5+10\,(7\,Bb^2c-5\,Abc^2)x^3-15\,(7\,Bb^3-5\,Ab^2c+(7\,Bb^2c-5\,Abc^2)x^2)\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)+15\,(7\,Bb^3-5\,Ab^2c)x}{30\,(c^5x^2+bc^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*B*c^3*x^7 - 4*(7*B*b*c^2 - 5*A*c^3)*x^5 + 20*(7*B*b^2*c - 5*A*b*c
^2)*x^3 - 15*(7*B*b^3 - 5*A*b^2*c + (7*B*b^2*c - 5*A*b*c^2)*x^2)*sqrt(-b/c)
*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 30*(7*B*b^3 - 5*A*b^2*c)
*x)/(c^5*x^2 + b*c^4), 1/30*(6*B*c^3*x^7 - 2*(7*B*b*c^2 - 5*A*c^3)*x^5 + 10
*(7*B*b^2*c - 5*A*b*c^2)*x^3 - 15*(7*B*b^3 - 5*A*b^2*c + (7*B*b^2*c - 5*A*b
```

```
*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) + 15*(7*B*b^3 - 5*A*b^2*c)*x)/
(c^5*x^2 + b*c^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(104) = 208.
time = 0.38, size = 211, normalized size = 1.92

$$\frac{Bx^5}{5c^2}+x^3\left(\frac{A}{3c^2}-\frac{2Bb}{3c^3}\right)+x\left(-\frac{2Ab}{c^3}+\frac{3Bb^2}{c^4}\right)+\frac{x(-Ab^2c+Bb^3)}{2bc^4+2c^5x^2}+\frac{\sqrt{-\frac{b^3}{c^9}}\,(-5Ac+7Bb)\log\left(-\frac{c^4\sqrt{-\frac{b^3}{c^9}}\,(-5Ac+7Bb)}{-5Abc+7Bb^2}+x\right)}{4}-\frac{\sqrt{-\frac{b^3}{c^9}}\,(-5Ac+7Bb)\log\left(\frac{c^4\sqrt{-\frac{b^3}{c^9}}\,(-5Ac+7Bb)}{-5Abc+7Bb^2}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**5/(5*c**2) + x**3*(A/(3*c**2) - 2*B*b/(3*c**3)) + x*(-2*A*b/c**3 + 3*B
*b**2/c**4) + x*(-A*b**2*c + B*b**3)/(2*b*c**4 + 2*c**5*x**2) + sqrt(-b**3/
c**9)*(-5*A*c + 7*B*b)*log(-c**4*sqrt(-b**3/c**9)*(-5*A*c + 7*B*b)/(-5*A*b*
c + 7*B*b**2) + x)/4 - sqrt(-b**3/c**9)*(-5*A*c + 7*B*b)*log(c**4*sqrt(-b**
3/c**9)*(-5*A*c + 7*B*b)/(-5*A*b*c + 7*B*b**2) + x)/4
```

Giac [A]
time = 0.89, size = 115, normalized size = 1.05

$$-\frac{(7\,Bb^3-5\,Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^4}+\frac{Bb^3x-Ab^2cx}{2\,(cx^2+b)c^4}+\frac{3\,Bc^8x^5-10\,Bbc^7x^3+5\,Ac^8x^3+45\,Bb^2c^6x-30\,Abc^7x}{15\,c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*B*b^3 - 5*A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/2*(B*b
^3*x - A*b^2*c*x)/((c*x^2 + b)*c^4) + 1/15*(3*B*c^8*x^5 - 10*B*b*c^7*x^3 +
5*A*c^8*x^3 + 45*B*b^2*c^6*x - 30*A*b*c^7*x)/c^10
```

Mupad [B]
time = 0.09, size = 141, normalized size = 1.28

$$x^3\left(\frac{A}{3\,c^2}-\frac{2\,B\,b}{3\,c^3}\right)-x\left(\frac{2\,b\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{c}+\frac{B\,b^2}{c^4}\right)+\frac{B\,x^5}{5\,c^2}+\frac{x\left(\frac{B\,b^3}{2}-\frac{A\,b^2\,c}{2}\right)}{c^5\,x^2+b\,c^4}-\frac{b^{3/2}\,\mathrm{atan}\left(\frac{b^{3/2}\,\sqrt{c}\,x\,(5\,A\,c-7\,B\,b)}{7\,B\,b^3-5\,A\,b^2\,c}\right)\,(5\,A\,c-7\,B\,b)}{2\,c^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^3*(A/(3*c^2) - (2*B*b)/(3*c^3)) - x*((2*b*(A/c^2 - (2*B*b)/c^3))/c + (B*b
^2)/c^4) + (B*x^5)/(5*c^2) + (x*((B*b^3)/2 - (A*b^2*c)/2))/(b*c^4 + c^5*x^2
) - (b^(3/2)*atan((b^(3/2)*c^(1/2)*x*(5*A*c - 7*B*b))/(7*B*b^3 - 5*A*b^2*c)
)*(5*A*c - 7*B*b))/(2*c^(9/2))
```

3.61 $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=83

$$-\frac{(2bB-Ac)x^2}{2c^3}+\frac{Bx^4}{4c^2}+\frac{b^2(bB-Ac)}{2c^4(b+cx^2)}+\frac{b(3bB-2Ac)\log(b+cx^2)}{2c^4}$$

```
[Out] -1/2*(-A*c+2*B*b)*x^2/c^3+1/4*B*x^4/c^2+1/2*b^2*(-A*c+B*b)/c^4/(c*x^2+b)+1/
2*b*(-2*A*c+3*B*b)*ln(c*x^2+b)/c^4
```

Rubi [A]
time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$\frac{b^2(bB-Ac)}{2c^4(b+cx^2)}+\frac{b(3bB-2Ac)\log(b+cx^2)}{2c^4}-\frac{x^2(2bB-Ac)}{2c^3}+\frac{Bx^4}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((2*b*B - A*c)*x^2)/c^3 + (B*x^4)/(4*c^2) + (b^2*(b*B - A*c))/(2*c^4*(
b + c*x^2)) + (b*(3*b*B - 2*A*c)*Log[b + c*x^2])/(2*c^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \int \frac{x^5(A+Bx^2)}{(b+cx^2)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(b+cx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{-2bB+Ac}{c^3}+\frac{Bx}{c^2}-\frac{b^2(bB-Ac)}{c^3(b+cx)^2}+\frac{b(3bB-2Ac)}{c^3(b+cx)}\right)dx, x, x^2\right) \\
&= -\frac{(2bB-Ac)x^2}{2c^3}+\frac{Bx^4}{4c^2}+\frac{b^2(bB-Ac)}{2c^4(b+cx^2)}+\frac{b(3bB-2Ac)\log(b+cx^2)}{2c^4}
\end{aligned}
$$

Mathematica [**A**]
time = 0.04, size = 72, normalized size = 0.87

$$
\frac{2c(-2bB+Ac)x^2+Bc^2x^4+\frac{2b^2(bB-Ac)}{b+cx^2}+2b(3bB-2Ac)\log(b+cx^2)}{4c^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*c*(-2*b*B + A*c)*x^2 + B*c^2*x^4 + (2*b^2*(b*B - A*c))/(b + c*x^2) + 2*b
*(3*b*B - 2*A*c)*Log[b + c*x^2])/(4*c^4)
```

Maple [**A**]
time = 0.41, size = 76, normalized size = 0.92

method	result	size
default	$\frac{(Bcx^2+Ac-2Bb)^2}{4c^4B}-\frac{b\left(\frac{(2Ac-3Bb)\ln(cx^2+b)}{c}+\frac{b(Ac-Bb)}{c(cx^2+b)}\right)}{2c^3}$	76
norman	$\frac{\frac{Bx^9}{4c}+\frac{(2Ac-3Bb)x^7}{4c^2}-\frac{b(2Abc-3b^2B)x^3}{2c^4}}{x^3(cx^2+b)}-\frac{b(2Ac-3Bb)\ln(cx^2+b)}{2c^4}$	86
risch	$\frac{Bx^4}{4c^2}+\frac{Ax^2}{2c^2}-\frac{Bbx^2}{c^3}+\frac{A^2}{4c^2B}-\frac{Ab}{c^3}+\frac{Bb^2}{c^4}-\frac{b^2A}{2c^3(cx^2+b)}+\frac{b^3B}{2c^4(cx^2+b)}-\frac{b\ln(cx^2+b)A}{c^3}+\frac{3b^2\ln(cx^2+b)B}{2c^4}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(B*c*x^2+A*c-2*B*b)^2/c^4/B-1/2*b/c^3*((2*A*c-3*B*b)/c*ln(c*x^2+b)+b*(A
*c-B*b)/c/(c*x^2+b))
```

Maxima [A]
time = 0.28, size = 82, normalized size = 0.99

$$\frac{Bb^3 - Ab^2c}{2\left(c^5x^2 + bc^4\right)} + \frac{Bcx^4 - 2\left(2\,Bb - Ac\right)x^2}{4\,c^3} + \frac{\left(3\,Bb^2 - 2\,Abc\right)\log\left(cx^2 + b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b^3 - A*b^2*c)/(c^5*x^2 + b*c^4) + 1/4*(B*c*x^4 - 2*(2*B*b - A*c)*x^
2)/c^3 + 1/2*(3*B*b^2 - 2*A*b*c)*log(c*x^2 + b)/c^4
```

Fricas [A]
time = 1.31, size = 121, normalized size = 1.46

$$\frac{Bc^3x^6 - \left(3\,Bbc^2 - 2\,Ac^3\right)x^4 + 2\,Bb^3 - 2\,Ab^2c - 2\left(2\,Bb^2c - Abc^2\right)x^2 + 2\left(3\,Bb^3 - 2\,Ab^2c + \left(3\,Bb^2c - 2\,Abc^2\right)x^2\right)\log\left(cx^2 + b\right)}{4\left(c^5x^2 + bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^3*x^6 - (3*B*b*c^2 - 2*A*c^3)*x^4 + 2*B*b^3 - 2*A*b^2*c - 2*(2*B*b
^2*c - A*b*c^2)*x^2 + 2*(3*B*b^3 - 2*A*b^2*c + (3*B*b^2*c - 2*A*b*c^2)*x^2)
*log(c*x^2 + b))/(c^5*x^2 + b*c^4)
```

Sympy [A]
time = 0.36, size = 78, normalized size = 0.94

$$\frac{Bx^4}{4c^2} + \frac{b(-2Ac + 3Bb)\log\left(b + cx^2\right)}{2c^4} + x^2\left(\frac{A}{2c^2} - \frac{Bb}{c^3}\right) + \frac{-Ab^2c + Bb^3}{2bc^4 + 2c^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**4/(4*c**2) + b*(-2*A*c + 3*B*b)*log(b + c*x**2)/(2*c**4) + x**2*(A/(2*
c**2) - B*b/c**3) + (-A*b**2*c + B*b**3)/(2*b*c**4 + 2*c**5*x**2)
```

Giac [A]
time = 0.82, size = 106, normalized size = 1.28

$$\frac{\left(3\,Bb^2 - 2\,Abc\right)\log\left(\left|cx^2 + b\right|\right)}{2\,c^4} + \frac{Bc^2x^4 - 4\,Bbcx^2 + 2\,Ac^2x^2}{4\,c^4} - \frac{3\,Bb^2cx^2 - 2\,Abc^2x^2 + 2\,Bb^3 - Ab^2c}{2\left(cx^2 + b\right)c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*b^2 - 2*A*b*c)*log(abs(c*x^2 + b))/c^4 + 1/4*(B*c^2*x^4 - 4*B*b*c*
x^2 + 2*A*c^2*x^2)/c^4 - 1/2*(3*B*b^2*c*x^2 - 2*A*b*c^2*x^2 + 2*B*b^3 - A*b
^2*c)/((c*x^2 + b)*c^4)
```

Mupad [B]
time = 0.07, size = 86, normalized size = 1.04

$$x^2\left(\frac{A}{2\,c^2}-\frac{B\,b}{c^3}\right)+\frac{\ln\left(c\,x^2+b\right)\,\left(3\,B\,b^2-2\,A\,b\,c\right)}{2\,c^4}+\frac{B\,x^4}{4\,c^2}+\frac{B\,b^3-A\,b^2\,c}{2\,c\,\left(c^4\,x^2+b\,c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^2*(A/(2*c^2) - (B*b)/c^3) + (log(b + c*x^2)*(3*B*b^2 - 2*A*b*c))/(2*c^4)
+ (B*x^4)/(4*c^2) + (B*b^3 - A*b^2*c)/(2*c*(b*c^3 + c^4*x^2))
```

3.62 $\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=89

$$-\frac{(2bB-Ac)x}{c^3}+\frac{Bx^3}{3c^2}-\frac{b(bB-Ac)x}{2c^3(b+cx^2)}+\frac{\sqrt{b}\,(5bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}$$

```
[Out] -(-A*c+2*B*b)*x/c^3+1/3*B*x^3/c^2-1/2*b*(-A*c+B*b)*x/c^3/(c*x^2+b)+1/2*(-3*
A*c+5*B*b)*arctan(x*c^(1/2)/b^(1/2))*b^(1/2)/c^(7/2)
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1598, 466, 1167, 211}

$$\frac{\sqrt{b}\,(5bB-3Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}-\frac{bx(bB-Ac)}{2c^3(b+cx^2)}-\frac{x(2bB-Ac)}{c^3}+\frac{Bx^3}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(((2*b*B - A*c)*x)/c^3) + (B*x^3)/(3*c^2) - (b*(b*B - A*c)*x)/(2*c^3*(b +
c*x^2)) + (Sqrt[b]*(5*b*B - 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \int \frac{x^4(A+Bx^2)}{(b+cx^2)^2}\,dx\\ &= -\frac{b(bB-Ac)x}{2c^3(b+cx^2)}-\frac{\int \frac{-b(bB-Ac)+2c(bB-Ac)x^2-2Bc^2x^4}{b+cx^2}\,dx}{2c^3}\\ &= -\frac{b(bB-Ac)x}{2c^3(b+cx^2)}-\frac{\int \left(2(2bB-Ac)-2Bcx^2+\frac{-5b^2B+3Abc}{b+cx^2}\right)dx}{2c^3}\\ &= -\frac{(2bB-Ac)x}{c^3}+\frac{Bx^3}{3c^2}-\frac{b(bB-Ac)x}{2c^3(b+cx^2)}+\frac{(b(5bB-3Ac))\int \frac{1}{b+cx^2}\,dx}{2c^3}\\ &= -\frac{(2bB-Ac)x}{c^3}+\frac{Bx^3}{3c^2}-\frac{b(bB-Ac)x}{2c^3(b+cx^2)}+\frac{\sqrt{b}\,(5bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 89, normalized size = 1.00

$$\frac{(-2bB+Ac)x}{c^3}+\frac{Bx^3}{3c^2}+\frac{(-b^2B+Abc)\,x}{2c^3(b+cx^2)}+\frac{\sqrt{b}\,(5bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*b*B + A*c)*x)/c^3 + (B*x^3)/(3*c^2) + ((-(b^2*B) + A*b*c)*x)/(2*c^3*(b
 + c*x^2)) + (Sqrt[b]*(5*b*B - 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/
2))
```

Maple [A]
time = 0.40, size = 75, normalized size = 0.84

method	result
default	$\frac{\frac{1}{3}Bc\,x^3+Acx-2bBx}{c^3}-\frac{b\left(\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)x}{c\,x^2+b}+\frac{(3Ac-5Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^3}$
risch	$\frac{B\,x^3}{3c^2}+\frac{Ax}{c^2}-\frac{2bBx}{c^3}+\frac{\left(\frac{1}{2}Abc-\frac{1}{2}b^2B\right)x}{c^3(c\,x^2+b)}+\frac{3\sqrt{-bc}\,\ln\left(-\sqrt{-bc}\,x-b\right)A}{4c^3}-\frac{5\sqrt{-bc}\,\ln\left(-\sqrt{-bc}\,x-b\right)Bb}{4c^4}-$ 3√

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/3*B*c*x^3+A*c*x-2*b*B*x)-b/c^3*((-1/2*A*c+1/2*B*b)*x/(c*x^2+b)+1/2
*(3*A*c-5*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Maxima [A]
time = 0.50, size = 85, normalized size = 0.96

$$-\frac{(Bb^2-Abc)x}{2\,(c^4x^2+bc^3)}+\frac{(5\,Bb^2-3\,Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^3}+\frac{Bcx^3-3\,(2\,Bb-Ac)x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b^2 - A*b*c)*x/(c^4*x^2 + b*c^3) + 1/2*(5*B*b^2 - 3*A*b*c)*arctan(c
*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/3*(B*c*x^3 - 3*(2*B*b - A*c)*x)/c^3
```

Fricas [A]
time = 1.63, size = 240, normalized size = 2.70

$$\left[\frac{4\,Bc^2x^5-4\,(5\,Bbc-3\,Ac^2)x^3-3\,(5\,Bb^2-3\,Abc+(5\,Bbc-3\,Ac^2)x^2)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-6\,(5\,Bb^2-3\,Abc)x}{12\,(c^4x^2+bc^3)},\ \frac{2\,Bc^2x^5-2\,(5\,Bbc-3\,Ac^2)x^3+3\,(5\,Bb^2-3\,Abc+(5\,Bbc-3\,Ac^2)x^2)\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)-3\,(5\,Bb^2-3\,Abc)x}{6\,(c^4x^2+bc^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*B*c^2*x^5 - 4*(5*B*b*c - 3*A*c^2)*x^3 - 3*(5*B*b^2 - 3*A*b*c + (5*
B*b*c - 3*A*c^2)*x^2)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2
+ b)) - 6*(5*B*b^2 - 3*A*b*c)*x)/(c^4*x^2 + b*c^3), 1/6*(2*B*c^2*x^5 - 2*(5
*B*b*c - 3*A*c^2)*x^3 + 3*(5*B*b^2 - 3*A*b*c + (5*B*b*c - 3*A*c^2)*x^2)*sqr
t(b/c)*arctan(c*x*sqrt(b/c)/b) - 3*(5*B*b^2 - 3*A*b*c)*x)/(c^4*x^2 + b*c^3)
]
```

Sympy [A]
time = 0.35, size = 129, normalized size = 1.45

$$\frac{Bx^3}{3c^2}+x\left(\frac{A}{c^2}-\frac{2Bb}{c^3}\right)+\frac{x(Abc-Bb^2)}{2bc^3+2c^4x^2}-\frac{\sqrt{-\frac{b}{c^7}}\,(-3Ac+5Bb)\log\left(-c^3\sqrt{-\frac{b}{c^7}}+x\right)}{4}+\frac{\sqrt{-\frac{b}{c^7}}\,(-3Ac+5Bb)\log\left(c^3\sqrt{-\frac{b}{c^7}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**3/(3*c**2) + x*(A/c**2 - 2*B*b/c**3) + x*(A*b*c - B*b**2)/(2*b*c**3 +
2*c**4*x**2) - sqrt(-b/c**7)*(-3*A*c + 5*B*b)*log(-c**3*sqrt(-b/c**7) + x)/
4 + sqrt(-b/c**7)*(-3*A*c + 5*B*b)*log(c**3*sqrt(-b/c**7) + x)/4
```

Giac [A]
time = 0.70, size = 88, normalized size = 0.99

$$\frac{(5\,Bb^2-3\,Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^3}-\frac{Bb^2x-Abcx}{2\,(cx^2+b)c^3}+\frac{Bc^4x^3-6\,Bbc^3x+3\,Ac^4x}{3\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*b^2 - 3*A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) - 1/2*(B*b^2*
x - A*b*c*x)/((c*x^2 + b)*c^3) + 1/3*(B*c^4*x^3 - 6*B*b*c^3*x + 3*A*c^4*x)/
c^6
```

Mupad [B]
time = 0.11, size = 104, normalized size = 1.17

$$x\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)-\frac{x\left(\frac{B\,b^2}{2}-\frac{A\,b\,c}{2}\right)}{c^4\,x^2+b\,c^3}+\frac{B\,x^3}{3\,c^2}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c}\,x\,(3\,A\,c-5\,B\,b)}{5\,B\,b^2-3\,A\,b\,c}\right)\,(3\,A\,c-5\,B\,b)}{2\,c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x*(A/c^2 - (2*B*b)/c^3) - (x*((B*b^2)/2 - (A*b*c)/2))/(b*c^3 + c^4*x^2) + (
B*x^3)/(3*c^2) + (b^(1/2)*atan((b^(1/2)*c^(1/2)*x*(3*A*c - 5*B*b))/(5*B*b^2
 - 3*A*b*c))*(3*A*c - 5*B*b))/(2*c^(7/2))
```

3.63 $\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=61

$$\frac{Bx^2}{2c^2}-\frac{b(bB-Ac)}{2c^3\left(b+cx^2\right)}-\frac{(2bB-Ac)\log\left(b+cx^2\right)}{2c^3}$$

```
[Out] 1/2*B*x^2/c^2-1/2*b*(-A*c+B*b)/c^3/(c*x^2+b)-1/2*(-A*c+2*B*b)*ln(c*x^2+b)/c
^3
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$-\frac{b(bB-Ac)}{2c^3\left(b+cx^2\right)}-\frac{(2bB-Ac)\log\left(b+cx^2\right)}{2c^3}+\frac{Bx^2}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*x^2)/(2*c^2) - (b*(b*B - A*c))/(2*c^3*(b + c*x^2)) - ((2*b*B - A*c)*Log[
b + c*x^2])/(2*c^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \int \frac{x^3(A+Bx^2)}{(b+cx^2)^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(b+cx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{B}{c^2}+\frac{b(bB-Ac)}{c^2(b+cx)^2}+\frac{-2bB+Ac}{c^2(b+cx)}\right)dx, x, x^2\right)\\ &= \frac{Bx^2}{2c^2}-\frac{b(bB-Ac)}{2c^3(b+cx^2)}-\frac{(2bB-Ac)\log(b+cx^2)}{2c^3}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 50, normalized size = 0.82

$$\frac{Bcx^2+\frac{b(-bB+Ac)}{b+cx^2}+(-2bB+Ac)\log(b+cx^2)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*c*x^2 + (b*(-(b*B) + A*c))/(b + c*x^2) + (-2*b*B + A*c)*Log[b + c*x^2])/
(2*c^3)
```

Maple [**A**]
time = 0.38, size = 59, normalized size = 0.97

method	result	size
default	$\frac{B\,x^2}{2c^2}+\frac{\frac{(Ac-2Bb)\ln\left(c\,x^2+b\right)}{c}+\frac{b(Ac-Bb)}{c\left(c\,x^2+b\right)}}{2c^2}$	59
norman	$\frac{\frac{B\,x^7}{2c}+\frac{b(Ac-2Bb)x^3}{2c^3}}{x^3(c\,x^2+b)}+\frac{(Ac-2Bb)\ln\left(c\,x^2+b\right)}{2c^3}$	63
risch	$\frac{B\,x^2}{2c^2}+\frac{bA}{2c^2(c\,x^2+b)}-\frac{b^2B}{2c^3(c\,x^2+b)}+\frac{\ln(c\,x^2+b)A}{2c^2}-\frac{\ln(c\,x^2+b)Bb}{c^3}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/c^2+1/2/c^2*(1/c*(A*c-2*B*b)*ln(c*x^2+b)+b*(A*c-B*b)/c/(c*x^2+b))
```

Maxima [**A**]
time = 0.28, size = 60, normalized size = 0.98

$$\frac{Bx^2}{2\,c^2}-\frac{Bb^2-Abc}{2\,(c^4x^2+bc^3)}-\frac{(2\,Bb-Ac)\log(cx^2+b)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*B*x^2/c^2 - 1/2*(B*b^2 - A*b*c)/(c^4*x^2 + b*c^3) - 1/2*(2*B*b - A*c)*l
og(c*x^2 + b)/c^3
```

Fricas [A]
time = 1.41, size = 81, normalized size = 1.33

$$\frac{Bc^2x^4 + Bbcx^2 - Bb^2 + Abc - (2\,Bb^2 - Abc + (2\,Bbc - Ac^2)x^2)\log{(cx^2 + b)}}{2\,(c^4x^2 + bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(B*c^2*x^4 + B*b*c*x^2 - B*b^2 + A*b*c - (2*B*b^2 - A*b*c + (2*B*b*c -
A*c^2)*x^2)*log(c*x^2 + b))/(c^4*x^2 + b*c^3)
```

Sympy [A]
time = 0.31, size = 56, normalized size = 0.92

$$\frac{Bx^2}{2c^2} + \frac{Abc - Bb^2}{2bc^3 + 2c^4x^2} - \frac{(-Ac + 2Bb)\log{(b + cx^2)}}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**2/(2*c**2) + (A*b*c - B*b**2)/(2*b*c**3 + 2*c**4*x**2) - (-A*c + 2*B*b
)*log(b + c*x**2)/(2*c**3)
```

Giac [A]
time = 1.38, size = 70, normalized size = 1.15

$$\frac{Bx^2}{2\,c^2} - \frac{(2\,Bb - Ac)\log{(|cx^2 + b|)}}{2\,c^3} + \frac{2\,Bbcx^2 - Ac^2x^2 + Bb^2}{2\,(cx^2 + b)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c^2 - 1/2*(2*B*b - A*c)*log(abs(c*x^2 + b))/c^3 + 1/2*(2*B*b*c*x^
2 - A*c^2*x^2 + B*b^2)/((c*x^2 + b)*c^3)
```

Mupad [B]
time = 0.12, size = 62, normalized size = 1.02

$$\frac{B\,x^2}{2\,c^2} + \frac{\ln{(c\,x^2 + b)}\,\left(A\,c - 2\,B\,b\right)}{2\,c^3} - \frac{B\,b^2 - A\,b\,c}{2\,c\,\left(c^3\,x^2 + b\,c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (B*x^2)/(2*c^2) + (log(b + c*x^2)*(A*c - 2*B*b))/(2*c^3) - (B*b^2 - A*b*c)/
(2*c*(b*c^2 + c^3*x^2))
```

3.64 $\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=68

$$\frac{Bx}{c^2}+\frac{(bB-Ac)x}{2c^2(b+cx^2)}-\frac{(3bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{5/2}}$$

```
[Out] B*x/c^2+1/2*(-A*c+B*b)*x/c^2/(c*x^2+b)-1/2*(-A*c+3*B*b)*arctan(x*c^(1/2)/b^
(1/2))/c^(5/2)/b^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1598, 466, 396, 211}

$$-\frac{(3bB-Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{5/2}}+\frac{x(bB-Ac)}{2c^2(b+cx^2)}+\frac{Bx}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*x)/c^2 + ((b*B - A*c)*x)/(2*c^2*(b + c*x^2)) - ((3*b*B - A*c)*ArcTan[(Sq
rt[c]*x)/Sqrt[b]])/(2*Sqrt[b]*c^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
```

```
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \int \frac{x^2(A+Bx^2)}{(b+cx^2)^2}\,dx\\ &= \frac{(bB-Ac)x}{2c^2(b+cx^2)}-\frac{\int \frac{bB-Ac-2Bcx^2}{b+cx^2}\,dx}{2c^2}\\ &= \frac{Bx}{c^2}+\frac{(bB-Ac)x}{2c^2(b+cx^2)}-\frac{(3bB-Ac)\int \frac{1}{b+cx^2}\,dx}{2c^2}\\ &= \frac{Bx}{c^2}+\frac{(bB-Ac)x}{2c^2(b+cx^2)}-\frac{(3bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 68, normalized size = 1.00

$$\frac{Bx}{c^2}-\frac{(-bB+Ac)x}{2c^2(b+cx^2)}-\frac{(3bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*x)/c^2 - ((-(b*B) + A*c)*x)/(2*c^2*(b + c*x^2)) - ((3*b*B - A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*Sqrt[b]*c^(5/2))
```

Maple [A]
time = 0.39, size = 57, normalized size = 0.84

method	result

default	$\frac{Bx}{c^2}+\frac{\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)x}{c\,x^2+b}+\frac{(Ac-3Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}}{c^2}$
risch	$\frac{Bx}{c^2}+\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)x}{c^2(c\,x^2+b)}-\frac{\ln\left(cx+\sqrt{-bc}\right)A}{4c\sqrt{-bc}}+\frac{3\ln\left(cx+\sqrt{-bc}\right)Bb}{4c^2\sqrt{-bc}}+\frac{\ln\left(-cx+\sqrt{-bc}\right)A}{4c\sqrt{-bc}}-\frac{3\ln\left(-cx+\sqrt{-bc}\right)B}{4c^2\sqrt{-bc}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] B*x/c^2+1/c^2*((-1/2*A*c+1/2*B*b)*x/(c*x^2+b)+1/2*(A*c-3*B*b)/(b*c)^(1/2)*a
rctan(c*x/(b*c)^(1/2)))
```

Maxima [A]
time = 0.50, size = 61, normalized size = 0.90

$$\frac{(Bb-Ac)x}{2\,(c^3x^2+bc^2)}+\frac{Bx}{c^2}-\frac{(3\,Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b - A*c)*x/(c^3*x^2 + b*c^2) + B*x/c^2 - 1/2*(3*B*b - A*c)*arctan(c*
x/sqrt(b*c))/(sqrt(b*c)*c^2)
```

Fricas [A]
time = 1.73, size = 208, normalized size = 3.06

$$\left[\frac{4\,Bbc^2x^3+(3\,Bb^2-Abc+(3\,Bbc-Ac^2)x^2)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)+2\,(3\,Bb^2c-Abc^2)x}{4\,(bc^4x^2+b^2c^3)},\frac{2\,Bbc^2x^3-(3\,Bb^2-Abc+(3\,Bbc-Ac^2)x^2)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)+(3\,Bb^2c-Abc^2)x}{2\,(bc^4x^2+b^2c^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*B*b*c^2*x^3 + (3*B*b^2 - A*b*c + (3*B*b*c - A*c^2)*x^2)*sqrt(-b*c)*
log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)) + 2*(3*B*b^2*c - A*b*c^2)*x)/
(b*c^4*x^2 + b^2*c^3), 1/2*(2*B*b*c^2*x^3 - (3*B*b^2 - A*b*c + (3*B*b*c - A
*c^2)*x^2)*sqrt(b*c)*arctan(sqrt(b*c)*x/b) + (3*B*b^2*c - A*b*c^2)*x)/(b*c^
4*x^2 + b^2*c^3)]
```

Sympy [A]
time = 0.28, size = 114, normalized size = 1.68

$$\frac{Bx}{c^2}+\frac{x(-Ac+Bb)}{2bc^2+2c^3x^2}+\frac{\sqrt{-\frac{1}{bc^5}}\,(-Ac+3Bb)\log\left(-bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{bc^5}}\,(-Ac+3Bb)\log\left(bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x/c**2 + x*(-A*c + B*b)/(2*b*c**2 + 2*c**3*x**2) + sqrt(-1/(b*c**5))*(-A*
c + 3*B*b)*log(-b*c**2*sqrt(-1/(b*c**5)) + x)/4 - sqrt(-1/(b*c**5))*(-A*c +
 3*B*b)*log(b*c**2*sqrt(-1/(b*c**5)) + x)/4
```

Giac [A]
time = 1.47, size = 59, normalized size = 0.87

$$\frac{Bx}{c^2} - \frac{(3\,Bb - Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^2} + \frac{Bbx - Acx}{2\,(cx^2 + b)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] B*x/c^2 - 1/2*(3*B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/2*(B*
b*x - A*c*x)/((c*x^2 + b)*c^2)
```

Mupad [B]
time = 0.13, size = 59, normalized size = 0.87

$$\frac{B\,x}{c^2} - \frac{x\,\left(\frac{A\,c}{2} - \frac{B\,b}{2}\right)}{c^3\,x^2 + b\,c^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,\left(A\,c - 3\,B\,b\right)}{2\,\sqrt{b}\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (B*x)/c^2 - (x*((A*c)/2 - (B*b)/2))/(b*c^2 + c^3*x^2) + (atan((c^(1/2)*x)/b
^(1/2))*(A*c - 3*B*b))/(2*b^(1/2)*c^(5/2))
```

3.65 $\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=41

$$\frac{bB-Ac}{2c^2(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^2}$$

```
[Out] 1/2*(-A*c+B*b)/c^2/(c*x^2+b)+1/2*B*ln(c*x^2+b)/c^2
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 455, 45}

$$\frac{bB-Ac}{2c^2(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*B - A*c)/(2*c^2*(b + c*x^2)) + (B*Log[b + c*x^2])/(2*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{x(A+Bx^2)}{(b+cx^2)^2}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(b+cx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{-bB+Ac}{c(b+cx)^2}+\frac{B}{c(b+cx)}\right)dx, x, x^2\right)$$
$$= \frac{bB-Ac}{2c^2(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^2}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.00

$$\frac{bB-Ac}{2c^2(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*B - A*c)/(2*c^2*(b + c*x^2)) + (B*Log[b + c*x^2])/(2*c^2)
```

Maple [A]
time = 0.38, size = 38, normalized size = 0.93

method	result	size
default	$\frac{B\ln(cx^2+b)}{2c^2}-\frac{Ac-Bb}{2c^2(cx^2+b)}$	38
norman	$\frac{B\ln(cx^2+b)}{2c^2}-\frac{Ac-Bb}{2c^2(cx^2+b)}$	38
risch	$\frac{B\ln(cx^2+b)}{2c^2}-\frac{A}{2c(cx^2+b)}+\frac{Bb}{2c^2(cx^2+b)}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*ln(c*x^2+b)/c^2-1/2/c^2*(A*c-B*b)/(c*x^2+b)
```

Maxima [A]
time = 0.27, size = 40, normalized size = 0.98

$$\frac{Bb-Ac}{2(c^3x^2+bc^2)}+\frac{B\log(cx^2+b)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b - A*c)/(c^3*x^2 + b*c^2) + 1/2*B*log(c*x^2 + b)/c^2
```

Fricas [A]
time = 1.51, size = 44, normalized size = 1.07

$$\frac{Bb - Ac + (Bcx^2 + Bb)\log(cx^2 + b)}{2(c^3x^2 + bc^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(B*b - A*c + (B*c*x^2 + B*b)*log(c*x^2 + b))/(c^3*x^2 + b*c^2)
```

Sympy [A]
time = 0.18, size = 36, normalized size = 0.88

$$\frac{B\log(b + cx^2)}{2c^2} + \frac{-Ac + Bb}{2bc^2 + 2c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*log(b + c*x**2)/(2*c**2) + (-A*c + B*b)/(2*b*c**2 + 2*c**3*x**2)
```

Giac [A]
time = 1.33, size = 37, normalized size = 0.90

$$\frac{B\log(|cx^2 + b|)}{2c^2} - \frac{Bx^2 + A}{2(cx^2 + b)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*B*log(abs(c*x^2 + b))/c^2 - 1/2*(B*x^2 + A)/((c*x^2 + b)*c)
```

Mupad [B]
time = 0.09, size = 37, normalized size = 0.90

$$\frac{B\ln(c\,x^2 + b)}{2\,c^2} - \frac{A\,c - B\,b}{2\,c^2\,(c\,x^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (B*log(b + c*x^2))/(2*c^2) - (A*c - B*b)/(2*c^2*(b + c*x^2))
```

3.66 $\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=63

$$-\frac{(bB-Ac)x}{2bc\,(b+cx^2)}+\frac{(bB+Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}$$

```
[Out] -1/2*(-A*c+B*b)*x/b/c/(c*x^2+b)+1/2*(A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(
3/2)/c^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 393, 211}

$$\frac{(Ac+bB)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}-\frac{x(bB-Ac)}{2bc\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((b*B - A*c)*x)/(b*c*(b + c*x^2)) + ((b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/(2*b^(3/2)*c^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \int \frac{A+Bx^2}{(b+cx^2)^2}\,dx\\ &= -\frac{(bB-Ac)x}{2bc\,(b+cx^2)}+\frac{(bB+Ac)\int \frac{1}{b+cx^2}\,dx}{2bc}\\ &= -\frac{(bB-Ac)x}{2bc\,(b+cx^2)}+\frac{(bB+Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 1.00

$$-\frac{(bB-Ac)x}{2bc\,(b+cx^2)}+\frac{(bB+Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
[Out] -1/2*((b*B - A*c)*x)/(b*c*(b + c*x^2)) + ((b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/(2*b^(3/2)*c^(3/2))
```

Maple [A]
time = 0.42, size = 57, normalized size = 0.90

method	result	size
default	$\frac{(Ac-Bb)x}{2bc(c\,x^2+b)}+\frac{(Ac+Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2bc\sqrt{bc}}$	57
risch	$\frac{(Ac-Bb)x}{2bc(c\,x^2+b)}-\frac{\ln\left(cx+\sqrt{-bc}\right)A}{4\sqrt{-bc}\;b}-\frac{\ln\left(cx+\sqrt{-bc}\right)B}{4\sqrt{-bc}\;c}+\frac{\ln\left(-cx+\sqrt{-bc}\right)A}{4\sqrt{-bc}\;b}+\frac{\ln\left(-cx+\sqrt{-bc}\right)B}{4\sqrt{-bc}\;c}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*(A*c-B*b)/b/c*x/(c*x^2+b)+1/2*(A*c+B*b)/b/c/(b*c)^(1/2)*arctan(c*x/(b*c
)^(1/2))
```

Maxima [A]
time = 0.49, size = 57, normalized size = 0.90

$$-\frac{(Bb-Ac)x}{2\,(bc^2x^2+b^2c)}+\frac{(Bb+Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b - A*c)*x/(b*c^2*x^2 + b^2*c) + 1/2*(B*b + A*c)*arctan(c*x/sqrt(b*
c))/(sqrt(b*c)*b*c)
```

Fricas [A]
time = 1.89, size = 182, normalized size = 2.89

$$\left[-\frac{(Bb^2 + Abc + (Bbc + Ac^2)x^2)\sqrt{-bc}\,\log\left(\frac{cx^2 - 2\sqrt{-bc}\,x - b}{cx^2 + b}\right) + 2\,(Bb^2c - Abc^2)x}{4\,(b^2c^3x^2 + b^3c^2)}, \frac{(Bb^2 + Abc + (Bbc + Ac^2)x^2)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right) - (Bb^2c - Abc^2)x}{2\,(b^2c^3x^2 + b^3c^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((B*b^2 + A*b*c + (B*b*c + A*c^2)*x^2)*sqrt(-b*c)*log((c*x^2 - 2*sqrt
(-b*c)*x - b)/(c*x^2 + b)) + 2*(B*b^2*c - A*b*c^2)*x)/(b^2*c^3*x^2 + b^3*c^
2), 1/2*((B*b^2 + A*b*c + (B*b*c + A*c^2)*x^2)*sqrt(b*c)*arctan(sqrt(b*c)*x
/b) - (B*b^2*c - A*b*c^2)*x)/(b^2*c^3*x^2 + b^3*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(54) = 108$.
time = 0.21, size = 112, normalized size = 1.78

$$\frac{x(Ac - Bb)}{2b^2c + 2bc^2x^2} - \frac{\sqrt{-\frac{1}{b^3c^3}}\,(Ac + Bb)\log\left(-b^2c\sqrt{-\frac{1}{b^3c^3}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{b^3c^3}}\,(Ac + Bb)\log\left(b^2c\sqrt{-\frac{1}{b^3c^3}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] x*(A*c - B*b)/(2*b**2*c + 2*b*c**2*x**2) - sqrt(-1/(b**3*c**3))*(A*c + B*b)
*log(-b**2*c*sqrt(-1/(b**3*c**3)) + x)/4 + sqrt(-1/(b**3*c**3))*(A*c + B*b)
*log(b**2*c*sqrt(-1/(b**3*c**3)) + x)/4
```

Giac [A]
time = 0.96, size = 57, normalized size = 0.90

$$\frac{(Bb + Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,bc} - \frac{Bbx - Acx}{2\,(cx^2 + b)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b + A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c) - 1/2*(B*b*x - A*c*x)
/((c*x^2 + b)*b*c)
```

Mupad [B]
time = 0.12, size = 51, normalized size = 0.81

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c+B\,b)}{2\,b^{3/2}\,c^{3/2}}+\frac{x\,(A\,c-B\,b)}{2\,b\,c\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (atan((c^(1/2)*x)/b^(1/2))*(A*c + B*b))/(2*b^(3/2)*c^(3/2)) + (x*(A*c - B*b
))/(2*b*c*(b + c*x^2))
```

3.67 $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=51

$$-\frac{bB-Ac}{2bc\,(b+cx^2)}+\frac{A\log(x)}{b^2}-\frac{A\log(b+cx^2)}{2b^2}$$

```
[Out] 1/2*(A*c-B*b)/b/c/(c*x^2+b)+A*ln(x)/b^2-1/2*A*ln(c*x^2+b)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$-\frac{A\log(b+cx^2)}{2b^2}+\frac{A\log(x)}{b^2}-\frac{bB-Ac}{2bc\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(b*B - A*c)/(b*c*(b + c*x^2)) + (A*Log[x])/b^2 - (A*Log[b + c*x^2])/(2
*b^2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \int \frac{A+Bx^2}{x\,(b+cx^2)^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x(b+cx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{b^2x}+\frac{bB-Ac}{b(b+cx)^2}-\frac{Ac}{b^2(b+cx)}\right)\,dx,x,x^2\right)\\ &= -\frac{bB-Ac}{2bc\,(b+cx^2)}+\frac{A\log(x)}{b^2}-\frac{A\log\left(b+cx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.90

$$\frac{\frac{b(-bB+Ac)}{c(b+cx^2)}+2A\log(x)-A\log\left(b+cx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
[Out] ((b*(-(b*B) + A*c))/(c*(b + c*x^2)) + 2*A*Log[x] - A*Log[b + c*x^2])/(2*b^2
)
```

Maple [A]
time = 0.37, size = 48, normalized size = 0.94

method	result	size
default	$-\frac{A\ln\left(c\,x^2+b\right)-\frac{b(Ac-Bb)}{c\left(c\,x^2+b\right)}}{2b^2}+\frac{A\ln(x)}{b^2}$	48
norman	$-\frac{(Ac-Bb)x^2}{2b^2(c\,x^2+b)}+\frac{A\ln(x)}{b^2}-\frac{A\ln\left(c\,x^2+b\right)}{2b^2}$	48
risch	$\frac{A}{2b(c\,x^2+b)}-\frac{B}{2c(c\,x^2+b)}+\frac{A\ln(x)}{b^2}-\frac{A\ln\left(c\,x^2+b\right)}{2b^2}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
[Out] -1/2/b^2*(A*ln(c*x^2+b)-b*(A*c-B*b)/c/(c*x^2+b))+A*ln(x)/b^2
```

Maxima [A]
time = 0.27, size = 51, normalized size = 1.00

$$-\frac{Bb-Ac}{2\left(bc^2x^2+b^2c\right)}-\frac{A\log\left(cx^2+b\right)}{2\,b^2}+\frac{A\log\left(x^2\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b - A*c)/(b*c^2*x^2 + b^2*c) - 1/2*A*log(c*x^2 + b)/b^2 + 1/2*A*log
(x^2)/b^2
```

Fricas [A]
time = 1.48, size = 70, normalized size = 1.37

$$-\frac{Bb^2 - Abc + (Ac^2x^2 + Abc)\log(cx^2 + b) - 2(Ac^2x^2 + Abc)\log(x)}{2(b^2c^2x^2 + b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(B*b^2 - A*b*c + (A*c^2*x^2 + A*b*c)*log(c*x^2 + b) - 2*(A*c^2*x^2 + A
*b*c)*log(x))/(b^2*c^2*x^2 + b^3*c)
```

Sympy [A]
time = 0.21, size = 46, normalized size = 0.90

$$\frac{A\log(x)}{b^2} - \frac{A\log\left(\frac{b}{c} + x^2\right)}{2b^2} + \frac{Ac - Bb}{2b^2c + 2bc^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] A*log(x)/b**2 - A*log(b/c + x**2)/(2*b**2) + (A*c - B*b)/(2*b**2*c + 2*b*c*
*2*x**2)
```

Giac [A]
time = 1.20, size = 52, normalized size = 1.02

$$-\frac{A\log(|cx^2 + b|)}{2b^2} + \frac{A\log(|x|)}{b^2} - \frac{Bb^2 - Abc}{2(cx^2 + b)b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*A*log(abs(c*x^2 + b))/b^2 + A*log(abs(x))/b^2 - 1/2*(B*b^2 - A*b*c)/((
c*x^2 + b)*b^2*c)
```

Mupad [B]
time = 0.16, size = 47, normalized size = 0.92

$$\frac{A\ln(x)}{b^2} - \frac{A\ln(c\,x^2 + b)}{2\,b^2} + \frac{A\,c - B\,b}{2\,b\,c\,(c\,x^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (A*log(x))/b^2 - (A*log(b + c*x^2))/(2*b^2) + (A*c - B*b)/(2*b*c*(b + c*x^2
))
```

3.68 $\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=70

$$-\frac{A}{b^2x}+\frac{(bB-Ac)x}{2b^2(b+cx^2)}+\frac{(bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}}$$

```
[Out] -A/b^2/x+1/2*(-A*c+B*b)*x/b^2/(c*x^2+b)+1/2*(-3*A*c+B*b)*arctan(x*c^(1/2)/b
^(1/2))/b^(5/2)/c^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1598, 467, 464, 211}

$$\frac{(bB-3Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}}+\frac{x(bB-Ac)}{2b^2(b+cx^2)}-\frac{A}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(A/(b^2*x)) + ((b*B - A*c)*x)/(2*b^2*(b + c*x^2)) + ((b*B - 3*A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*b^(5/2)*Sqrt[c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
```

```
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \int \frac{A+Bx^2}{x^2\,(b+cx^2)^2}\,dx \\
&= \frac{(bB-Ac)x}{2b^2\,(b+cx^2)} - \frac{1}{2}\int \frac{-\frac{2A}{b}-\frac{(bB-Ac)x^2}{b^2}}{x^2\,(b+cx^2)}\,dx \\
&= -\frac{A}{b^2x} + \frac{(bB-Ac)x}{2b^2\,(b+cx^2)} + \frac{(bB-3Ac)\int \frac{1}{b+cx^2}\,dx}{2b^2} \\
&= -\frac{A}{b^2x} + \frac{(bB-Ac)x}{2b^2\,(b+cx^2)} + \frac{(bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 1.00

$$-\frac{A}{b^2x} + \frac{(bB-Ac)x}{2b^2\,(b+cx^2)} + \frac{(bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(A/(b^2*x)) + ((b*B - A*c)*x)/(2*b^2*(b + c*x^2)) + ((b*B - 3*A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*b^(5/2)*Sqrt[c])
```

Maple [A]
time = 0.40, size = 62, normalized size = 0.89

method	result

default	$-\frac{\frac{\left(\frac{Ac}{2}-\frac{Bb}{2}\right)x}{c\,x^{2}+b}+\frac{(3Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}}{b^{2}}-\frac{A}{b^{2}x}$
risch	$\frac{-\frac{(3Ac-Bb)x^{2}}{2b^{2}}-\frac{A}{b}}{x(c\,x^{2}+b)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{5}c\_Z^{2}+9A^{2}c^{2}-6ABbc+B^{2}b^{2}\right)}\_R\ln\left(\left(3\_R^{2}b^{5}c+18A^{2}c^{2}-12ABbc+2B^{2}b^{2}\right)x+\left(3A\,b^{3}c-B\,b^{4}\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2*((1/2*A*c-1/2*B*b)*x/(c*x^2+b)+1/2*(3*A*c-B*b)/(b*c)^(1/2)*arctan(c*
x/(b*c)^(1/2)))-A/b^2/x
```

Maxima [A]
time = 0.49, size = 63, normalized size = 0.90

$$\frac{(Bb-3\,Ac)x^{2}-2\,Ab}{2\,(b^{2}cx^{3}+b^{3}x)}+\frac{(Bb-3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b - 3*A*c)*x^2 - 2*A*b)/(b^2*c*x^3 + b^3*x) + 1/2*(B*b - 3*A*c)*arc
tan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2)
```

Fricas [A]
time = 1.87, size = 210, normalized size = 3.00

$$\left[-\frac{4\,Ab^{2}c-2\,(Bb^{2}c-3\,Abc^{2})x^{2}-((Bbc-3\,Ac^{2})x^{3}+(Bb^{2}-3\,Abc)x)\sqrt{-bc}\,\log\left(\frac{cx^{2}+2\,\sqrt{-bc}\,x-b}{cx^{2}+b}\right)}{4\,(b^{3}c^{2}x^{3}+b^{4}cx)},-\frac{2\,Ab^{2}c-(Bb^{2}c-3\,Abc^{2})x^{2}-((Bbc-3\,Ac^{2})x^{3}+(Bb^{2}-3\,Abc)x)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{2\,(b^{3}c^{2}x^{3}+b^{4}cx)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*A*b^2*c - 2*(B*b^2*c - 3*A*b*c^2)*x^2 - ((B*b*c - 3*A*c^2)*x^3 + (
B*b^2 - 3*A*b*c)*x)*sqrt(-b*c)*log((c*x^2 + 2*sqrt(-b*c)*x - b)/(c*x^2 + b)
))/(b^3*c^2*x^3 + b^4*c*x), -1/2*(2*A*b^2*c - (B*b^2*c - 3*A*b*c^2)*x^2 - (
(B*b*c - 3*A*c^2)*x^3 + (B*b^2 - 3*A*b*c)*x)*sqrt(b*c)*arctan(sqrt(b*c)*x/b
))/(b^3*c^2*x^3 + b^4*c*x)]
```

Sympy [A]
time = 0.25, size = 114, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{b^{5}c}}\,(-3Ac+Bb)\log\left(-b^{3}\sqrt{-\frac{1}{b^{5}c}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{b^{5}c}}\,(-3Ac+Bb)\log\left(b^{3}\sqrt{-\frac{1}{b^{5}c}}+x\right)}{4}+\frac{-2Ab+x^{2}(-3Ac+Bb)}{2b^{3}x+2b^{2}cx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] -sqrt(-1/(b**5*c))*(-3*A*c + B*b)*log(-b**3*sqrt(-1/(b**5*c)) + x)/4 + sqrt
(-1/(b**5*c))*(-3*A*c + B*b)*log(b**3*sqrt(-1/(b**5*c)) + x)/4 + (-2*A*b +
x**2*(-3*A*c + B*b))/(2*b**3*x + 2*b**2*c*x**3)
```

Giac [A]
time = 1.63, size = 62, normalized size = 0.89

$$\frac{(Bb - 3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^2} + \frac{Bbx^2 - 3\,Acx^2 - 2\,Ab}{2\,(cx^3 + bx)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b - 3*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/2*(B*b*x^2 - 3*
A*c*x^2 - 2*A*b)/((c*x^3 + b*x)*b^2)
```

Mupad [B]
time = 0.13, size = 63, normalized size = 0.90

$$-\frac{\frac{A}{b} + \frac{x^2\,(3\,A\,c-B\,b)}{2\,b^2}}{c\,x^3 + b\,x} - \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(3\,A\,c - B\,b)}{2\,b^{5/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] - (A/b + (x^2*(3*A*c - B*b))/(2*b^2))/(b*x + c*x^3) - (atan((c^(1/2)*x)/b^(
1/2))*(3*A*c - B*b))/(2*b^(5/2)*c^(1/2))
```

3.69 $\int \frac{x\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=73

$$-\frac{A}{2b^2x^2}+\frac{bB-Ac}{2b^2\left(b+cx^2\right)}+\frac{(bB-2Ac)\log(x)}{b^3}-\frac{(bB-2Ac)\log\left(b+cx^2\right)}{2b^3}$$

```
[Out] -1/2*A/b^2/x^2+1/2*(-A*c+B*b)/b^2/(c*x^2+b)+(-2*A*c+B*b)*ln(x)/b^3-1/2*(-2*
A*c+B*b)*ln(c*x^2+b)/b^3
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1598, 457, 78}

$$-\frac{(bB-2Ac)\log\left(b+cx^2\right)}{2b^3}+\frac{\log(x)(bB-2Ac)}{b^3}+\frac{bB-Ac}{2b^2\left(b+cx^2\right)}-\frac{A}{2b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*A/(b^2*x^2) + (b*B - A*c)/(2*b^2*(b + c*x^2)) + ((b*B - 2*A*c)*Log[x])
/b^3 - ((b*B - 2*A*c)*Log[b + c*x^2])/(2*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \int \frac{A+Bx^2}{x^3\,(b+cx^2)^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2(b+cx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{b^2x^2}+\frac{bB-2Ac}{b^3x}-\frac{c(bB-Ac)}{b^2(b+cx)^2}-\frac{c(bB-2Ac)}{b^3(b+cx)}\right)dx, x, x^2\right)\\ &= -\frac{A}{2b^2x^2}+\frac{bB-Ac}{2b^2\,(b+cx^2)}+\frac{(bB-2Ac)\log(x)}{b^3}-\frac{(bB-2Ac)\log\left(b+cx^2\right)}{2b^3}\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 64, normalized size = 0.88

$$\frac{-\frac{Ab}{x^2}+\frac{b(bB-Ac)}{b+cx^2}+2(bB-2Ac)\log(x)+(-bB+2Ac)\log\left(b+cx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-((A*b)/x^2) + (b*(b*B - A*c))/(b + c*x^2) + 2*(b*B - 2*A*c)*Log[x] + (-(b
*B) + 2*A*c)*Log[b + c*x^2])/(2*b^3)
```

Maple [**A**]
time = 0.38, size = 76, normalized size = 1.04

method	result	size
default	$\frac{c\left(\frac{(2Ac-Bb)\ln\left(cx^2+b\right)}{c}-\frac{b(Ac-Bb)}{c\left(cx^2+b\right)}\right)}{2b^3}-\frac{A}{2b^2x^2}+\frac{(-2Ac+Bb)\ln(x)}{b^3}$	76
norman	$\frac{-\frac{Ax}{2b}+\frac{c(2Ac-Bb)x^5}{2b^3}}{x^3(cx^2+b)}-\frac{(2Ac-Bb)\ln(x)}{b^3}+\frac{(2Ac-Bb)\ln(cx^2+b)}{2b^3}$	79
risch	$\frac{-\frac{(2Ac-Bb)x^2}{2b^2}-\frac{A}{2b}}{x^2(cx^2+b)}-\frac{2\ln(x)Ac}{b^3}+\frac{\ln(x)B}{b^2}+\frac{\ln(-cx^2-b)Ac}{b^3}-\frac{\ln(-cx^2-b)B}{2b^2}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^3*c*((2*A*c-B*b)/c*ln(c*x^2+b)-b*(A*c-B*b)/c/(c*x^2+b))-1/2*A/b^2/x^2
+(-2*A*c+B*b)*ln(x)/b^3
```

Maxima [A]
time = 0.28, size = 76, normalized size = 1.04

$$\frac{(Bb-2\,Ac)x^2-Ab}{2\,(b^2cx^4+b^3x^2)}-\frac{(Bb-2\,Ac)\log\left(cx^2+b\right)}{2\,b^3}+\frac{(Bb-2\,Ac)\log\left(x^2\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b - 2*A*c)*x^2 - A*b)/(b^2*c*x^4 + b^3*x^2) - 1/2*(B*b - 2*A*c)*log
(c*x^2 + b)/b^3 + 1/2*(B*b - 2*A*c)*log(x^2)/b^3
```

Fricas [A]
time = 2.02, size = 117, normalized size = 1.60

$$-\frac{Ab^2-(Bb^2-2\,Abc)x^2+\left((Bbc-2\,Ac^2)x^4+(Bb^2-2\,Abc)x^2\right)\log\left(cx^2+b\right)-2\left((Bbc-2\,Ac^2)x^4+(Bb^2-2\,Abc)x^2\right)\log\left(x\right)}{2\,(b^3cx^4+b^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(A*b^2 - (B*b^2 - 2*A*b*c)*x^2 + ((B*b*c - 2*A*c^2)*x^4 + (B*b^2 - 2*A
*b*c)*x^2)*log(c*x^2 + b) - 2*((B*b*c - 2*A*c^2)*x^4 + (B*b^2 - 2*A*b*c)*x^
2)*log(x))/(b^3*c*x^4 + b^4*x^2)
```

Sympy [A]
time = 0.49, size = 70, normalized size = 0.96

$$\frac{-Ab+x^2(-2Ac+Bb)}{2b^3x^2+2b^2cx^4}+\frac{(-2Ac+Bb)\log\left(x\right)}{b^3}-\frac{(-2Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] (-A*b + x**2*(-2*A*c + B*b))/(2*b**3*x**2 + 2*b**2*c*x**4) + (-2*A*c + B*b)
*log(x)/b**3 - (-2*A*c + B*b)*log(b/c + x**2)/(2*b**3)
```

Giac [A]
time = 1.41, size = 80, normalized size = 1.10

$$\frac{(Bb-2\,Ac)\log\left(|x|\right)}{b^3}+\frac{Bbx^2-2\,Acx^2-Ab}{2\,(cx^4+bx^2)b^2}-\frac{(Bbc-2\,Ac^2)\log\left(|cx^2+b|\right)}{2\,b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] (B*b - 2*A*c)*log(abs(x))/b^3 + 1/2*(B*b*x^2 - 2*A*c*x^2 - A*b)/((c*x^4 + b
*x^2)*b^2) - 1/2*(B*b*c - 2*A*c^2)*log(abs(c*x^2 + b))/(b^3*c)
```

Mupad [B]
time = 0.15, size = 78, normalized size = 1.07

$$\frac{\ln\left(c\,x^2+b\right)\,\left(2\,A\,c-B\,b\right)}{2\,b^3}-\frac{\frac{A}{2\,b}+\frac{x^2\,\left(2\,A\,c-B\,b\right)}{2\,b^2}}{c\,x^4+b\,x^2}-\frac{\ln\left(x\right)\,\left(2\,A\,c-B\,b\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (log(b + c*x^2)*(2*A*c - B*b))/(2*b^3) - (A/(2*b) + (x^2*(2*A*c - B*b))/(2*
b^2))/(b*x^2 + c*x^4) - (log(x)*(2*A*c - B*b))/b^3
```

3.70 $\int \frac{A+Bx^2}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=90

$$-\frac{A}{3b^2x^3}-\frac{bB-2Ac}{b^3x}-\frac{c(bB-Ac)x}{2b^3\left(b+cx^2\right)}-\frac{\sqrt{c}\,(3bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}$$

```
[Out] -1/3*A/b^2/x^3+(2*A*c-B*b)/b^3/x-1/2*c*(-A*c+B*b)*x/b^3/(c*x^2+b)-1/2*(-5*A
*c+3*B*b)*arctan(x*c^(1/2)/b^(1/2))*c^(1/2)/b^(7/2)
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1607, 467, 1275, 211}

$$-\frac{\sqrt{c}\,(3bB-5Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}-\frac{cx(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{bB-2Ac}{b^3x}-\frac{A}{3b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/3*A/(b^2*x^3) - (b*B - 2*A*c)/(b^3*x) - (c*(b*B - A*c)*x)/(2*b^3*(b + c*
x^2)) - (Sqrt[c]*(3*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
```

```
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{(bx^2+cx^4)^2}\,dx &= \int \frac{A+Bx^2}{x^4(b+cx^2)^2}\,dx\\
&= -\frac{c(bB-Ac)x}{2b^3(b+cx^2)} - \frac{1}{2}c\int \frac{-\frac{2A}{bc} - \frac{2(bB-Ac)x^2}{b^2c} + \frac{(bB-Ac)x^4}{b^3}}{x^4(b+cx^2)}\,dx\\
&= -\frac{c(bB-Ac)x}{2b^3(b+cx^2)} - \frac{1}{2}c\int \left(-\frac{2A}{b^2cx^4} - \frac{2(bB-2Ac)}{b^3cx^2} + \frac{3bB-5Ac}{b^3(b+cx^2)}\right)dx\\
&= -\frac{A}{3b^2x^3} - \frac{bB-2Ac}{b^3x} - \frac{c(bB-Ac)x}{2b^3(b+cx^2)} - \frac{(c(3bB-5Ac))\int \frac{1}{b+cx^2}\,dx}{2b^3}\\
&= -\frac{A}{3b^2x^3} - \frac{bB-2Ac}{b^3x} - \frac{c(bB-Ac)x}{2b^3(b+cx^2)} - \frac{\sqrt{c}\,(3bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 90, normalized size = 1.00

$$-\frac{A}{3b^2x^3} + \frac{-bB+2Ac}{b^3x} - \frac{c(bB-Ac)x}{2b^3(b+cx^2)} - \frac{\sqrt{c}\,(3bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/3*A/(b^2*x^3) + (-(b*B) + 2*A*c)/(b^3*x) - (c*(b*B - A*c)*x)/(2*b^3*(b +
 c*x^2)) - (Sqrt[c]*(3*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2)
)
```

Maple [A]
time = 0.47, size = 78, normalized size = 0.87

method	result

default	$\frac{c\left(\frac{\left(\frac{Ac}{2}-\frac{Bb}{2}\right)x}{c\,x^2+b}+\frac{(5Ac-3Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^3}-\frac{A}{3b^2x^3}-\frac{-2Ac+Bb}{b^3x}$
risch	$\frac{\frac{c(5Ac-3Bb)x^4}{2b^3}+\frac{(5Ac-3Bb)x^2}{3b^2}-\frac{A}{3b}}{x^3(c\,x^2+b)}+\frac{5\sqrt{-bc}\,\ln\left(-cx-\sqrt{-bc}\right)Ac}{4b^4}-\frac{3\sqrt{-bc}\,\ln\left(-cx-\sqrt{-bc}\right)B}{4b^3}-\frac{5\sqrt{-bc}\,\ln\left(-c\right.}{4\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*c*((1/2*A*c-1/2*B*b)*x/(c*x^2+b)+1/2*(5*A*c-3*B*b)/(b*c)^(1/2)*arctan
(c*x/(b*c)^(1/2)))-1/3*A/b^2/x^3-(-2*A*c+B*b)/b^3/x
```

Maxima [A]
time = 0.50, size = 93, normalized size = 1.03

$$-\frac{3\,(3\,Bbc-5\,Ac^2)x^4+2\,Ab^2+2\,(3\,Bb^2-5\,Abc)x^2}{6\,(b^3cx^5+b^4x^3)}-\frac{(3\,Bbc-5\,Ac^2)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*B*b*c - 5*A*c^2)*x^4 + 2*A*b^2 + 2*(3*B*b^2 - 5*A*b*c)*x^2)/(b^3
*c*x^5 + b^4*x^3) - 1/2*(3*B*b*c - 5*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c
)*b^3)
```

Fricas [A]
time = 2.16, size = 250, normalized size = 2.78

$$\left[-\frac{6\,(3\,Bbc-5\,Ac^2)x^4+4\,Ab^2+4\,(3\,Bb^2-5\,Abc)x^2+3\,((3\,Bbc-5\,Ac^2)x^5+(3\,Bb^2-5\,Abc)x^3)\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)}{12\,(b^3cx^5+b^4x^3)},\,-\frac{3\,(3\,Bbc-5\,Ac^2)x^4+2\,Ab^2+2\,(3\,Bb^2-5\,Abc)x^2+3\,((3\,Bbc-5\,Ac^2)x^5+(3\,Bb^2-5\,Abc)x^3)\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)}{6\,(b^3cx^5+b^4x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(3*B*b*c - 5*A*c^2)*x^4 + 4*A*b^2 + 4*(3*B*b^2 - 5*A*b*c)*x^2 + 3
*((3*B*b*c - 5*A*c^2)*x^5 + (3*B*b^2 - 5*A*b*c)*x^3)*sqrt(-c/b)*log((c*x^2
 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^3*c*x^5 + b^4*x^3), -1/6*(3*(3*B*b
*c - 5*A*c^2)*x^4 + 2*A*b^2 + 2*(3*B*b^2 - 5*A*b*c)*x^2 + 3*((3*B*b*c - 5*A
*c^2)*x^5 + (3*B*b^2 - 5*A*b*c)*x^3)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^3*c*
x^5 + b^4*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(82) = 164$.
time = 0.30, size = 184, normalized size = 2.04

$$\frac{\sqrt{-\frac{c}{b^7}}\,(-5Ac+3Bb)\log\left(-\frac{b^4\sqrt{-\frac{c}{b^7}}\,(-5Ac+3Bb)}{-5Ac^2+3Bbc}+x\right)}{4}-\frac{\sqrt{-\frac{c}{b^7}}\,(-5Ac+3Bb)\log\left(\frac{b^4\sqrt{-\frac{c}{b^7}}\,(-5Ac+3Bb)}{-5Ac^2+3Bbc}+x\right)}{4}+\frac{-2Ab^2+x^4\cdot(15Ac^2-9Bbc)+x^2\cdot(10Abc-6Bb^2)}{6b^4x^3+6b^3cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] sqrt(-c/b**7)*(-5*A*c + 3*B*b)*log(-b**4*sqrt(-c/b**7)*(-5*A*c + 3*B*b)/(-5
*A*c**2 + 3*B*b*c) + x)/4 - sqrt(-c/b**7)*(-5*A*c + 3*B*b)*log(b**4*sqrt(-c
/b**7)*(-5*A*c + 3*B*b)/(-5*A*c**2 + 3*B*b*c) + x)/4 + (-2*A*b**2 + x**4*(1
5*A*c**2 - 9*B*b*c) + x**2*(10*A*b*c - 6*B*b**2))/(6*b**4*x**3 + 6*b**3*c*x
**5)
```

Giac [A]
time = 1.68, size = 85, normalized size = 0.94

$$-\frac{(3\,Bbc-5\,Ac^2)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^3}-\frac{Bbcx-Ac^2x}{2\,(cx^2+b)b^3}-\frac{3\,Bbx^2-6\,Acx^2+Ab}{3\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*B*b*c - 5*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/2*(B*b*c
*x - A*c^2*x)/((c*x^2 + b)*b^3) - 1/3*(3*B*b*x^2 - 6*A*c*x^2 + A*b)/(b^3*x^
3)
```

Mupad [B]
time = 0.14, size = 83, normalized size = 0.92

$$\frac{\frac{x^2\,(5\,A\,c-3\,B\,b)}{3\,b^2}-\frac{A}{3\,b}+\frac{c\,x^4\,(5\,A\,c-3\,B\,b)}{2\,b^3}}{c\,x^5+b\,x^3}+\frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(5\,A\,c-3\,B\,b)}{2\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x^2*(5*A*c - 3*B*b))/(3*b^2) - A/(3*b) + (c*x^4*(5*A*c - 3*B*b))/(2*b^3))
/(b*x^3 + c*x^5) + (c^(1/2)*atan((c^(1/2)*x)/b^(1/2))*(5*A*c - 3*B*b))/(2*b
^(7/2))
```

3.71 $\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{A}{4b^2x^4}-\frac{bB-2Ac}{2b^3x^2}-\frac{c(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{c(2bB-3Ac)\log(x)}{b^4}+\frac{c(2bB-3Ac)\log\left(b+cx^2\right)}{2b^4}$$

```
[Out] -1/4*A/b^2/x^4+1/2*(2*A*c-B*b)/b^3/x^2-1/2*c*(-A*c+B*b)/b^3/(c*x^2+b)-c*(-3
*A*c+2*B*b)*ln(x)/b^4+1/2*c*(-3*A*c+2*B*b)*ln(c*x^2+b)/b^4
```

Rubi [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$\frac{c(2bB-3Ac)\log\left(b+cx^2\right)}{2b^4}-\frac{c\log(x)(2bB-3Ac)}{b^4}-\frac{c(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{bB-2Ac}{2b^3x^2}-\frac{A}{4b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*A/(b^2*x^4) - (b*B - 2*A*c)/(2*b^3*x^2) - (c*(b*B - A*c))/(2*b^3*(b +
c*x^2)) - (c*(2*b*B - 3*A*c)*Log[x])/b^4 + (c*(2*b*B - 3*A*c)*Log[b + c*x^2
])/(2*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2}\,dx = \int \frac{A+Bx^2}{x^5\left(b+cx^2\right)^2}\,dx$$
$$= \frac{1}{2}\mathrm{Subst}\left(\int \frac{A+Bx}{x^3(b+cx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\mathrm{Subst}\left(\int \left(\frac{A}{b^2x^3}+\frac{bB-2Ac}{b^3x^2}-\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)}{b^3(b+cx)^2}+\frac{c^2(2bB-3Ac)}{b^4(b+cx)}\right)\right.$$
$$= -\frac{A}{4b^2x^4}-\frac{bB-2Ac}{2b^3x^2}-\frac{c(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{c(2bB-3Ac)\log(x)}{b^4}+\frac{c(2bB-3Ac)\log\left(b\right.}{2b^4}$$

Mathematica [A]
time = 0.07, size = 85, normalized size = 0.88

$$-\frac{\frac{Ab^2}{x^4}+\frac{2b(bB-2Ac)}{x^2}+\frac{2bc(bB-Ac)}{b+cx^2}-4c(-2bB+3Ac)\log(x)+2c(-2bB+3Ac)\log\left(b+cx^2\right)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*((A*b^2)/x^4 + (2*b*(b*B - 2*A*c))/x^2 + (2*b*c*(b*B - A*c))/(b + c*x^
2) - 4*c*(-2*b*B + 3*A*c)*Log[x] + 2*c*(-2*b*B + 3*A*c)*Log[b + c*x^2])/b^4
```

Maple [A]
time = 0.45, size = 96, normalized size = 0.99

method	result	size
default	$-\frac{c^2\left(\frac{(3Ac-2Bb)\ln\left(c\,x^2+b\right)}{c}-\frac{b(Ac-Bb)}{c\left(c\,x^2+b\right)}\right)}{2b^4}-\frac{A}{4b^2x^4}-\frac{-2Ac+Bb}{2b^3x^2}+\frac{c(3Ac-2Bb)\ln(x)}{b^4}$	96
norman	$\frac{-\frac{A}{4b}+\frac{(3Ac-2Bb)x^2}{4b^2}-\frac{c\left(3A\,c^2-2bBc\right)x^6}{2b^4}}{x^4\left(c\,x^2+b\right)}+\frac{c(3Ac-2Bb)\ln(x)}{b^4}-\frac{c(3Ac-2Bb)\ln\left(c\,x^2+b\right)}{2b^4}$	99
risch	$\frac{\frac{c(3Ac-2Bb)x^4}{2b^3}+\frac{(3Ac-2Bb)x^2}{4b^2}-\frac{A}{4b}}{x^4\left(c\,x^2+b\right)}+\frac{3c^2\ln(x)A}{b^4}-\frac{2c\ln(x)B}{b^3}-\frac{3c^2\ln\left(c\,x^2+b\right)A}{2b^4}+\frac{c\ln\left(c\,x^2+b\right)B}{b^3}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^4*c^2*((3*A*c-2*B*b)/c*ln(c*x^2+b)-b*(A*c-B*b)/c/(c*x^2+b))-1/4*A/b^
2/x^4-1/2*(-2*A*c+B*b)/b^3/x^2+c*(3*A*c-2*B*b)/b^4*ln(x)
```

Maxima [A]
time = 0.28, size = 106, normalized size = 1.09

$$-\frac{2\,(2\,Bbc-3\,Ac^{2})x^{4}+Ab^{2}+(2\,Bb^{2}-3\,Abc)x^{2}}{4\,(b^{3}cx^{6}+b^{4}x^{4})}+\frac{(2\,Bbc-3\,Ac^{2})\log\left(cx^{2}+b\right)}{2\,b^{4}}-\frac{(2\,Bbc-3\,Ac^{2})\log\left(x^{2}\right)}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(2*B*b*c - 3*A*c^2)*x^4 + A*b^2 + (2*B*b^2 - 3*A*b*c)*x^2)/(b^3*c*x
^6 + b^4*x^4) + 1/2*(2*B*b*c - 3*A*c^2)*log(c*x^2 + b)/b^4 - 1/2*(2*B*b*c -
 3*A*c^2)*log(x^2)/b^4
```

Fricas [A]
time = 2.10, size = 154, normalized size = 1.59

$$-\frac{2\,(2\,Bb^{2}c-3\,Abc^{2})x^{4}+Ab^{3}+(2\,Bb^{3}-3\,Ab^{2}c)x^{2}-2\,((2\,Bbc^{2}-3\,Ac^{3})x^{6}+(2\,Bb^{2}c-3\,Abc^{2})x^{4})\log\left(cx^{2}+b\right)+4\,((2\,Bbc^{2}-3\,Ac^{3})x^{6}+(2\,Bb^{2}c-3\,Abc^{2})x^{4})\log\left(x\right)}{4\,(b^{4}cx^{6}+b^{5}x^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*B*b^2*c - 3*A*b*c^2)*x^4 + A*b^3 + (2*B*b^3 - 3*A*b^2*c)*x^2 - 2
*((2*B*b*c^2 - 3*A*c^3)*x^6 + (2*B*b^2*c - 3*A*b*c^2)*x^4)*log(c*x^2 + b) +
 4*((2*B*b*c^2 - 3*A*c^3)*x^6 + (2*B*b^2*c - 3*A*b*c^2)*x^4)*log(x))/(b^4*c
*x^6 + b^5*x^4)
```

Sympy [A]
time = 0.56, size = 100, normalized size = 1.03

$$\frac{-Ab^{2}+x^{4}\cdot(6Ac^{2}-4Bbc)+x^{2}\cdot(3Abc-2Bb^{2})}{4b^{4}x^{4}+4b^{3}cx^{6}}-\frac{c(-3Ac+2Bb)\log\left(x\right)}{b^{4}}+\frac{c(-3Ac+2Bb)\log\left(\frac{b}{c}+x^{2}\right)}{2b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2)**2,x)
```

```
[Out] (-A*b**2 + x**4*(6*A*c**2 - 4*B*b*c) + x**2*(3*A*b*c - 2*B*b**2))/(4*b**4*x
**4 + 4*b**3*c*x**6) - c*(-3*A*c + 2*B*b)*log(x)/b**4 + c*(-3*A*c + 2*B*b)*
log(b/c + x**2)/(2*b**4)
```

Giac [A]
time = 0.85, size = 150, normalized size = 1.55

$$-\frac{(2\,Bbc-3\,Ac^{2})\log\left(x^{2}\right)}{2\,b^{4}}+\frac{(2\,Bbc^{2}-3\,Ac^{3})\log\left(|cx^{2}+b|\right)}{2\,b^{4}c}-\frac{2\,Bbc^{2}x^{2}-3\,Ac^{3}x^{2}+3\,Bb^{2}c-4\,Abc^{2}}{2\,(cx^{2}+b)b^{4}}+\frac{6\,Bbcx^{4}-9\,Ac^{2}x^{4}-2\,Bb^{2}x^{2}+4\,Abcx^{2}-Ab^{2}}{4\,b^{4}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*B*b*c - 3*A*c^2)*log(x^2)/b^4 + 1/2*(2*B*b*c^2 - 3*A*c^3)*log(abs(c
*x^2 + b))/(b^4*c) - 1/2*(2*B*b*c^2*x^2 - 3*A*c^3*x^2 + 3*B*b^2*c - 4*A*b*c
^2)/((c*x^2 + b)*b^4) + 1/4*(6*B*b*c*x^4 - 9*A*c^2*x^4 - 2*B*b^2*x^2 + 4*A*
b*c*x^2 - A*b^2)/(b^4*x^4)
```

Mupad [B]
time = 0.14, size = 100, normalized size = 1.03

$$\frac{\frac{x^2\,\left(3\,A\,c-2\,B\,b\right)}{4\,b^2}-\frac{A}{4\,b}+\frac{c\,x^4\,\left(3\,A\,c-2\,B\,b\right)}{2\,b^3}}{c\,x^6+b\,x^4}-\frac{\ln\left(c\,x^2+b\right)\,\left(3\,A\,c^2-2\,B\,b\,c\right)}{2\,b^4}+\frac{\ln\left(x\right)\,\left(3\,A\,c^2-2\,B\,b\,c\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(b*x^2 + c*x^4)^2),x)
```

```
[Out] ((x^2*(3*A*c - 2*B*b))/(4*b^2) - A/(4*b) + (c*x^4*(3*A*c - 2*B*b))/(2*b^3))
/(b*x^4 + c*x^6) - (log(b + c*x^2)*(3*A*c^2 - 2*B*b*c))/(2*b^4) + (log(x)*(
3*A*c^2 - 2*B*b*c))/b^4
```

3.72 $\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=111

$$-\frac{A}{5b^2x^5}-\frac{bB-2Ac}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}+\frac{c^{3/2}(5bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}$$

```
[Out] -1/5*A/b^2/x^5+1/3*(2*A*c-B*b)/b^3/x^3+c*(-3*A*c+2*B*b)/b^4/x+1/2*c^2*(-A*c
+B*b)*x/b^4/(c*x^2+b)+1/2*c^(3/2)*(-7*A*c+5*B*b)*arctan(x*c^(1/2)/b^(1/2))/
b^(9/2)
```

Rubi [A]
time = 0.13, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 467, 1816, 211}

$$\frac{c^{3/2}(5bB-7Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}+\frac{c^2x(bB-Ac)}{2b^4\left(b+cx^2\right)}+\frac{c(2bB-3Ac)}{b^4x}-\frac{bB-2Ac}{3b^3x^3}-\frac{A}{5b^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/5*A/(b^2*x^5) - (b*B - 2*A*c)/(3*b^3*x^3) + (c*(2*b*B - 3*A*c))/(b^4*x)
+ (c^2*(b*B - A*c)*x)/(2*b^4*(b + c*x^2)) + (c^(3/2)*(5*b*B - 7*A*c)*ArcTan
[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx = \int \frac{A+Bx^2}{x^6\left(b+cx^2\right)^2}\,dx$$
$$= \frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)} - \frac{1}{2}c^2\int \frac{-\frac{2A}{bc^2}-\frac{2(bB-Ac)x^2}{b^2c^2}+\frac{2(bB-Ac)x^4}{b^3c}-\frac{(bB-Ac)x^6}{b^4}}{x^6\left(b+cx^2\right)}\,dx$$
$$= \frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)} - \frac{1}{2}c^2\int \left(-\frac{2A}{b^2c^2x^6}-\frac{2(bB-2Ac)}{b^3c^2x^4}+\frac{2(2bB-3Ac)}{b^4cx^2}+\frac{-5bB+7A}{b^4\left(b+cx^2\right.}\right.$$
$$= -\frac{A}{5b^2x^5}-\frac{bB-2Ac}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}+\frac{\left(c^2(5bB-7Ac)\right)\int\frac{1}{b+cx^2}}{2b^4}$$
$$= -\frac{A}{5b^2x^5}-\frac{bB-2Ac}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}+\frac{c^{3/2}(5bB-7Ac)\tan^{-1}\left(\right.}{2b^{9/2}}$$

Mathematica [A]
time = 0.06, size = 112, normalized size = 1.01

$$-\frac{A}{5b^2x^5}+\frac{-bB+2Ac}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}+\frac{c^{3/2}(5bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/5*A/(b^2*x^5) + (-(b*B) + 2*A*c)/(3*b^3*x^3) + (c*(2*b*B - 3*A*c))/(b^4*
x) + (c^2*(b*B - A*c)*x)/(2*b^4*(b + c*x^2)) + (c^(3/2)*(5*b*B - 7*A*c)*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

Maple [A]
time = 0.42, size = 99, normalized size = 0.89

method	result
default	$-\frac{c^{2}\left(\frac{\left(\frac{Ac}{2}-\frac{Bb}{2}\right)x}{c\,x^{2}+b}+\frac{(7Ac-5Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^{4}}-\frac{A}{5b^{2}x^{5}}-\frac{-2Ac+Bb}{3b^{3}x^{3}}-\frac{c(3Ac-2Bb)}{b^{4}x}$
risch	$\frac{-\frac{c^{2}(7Ac-5Bb)x^{6}}{2b^{4}}-\frac{c(7Ac-5Bb)x^{4}}{3b^{3}}+\frac{(7Ac-5Bb)x^{2}}{15b^{2}}-\frac{A}{5b}}{x^{5}(c\,x^{2}+b)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{9}\_Z^{2}+49A^{2}c^{5}-70ABb\,c^{4}+25B^{2}b^{2}c^{3}\right)}\_R\ln\left(\left(3\_R^{2}b^{9}+98A\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*c^2*((1/2*A*c-1/2*B*b)*x/(c*x^2+b)+1/2*(7*A*c-5*B*b)/(b*c)^(1/2)*arc
tan(c*x/(b*c)^(1/2)))-1/5*A/b^2/x^5-1/3*(-2*A*c+B*b)/b^3/x^3-c*(3*A*c-2*B*b
)/b^4/x
```

Maxima [A]
time = 0.50, size = 119, normalized size = 1.07

$$\frac{15\,(5\,Bbc^{2}-7\,Ac^{3})x^{6}+10\,(5\,Bb^{2}c-7\,Abc^{2})x^{4}-6\,Ab^{3}-2\,(5\,Bb^{3}-7\,Ab^{2}c)x^{2}}{30\,(b^{4}cx^{7}+b^{5}x^{5})}+\frac{(5\,Bbc^{2}-7\,Ac^{3})\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/30*(15*(5*B*b*c^2 - 7*A*c^3)*x^6 + 10*(5*B*b^2*c - 7*A*b*c^2)*x^4 - 6*A*b
^3 - 2*(5*B*b^3 - 7*A*b^2*c)*x^2)/(b^4*c*x^7 + b^5*x^5) + 1/2*(5*B*b*c^2 -
7*A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4)
```

Fricas [A]
time = 1.37, size = 308, normalized size = 2.77

$$\left[\frac{30\,(5\,Bbc^{2}-7\,Ac^{3})x^{6}+20\,(5\,Bb^{2}c-7\,Abc^{2})x^{4}-12\,Ab^{3}-4\,(5\,Bb^{3}-7\,Ab^{2}c)x^{2}-15\,((5\,Bbc^{2}-7\,Ac^{3})x^{7}+(5\,Bb^{2}c-7\,Abc^{2})x^{5})\sqrt{-\frac{c}{b}}\log\left(\frac{cx^{2}-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^{2}+b}\right)}{60\,(b^{4}cx^{7}+b^{5}x^{5})},\ \frac{15\,(5\,Bbc^{2}-7\,Ac^{3})x^{6}+10\,(5\,Bb^{2}c-7\,Abc^{2})x^{4}-6\,Ab^{3}-2\,(5\,Bb^{3}-7\,Ab^{2}c)x^{2}+15\,((5\,Bbc^{2}-7\,Ac^{3})x^{7}+(5\,Bb^{2}c-7\,Abc^{2})x^{5})\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)}{30\,(b^{4}cx^{7}+b^{5}x^{5})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(30*(5*B*b*c^2 - 7*A*c^3)*x^6 + 20*(5*B*b^2*c - 7*A*b*c^2)*x^4 - 12*A
*b^3 - 4*(5*B*b^3 - 7*A*b^2*c)*x^2 - 15*((5*B*b*c^2 - 7*A*c^3)*x^7 + (5*B*b
^2*c - 7*A*b*c^2)*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2
 + b)))/(b^4*c*x^7 + b^5*x^5), 1/30*(15*(5*B*b*c^2 - 7*A*c^3)*x^6 + 10*(5*B
*b^2*c - 7*A*b*c^2)*x^4 - 6*A*b^3 - 2*(5*B*b^3 - 7*A*b^2*c)*x^2 + 15*((5*B*
```

```
b*c^2 - 7*A*c^3)*x^7 + (5*B*b^2*c - 7*A*b*c^2)*x^5)*sqrt(c/b)*arctan(x*sqrt
(c/b)))/(b^4*c*x^7 + b^5*x^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(104) = 208$.
time = 0.34, size = 218, normalized size = 1.96

$$-\frac{\sqrt{-\frac{c^3}{b^9}}\,(-7Ac+5Bb)\log\left(-\frac{b^5\sqrt{-\frac{c^3}{b^9}}\,(-7Ac+5Bb)}{-7Ac^3+5Bbc^2}+x\right)}{4}+\frac{\sqrt{-\frac{c^3}{b^9}}\,(-7Ac+5Bb)\log\left(\frac{b^5\sqrt{-\frac{c^3}{b^9}}\,(-7Ac+5Bb)}{-7Ac^3+5Bbc^2}+x\right)}{4}+\frac{-6Ab^3+x^6(-105Ac^3+75Bbc^2)+x^4(-70Abc^2+50Bb^2c)+x^2\cdot(14Ab^2c-10Bb^3)}{30b^5x^5+30b^4cx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2)**2,x)
```

```
[Out] -sqrt(-c**3/b**9)*(-7*A*c + 5*B*b)*log(-b**5*sqrt(-c**3/b**9)*(-7*A*c + 5*B
*b)/(-7*A*c**3 + 5*B*b*c**2) + x)/4 + sqrt(-c**3/b**9)*(-7*A*c + 5*B*b)*log
(b**5*sqrt(-c**3/b**9)*(-7*A*c + 5*B*b)/(-7*A*c**3 + 5*B*b*c**2) + x)/4 + (
-6*A*b**3 + x**6*(-105*A*c**3 + 75*B*b*c**2) + x**4*(-70*A*b*c**2 + 50*B*b*
*2*c) + x**2*(14*A*b**2*c - 10*B*b**3))/(30*b**5*x**5 + 30*b**4*c*x**7)
```

Giac [A]
time = 0.78, size = 112, normalized size = 1.01

$$\frac{(5\,Bbc^2-7\,Ac^3)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^4}+\frac{Bbc^2x-Ac^3x}{2\,(cx^2+b)b^4}+\frac{30\,Bbcx^4-45\,Ac^2x^4-5\,Bb^2x^2+10\,Abcx^2-3\,Ab^2}{15\,b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*b*c^2 - 7*A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) + 1/2*(B*b*
c^2*x - A*c^3*x)/((c*x^2 + b)*b^4) + 1/15*(30*B*b*c*x^4 - 45*A*c^2*x^4 - 5*
B*b^2*x^2 + 10*A*b*c*x^2 - 3*A*b^2)/(b^4*x^5)
```

Mupad [B]
time = 0.16, size = 104, normalized size = 0.94

$$-\frac{\frac{A}{5\,b}-\frac{x^2\,(7\,A\,c-5\,B\,b)}{15\,b^2}+\frac{c^2\,x^6\,(7\,A\,c-5\,B\,b)}{2\,b^4}+\frac{c\,x^4\,(7\,A\,c-5\,B\,b)}{3\,b^3}}{c\,x^7+b\,x^5}-\frac{c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(7\,A\,c-5\,B\,b)}{2\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^2),x)
```

```
[Out] - (A/(5*b) - (x^2*(7*A*c - 5*B*b))/(15*b^2) + (c^2*x^6*(7*A*c - 5*B*b))/(2*
b^4) + (c*x^4*(7*A*c - 5*B*b))/(3*b^3))/(b*x^5 + c*x^7) - (c^(3/2)*atan((c^
(1/2)*x)/b^(1/2))*(7*A*c - 5*B*b))/(2*b^(9/2))
```

3.73 $\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=140

$$\frac{3b(2bB-Ac)x}{c^5}-\frac{(3bB-Ac)x^3}{3c^4}+\frac{Bx^5}{5c^3}-\frac{b^3(bB-Ac)x}{4c^5\left(b+cx^2\right)^2}+\frac{b^2(17bB-13Ac)x}{8c^5\left(b+cx^2\right)}-\frac{7b^{3/2}(9bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{11/2}}$$

```
[Out] 3*b*(-A*c+2*B*b)*x/c^5-1/3*(-A*c+3*B*b)*x^3/c^4+1/5*B*x^5/c^3-1/4*b^3*(-A*c
+B*b)*x/c^5/(c*x^2+b)^2+1/8*b^2*(-13*A*c+17*B*b)*x/c^5/(c*x^2+b)-7/8*b^(3/2
)*(-5*A*c+9*B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(11/2)
```

Rubi [A]
time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1598, 466, 1828, 1824, 211}

$$-\frac{7b^{3/2}(9bB-5Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{11/2}}-\frac{b^3x(bB-Ac)}{4c^5\left(b+cx^2\right)^2}+\frac{b^2x(17bB-13Ac)}{8c^5\left(b+cx^2\right)}+\frac{3bx(2bB-Ac)}{c^5}-\frac{x^3(3bB-Ac)}{3c^4}+\frac{Bx^5}{5c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^14*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (3*b*(2*b*B - A*c)*x)/c^5 - ((3*b*B - A*c)*x^3)/(3*c^4) + (B*x^5)/(5*c^3) -
 (b^3*(b*B - A*c)*x)/(4*c^5*(b + c*x^2)^2) + (b^2*(17*b*B - 13*A*c)*x)/(8*c
^5*(b + c*x^2)) - (7*b^(3/2)*(9*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(
8*c^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \int \frac{x^8(A+Bx^2)}{(b+cx^2)^3}\,dx$$
$$= -\frac{b^3(bB-Ac)x}{4c^5(b+cx^2)^2} - \frac{\int \frac{-b^3(bB-Ac)+4b^2c(bB-Ac)x^2-4bc^2(bB-Ac)x^4+4c^3(bB-Ac)x^6-4Bc^4x^8}{(b+cx^2)^2}\,dx}{4c^5}$$
$$= -\frac{b^3(bB-Ac)x}{4c^5(b+cx^2)^2} + \frac{b^2(17bB-13Ac)x}{8c^5(b+cx^2)} + \frac{\int \frac{-b^3(15bB-11Ac)+8b^2c(3bB-2Ac)x^2-8bc^2(2bB-Ac)x}{b+cx^2}}{8bc^5}$$
$$= -\frac{b^3(bB-Ac)x}{4c^5(b+cx^2)^2} + \frac{b^2(17bB-13Ac)x}{8c^5(b+cx^2)} + \frac{\int \Big(24b^2(2bB-Ac)-8bc(3bB-Ac)x^2+8}{8bc^5}$$
$$= \frac{3b(2bB-Ac)x}{c^5} - \frac{(3bB-Ac)x^3}{3c^4} + \frac{Bx^5}{5c^3} - \frac{b^3(bB-Ac)x}{4c^5(b+cx^2)^2} + \frac{b^2(17bB-13Ac)x}{8c^5(b+cx^2)} - \frac{(}{}$$
$$= \frac{3b(2bB-Ac)x}{c^5} - \frac{(3bB-Ac)x^3}{3c^4} + \frac{Bx^5}{5c^3} - \frac{b^3(bB-Ac)x}{4c^5(b+cx^2)^2} + \frac{b^2(17bB-13Ac)x}{8c^5(b+cx^2)} - \frac{7}{}$$

Mathematica [A]
time = 0.07, size = 133, normalized size = 0.95

$$\frac{x(945b^4B-525b^3c(A-3Bx^2)+8c^4x^6(5A+3Bx^2)-8bc^3x^4(35A+9Bx^2)+7b^2c^2x^2(-125A+72Bx^2))}{120c^5(b+cx^2)^2} - \frac{7b^{3/2}(9bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^14*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(945*b^4*B - 525*b^3*c*(A - 3*B*x^2) + 8*c^4*x^6*(5*A + 3*B*x^2) - 8*b*c
^3*x^4*(35*A + 9*B*x^2) + 7*b^2*c^2*x^2*(-125*A + 72*B*x^2)))/(120*c^5*(b +
 c*x^2)^2) - (7*b^(3/2)*(9*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(
11/2))
```

Maple [A]
time = 0.42, size = 119, normalized size = 0.85

method	result
default	$-\frac{-\frac{1}{5} B c^2 x^5-\frac{1}{3} A c^2 x^3+B b c x^3+3 A b c x-6 b^2 B x}{c^5}+\frac{b^2 \left(\frac{\left(-\frac{13}{8} A c^2+\frac{17}{8} b B c\right) x^3-\frac{b(11 A c-15 B b) x}{8}}{\left(c x^2+b\right)^2}+\frac{7(5 A c-9 B b) \arctan \left(\frac{c x}{\sqrt{b c}}\right)}{8 \sqrt{b c}}\right)}{c^5}$
risch	$\frac{B x^5}{5 c^3}+\frac{A x^3}{3 c^3}-\frac{B b x^3}{c^4}-\frac{3 A b x}{c^4}+\frac{6 b^2 B x}{c^5}+\frac{\left(-\frac{13}{8} b^2 A c^2+\frac{17}{8} B b^3 c\right) x^3-\frac{b^3(11 A c-15 B b) x}{8}}{c^5\left(c x^2+b\right)^2}+\frac{35 \sqrt{-b c}\, b \ln \left(-\sqrt{-b c}\, x+b\right)}{16 c^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^5*(-1/5*B*c^2*x^5-1/3*A*c^2*x^3+B*b*c*x^3+3*A*b*c*x-6*b^2*B*x)+b^2/c^5
*(((-13/8*A*c^2+17/8*b*B*c)*x^3-1/8*b*(11*A*c-15*B*b)*x)/(c*x^2+b)^2+7/8*(5
*A*c-9*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Maxima [A]
time = 0.50, size = 147, normalized size = 1.05

$$\frac{(17 B b^3 c - 13 A b^2 c^2)x^3 + (15 B b^4 - 11 A b^3 c)x}{8 (c^7 x^4 + 2 b c^6 x^2 + b^2 c^5)} - \frac{7 (9 B b^3 - 5 A b^2 c) \arctan\left(\frac{c x}{\sqrt{b c}}\right)}{8 \sqrt{b c}\, c^5} + \frac{3 B c^2 x^5 - 5 (3 B b c - A c^2)x^3 + 45 (2 B b^2 - A b c)x}{15 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((17*B*b^3*c - 13*A*b^2*c^2)*x^3 + (15*B*b^4 - 11*A*b^3*c)*x)/(c^7*x^4
+ 2*b*c^6*x^2 + b^2*c^5) - 7/8*(9*B*b^3 - 5*A*b^2*c)*arctan(c*x/sqrt(b*c))/
(sqrt(b*c)*c^5) + 1/15*(3*B*c^2*x^5 - 5*(3*B*b*c - A*c^2)*x^3 + 45*(2*B*b^2
 - A*b*c)*x)/c^5
```

Fricas [A]
time = 2.23, size = 416, normalized size = 2.97

$$\left[\frac{48 B c^4 x^9 - 16 (9 B b c^3 - 5 A c^4)x^7 + 112 (9 B b^2 c^2 - 5 A b c^3)x^5 + 350 (9 B b^3 c - 5 A b^2 c^2)x^3 - 105 (9 B b^4 - 5 A b^3 c + (9 B b^2 c^2 - 5 A b c^3)x^4 + 2 (9 B b^3 c - 5 A b^2 c^2)x^2)\sqrt{-\frac{b}{c}} \log\left(\frac{c x^2 + 2 c x \sqrt{-\frac{b}{c}} - b}{c x^2 + b}\right) + 210 (9 B b^4 - 5 A b^3 c)x}{240 (c^7 x^4 + 2 b c^6 x^2 + b^2 c^5)}, \frac{24 B c^4 x^9 - 8 (9 B b c^3 - 5 A c^4)x^7 + 56 (9 B b^2 c^2 - 5 A b c^3)x^5 + 175 (9 B b^3 c - 5 A b^2 c^2)x^3 - 105 (9 B b^4 - 5 A b^3 c + (9 B b^2 c^2 - 5 A b c^3)x^4 + 2 (9 B b^3 c - 5 A b^2 c^2)x^2)\sqrt{\frac{b}{c}} \arctan\left(\frac{c x \sqrt{\frac{b}{c}}}{b}\right) + 105 (9 B b^4 - 5 A b^3 c)x}{120 (c^7 x^4 + 2 b c^6 x^2 + b^2 c^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*B*c^4*x^9 - 16*(9*B*b*c^3 - 5*A*c^4)*x^7 + 112*(9*B*b^2*c^2 - 5*
A*b*c^3)*x^5 + 350*(9*B*b^3*c - 5*A*b^2*c^2)*x^3 - 105*(9*B*b^4 - 5*A*b^3*c
 + (9*B*b^2*c^2 - 5*A*b*c^3)*x^4 + 2*(9*B*b^3*c - 5*A*b^2*c^2)*x^2)*sqrt(-b
/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 210*(9*B*b^4 - 5*A*b^
3*c)*x)/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5), 1/120*(24*B*c^4*x^9 - 8*(9*B*b*c
^3 - 5*A*c^4)*x^7 + 56*(9*B*b^2*c^2 - 5*A*b*c^3)*x^5 + 175*(9*B*b^3*c - 5*A
*b^2*c^2)*x^3 - 105*(9*B*b^4 - 5*A*b^3*c + (9*B*b^2*c^2 - 5*A*b*c^3)*x^4 +
2*(9*B*b^3*c - 5*A*b^2*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) + 105*(9
*B*b^4 - 5*A*b^3*c)*x)/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5)]
```

Sympy [A]
time = 0.77, size = 252, normalized size = 1.80

$$\frac{Bx^5}{5c^3}+x^3\left(\frac{A}{3c^3}-\frac{Bb}{c^4}\right)+x\left(-\frac{3Ab}{c^4}+\frac{6Bb^2}{c^5}\right)+\frac{7\sqrt{-\frac{b^3}{c^{11}}}\,(-5Ac+9Bb)\log\left(-\frac{7c^5\sqrt{-\frac{b^3}{c^{11}}}\,(-5Ac+9Bb)}{-35Abc+63Bb^2}+x\right)}{16}-\frac{7\sqrt{-\frac{b^3}{c^{11}}}\,(-5Ac+9Bb)\log\left(\frac{7c^5\sqrt{-\frac{b^3}{c^{11}}}\,(-5Ac+9Bb)}{-35Abc+63Bb^2}+x\right)}{16}+\frac{x^3(-13Ab^2c^2+17Bb^3c)+x(-11Ab^3c+15Bb^4)}{8b^2c^5+16bc^6x^2+8c^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**5/(5*c**3) + x**3*(A/(3*c**3) - B*b/c**4) + x*(-3*A*b/c**4 + 6*B*b**2/
c**5) + 7*sqrt(-b**3/c**11)*(-5*A*c + 9*B*b)*log(-7*c**5*sqrt(-b**3/c**11)*
(-5*A*c + 9*B*b)/(-35*A*b*c + 63*B*b**2) + x)/16 - 7*sqrt(-b**3/c**11)*(-5*
A*c + 9*B*b)*log(7*c**5*sqrt(-b**3/c**11)*(-5*A*c + 9*B*b)/(-35*A*b*c + 63*
B*b**2) + x)/16 + (x**3*(-13*A*b**2*c**2 + 17*B*b**3*c) + x*(-11*A*b**3*c +
 15*B*b**4))/(8*b**2*c**5 + 16*b*c**6*x**2 + 8*c**7*x**4)
```

Giac [A]
time = 0.83, size = 138, normalized size = 0.99

$$-\frac{7\,(9\,Bb^3-5\,Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^5}+\frac{17\,Bb^3cx^3-13\,Ab^2c^2x^3+15\,Bb^4x-11\,Ab^3cx}{8\,(cx^2+b)^2c^5}+\frac{3\,Bc^{12}x^5-15\,Bbc^{11}x^3+5\,Ac^{12}x^3+90\,Bb^2c^{10}x-45\,Abc^{11}x}{15\,c^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -7/8*(9*B*b^3 - 5*A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/8*(17*
B*b^3*c*x^3 - 13*A*b^2*c^2*x^3 + 15*B*b^4*x - 11*A*b^3*c*x)/((c*x^2 + b)^2*
c^5) + 1/15*(3*B*c^12*x^5 - 15*B*b*c^11*x^3 + 5*A*c^12*x^3 + 90*B*b^2*c^10*
x - 45*A*b*c^11*x)/c^15
```

Mupad [B]
time = 0.12, size = 177, normalized size = 1.26

$$\frac{x\,\left(\frac{15\,B\,b^4}{8}-\frac{11\,A\,b^3\,c}{8}\right)-x^3\,\left(\frac{13\,A\,b^2\,c^2}{8}-\frac{17\,B\,b^3\,c}{8}\right)}{b^2\,c^5+2\,b\,c^6\,x^2+c^7\,x^4}-x\,\left(\frac{3\,b\,\left(\frac{A}{c^3}-\frac{3\,B\,b}{c^4}\right)}{c}+\frac{3\,B\,b^2}{c^5}\right)+x^3\,\left(\frac{A}{3\,c^3}-\frac{B\,b}{c^4}\right)+\frac{B\,x^5}{5\,c^3}-\frac{7\,b^{3/2}\,\mathrm{atan}\left(\frac{b^{3/2}\,\sqrt{c}\,x\,(5\,A\,c-9\,B\,b)}{9\,B\,b^3-5\,A\,b^2\,c}\right)\,(5\,A\,c-9\,B\,b)}{8\,c^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^14*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x*((15*B*b^4)/8 - (11*A*b^3*c)/8) - x^3*((13*A*b^2*c^2)/8 - (17*B*b^3*c)/8
))/(b^2*c^5 + c^7*x^4 + 2*b*c^6*x^2) - x*((3*b*(A/c^3 - (3*B*b)/c^4))/c + (
3*B*b^2)/c^5) + x^3*(A/(3*c^3) - (B*b)/c^4) + (B*x^5)/(5*c^3) - (7*b^(3/2)*
atan((b^(3/2)*c^(1/2)*x*(5*A*c - 9*B*b))/(9*B*b^3 - 5*A*b^2*c))*(5*A*c - 9*
B*b))/(8*c^(11/2))
```

3.74 $\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=111

$$-\frac{(3bB-Ac)x^2}{2c^4}+\frac{Bx^4}{4c^3}-\frac{b^3(bB-Ac)}{4c^5\left(b+cx^2\right)^2}+\frac{b^2(4bB-3Ac)}{2c^5\left(b+cx^2\right)}+\frac{3b(2bB-Ac)\log\left(b+cx^2\right)}{2c^5}$$

```
[Out] -1/2*(-A*c+3*B*b)*x^2/c^4+1/4*B*x^4/c^3-1/4*b^3*(-A*c+B*b)/c^5/(c*x^2+b)^2+
1/2*b^2*(-3*A*c+4*B*b)/c^5/(c*x^2+b)+3/2*b*(-A*c+2*B*b)*ln(c*x^2+b)/c^5
```

Rubi [A]

time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$-\frac{b^3(bB-Ac)}{4c^5\left(b+cx^2\right)^2}+\frac{b^2(4bB-3Ac)}{2c^5\left(b+cx^2\right)}+\frac{3b(2bB-Ac)\log\left(b+cx^2\right)}{2c^5}-\frac{x^2(3bB-Ac)}{2c^4}+\frac{Bx^4}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^13*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/2*((3*b*B - A*c)*x^2)/c^4 + (B*x^4)/(4*c^3) - (b^3*(b*B - A*c))/(4*c^5*(
b + c*x^2)^2) + (b^2*(4*b*B - 3*A*c))/(2*c^5*(b + c*x^2)) + (3*b*(2*b*B - A
*c)*Log[b + c*x^2])/(2*c^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \int \frac{x^7(A+Bx^2)}{(b+cx^2)^3}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{-3bB+Ac}{c^4}+\frac{Bx}{c^3}+\frac{b^3(bB-Ac)}{c^4(b+cx)^3}-\frac{b^2(4bB-3Ac)}{c^4(b+cx)^2}+\frac{3b(2bB-Ac)}{c^4(b+cx)}\right)$$
$$= -\frac{(3bB-Ac)x^2}{2c^4}+\frac{Bx^4}{4c^3}-\frac{b^3(bB-Ac)}{4c^5(b+cx^2)^2}+\frac{b^2(4bB-3Ac)}{2c^5(b+cx^2)}+\frac{3b(2bB-Ac)\log(b+c}{2c^5}$$

Mathematica [A]
time = 0.04, size = 94, normalized size = 0.85

$$\frac{2c(-3bB+Ac)x^2+Bc^2x^4+\frac{b^3(-bB+Ac)}{(b+cx^2)^2}+\frac{2b^2(4bB-3Ac)}{b+cx^2}+6b(2bB-Ac)\log(b+cx^2)}{4c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^13*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*c*(-3*b*B + A*c)*x^2 + B*c^2*x^4 + (b^3*(-(b*B) + A*c))/(b + c*x^2)^2 +
(2*b^2*(4*b*B - 3*A*c))/(b + c*x^2) + 6*b*(2*b*B - A*c)*Log[b + c*x^2])/(4*
c^5)
```

Maple [A]
time = 0.43, size = 102, normalized size = 0.92

method	result
default	$\frac{(Bc\,x^2+Ac-3Bb)^2}{4c^5B}-\frac{b\left(\frac{(3Ac-6Bb)\ln(c\,x^2+b)}{c}-\frac{b^2(Ac-Bb)}{2c(c\,x^2+b)^2}+\frac{b(3Ac-4Bb)}{c(c\,x^2+b)}\right)}{2c^4}$
norman	$\frac{\frac{B\,x^{13}}{4c}+\frac{(Ac-2Bb)x^{11}}{2c^2}-\frac{b(3Abc-6b^2B)x^7}{c^4}-\frac{b^2(9Abc-18b^2B)x^5}{4c^5}}{x^5(c\,x^2+b)^2}-\frac{3b(Ac-2Bb)\ln(c\,x^2+b)}{2c^5}$
risch	$\frac{B\,x^4}{4c^3}+\frac{A\,x^2}{2c^3}-\frac{3Bb\,x^2}{2c^4}+\frac{A^2}{4c^3B}-\frac{3Ab}{2c^4}+\frac{9B\,b^2}{4c^5}+\frac{(-\frac{3}{2}A\,b^2c+2B\,b^3)x^2-\frac{b^3(5Ac-7Bb)}{4c}}{c^4(c\,x^2+b)^2}-\frac{3b\ln(c\,x^2+b)A}{2c^4}+\frac{3b^2\ln(c\,x^2+b)}{c^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(B*c*x^2+A*c-3*B*b)^2/c^5/B-1/2*b/c^4*((3*A*c-6*B*b)/c*ln(c*x^2+b)-1/2*
b^2*(A*c-B*b)/c/(c*x^2+b)^2+b*(3*A*c-4*B*b)/c/(c*x^2+b))
```

Maxima [A]
time = 0.28, size = 116, normalized size = 1.05

$$\frac{7\,Bb^4-5\,Ab^3c+2\,(4\,Bb^3c-3\,Ab^2c^2)x^2}{4\,(c^7x^4+2\,bc^6x^2+b^2c^5)}+\frac{Bcx^4-2\,(3\,Bb-Ac)x^2}{4\,c^4}+\frac{3\,(2\,Bb^2-Abc)\log\left(cx^2+b\right)}{2\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(7*B*b^4 - 5*A*b^3*c + 2*(4*B*b^3*c - 3*A*b^2*c^2)*x^2)/(c^7*x^4 + 2*b*
c^6*x^2 + b^2*c^5) + 1/4*(B*c*x^4 - 2*(3*B*b - A*c)*x^2)/c^4 + 3/2*(2*B*b^2
 - A*b*c)*log(c*x^2 + b)/c^5
```

Fricas [A]
time = 1.46, size = 179, normalized size = 1.61

$$\frac{Bc^4x^8-2\,(2\,Bbc^3-Ac^4)x^6+7\,Bb^4-5\,Ab^3c-(11\,Bb^2c^2-4\,Abc^3)x^4+2\,(Bb^3c-2\,Ab^2c^2)x^2+6\,(2\,Bb^4-Ab^3c+(2\,Bb^2c^2-Abc^3)x^4+2\,(2\,Bb^3c-Ab^2c^2)x^2)\log\left(cx^2+b\right)}{4\,(c^7x^4+2\,bc^6x^2+b^2c^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^4*x^8 - 2*(2*B*b*c^3 - A*c^4)*x^6 + 7*B*b^4 - 5*A*b^3*c - (11*B*b^
2*c^2 - 4*A*b*c^3)*x^4 + 2*(B*b^3*c - 2*A*b^2*c^2)*x^2 + 6*(2*B*b^4 - A*b^3
*c + (2*B*b^2*c^2 - A*b*c^3)*x^4 + 2*(2*B*b^3*c - A*b^2*c^2)*x^2)*log(c*x^2
 + b))/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5)
```

Sympy [A]
time = 0.80, size = 119, normalized size = 1.07

$$\frac{Bx^4}{4c^3}+\frac{3b(-Ac+2Bb)\log\left(b+cx^2\right)}{2c^5}+x^2\left(\frac{A}{2c^3}-\frac{3Bb}{2c^4}\right)+\frac{-5Ab^3c+7Bb^4+x^2(-6Ab^2c^2+8Bb^3c)}{4b^2c^5+8bc^6x^2+4c^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**4/(4*c**3) + 3*b*(-A*c + 2*B*b)*log(b + c*x**2)/(2*c**5) + x**2*(A/(2*
c**3) - 3*B*b/(2*c**4)) + (-5*A*b**3*c + 7*B*b**4 + x**2*(-6*A*b**2*c**2 +
8*B*b**3*c))/(4*b**2*c**5 + 8*b*c**6*x**2 + 4*c**7*x**4)
```

Giac [A]
time = 0.83, size = 132, normalized size = 1.19

$$\frac{3\,(2\,Bb^2-Abc)\log\left(|cx^2+b|\right)}{2\,c^5}+\frac{Bc^3x^4-6\,Bbc^2x^2+2\,Ac^3x^2}{4\,c^6}-\frac{18\,Bb^2c^2x^4-9\,Abc^3x^4+28\,Bb^3cx^2-12\,Ab^2c^2x^2+11\,Bb^4-4\,Ab^3c}{4\,(cx^2+b)^2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/2*(2*B*b^2 - A*b*c)*log(abs(c*x^2 + b))/c^5 + 1/4*(B*c^3*x^4 - 6*B*b*c^2*
x^2 + 2*A*c^3*x^2)/c^6 - 1/4*(18*B*b^2*c^2*x^4 - 9*A*b*c^3*x^4 + 28*B*b^3*c
*x^2 - 12*A*b^2*c^2*x^2 + 11*B*b^4 - 4*A*b^3*c)/((c*x^2 + b)^2*c^5)
```

Mupad [B]
time = 0.12, size = 118, normalized size = 1.06

$$\frac{\frac{7\,B\,b^4-5\,A\,b^3\,c}{4\,c}+x^2\,\left(2\,B\,b^3-\frac{3\,A\,b^2\,c}{2}\right)}{b^2\,c^4+2\,b\,c^5\,x^2+c^6\,x^4}+x^2\,\left(\frac{A}{2\,c^3}-\frac{3\,B\,b}{2\,c^4}\right)+\frac{\ln\left(c\,x^2+b\right)\,\left(6\,B\,b^2-3\,A\,b\,c\right)}{2\,c^5}+\frac{B\,x^4}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^13*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((7*B*b^4 - 5*A*b^3*c)/(4*c) + x^2*(2*B*b^3 - (3*A*b^2*c)/2))/(b^2*c^4 + c^
6*x^4 + 2*b*c^5*x^2) + x^2*(A/(2*c^3) - (3*B*b)/(2*c^4)) + (log(b + c*x^2)*
(6*B*b^2 - 3*A*b*c))/(2*c^5) + (B*x^4)/(4*c^3)
```

3.75 $\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=118

$$-\frac{(3bB-Ac)x}{c^4}+\frac{Bx^3}{3c^3}+\frac{b^2(bB-Ac)x}{4c^4(b+cx^2)^2}-\frac{b(13bB-9Ac)x}{8c^4(b+cx^2)}+\frac{5\sqrt{b}\,(7bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}}$$

```
[Out] -(-A*c+3*B*b)*x/c^4+1/3*B*x^3/c^3+1/4*b^2*(-A*c+B*b)*x/c^4/(c*x^2+b)^2-1/8*
b*(-9*A*c+13*B*b)*x/c^4/(c*x^2+b)+5/8*(-3*A*c+7*B*b)*arctan(x*c^(1/2)/b^(1/
2))*b^(1/2)/c^(9/2)
```

Rubi [A]
time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1598, 466, 1828, 1167, 211}

$$\frac{5\sqrt{b}\,(7bB-3Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}}+\frac{b^2x(bB-Ac)}{4c^4(b+cx^2)^2}-\frac{bx(13bB-9Ac)}{8c^4(b+cx^2)}-\frac{x(3bB-Ac)}{c^4}+\frac{Bx^3}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(((3*b*B - A*c)*x)/c^4) + (B*x^3)/(3*c^3) + (b^2*(b*B - A*c)*x)/(4*c^4*(b
+ c*x^2)^2) - (b*(13*b*B - 9*A*c)*x)/(8*c^4*(b + c*x^2)) + (5*Sqrt[b]*(7*b*
B - 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \int \frac{x^6(A+Bx^2)}{(b+cx^2)^3}\,dx$$

$$= \frac{b^2(bB-Ac)x}{4c^4(b+cx^2)^2} - \frac{\int \frac{b^2(bB-Ac)-4bc(bB-Ac)x^2+4c^2(bB-Ac)x^4-4Bc^3x^6}{(b+cx^2)^2}\,dx}{4c^4}$$

$$= \frac{b^2(bB-Ac)x}{4c^4(b+cx^2)^2} - \frac{b(13bB-9Ac)x}{8c^4(b+cx^2)} + \frac{\int \frac{b^2(11bB-7Ac)-8bc(2bB-Ac)x^2+8bBc^2x^4}{b+cx^2}\,dx}{8bc^4}$$

$$= \frac{b^2(bB-Ac)x}{4c^4(b+cx^2)^2} - \frac{b(13bB-9Ac)x}{8c^4(b+cx^2)} + \frac{\int \left(-8b(3bB-Ac)+8bBcx^2+\frac{5(7b^3B-3Ab^2c)}{b+cx^2}\right) d}{8bc^4}$$

$$= -\frac{(3bB-Ac)x}{c^4} + \frac{Bx^3}{3c^3} + \frac{b^2(bB-Ac)x}{4c^4(b+cx^2)^2} - \frac{b(13bB-9Ac)x}{8c^4(b+cx^2)} + \frac{(5b(7bB-3Ac))\int \frac{1}{b+c}}{8c^4}$$

$$= -\frac{(3bB-Ac)x}{c^4} + \frac{Bx^3}{3c^3} + \frac{b^2(bB-Ac)x}{4c^4(b+cx^2)^2} - \frac{b(13bB-9Ac)x}{8c^4(b+cx^2)} + \frac{5\sqrt{b}\,(7bB-3Ac)\tan}{8c^{9/2}}$$

Mathematica [A]

time = 0.06, size = 113, normalized size = 0.96

$$\frac{-105b^3Bx+bc^2x^3(75A-56Bx^2)+5b^2cx(9A-35Bx^2)+8c^3x^5(3A+Bx^2)}{24c^4\left(b+cx^2\right)^2}+\frac{5\sqrt{b}\,(7bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-105*b^3*B*x + b*c^2*x^3*(75*A - 56*B*x^2) + 5*b^2*c*x*(9*A - 35*B*x^2) +
8*c^3*x^5*(3*A + B*x^2))/(24*c^4*(b + c*x^2)^2) + (5*Sqrt[b]*(7*b*B - 3*A*c
)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

Maple [A]
time = 0.42, size = 95, normalized size = 0.81

method	result
default	$\frac{\frac{1}{3}Bc\,x^3+Acx-3bBx}{c^4}-\frac{b\left(\frac{\left(-\frac{9}{8}A\,c^2+\frac{13}{8}bBc\right)x^3-\frac{b(7Ac-11Bb)x}{8}}{\left(c\,x^2+b\right)^2}+\frac{5(3Ac-7Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{c^4}$
risch	$\frac{B\,x^3}{3c^3}+\frac{Ax}{c^3}-\frac{3bBx}{c^4}+\frac{\left(\frac{9}{8}Ab\,c^2-\frac{13}{8}B\,b^2c\right)x^3+\frac{b^2(7Ac-11Bb)x}{8}}{c^4(c\,x^2+b)^2}+\frac{15\sqrt{-bc}\,\ln\left(-\sqrt{-bc}\,x-b\right)A}{16c^4}-\frac{35\sqrt{-bc}\,\ln\left(-\sqrt{\right.}\right.}{16c^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/3*B*c*x^3+A*c*x-3*b*B*x)-b/c^4*(((-9/8*A*c^2+13/8*b*B*c)*x^3-1/8*b
*(7*A*c-11*B*b)*x)/(c*x^2+b)^2+5/8*(3*A*c-7*B*b)/(b*c)^(1/2)*arctan(c*x/(b*
c)^(1/2)))
```

Maxima [A]
time = 0.49, size = 120, normalized size = 1.02

$$-\frac{(13\,Bb^2c-9\,Abc^2)x^3+(11\,Bb^3-7\,Ab^2c)x}{8\,(c^6x^4+2\,bc^5x^2+b^2c^4)}+\frac{5\,(7\,Bb^2-3\,Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^4}+\frac{Bcx^3-3\,(3\,Bb-Ac)x}{3\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((13*B*b^2*c - 9*A*b*c^2)*x^3 + (11*B*b^3 - 7*A*b^2*c)*x)/(c^6*x^4 + 2
*b*c^5*x^2 + b^2*c^4) + 5/8*(7*B*b^2 - 3*A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt
(b*c)*c^4) + 1/3*(B*c*x^3 - 3*(3*B*b - A*c)*x)/c^4
```

Fricas [A]
time = 2.38, size = 358, normalized size = 3.03

$$\left[\frac{16 \, B c^{3} x^{7} - 16 \, {\left(7 \, B b c^{2} - 3 \, A c^{3}\right)} x^{5} - 50 \, {\left(7 \, B b^{2} c - 3 \, A b c^{2}\right)} x^{3} - 15 \, {\left({\left(7 \, B b c^{2} - 3 \, A c^{3}\right)} x^{4} + 7 \, B b^{3} - 3 \, A b^{2} c + 2 \, {\left(7 \, B b^{2} c - 3 \, A b c^{2}\right)} x^{2}\right)} \sqrt{-\frac{b}{c}} \log\left(\frac{c x^{2} - 2 \, c x \sqrt{-\frac{b}{c}} - b}{c x^{2} + b}\right) - 30 \, {\left(7 \, B b^{3} - 3 \, A b^{2} c\right)} x}{48 \, {\left(c^{6} x^{4} + 2 \, b c^{5} x^{2} + b^{2} c^{4}\right)}}, \frac{8 \, B c^{3} x^{7} - 8 \, {\left(7 \, B b c^{2} - 3 \, A c^{3}\right)} x^{5} - 25 \, {\left(7 \, B b^{2} c - 3 \, A b c^{2}\right)} x^{3} + 15 \, {\left({\left(7 \, B b c^{2} - 3 \, A c^{3}\right)} x^{4} + 7 \, B b^{3} - 3 \, A b^{2} c + 2 \, {\left(7 \, B b^{2} c - 3 \, A b c^{2}\right)} x^{2}\right)} \sqrt{\frac{b}{c}} \arctan\left(\frac{c x \sqrt{\frac{b}{c}}}{b}\right) - 15 \, {\left(7 \, B b^{3} - 3 \, A b^{2} c\right)} x}{24 \, {\left(c^{6} x^{4} + 2 \, b c^{5} x^{2} + b^{2} c^{4}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*B*c^3*x^7 - 16*(7*B*b*c^2 - 3*A*c^3)*x^5 - 50*(7*B*b^2*c - 3*A*b*
c^2)*x^3 - 15*((7*B*b*c^2 - 3*A*c^3)*x^4 + 7*B*b^3 - 3*A*b^2*c + 2*(7*B*b^2
*c - 3*A*b*c^2)*x^2)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 +
 b)) - 30*(7*B*b^3 - 3*A*b^2*c)*x)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4), 1/24*
(8*B*c^3*x^7 - 8*(7*B*b*c^2 - 3*A*c^3)*x^5 - 25*(7*B*b^2*c - 3*A*b*c^2)*x^3
 + 15*((7*B*b*c^2 - 3*A*c^3)*x^4 + 7*B*b^3 - 3*A*b^2*c + 2*(7*B*b^2*c - 3*A
*b*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - 15*(7*B*b^3 - 3*A*b^2*c)*x
)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)]
```

Sympy [A]
time = 0.71, size = 214, normalized size = 1.81

$$\frac{Bx^{3}}{3c^{3}} + x\left(\frac{A}{c^{3}} - \frac{3Bb}{c^{4}}\right) - \frac{5\sqrt{-\frac{b}{c^{9}}}\,(-3Ac+7Bb)\log\left(-\frac{5c^{4}\sqrt{-\frac{b}{c^{9}}}\,(-3Ac+7Bb)}{-15Ac+35Bb}+x\right)}{16} + \frac{5\sqrt{-\frac{b}{c^{9}}}\,(-3Ac+7Bb)\log\left(\frac{5c^{4}\sqrt{-\frac{b}{c^{9}}}\,(-3Ac+7Bb)}{-15Ac+35Bb}+x\right)}{16} + \frac{x^{3}\cdot(9Abc^{2}-13Bb^{2}c)+x(7Ab^{2}c-11Bb^{3})}{8b^{2}c^{4}+16bc^{5}x^{2}+8c^{6}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**3/(3*c**3) + x*(A/c**3 - 3*B*b/c**4) - 5*sqrt(-b/c**9)*(-3*A*c + 7*B*b
)*log(-5*c**4*sqrt(-b/c**9)*(-3*A*c + 7*B*b)/(-15*A*c + 35*B*b) + x)/16 + 5
*sqrt(-b/c**9)*(-3*A*c + 7*B*b)*log(5*c**4*sqrt(-b/c**9)*(-3*A*c + 7*B*b)/(
-15*A*c + 35*B*b) + x)/16 + (x**3*(9*A*b*c**2 - 13*B*b**2*c) + x*(7*A*b**2*
c - 11*B*b**3))/(8*b**2*c**4 + 16*b*c**5*x**2 + 8*c**6*x**4)
```

Giac [A]
time = 0.70, size = 111, normalized size = 0.94

$$\frac{5 \, {\left(7 \, B b^{2} - 3 \, A b c\right)} \arctan\left(\frac{c x}{\sqrt{b c}}\right)}{8 \, \sqrt{b c} c^{4}} - \frac{13 \, B b^{2} c x^{3} - 9 \, A b c^{2} x^{3} + 11 \, B b^{3} x - 7 \, A b^{2} c x}{8 \, {\left(c x^{2} + b\right)}^{2} c^{4}} + \frac{B c^{6} x^{3} - 9 \, B b c^{5} x + 3 \, A c^{6} x}{3 \, c^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 5/8*(7*B*b^2 - 3*A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) - 1/8*(13*B*b
^2*c*x^3 - 9*A*b*c^2*x^3 + 11*B*b^3*x - 7*A*b^2*c*x)/((c*x^2 + b)^2*c^4) +
1/3*(B*c^6*x^3 - 9*B*b*c^5*x + 3*A*c^6*x)/c^9
```

Mupad [B]
time = 0.08, size = 138, normalized size = 1.17

$$\frac{x^3\,\left(\frac{9\,A\,b\,c^2}{8}-\frac{13\,B\,b^2\,c}{8}\right)-x\,\left(\frac{11\,B\,b^3}{8}-\frac{7\,A\,b^2\,c}{8}\right)}{b^2\,c^4+2\,b\,c^5\,x^2+c^6\,x^4}+x\,\left(\frac{A}{c^3}-\frac{3\,B\,b}{c^4}\right)+\frac{B\,x^3}{3\,c^3}+\frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c}\,x\,(3\,A\,c-7\,B\,b)}{7\,B\,b^2-3\,A\,b\,c}\right)\,(3\,A\,c-7\,B\,b)}{8\,c^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^12*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^3*((9*A*b*c^2)/8 - (13*B*b^2*c)/8) - x*((11*B*b^3)/8 - (7*A*b^2*c)/8))/(
b^2*c^4 + c^6*x^4 + 2*b*c^5*x^2) + x*(A/c^3 - (3*B*b)/c^4) + (B*x^3)/(3*c^3
) + (5*b^(1/2)*atan((b^(1/2)*c^(1/2)*x*(3*A*c - 7*B*b))/(7*B*b^2 - 3*A*b*c)
)*(3*A*c - 7*B*b))/(8*c^(9/2))
```

3.76 $\int \frac{x^{11}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=89

$$\frac{Bx^2}{2c^3}+\frac{b^2(bB-Ac)}{4c^4\left(b+cx^2\right)^2}-\frac{b(3bB-2Ac)}{2c^4\left(b+cx^2\right)}-\frac{(3bB-Ac)\log\left(b+cx^2\right)}{2c^4}$$

```
[Out] 1/2*B*x^2/c^3+1/4*b^2*(-A*c+B*b)/c^4/(c*x^2+b)^2-1/2*b*(-2*A*c+3*B*b)/c^4/(
c*x^2+b)-1/2*(-A*c+3*B*b)*ln(c*x^2+b)/c^4
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 78}

$$\frac{b^2(bB-Ac)}{4c^4\left(b+cx^2\right)^2}-\frac{b(3bB-2Ac)}{2c^4\left(b+cx^2\right)}-\frac{(3bB-Ac)\log\left(b+cx^2\right)}{2c^4}+\frac{Bx^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (B*x^2)/(2*c^3) + (b^2*(b*B - A*c))/(4*c^4*(b + c*x^2)^2) - (b*(3*b*B - 2*A
*c))/(2*c^4*(b + c*x^2)) - ((3*b*B - A*c)*Log[b + c*x^2])/(2*c^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= \int \frac{x^5(A+Bx^2)}{(b+cx^2)^3}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(b+cx)^3}\,dx,x,x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{B}{c^3}-\frac{b^2(bB-Ac)}{c^3(b+cx)^3}+\frac{b(3bB-2Ac)}{c^3(b+cx)^2}+\frac{-3bB+Ac}{c^3(b+cx)}\right)dx,x,x^2\right) \\
&= \frac{Bx^2}{2c^3}+\frac{b^2(bB-Ac)}{4c^4\left(b+cx^2\right)^2}-\frac{b(3bB-2Ac)}{2c^4\left(b+cx^2\right)}-\frac{(3bB-Ac)\log\left(b+cx^2\right)}{2c^4}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 92, normalized size = 1.03

$$\frac{Bx^2}{2c^3}+\frac{b^3B-Ab^2c}{4c^4\left(b+cx^2\right)^2}+\frac{-3b^2B+2Abc}{2c^4\left(b+cx^2\right)}+\frac{(-3bB+Ac)\log\left(b+cx^2\right)}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (B*x^2)/(2*c^3) + (b^3*B - A*b^2*c)/(4*c^4*(b + c*x^2)^2) + (-3*b^2*B + 2*A
*b*c)/(2*c^4*(b + c*x^2)) + ((-3*b*B + A*c)*Log[b + c*x^2])/(2*c^4)
```

Maple [A]
time = 0.40, size = 85, normalized size = 0.96

method	result	size
norman	$\frac{\frac{b(Ac-3Bb)x^7}{c^3}+\frac{Bx^{11}}{2c}+\frac{b^2(3Ac-9Bb)x^5}{4c^4}}{x^5(cx^2+b)^2}+\frac{(Ac-3Bb)\ln\left(cx^2+b\right)}{2c^4}$	82
default	$\frac{Bx^2}{2c^3}+\frac{\frac{(Ac-3Bb)\ln\left(cx^2+b\right)}{c}-\frac{b^2(Ac-Bb)}{2c\left(cx^2+b\right)^2}+\frac{b(2Ac-3Bb)}{c\left(cx^2+b\right)}}{2c^3}$	85
risch	$\frac{Bx^2}{2c^3}+\frac{\left(Abc-\frac{3}{2}b^2B\right)x^2+\frac{b^2(3Ac-5Bb)}{4c}}{c^3(cx^2+b)^2}+\frac{\ln\left(cx^2+b\right)A}{2c^3}-\frac{3\ln\left(cx^2+b\right)Bb}{2c^4}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/c^3+1/2/c^3*(1/c*(A*c-3*B*b)*ln(c*x^2+b)-1/2*b^2*(A*c-B*b)/c/(c*x
^2+b)^2+b*(2*A*c-3*B*b)/c/(c*x^2+b))
```

Maxima [A]
time = 0.29, size = 94, normalized size = 1.06

$$-\frac{5\,Bb^3-3\,Ab^2c+2\,(3\,Bb^2c-2\,Abc^2)x^2}{4\,(c^6x^4+2\,bc^5x^2+b^2c^4)}+\frac{Bx^2}{2\,c^3}-\frac{(3\,Bb-Ac)\log{(cx^2+b)}}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*B*b^3 - 3*A*b^2*c + 2*(3*B*b^2*c - 2*A*b*c^2)*x^2)/(c^6*x^4 + 2*b*c
^5*x^2 + b^2*c^4) + 1/2*B*x^2/c^3 - 1/2*(3*B*b - A*c)*log(c*x^2 + b)/c^4
```

Fricas [A]
time = 1.19, size = 142, normalized size = 1.60

$$\frac{2\,Bc^3x^6+4\,Bbc^2x^4-5\,Bb^3+3\,Ab^2c-4\,(Bb^2c-Abc^2)x^2-2\,((3\,Bbc^2-Ac^3)x^4+3\,Bb^3-Ab^2c+2\,(3\,Bb^2c-Abc^2)x^2)\log{(cx^2+b)}}{4\,(c^6x^4+2\,bc^5x^2+b^2c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*c^3*x^6 + 4*B*b*c^2*x^4 - 5*B*b^3 + 3*A*b^2*c - 4*(B*b^2*c - A*b*c
^2)*x^2 - 2*((3*B*b*c^2 - A*c^3)*x^4 + 3*B*b^3 - A*b^2*c + 2*(3*B*b^2*c - A
*b*c^2)*x^2)*log(c*x^2 + b))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)
```

Sympy [A]
time = 0.71, size = 94, normalized size = 1.06

$$\frac{Bx^2}{2c^3}+\frac{3Ab^2c-5Bb^3+x^2\cdot(4Abc^2-6Bb^2c)}{4b^2c^4+8bc^5x^2+4c^6x^4}-\frac{(-Ac+3Bb)\log{(b+cx^2)}}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**2/(2*c**3) + (3*A*b**2*c - 5*B*b**3 + x**2*(4*A*b*c**2 - 6*B*b**2*c))/
(4*b**2*c**4 + 8*b*c**5*x**2 + 4*c**6*x**4) - (-A*c + 3*B*b)*log(b + c*x**2
)/(2*c**4)
```

Giac [A]
time = 0.82, size = 93, normalized size = 1.04

$$\frac{Bx^2}{2\,c^3}-\frac{(3\,Bb-Ac)\log{(|cx^2+b|)}}{2\,c^4}+\frac{9\,Bbc^2x^4-3\,Ac^3x^4+12\,Bb^2cx^2-2\,Abc^2x^2+4\,Bb^3}{4\,(cx^2+b)^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c^3 - 1/2*(3*B*b - A*c)*log(abs(c*x^2 + b))/c^4 + 1/4*(9*B*b*c^2*
x^4 - 3*A*c^3*x^4 + 12*B*b^2*c*x^2 - 2*A*b*c^2*x^2 + 4*B*b^3)/((c*x^2 + b)^
2*c^4)
```

Mupad [B]
time = 0.09, size = 95, normalized size = 1.07

$$\frac{B\,x^2}{2\,c^3}-\frac{x^2\,\left(\frac{3\,B\,b^2}{2}-A\,b\,c\right)+\frac{5\,B\,b^3-3\,A\,b^2\,c}{4\,c}}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}+\frac{\ln\left(c\,x^2+b\right)\,\left(A\,c-3\,B\,b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (B*x^2)/(2*c^3) - (x^2*((3*B*b^2)/2 - A*b*c) + (5*B*b^3 - 3*A*b^2*c)/(4*c))
/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (log(b + c*x^2)*(A*c - 3*B*b))/(2*c^4)
```

3.77 $\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=95

$$\frac{Bx}{c^3}-\frac{b(bB-Ac)x}{4c^3(b+cx^2)^2}+\frac{(9bB-5Ac)x}{8c^3(b+cx^2)}-\frac{3(5bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{7/2}}$$

```
[Out] B*x/c^3-1/4*b*(-A*c+B*b)*x/c^3/(c*x^2+b)^2+1/8*(-5*A*c+9*B*b)*x/c^3/(c*x^2+
b)-3/8*(-A*c+5*B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(7/2)/b^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.208$, Rules used = {1598, 466, 1171, 396, 211}

$$-\frac{3(5bB-Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{7/2}}+\frac{x(9bB-5Ac)}{8c^3(b+cx^2)}-\frac{bx(bB-Ac)}{4c^3(b+cx^2)^2}+\frac{Bx}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (B*x)/c^3 - (b*(b*B - A*c)*x)/(4*c^3*(b + c*x^2)^2) + ((9*b*B - 5*A*c)*x)/(
8*c^3*(b + c*x^2)) - (3*(5*b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*Sqrt[
b]*c^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
```

```
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= \int \frac{x^4(A+Bx^2)}{(b+cx^2)^3}\,dx \\
&= -\frac{b(bB-Ac)x}{4c^3(b+cx^2)^2} - \frac{\int \frac{-b(bB-Ac)+4c(bB-Ac)x^2-4Bc^2x^4}{(b+cx^2)^2}\,dx}{4c^3} \\
&= -\frac{b(bB-Ac)x}{4c^3(b+cx^2)^2} + \frac{(9bB-5Ac)x}{8c^3(b+cx^2)} + \frac{\int \frac{-b(7bB-3Ac)+8bBcx^2}{b+cx^2}\,dx}{8bc^3} \\
&= \frac{Bx}{c^3} - \frac{b(bB-Ac)x}{4c^3(b+cx^2)^2} + \frac{(9bB-5Ac)x}{8c^3(b+cx^2)} - \frac{(3(5bB-Ac))\int \frac{1}{b+cx^2}\,dx}{8c^3} \\
&= \frac{Bx}{c^3} - \frac{b(bB-Ac)x}{4c^3(b+cx^2)^2} + \frac{(9bB-5Ac)x}{8c^3(b+cx^2)} - \frac{3(5bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 92, normalized size = 0.97

$$\frac{x(15b^2B+c^2x^2(-5A+8Bx^2)+b(-3Ac+25Bcx^2))}{8c^3(b+cx^2)^2} - \frac{3(5bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(15*b^2*B + c^2*x^2*(-5*A + 8*B*x^2) + b*(-3*A*c + 25*B*c*x^2)))/(8*c^3*
(b + c*x^2)^2) - (3*(5*b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*Sqrt[b]*c
^(7/2))
```

Maple [A]
time = 0.41, size = 77, normalized size = 0.81

method	result
default	$\frac{Bx}{c^3}+\frac{\frac{\left(-\frac{5}{8}A\,c^2+\frac{9}{8}bBc\right)x^3-\frac{b(3Ac-7Bb)x}{8}}{\left(c\,x^2+b\right)^2}+\frac{3(Ac-5Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}}{c^3}$
risch	$\frac{Bx}{c^3}+\frac{\left(-\frac{5}{8}A\,c^2+\frac{9}{8}bBc\right)x^3-\frac{b(3Ac-7Bb)x}{8}}{c^3\left(c\,x^2+b\right)^2}-\frac{3\ln\left(cx+\sqrt{-bc}\right)A}{16c^2\sqrt{-bc}}+\frac{15\ln\left(cx+\sqrt{-bc}\right)Bb}{16c^3\sqrt{-bc}}+\frac{3\ln\left(-cx+\sqrt{-bc}\right)A}{16c^2\sqrt{-bc}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] B*x/c^3+1/c^3*(((-5/8*A*c^2+9/8*b*B*c)*x^3-1/8*b*(3*A*c-7*B*b)*x)/(c*x^2+b)
^2+3/8*(A*c-5*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Maxima [A]
time = 0.50, size = 94, normalized size = 0.99

$$\frac{(9\,Bbc-5\,Ac^2)x^3+(7\,Bb^2-3\,Abc)x}{8\,(c^5x^4+2\,bc^4x^2+b^2c^3)}+\frac{Bx}{c^3}-\frac{3\,(5\,Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((9*B*b*c - 5*A*c^2)*x^3 + (7*B*b^2 - 3*A*b*c)*x)/(c^5*x^4 + 2*b*c^4*x^
2 + b^2*c^3) + B*x/c^3 - 3/8*(5*B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)
*c^3)
```

Fricas [A]
time = 2.07, size = 328, normalized size = 3.45

$$\left[\frac{16\,Bbc^3x^5+10\,(5\,Bb^2c^2-Abc^3)x^3+3\,((5\,Bbc^2-Ac^3)x^4+5\,Bb^3-Ab^2c+2\,(5\,Bb^2c-Abc^2)x^2)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)+6\,(5\,Bb^3c-Ab^2c^2)x}{16\,(bc^6x^4+2\,b^2c^5x^2+b^3c^4)},\ \frac{8\,Bbc^3x^5+5\,(5\,Bb^2c^2-Abc^3)x^3-3\,((5\,Bbc^2-Ac^3)x^4+5\,Bb^3-Ab^2c+2\,(5\,Bb^2c-Abc^2)x^2)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)+3\,(5\,Bb^3c-Ab^2c^2)x}{8\,(bc^6x^4+2\,b^2c^5x^2+b^3c^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*B*b*c^3*x^5 + 10*(5*B*b^2*c^2 - A*b*c^3)*x^3 + 3*((5*B*b*c^2 - A*
c^3)*x^4 + 5*B*b^3 - A*b^2*c + 2*(5*B*b^2*c - A*b*c^2)*x^2)*sqrt(-b*c)*log(
(c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)) + 6*(5*B*b^3*c - A*b^2*c^2)*x)/(b
*c^6*x^4 + 2*b^2*c^5*x^2 + b^3*c^4), 1/8*(8*B*b*c^3*x^5 + 5*(5*B*b^2*c^2 -
A*b*c^3)*x^3 - 3*((5*B*b*c^2 - A*c^3)*x^4 + 5*B*b^3 - A*b^2*c + 2*(5*B*b^2*
c - A*b*c^2)*x^2)*sqrt(b*c)*arctan(sqrt(b*c)*x/b) + 3*(5*B*b^3*c - A*b^2*c^
2)*x)/(b*c^6*x^4 + 2*b^2*c^5*x^2 + b^3*c^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(92) = 184$.
time = 0.59, size = 194, normalized size = 2.04

$$\frac{Bx}{c^3}+\frac{3\sqrt{-\frac{1}{bc^7}}\,(-Ac+5Bb)\log\left(-\frac{3bc^3\sqrt{-\frac{1}{bc^7}}\,(-Ac+5Bb)}{-3Ac+15Bb}+x\right)}{16}-\frac{3\sqrt{-\frac{1}{bc^7}}\,(-Ac+5Bb)\log\left(\frac{3bc^3\sqrt{-\frac{1}{bc^7}}\,(-Ac+5Bb)}{-3Ac+15Bb}+x\right)}{16}+\frac{x^3(-5Ac^2+9Bbc)+x(-3Abc+7Bb^2)}{8b^2c^3+16bc^4x^2+8c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x/c**3 + 3*sqrt(-1/(b*c**7))*(-A*c + 5*B*b)*log(-3*b*c**3*sqrt(-1/(b*c**7
))*(-A*c + 5*B*b)/(-3*A*c + 15*B*b) + x)/16 - 3*sqrt(-1/(b*c**7))*(-A*c + 5
*B*b)*log(3*b*c**3*sqrt(-1/(b*c**7))*(-A*c + 5*B*b)/(-3*A*c + 15*B*b) + x)/
16 + (x**3*(-5*A*c**2 + 9*B*b*c) + x*(-3*A*b*c + 7*B*b**2))/(8*b**2*c**3 +
16*b*c**4*x**2 + 8*c**5*x**4)
```

Giac [A]
time = 0.99, size = 80, normalized size = 0.84

$$\frac{Bx}{c^3}-\frac{3\,(5\,Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^3}+\frac{9\,Bbcx^3-5\,Ac^2x^3+7\,Bb^2x-3\,Abcx}{8\,(cx^2+b)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] B*x/c^3 - 3/8*(5*B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/8*(9*
B*b*c*x^3 - 5*A*c^2*x^3 + 7*B*b^2*x - 3*A*b*c*x)/((c*x^2 + b)^2*c^3)
```

Mupad [B]
time = 0.15, size = 92, normalized size = 0.97

$$\frac{B\,x}{c^3}-\frac{x^3\,\left(\frac{5\,A\,c^2}{8}-\frac{9\,B\,b\,c}{8}\right)-x\,\left(\frac{7\,B\,b^2}{8}-\frac{3\,A\,b\,c}{8}\right)}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,\left(A\,c-5\,B\,b\right)}{8\,\sqrt{b}\,c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)

[Out] (B*x)/c^3 - (x^3*((5*A*c^2)/8 - (9*B*b*c)/8) - x*((7*B*b^2)/8 - (3*A*b*c)/8
))/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (3*atan((c^(1/2)*x)/b^(1/2))*(A*c -
5*B*b))/(8*b^(1/2)*c^(7/2))
```

3.78 $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=67

$$-\frac{b(bB-Ac)}{4c^3(b+cx^2)^2}+\frac{2bB-Ac}{2c^3(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^3}$$

```
[Out] -1/4*b*(-A*c+B*b)/c^3/(c*x^2+b)^2+1/2*(-A*c+2*B*b)/c^3/(c*x^2+b)+1/2*B*ln(c
*x^2+b)/c^3
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 78}

$$-\frac{b(bB-Ac)}{4c^3(b+cx^2)^2}+\frac{2bB-Ac}{2c^3(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(b*(b*B - A*c))/(c^3*(b + c*x^2)^2) + (2*b*B - A*c)/(2*c^3*(b + c*x^2)
) + (B*Log[b + c*x^2])/(2*c^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \int \frac{x^3(A+Bx^2)}{(b+cx^2)^3}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b(bB-Ac)}{c^2(b+cx)^3} + \frac{-2bB+Ac}{c^2(b+cx)^2} + \frac{B}{c^2(b+cx)}\right)\,dx, x, x^2\right)$$
$$= -\frac{b(bB-Ac)}{4c^3(b+cx^2)^2} + \frac{2bB-Ac}{2c^3(b+cx^2)} + \frac{B\log(b+cx^2)}{2c^3}$$

Mathematica [A]
time = 0.02, size = 64, normalized size = 0.96

$$\frac{3b^2B-2Ac^2x^2-bc(A-4Bx^2)+2B(b+cx^2)^2\log(b+cx^2)}{4c^3(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (3*b^2*B - 2*A*c^2*x^2 - b*c*(A - 4*B*x^2) + 2*B*(b + c*x^2)^2*Log[b + c*x^
2])/(4*c^3*(b + c*x^2)^2)
```

Maple [A]
time = 0.39, size = 61, normalized size = 0.91

method	result	size
risch	$\frac{-\frac{(Ac-2Bb)x^2}{2c^2}-\frac{b(Ac-3Bb)}{4c^3}}{(cx^2+b)^2}+\frac{B\ln(cx^2+b)}{2c^3}$	57
default	$\frac{B\ln(cx^2+b)}{2c^3}+\frac{b(Ac-Bb)}{4c^3(cx^2+b)^2}-\frac{Ac-2Bb}{2c^3(cx^2+b)}$	61
norman	$\frac{-\frac{(Ac-2Bb)x^7}{2c^2}-\frac{b(Ac-3Bb)x^5}{4c^3}}{x^5(cx^2+b)^2}+\frac{B\ln(cx^2+b)}{2c^3}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*ln(c*x^2+b)/c^3+1/4*b/c^3*(A*c-B*b)/(c*x^2+b)^2-1/2*(A*c-2*B*b)/c^3/(
c*x^2+b)
```

Maxima [A]
time = 0.27, size = 72, normalized size = 1.07

$$\frac{3\,Bb^2 - Abc + 2\,(2\,Bbc - Ac^2)x^2}{4\,(c^5x^4 + 2\,bc^4x^2 + b^2c^3)} + \frac{B\log\left(cx^2 + b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*B*b^2 - A*b*c + 2*(2*B*b*c - A*c^2)*x^2)/(c^5*x^4 + 2*b*c^4*x^2 + b^
2*c^3) + 1/2*B*log(c*x^2 + b)/c^3
```

Fricas [A]
time = 1.63, size = 89, normalized size = 1.33

$$\frac{3\,Bb^2 - Abc + 2\,(2\,Bbc - Ac^2)x^2 + 2\,(Bc^2x^4 + 2\,Bbcx^2 + Bb^2)\log\left(cx^2 + b\right)}{4\,(c^5x^4 + 2\,bc^4x^2 + b^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*B*b^2 - A*b*c + 2*(2*B*b*c - A*c^2)*x^2 + 2*(B*c^2*x^4 + 2*B*b*c*x^2
 + B*b^2)*log(c*x^2 + b))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

Sympy [A]
time = 0.49, size = 70, normalized size = 1.04

$$\frac{B\log\left(b + cx^2\right)}{2c^3} + \frac{-Abc + 3Bb^2 + x^2(-2Ac^2 + 4Bbc)}{4b^2c^3 + 8bc^4x^2 + 4c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*log(b + c*x**2)/(2*c**3) + (-A*b*c + 3*B*b**2 + x**2*(-2*A*c**2 + 4*B*b*c
))/(4*b**2*c**3 + 8*b*c**4*x**2 + 4*c**5*x**4)
```

Giac [A]
time = 1.39, size = 55, normalized size = 0.82

$$\frac{B\log\left(|cx^2 + b|\right)}{2\,c^3} - \frac{3\,Bcx^4 + 2\,Bbx^2 + 2\,Acx^2 + Ab}{4\,(cx^2 + b)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*log(abs(c*x^2 + b))/c^3 - 1/4*(3*B*c*x^4 + 2*B*b*x^2 + 2*A*c*x^2 + A*
b)/((c*x^2 + b)^2*c^2)
```

Mupad [B]
time = 0.11, size = 70, normalized size = 1.04

$$\frac{\frac{3\,B\,b^2-A\,b\,c}{4\,c^3}-\frac{x^2\,\left(A\,c-2\,B\,b\right)}{2\,c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}+\frac{B\,\ln\left(c\,x^2+b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
[Out] ((3*B*b^2 - A*b*c)/(4*c^3) - (x^2*(A*c - 2*B*b))/(2*c^2))/(b^2 + c^2*x^4 +
2*b*c*x^2) + (B*log(b + c*x^2))/(2*c^3)
```

3.79 $\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=90

$$\frac{(bB-Ac)x}{4c^2(b+cx^2)^2}-\frac{(5bB-Ac)x}{8bc^2(b+cx^2)}+\frac{(3bB+Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{5/2}}$$

```
[Out] 1/4*(-A*c+B*b)*x/c^2/(c*x^2+b)^2-1/8*(-A*c+5*B*b)*x/b/c^2/(c*x^2+b)+1/8*(A*
c+3*B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(3/2)/c^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 466, 393, 211}

$$\frac{(Ac+3bB)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{5/2}}-\frac{x(5bB-Ac)}{8bc^2(b+cx^2)}+\frac{x(bB-Ac)}{4c^2(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*B - A*c)*x)/(4*c^2*(b + c*x^2)^2) - ((5*b*B - A*c)*x)/(8*b*c^2*(b + c*x
^2)) + ((3*b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(3/2)*c^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
 + 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
```

```
1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= \int \frac{x^2(A+Bx^2)}{(b+cx^2)^3}\,dx\\ &= \frac{(bB-Ac)x}{4c^2(b+cx^2)^2} - \frac{\int \frac{bB-Ac-4Bcx^2}{(b+cx^2)^2}\,dx}{4c^2}\\ &= \frac{(bB-Ac)x}{4c^2(b+cx^2)^2} - \frac{(5bB-Ac)x}{8bc^2(b+cx^2)} + \frac{(3bB+Ac)\int \frac{1}{b+cx^2}\,dx}{8bc^2}\\ &= \frac{(bB-Ac)x}{4c^2(b+cx^2)^2} - \frac{(5bB-Ac)x}{8bc^2(b+cx^2)} + \frac{(3bB+Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{5/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 83, normalized size = 0.92

$$\frac{\frac{\sqrt{c}\,x(-3b^2B+Ac^2x^2-bc(A+5Bx^2))}{b(b+cx^2)^2}+\frac{(3bB+Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}}}{8c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((Sqrt[c]*x*(-3*b^2*B + A*c^2*x^2 - b*c*(A + 5*B*x^2)))/(b*(b + c*x^2)^2) +
 ((3*b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2))/(8*c^(5/2))
```

Maple [**A**]
time = 0.44, size = 76, normalized size = 0.84

method	result

default	$\frac{\frac{(Ac-5Bb)x^3}{8bc}-\frac{(Ac+3Bb)x}{8c^2}}{(c\,x^2+b)^2}+\frac{(Ac+3Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8c^2b\sqrt{bc}}$
risch	$\frac{\frac{(Ac-5Bb)x^3}{8bc}-\frac{(Ac+3Bb)x}{8c^2}}{(c\,x^2+b)^2}-\frac{\ln\left(cx+\sqrt{-bc}\right)A}{16\sqrt{-bc}\,cb}-\frac{3\ln\left(cx+\sqrt{-bc}\right)B}{16\sqrt{-bc}\,c^2}+\frac{\ln\left(-cx+\sqrt{-bc}\right)A}{16\sqrt{-bc}\,cb}+\frac{3\ln\left(-cx+\sqrt{-bc}\right)}{16\sqrt{-bc}\,c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(A*c-5*B*b)/b/c*x^3-1/8*(A*c+3*B*b)/c^2*x)/(c*x^2+b)^2+1/8*(A*c+3*B*b)
/c^2/b/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [A]
time = 0.50, size = 92, normalized size = 1.02

$$-\frac{(5\,Bbc-Ac^2)x^3+(3\,Bb^2+Abc)x}{8\,(bc^4x^4+2\,b^2c^3x^2+b^3c^2)}+\frac{(3\,Bb+Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*B*b*c - A*c^2)*x^3 + (3*B*b^2 + A*b*c)*x)/(b*c^4*x^4 + 2*b^2*c^3*x
^2 + b^3*c^2) + 1/8*(3*B*b + A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c^2)
```

Fricas [A]
time = 2.27, size = 301, normalized size = 3.34

$$\left[-\frac{2\,(5\,Bb^2c^2-Abc^3)x^3+((3\,Bbc^2+Ac^3)x^4+3\,Bb^3+Ab^2c+2\,(3\,Bb^2c+Abc^2)x^2)\sqrt{-bc}\,\log\left(\frac{cx^2-2\sqrt{-bc}\,x-b}{cx^2+b}\right)+2\,(3\,Bb^3c+Ab^2c^2)x}{16\,(b^2c^5x^4+2\,b^3c^4x^2+b^4c^3)},-\frac{(5\,Bb^2c^2-Abc^3)x^3-((3\,Bbc^2+Ac^3)x^4+3\,Bb^3+Ab^2c+2\,(3\,Bb^2c+Abc^2)x^2)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)+(3\,Bb^3c+Ab^2c^2)x}{8\,(b^2c^5x^4+2\,b^3c^4x^2+b^4c^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*B*b^2*c^2 - A*b*c^3)*x^3 + ((3*B*b*c^2 + A*c^3)*x^4 + 3*B*b^3
+ A*b^2*c + 2*(3*B*b^2*c + A*b*c^2)*x^2)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*
c)*x - b)/(c*x^2 + b)) + 2*(3*B*b^3*c + A*b^2*c^2)*x)/(b^2*c^5*x^4 + 2*b^3*
c^4*x^2 + b^4*c^3), -1/8*((5*B*b^2*c^2 - A*b*c^3)*x^3 - ((3*B*b*c^2 + A*c^3
)*x^4 + 3*B*b^3 + A*b^2*c + 2*(3*B*b^2*c + A*b*c^2)*x^2)*sqrt(b*c)*arctan(s
qrt(b*c)*x/b) + (3*B*b^3*c + A*b^2*c^2)*x)/(b^2*c^5*x^4 + 2*b^3*c^4*x^2 + b
^4*c^3)]
```

Sympy [A]
time = 0.38, size = 155, normalized size = 1.72

$$-\frac{\sqrt{-\frac{1}{b^3c^5}}\,(Ac+3Bb)\log\left(-b^2c^2\sqrt{-\frac{1}{b^3c^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{b^3c^5}}\,(Ac+3Bb)\log\left(b^2c^2\sqrt{-\frac{1}{b^3c^5}}+x\right)}{16}+\frac{x^3(Ac^2-5Bbc)+x(-Abc-3Bb^2)}{8b^3c^2+16b^2c^3x^2+8bc^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -sqrt(-1/(b**3*c**5))*(A*c + 3*B*b)*log(-b**2*c**2*sqrt(-1/(b**3*c**5)) + x
)/16 + sqrt(-1/(b**3*c**5))*(A*c + 3*B*b)*log(b**2*c**2*sqrt(-1/(b**3*c**5)
) + x)/16 + (x**3*(A*c**2 - 5*B*b*c) + x*(-A*b*c - 3*B*b**2))/(8*b**3*c**2
+ 16*b**2*c**3*x**2 + 8*b*c**4*x**4)
```

Giac [A]
time = 1.34, size = 78, normalized size = 0.87

$$\frac{(3\,Bb+Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,bc^{2}}-\frac{5\,Bbcx^{3}-Ac^{2}x^{3}+3\,Bb^{2}x+Abcx}{8\,(cx^{2}+b)^{2}bc^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*B*b + A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c^2) - 1/8*(5*B*b*c*x^
3 - A*c^2*x^3 + 3*B*b^2*x + A*b*c*x)/((c*x^2 + b)^2*b*c^2)
```

Mupad [B]
time = 0.15, size = 82, normalized size = 0.91

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,\left(A\,c+3\,B\,b\right)}{8\,b^{3/2}\,c^{5/2}}-\frac{\frac{x\,\left(A\,c+3\,B\,b\right)}{8\,c^2}-\frac{x^3\,\left(A\,c-5\,B\,b\right)}{8\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (atan((c^(1/2)*x)/b^(1/2))*(A*c + 3*B*b))/(8*b^(3/2)*c^(5/2)) - ((x*(A*c +
3*B*b))/(8*c^2) - (x^3*(A*c - 5*B*b))/(8*b*c))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

3.80 $\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=32

$$\frac{(A+Bx^2)^2}{4(bB-Ac)\,(b+cx^2)^2}$$

[Out] 1/4*(B*x^2+A)^2/(-A*c+B*b)/(c*x^2+b)^2

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 455, 37}

$$\frac{(A+Bx^2)^2}{4\,(b+cx^2)^2\,(bB-Ac)}$$

Antiderivative was successfully verified.

[In] Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]

[Out] (A + B*x^2)^2/(4*(b*B - A*c)*(b + c*x^2)^2)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \int \frac{x(A+Bx^2)}{(b+cx^2)^3}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{(A+Bx^2)^2}{4(bB-Ac)\,(b+cx^2)^2}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.94

$$-\frac{bB+c(A+2Bx^2)}{4c^2\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(b*B + c*(A + 2*B*x^2))/(c^2*(b + c*x^2)^2)
```

Maple [A]
time = 0.37, size = 39, normalized size = 1.22

method	result	size
gosper	$-\frac{2Bc\,x^2+Ac+Bb}{4(c\,x^2+b)^2c^2}$	29
risch	$\frac{-\frac{B\,x^2}{2c}-\frac{Ac+Bb}{4c^2}}{(c\,x^2+b)^2}$	33
default	$-\frac{Ac-Bb}{4c^2(c\,x^2+b)^2}-\frac{B}{2c^2(c\,x^2+b)}$	39
norman	$\frac{-\frac{B\,x^7}{2c}-\frac{(Ac+Bb)x^5}{4c^2}}{x^5(c\,x^2+b)^2}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/c^2*(A*c-B*b)/(c*x^2+b)^2-1/2*B/c^2/(c*x^2+b)
```

Maxima [A]
time = 0.27, size = 42, normalized size = 1.31

$$-\frac{2\,Bcx^2+Bb+Ac}{4\,(c^4x^4+2\,bc^3x^2+b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*B*c*x^2 + B*b + A*c)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Fricas [A]
time = 1.26, size = 42, normalized size = 1.31

$$-\frac{2\,Bcx^2+Bb+Ac}{4\,(c^4x^4+2\,bc^3x^2+b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*B*c*x^2 + B*b + A*c)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Sympy [A]
time = 0.27, size = 42, normalized size = 1.31

$$\frac{-Ac-Bb-2Bcx^2}{4b^2c^2+8bc^3x^2+4c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-A*c - B*b - 2*B*c*x**2)/(4*b**2*c**2 + 8*b*c**3*x**2 + 4*c**4*x**4)
```

Giac [A]
time = 1.12, size = 28, normalized size = 0.88

$$-\frac{2\,Bcx^2+Bb+Ac}{4\,(cx^2+b)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*B*c*x^2 + B*b + A*c)/((c*x^2 + b)^2*c^2)
```

Mupad [B]
time = 0.08, size = 44, normalized size = 1.38

$$-\frac{\frac{A\,c+B\,b}{4\,c^2}+\frac{B\,x^2}{2\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] -((A*c + B*b)/(4*c^2) + (B*x^2)/(2*c))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

3.81 $\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=92

$$-\frac{(bB-Ac)x}{4bc\,(b+cx^2)^2}+\frac{(bB+3Ac)x}{8b^2c\,(b+cx^2)}+\frac{(bB+3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}$$

```
[Out] -1/4*(-A*c+B*b)*x/b/c/(c*x^2+b)^2+1/8*(3*A*c+B*b)*x/b^2/c/(c*x^2+b)+1/8*(3*
A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(5/2)/c^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 393, 205, 211}

$$\frac{(3Ac+bB)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}+\frac{x(3Ac+bB)}{8b^2c\,(b+cx^2)}-\frac{x(bB-Ac)}{4bc\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*((b*B - A*c)*x)/(b*c*(b + c*x^2)^2) + ((b*B + 3*A*c)*x)/(8*b^2*c*(b +
c*x^2)) + ((b*B + 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(5/2)*c^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
 + p, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= \int \frac{A+Bx^2}{(b+cx^2)^3}\,dx\\ &= -\frac{(bB-Ac)x}{4bc\,(b+cx^2)^2}+\frac{(bB+3Ac)\int \frac{1}{(b+cx^2)^2}\,dx}{4bc}\\ &= -\frac{(bB-Ac)x}{4bc\,(b+cx^2)^2}+\frac{(bB+3Ac)x}{8b^2c\,(b+cx^2)}+\frac{(bB+3Ac)\int \frac{1}{b+cx^2}\,dx}{8b^2c}\\ &= -\frac{(bB-Ac)x}{4bc\,(b+cx^2)^2}+\frac{(bB+3Ac)x}{8b^2c\,(b+cx^2)}+\frac{(bB+3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 84, normalized size = 0.91

$$\frac{x(-b^2B+3Ac^2x^2+bc(5A+Bx^2))}{8b^2c\,(b+cx^2)^2}+\frac{(bB+3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(-(b^2*B) + 3*A*c^2*x^2 + b*c*(5*A + B*x^2)))/(8*b^2*c*(b + c*x^2)^2) +
((b*B + 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(5/2)*c^(3/2))
```

Maple [A]
time = 0.48, size = 77, normalized size = 0.84

method	result
default	$\frac{\frac{(3Ac+Bb)x^3}{8b^2}+\frac{(5Ac-Bb)x}{8bc}}{(c\,x^2+b)^2}+\frac{(3Ac+Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8b^2c\sqrt{bc}}$

risch	$\frac{\frac{(3Ac+Bb)x^3}{8b^2}+\frac{(5Ac-Bb)x}{8bc}}{(cx^2+b)^2}-\frac{3\ln\left(cx+\sqrt{-bc}\right)A}{16\sqrt{-bc}\,b^2}-\frac{\ln\left(cx+\sqrt{-bc}\right)B}{16\sqrt{-bc}\,cb}+\frac{3\ln\left(-cx+\sqrt{-bc}\right)A}{16\sqrt{-bc}\,b^2}+\frac{\ln\left(-cx+\sqrt{-bc}\right)B}{16\sqrt{-bc}\,cb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*A*c+B*b)/b^2*x^3+1/8*(5*A*c-B*b)/b/c*x)/(c*x^2+b)^2+1/8*(3*A*c+B*b)
/b^2/c/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [A]
time = 0.50, size = 92, normalized size = 1.00

$$\frac{(Bbc+3\,Ac^2)x^3-(Bb^2-5\,Abc)x}{8\,(b^2c^3x^4+2\,b^3c^2x^2+b^4c)}+\frac{(Bb+3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((B*b*c + 3*A*c^2)*x^3 - (B*b^2 - 5*A*b*c)*x)/(b^2*c^3*x^4 + 2*b^3*c^2*
x^2 + b^4*c) + 1/8*(B*b + 3*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2*c)
```

Fricas [A]
time = 2.12, size = 300, normalized size = 3.26

$$\left[\frac{2\,(Bb^2c^2+3\,Abc^3)x^3-((Bbc^2+3\,Ac^3)x^4+Bb^3+3\,Ab^2c+2\,(Bb^2c+3\,Abc^2)x^2)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)-2\,(Bb^3c-5\,Ab^2c^2)x}{16\,(b^3c^4x^4+2\,b^4c^3x^2+b^5c^2)},\ \frac{(Bb^2c^2+3\,Abc^3)x^3+((Bbc^2+3\,Ac^3)x^4+Bb^3+3\,Ab^2c+2\,(Bb^2c+3\,Abc^2)x^2)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)-(Bb^3c-5\,Ab^2c^2)x}{8\,(b^3c^4x^4+2\,b^4c^3x^2+b^5c^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(B*b^2*c^2 + 3*A*b*c^3)*x^3 - ((B*b*c^2 + 3*A*c^3)*x^4 + B*b^3 + 3
*A*b^2*c + 2*(B*b^2*c + 3*A*b*c^2)*x^2)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c
)*x - b)/(c*x^2 + b)) - 2*(B*b^3*c - 5*A*b^2*c^2)*x)/(b^3*c^4*x^4 + 2*b^4*c
^3*x^2 + b^5*c^2), 1/8*((B*b^2*c^2 + 3*A*b*c^3)*x^3 + ((B*b*c^2 + 3*A*c^3)*
x^4 + B*b^3 + 3*A*b^2*c + 2*(B*b^2*c + 3*A*b*c^2)*x^2)*sqrt(b*c)*arctan(sqr
t(b*c)*x/b) - (B*b^3*c - 5*A*b^2*c^2)*x)/(b^3*c^4*x^4 + 2*b^4*c^3*x^2 + b^5
*c^2)]
```

Sympy [A]
time = 0.29, size = 150, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{b^5c^3}}\cdot(3Ac+Bb)\log\left(-b^3c\sqrt{-\frac{1}{b^5c^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{b^5c^3}}\cdot(3Ac+Bb)\log\left(b^3c\sqrt{-\frac{1}{b^5c^3}}+x\right)}{16}+\frac{x^3\cdot(3Ac^2+Bbc)+x(5Abc-Bb^2)}{8b^4c+16b^3c^2x^2+8b^2c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -sqrt(-1/(b**5*c**3))*(3*A*c + B*b)*log(-b**3*c*sqrt(-1/(b**5*c**3)) + x)/1
6 + sqrt(-1/(b**5*c**3))*(3*A*c + B*b)*log(b**3*c*sqrt(-1/(b**5*c**3)) + x)
/16 + (x**3*(3*A*c**2 + B*b*c) + x*(5*A*b*c - B*b**2))/(8*b**4*c + 16*b**3*
c**2*x**2 + 8*b**2*c**3*x**4)
```

Giac [A]
time = 2.37, size = 78, normalized size = 0.85

$$\frac{(Bb+3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^2c}+\frac{Bbcx^3+3\,Ac^2x^3-Bb^2x+5\,Abcx}{8\,(cx^2+b)^2b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(B*b + 3*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2*c) + 1/8*(B*b*c*x^3
+ 3*A*c^2*x^3 - B*b^2*x + 5*A*b*c*x)/((c*x^2 + b)^2*b^2*c)
```

Mupad [B]
time = 0.14, size = 82, normalized size = 0.89

$$\frac{\frac{x^3\,(3\,A\,c+B\,b)}{8\,b^2}+\frac{x\,(5\,A\,c-B\,b)}{8\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(3\,A\,c+B\,b)}{8\,b^{5/2}\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^3*(3*A*c + B*b))/(8*b^2) + (x*(5*A*c - B*b))/(8*b*c))/(b^2 + c^2*x^4 +
2*b*c*x^2) + (atan((c^(1/2)*x)/b^(1/2))*(3*A*c + B*b))/(8*b^(5/2)*c^(3/2))
```

3.82 $\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=68

$$-\frac{bB-Ac}{4bc\,(b+cx^2)^2}+\frac{A}{2b^2\,(b+cx^2)}+\frac{A\log(x)}{b^3}-\frac{A\log\left(b+cx^2\right)}{2b^3}$$

```
[Out] 1/4*(A*c-B*b)/b/c/(c*x^2+b)^2+1/2*A/b^2/(c*x^2+b)+A*ln(x)/b^3-1/2*A*ln(c*x^
2+b)/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$-\frac{A\log\left(b+cx^2\right)}{2b^3}+\frac{A\log(x)}{b^3}+\frac{A}{2b^2\,(b+cx^2)}-\frac{bB-Ac}{4bc\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(b*B - A*c)/(b*c*(b + c*x^2)^2) + A/(2*b^2*(b + c*x^2)) + (A*Log[x])/b
^3 - (A*Log[b + c*x^2])/(2*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= \int \frac{A+Bx^2}{x\,(b+cx^2)^3}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x(b+cx)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{b^3x}+\frac{bB-Ac}{b(b+cx)^3}-\frac{Ac}{b^2(b+cx)^2}-\frac{Ac}{b^3(b+cx)}\right)dx, x, x^2\right) \\
&= -\frac{bB-Ac}{4bc\,(b+cx^2)^2}+\frac{A}{2b^2\,(b+cx^2)}+\frac{A\log(x)}{b^3}-\frac{A\log\left(b+cx^2\right)}{2b^3}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 59, normalized size = 0.87

$$
\frac{\frac{b(-b^2B+3Abc+2Ac^2x^2)}{c(b+cx^2)^2}+4A\log(x)-2A\log\left(b+cx^2\right)}{4b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*(-(b^2*B) + 3*A*b*c + 2*A*c^2*x^2))/(c*(b + c*x^2)^2) + 4*A*Log[x] - 2*
A*Log[b + c*x^2])/(4*b^3)
```

Maple [A]
time = 0.38, size = 63, normalized size = 0.93

method	result	size
risch	$\frac{\frac{Ac\,x^2}{2b^2}+\frac{3Ac-Bb}{4bc}}{(c\,x^2+b)^2}+\frac{A\ln(x)}{b^3}-\frac{A\ln(c\,x^2+b)}{2b^3}$	61
default	$-\frac{A\ln(c\,x^2+b)-\frac{b^2(Ac-Bb)}{2c(c\,x^2+b)^2}-\frac{Ab}{c\,x^2+b}}{2b^3}+\frac{A\ln(x)}{b^3}$	63
norman	$\frac{-\frac{(2Ac-Bb)x^7}{2b^2}-\frac{c(3Ac-Bb)x^9}{4b^3}}{x^5(c\,x^2+b)^2}+\frac{A\ln(x)}{b^3}-\frac{A\ln(c\,x^2+b)}{2b^3}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^3*(A*ln(c*x^2+b)-1/2*b^2*(A*c-B*b)/c/(c*x^2+b)^2-A*b/(c*x^2+b))+A*ln
(x)/b^3
```

Maxima [A]
time = 0.28, size = 77, normalized size = 1.13

$$\frac{2\,Ac^2x^2 - Bb^2 + 3\,Abc}{4\,(b^2c^3x^4 + 2\,b^3c^2x^2 + b^4c)} - \frac{A\log(cx^2+b)}{2\,b^3} + \frac{A\log(x^2)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*A*c^2*x^2 - B*b^2 + 3*A*b*c)/(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c) -
 1/2*A*log(c*x^2 + b)/b^3 + 1/2*A*log(x^2)/b^3
```

Fricas [A]
time = 1.68, size = 119, normalized size = 1.75

$$\frac{2\,Abc^2x^2 - Bb^3 + 3\,Ab^2c - 2\,(Ac^3x^4 + 2\,Abc^2x^2 + Ab^2c)\log(cx^2+b) + 4\,(Ac^3x^4 + 2\,Abc^2x^2 + Ab^2c)\log(x)}{4\,(b^3c^3x^4 + 2\,b^4c^2x^2 + b^5c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*A*b*c^2*x^2 - B*b^3 + 3*A*b^2*c - 2*(A*c^3*x^4 + 2*A*b*c^2*x^2 + A*b
^2*c)*log(c*x^2 + b) + 4*(A*c^3*x^4 + 2*A*b*c^2*x^2 + A*b^2*c)*log(x))/(b^3
*c^3*x^4 + 2*b^4*c^2*x^2 + b^5*c)
```

Sympy [A]
time = 0.29, size = 75, normalized size = 1.10

$$\frac{A\log(x)}{b^3} - \frac{A\log\left(\frac{b}{c} + x^2\right)}{2b^3} + \frac{3Abc + 2Ac^2x^2 - Bb^2}{4b^4c + 8b^3c^2x^2 + 4b^2c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] A*log(x)/b**3 - A*log(b/c + x**2)/(2*b**3) + (3*A*b*c + 2*A*c**2*x**2 - B*b
**2)/(4*b**4*c + 8*b**3*c**2*x**2 + 4*b**2*c**3*x**4)
```

Giac [A]
time = 1.71, size = 76, normalized size = 1.12

$$\frac{A\log(x^2)}{2\,b^3} - \frac{A\log(|cx^2+b|)}{2\,b^3} + \frac{3\,Ac^3x^4 + 8\,Abc^2x^2 - Bb^3 + 6\,Ab^2c}{4\,(cx^2+b)^2b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(x^2)/b^3 - 1/2*A*log(abs(c*x^2 + b))/b^3 + 1/4*(3*A*c^3*x^4 + 8*A
*b*c^2*x^2 - B*b^3 + 6*A*b^2*c)/((c*x^2 + b)^2*b^3*c)
```

Mupad [B]
time = 0.18, size = 71, normalized size = 1.04

$$\frac{\frac{3\,A\,c-B\,b}{4\,b\,c}+\frac{A\,c\,x^2}{2\,b^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{A\,\ln\left(c\,x^2+b\right)}{2\,b^3}+\frac{A\,\ln\left(x\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*A*c - B*b)/(4*b*c) + (A*c*x^2)/(2*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) - (
A*log(b + c*x^2))/(2*b^3) + (A*log(x))/b^3
```

3.83 $\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=96

$$-\frac{A}{b^3x}+\frac{(bB-Ac)x}{4b^2(b+cx^2)^2}+\frac{(3bB-7Ac)x}{8b^3(b+cx^2)}+\frac{3(bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}$$

```
[Out] -A/b^3/x+1/4*(-A*c+B*b)*x/b^2/(c*x^2+b)^2+1/8*(-7*A*c+3*B*b)*x/b^3/(c*x^2+b
)+3/8*(-5*A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(7/2)/c^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1598, 467, 464, 211}

$$\frac{3(bB-5Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}+\frac{x(3bB-7Ac)}{8b^3(b+cx^2)}-\frac{A}{b^3x}+\frac{x(bB-Ac)}{4b^2(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(A/(b^3*x)) + ((b*B - A*c)*x)/(4*b^2*(b + c*x^2)^2) + ((3*b*B - 7*A*c)*x)/
(8*b^3*(b + c*x^2)) + (3*(b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7
/2)*Sqrt[c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
```

```
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= \int \frac{A+Bx^2}{x^2\,(b+cx^2)^3}\,dx \\
&= \frac{(bB-Ac)x}{4b^2\,(b+cx^2)^2} - \frac{1}{4}\int \frac{-\frac{4A}{b}-\frac{3(bB-Ac)x^2}{b^2}}{x^2\,(b+cx^2)^2}\,dx \\
&= \frac{(bB-Ac)x}{4b^2\,(b+cx^2)^2} + \frac{(3bB-7Ac)x}{8b^3\,(b+cx^2)} + \frac{1}{8}\int \frac{\frac{8A}{b^2}+\frac{(3bB-7Ac)x^2}{b^3}}{x^2\,(b+cx^2)}\,dx \\
&= -\frac{A}{b^3x} + \frac{(bB-Ac)x}{4b^2\,(b+cx^2)^2} + \frac{(3bB-7Ac)x}{8b^3\,(b+cx^2)} + \frac{(3(bB-5Ac))\int \frac{1}{b+cx^2}\,dx}{8b^3} \\
&= -\frac{A}{b^3x} + \frac{(bB-Ac)x}{4b^2\,(b+cx^2)^2} + \frac{(3bB-7Ac)x}{8b^3\,(b+cx^2)} + \frac{3(bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 96, normalized size = 1.00

$$-\frac{A}{b^3x} + \frac{(bB-Ac)x}{4b^2\,(b+cx^2)^2} + \frac{(3bB-7Ac)x}{8b^3\,(b+cx^2)} + \frac{3(bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(A/(b^3*x)) + ((b*B - A*c)*x)/(4*b^2*(b + c*x^2)^2) + ((3*b*B - 7*A*c)*x)/
(8*b^3*(b + c*x^2)) + (3*(b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7
/2)*Sqrt[c])
```

Maple [A]
time = 0.40, size = 82, normalized size = 0.85

method	result
default	$-\frac{\frac{\left(\frac{7}{8}A\,c^{2}-\frac{3}{8}bBc\right)x^{3}+\frac{b(9Ac-5Bb)x}{8}}{\left(c\,x^{2}+b\right)^{2}}+\frac{3(5Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}}{b^{3}}-\frac{A}{b^{3}x}$
risch	$\frac{-\frac{3c(5Ac-Bb)x^{4}}{8b^{3}}-\frac{5(5Ac-Bb)x^{2}}{8b^{2}}-\frac{A}{b}}{x(c\,x^{2}+b)^{2}}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(b^{7}c\_Z^{2}+25A^{2}c^{2}-10ABbc+B^{2}b^{2}\right)}\_R\ln\left(\left(3\_R^{2}b^{7}c+50A^{2}c^{2}-20ABbc+2B^{2}\right.\right.\right.}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(((7/8*A*c^2-3/8*b*B*c)*x^3+1/8*b*(9*A*c-5*B*b)*x)/(c*x^2+b)^2+3/8*(
5*A*c-B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))-A/b^3/x
```

Maxima [A]
time = 0.50, size = 96, normalized size = 1.00

$$\frac{3\,(Bbc-5\,Ac^{2})x^{4}-8\,Ab^{2}+5\,(Bb^{2}-5\,Abc)x^{2}}{8\,(b^{3}c^{2}x^{5}+2\,b^{4}cx^{3}+b^{5}x)}+\frac{3\,(Bb-5\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(B*b*c - 5*A*c^2)*x^4 - 8*A*b^2 + 5*(B*b^2 - 5*A*b*c)*x^2)/(b^3*c^2*
x^5 + 2*b^4*c*x^3 + b^5*x) + 3/8*(B*b - 5*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(
b*c)*b^3)
```

Fricas [A]
time = 1.65, size = 324, normalized size = 3.38

$$\left[-\frac{16\,Ab^{3}c-6\,(Bb^{2}c^{2}-5\,Abc^{3})x^{4}-10\,(Bb^{3}c-5\,Ab^{2}c^{2})x^{2}-3\,((Bbc^{2}-5\,Ac^{3})x^{5}+2\,(Bb^{2}c-5\,Abc^{2})x^{3}+(Bb^{3}-5\,Ab^{2}c)x)\sqrt{-bc}\,\log\left(\frac{cx^{2}+2\,\sqrt{-bc}\,x-b}{cx^{2}+b}\right)}{16\,(b^{4}c^{3}x^{5}+2\,b^{5}c^{2}x^{3}+b^{6}cx)},-\frac{8\,Ab^{3}c-3\,(Bb^{2}c^{2}-5\,Abc^{3})x^{4}-5\,(Bb^{3}c-5\,Ab^{2}c^{2})x^{2}-3\,((Bbc^{2}-5\,Ac^{3})x^{5}+2\,(Bb^{2}c-5\,Abc^{2})x^{3}+(Bb^{3}-5\,Ab^{2}c)x)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{8\,(b^{4}c^{3}x^{5}+2\,b^{5}c^{2}x^{3}+b^{6}cx)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*A*b^3*c - 6*(B*b^2*c^2 - 5*A*b*c^3)*x^4 - 10*(B*b^3*c - 5*A*b^2*
c^2)*x^2 - 3*((B*b*c^2 - 5*A*c^3)*x^5 + 2*(B*b^2*c - 5*A*b*c^2)*x^3 + (B*b^
3 - 5*A*b^2*c)*x))*sqrt(-b*c)*log((c*x^2 + 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))
/(b^4*c^3*x^5 + 2*b^5*c^2*x^3 + b^6*c*x), -1/8*(8*A*b^3*c - 3*(B*b^2*c^2 -
5*A*b*c^3)*x^4 - 5*(B*b^3*c - 5*A*b^2*c^2)*x^2 - 3*((B*b*c^2 - 5*A*c^3)*x^5
```

```
 + 2*(B*b^2*c - 5*A*b*c^2)*x^3 + (B*b^3 - 5*A*b^2*c)*x)*sqrt(b*c)*arctan(sq
rt(b*c)*x/b))/(b^4*c^3*x^5 + 2*b^5*c^2*x^3 + b^6*c*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(92) = 184$.
time = 0.35, size = 194, normalized size = 2.02

$$-\frac{3\sqrt{-\frac{1}{b^7c}}\,(-5Ac+Bb)\log\left(-\frac{3b^4\sqrt{-\frac{1}{b^7c}}\,(-5Ac+Bb)}{-15Ac+3Bb}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{b^7c}}\,(-5Ac+Bb)\log\left(\frac{3b^4\sqrt{-\frac{1}{b^7c}}\,(-5Ac+Bb)}{-15Ac+3Bb}+x\right)}{16}+\frac{-8Ab^2+x^4(-15Ac^2+3Bbc)+x^2(-25Abc+5Bb^2)}{8b^5x+16b^4cx^3+8b^3c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*sqrt(-1/(b**7*c))*(-5*A*c + B*b)*log(-3*b**4*sqrt(-1/(b**7*c))*(-5*A*c +
 B*b)/(-15*A*c + 3*B*b) + x)/16 + 3*sqrt(-1/(b**7*c))*(-5*A*c + B*b)*log(3*
b**4*sqrt(-1/(b**7*c))*(-5*A*c + B*b)/(-15*A*c + 3*B*b) + x)/16 + (-8*A*b**
2 + x**4*(-15*A*c**2 + 3*B*b*c) + x**2*(-25*A*b*c + 5*B*b**2))/(8*b**5*x +
16*b**4*c*x**3 + 8*b**3*c**2*x**5)
```

Giac [A]
time = 1.05, size = 82, normalized size = 0.85

$$\frac{3\,(Bb-5\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^3}-\frac{A}{b^3x}+\frac{3\,Bbcx^3-7\,Ac^2x^3+5\,Bb^2x-9\,Abcx}{8\,(cx^2+b)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*(B*b - 5*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - A/(b^3*x) + 1/8*(
3*B*b*c*x^3 - 7*A*c^2*x^3 + 5*B*b^2*x - 9*A*b*c*x)/((c*x^2 + b)^2*b^3)
```

Mupad [B]
time = 0.18, size = 113, normalized size = 1.18

$$-\frac{\frac{A}{b}+\frac{5\,x^2\,(5\,A\,c-B\,b)}{8\,b^2}+\frac{3\,c\,x^4\,(5\,A\,c-B\,b)}{8\,b^3}}{b^2\,x+2\,b\,c\,x^3+c^2\,x^5}-\frac{3\,\mathrm{atan}\left(\frac{3\,\sqrt{c}\,x\,(5\,A\,c-B\,b)}{\sqrt{b}\,(15\,A\,c-3\,B\,b)}\right)\,(5\,A\,c-B\,b)}{8\,b^{7/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - (A/b + (5*x^2*(5*A*c - B*b))/(8*b^2) + (3*c*x^4*(5*A*c - B*b))/(8*b^3))/(
b^2*x + c^2*x^5 + 2*b*c*x^3) - (3*atan((3*c^(1/2)*x*(5*A*c - B*b))/(b^(1/2)
*(15*A*c - 3*B*b)))*(5*A*c - B*b))/(8*b^(7/2)*c^(1/2))
```

3.84 $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=97

$$-\frac{A}{2b^3x^2}+\frac{bB-Ac}{4b^2\left(b+cx^2\right)^2}+\frac{bB-2Ac}{2b^3\left(b+cx^2\right)}+\frac{(bB-3Ac)\log(x)}{b^4}-\frac{(bB-3Ac)\log\left(b+cx^2\right)}{2b^4}$$

```
[Out] -1/2*A/b^3/x^2+1/4*(-A*c+B*b)/b^2/(c*x^2+b)^2+1/2*(-2*A*c+B*b)/b^3/(c*x^2+b
)+(-3*A*c+B*b)*ln(x)/b^4-1/2*(-3*A*c+B*b)*ln(c*x^2+b)/b^4
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$-\frac{(bB-3Ac)\log\left(b+cx^2\right)}{2b^4}+\frac{\log(x)(bB-3Ac)}{b^4}+\frac{bB-2Ac}{2b^3\left(b+cx^2\right)}-\frac{A}{2b^3x^2}+\frac{bB-Ac}{4b^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/2*A/(b^3*x^2) + (b*B - A*c)/(4*b^2*(b + c*x^2)^2) + (b*B - 2*A*c)/(2*b^3
*(b + c*x^2)) + ((b*B - 3*A*c)*Log[x])/b^4 - ((b*B - 3*A*c)*Log[b + c*x^2])
/(2*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \int \frac{A+Bx^2}{x^3(b+cx^2)^3}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{b^3x^2} + \frac{bB-3Ac}{b^4x} - \frac{c(bB-Ac)}{b^2(b+cx)^3} - \frac{c(bB-2Ac)}{b^3(b+cx)^2} - \frac{c(bB-3Ac)}{b^4(b+cx)}\right) dx,$$
$$= -\frac{A}{2b^3x^2} + \frac{bB-Ac}{4b^2(b+cx^2)^2} + \frac{bB-2Ac}{2b^3(b+cx^2)} + \frac{(bB-3Ac)\log(x)}{b^4} - \frac{(bB-3Ac)\log(b+}{2b^4}$$

Mathematica [A]
time = 0.04, size = 86, normalized size = 0.89

$$\frac{-\frac{2Ab}{x^2} + \frac{b^2(bB-Ac)}{(b+cx^2)^2} + \frac{2b(bB-2Ac)}{b+cx^2} + 4(bB-3Ac)\log(x) - 2(bB-3Ac)\log(b+cx^2)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-2*A*b)/x^2 + (b^2*(b*B - A*c))/(b + c*x^2)^2 + (2*b*(b*B - 2*A*c))/(b +
c*x^2) + 4*(b*B - 3*A*c)*Log[x] - 2*(b*B - 3*A*c)*Log[b + c*x^2])/(4*b^4)
```

Maple [A]
time = 0.38, size = 102, normalized size = 1.05

method	result	size
norman	$\frac{\frac{c(3Ac-Bb)x^7}{b^3}-\frac{A x^3}{2b}+\frac{c^2(9Ac-3Bb)x^9}{4b^4}}{x^5(c x^2+b)^2}-\frac{(3Ac-Bb)\ln(x)}{b^4}+\frac{(3Ac-Bb)\ln(c x^2+b)}{2b^4}$	100
default	$\frac{c\left(\frac{(3Ac-Bb)\ln(c x^2+b)}{c}-\frac{b^2(Ac-Bb)}{2c(c x^2+b)^2}-\frac{b(2Ac-Bb)}{c(c x^2+b)}\right)}{2b^4}-\frac{A}{2b^3x^2}+\frac{(-3Ac+Bb)\ln(x)}{b^4}$	102
risch	$\frac{-\frac{c(3Ac-Bb)x^4}{2b^3}-\frac{3(3Ac-Bb)x^2}{4b^2}-\frac{A}{2b}}{x^2(c x^2+b)^2}-\frac{3\ln(x)Ac}{b^4}+\frac{\ln(x)B}{b^3}+\frac{3\ln(-c x^2-b)Ac}{2b^4}-\frac{\ln(-c x^2-b)B}{2b^3}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^4*c*((3*A*c-B*b)/c*ln(c*x^2+b)-1/2*b^2*(A*c-B*b)/c/(c*x^2+b)^2-b*(2*A
*c-B*b)/c/(c*x^2+b))-1/2*A/b^3/x^2+(-3*A*c+B*b)*ln(x)/b^4
```

Maxima [A]
time = 0.27, size = 109, normalized size = 1.12

$$\frac{2\,(Bbc - 3\,Ac^2)x^4 - 2\,Ab^2 + 3\,(Bb^2 - 3\,Abc)x^2}{4\,(b^3c^2x^6 + 2\,b^4cx^4 + b^5x^2)} - \frac{(Bb - 3\,Ac)\log\left(cx^2 + b\right)}{2\,b^4} + \frac{(Bb - 3\,Ac)\log\left(x^2\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(B*b*c - 3*A*c^2)*x^4 - 2*A*b^2 + 3*(B*b^2 - 3*A*b*c)*x^2)/(b^3*c^2*
x^6 + 2*b^4*c*x^4 + b^5*x^2) - 1/2*(B*b - 3*A*c)*log(c*x^2 + b)/b^4 + 1/2*(
B*b - 3*A*c)*log(x^2)/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(89) = 178.
time = 1.80, size = 197, normalized size = 2.03

$$\frac{2\,(Bb^2c - 3\,Abc^2)x^4 - 2\,Ab^3 + 3\,(Bb^3 - 3\,Ab^2c)x^2 - 2\,((Bbc^2 - 3\,Ac^3)x^6 + 2\,(Bb^2c - 3\,Abc^2)x^4 + (Bb^3 - 3\,Ab^2c)x^2)\log\left(cx^2 + b\right) + 4\,((Bbc^2 - 3\,Ac^3)x^6 + 2\,(Bb^2c - 3\,Abc^2)x^4 + (Bb^3 - 3\,Ab^2c)x^2)\log\left(x\right)}{4\,(b^4c^2x^6 + 2\,b^5cx^4 + b^6x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*b^2*c - 3*A*b*c^2)*x^4 - 2*A*b^3 + 3*(B*b^3 - 3*A*b^2*c)*x^2 - 2*
((B*b*c^2 - 3*A*c^3)*x^6 + 2*(B*b^2*c - 3*A*b*c^2)*x^4 + (B*b^3 - 3*A*b^2*c
)*x^2)*log(c*x^2 + b) + 4*((B*b*c^2 - 3*A*c^3)*x^6 + 2*(B*b^2*c - 3*A*b*c^2
)*x^4 + (B*b^3 - 3*A*b^2*c)*x^2)*log(x))/(b^4*c^2*x^6 + 2*b^5*c*x^4 + b^6*x
^2)
```

Sympy [A]
time = 0.59, size = 107, normalized size = 1.10

$$\frac{-2Ab^2 + x^4(-6Ac^2 + 2Bbc) + x^2(-9Abc + 3Bb^2)}{4b^5x^2 + 8b^4cx^4 + 4b^3c^2x^6} + \frac{(-3Ac + Bb)\log\left(x\right)}{b^4} - \frac{(-3Ac + Bb)\log\left(\frac{b}{c} + x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-2*A*b**2 + x**4*(-6*A*c**2 + 2*B*b*c) + x**2*(-9*A*b*c + 3*B*b**2))/(4*b*
*5*x**2 + 8*b**4*c*x**4 + 4*b**3*c**2*x**6) + (-3*A*c + B*b)*log(x)/b**4 -
(-3*A*c + B*b)*log(b/c + x**2)/(2*b**4)
```

Giac [A]
time = 1.36, size = 105, normalized size = 1.08

$$\frac{(Bb - 3\,Ac)\log\left(|x|\right)}{b^4} - \frac{(Bbc - 3\,Ac^2)\log\left(|cx^2 + b|\right)}{2\,b^4c} + \frac{2\,(Bb^2c - 3\,Abc^2)x^4 - 2\,Ab^3 + 3\,(Bb^3 - 3\,Ab^2c)x^2}{4\,{(cx^2 + b)}^2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] (B*b - 3*A*c)*log(abs(x))/b^4 - 1/2*(B*b*c - 3*A*c^2)*log(abs(c*x^2 + b))/(
b^4*c) + 1/4*(2*(B*b^2*c - 3*A*b*c^2)*x^4 - 2*A*b^3 + 3*(B*b^3 - 3*A*b^2*c)
*x^2)/((c*x^2 + b)^2*b^4*x^2)
```

Mupad [B]
time = 0.15, size = 107, normalized size = 1.10

$$\frac{\ln\left(c\,x^2+b\right)\,\left(3\,A\,c-B\,b\right)}{2\,b^4}-\frac{\frac{A}{2\,b}+\frac{3\,x^2\,\left(3\,A\,c-B\,b\right)}{4\,b^2}+\frac{c\,x^4\,\left(3\,A\,c-B\,b\right)}{2\,b^3}}{b^2\,x^2+2\,b\,c\,x^4+c^2\,x^6}-\frac{\ln\left(x\right)\,\left(3\,A\,c-B\,b\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (log(b + c*x^2)*(3*A*c - B*b))/(2*b^4) - (A/(2*b) + (3*x^2*(3*A*c - B*b))/(
4*b^2) + (c*x^4*(3*A*c - B*b))/(2*b^3))/(b^2*x^2 + c^2*x^6 + 2*b*c*x^4) - (
log(x)*(3*A*c - B*b))/b^4
```

3.85 $\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=117

$$-\frac{A}{3b^3x^3}-\frac{bB-3Ac}{b^4x}-\frac{c(bB-Ac)x}{4b^3(b+cx^2)^2}-\frac{c(7bB-11Ac)x}{8b^4(b+cx^2)}-\frac{5\sqrt{c}\,(3bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}}$$

```
[Out] -1/3*A/b^3/x^3+(3*A*c-B*b)/b^4/x-1/4*c*(-A*c+B*b)*x/b^3/(c*x^2+b)^2-1/8*c*(
-11*A*c+7*B*b)*x/b^4/(c*x^2+b)-5/8*(-7*A*c+3*B*b)*arctan(x*c^(1/2)/b^(1/2))
*c^(1/2)/b^(9/2)
```

Rubi [A]
time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1598, 467, 1273, 1275, 211}

$$-\frac{5\sqrt{c}\,(3bB-7Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}}-\frac{cx(7bB-11Ac)}{8b^4(b+cx^2)}-\frac{bB-3Ac}{b^4x}-\frac{cx(bB-Ac)}{4b^3(b+cx^2)^2}-\frac{A}{3b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/3*A/(b^3*x^3) - (b*B - 3*A*c)/(b^4*x) - (c*(b*B - A*c)*x)/(4*b^3*(b + c*
x^2)^2) - (c*(7*b*B - 11*A*c)*x)/(8*b^4*(b + c*x^2)) - (5*Sqrt[c]*(3*b*B -
7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= \int \frac{A+Bx^2}{x^4\left(b+cx^2\right)^3}\,dx \\
&= -\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2}-\frac{1}{4}c\int \frac{-\frac{4A}{bc}-\frac{4(bB-Ac)x^2}{b^2c}+\frac{3(bB-Ac)x^4}{b^3}}{x^4\left(b+cx^2\right)^2}\,dx \\
&= -\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2}-\frac{c(7bB-11Ac)x}{8b^4\left(b+cx^2\right)}-\frac{\int \frac{-8Abc-8c(bB-2Ac)x^2+\frac{c^2(7bB-11Ac)x^4}{b}}{x^4(b+cx^2)}\,dx}{8b^3c} \\
&= -\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2}-\frac{c(7bB-11Ac)x}{8b^4\left(b+cx^2\right)}-\frac{\int \left(-\frac{8Ac}{x^4}-\frac{8c(bB-3Ac)}{bx^2}+\frac{5c^2(3bB-7Ac)}{b(b+cx^2)}\right)dx}{8b^3c} \\
&= -\frac{A}{3b^3x^3}-\frac{bB-3Ac}{b^4x}-\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2}-\frac{c(7bB-11Ac)x}{8b^4\left(b+cx^2\right)}-\frac{(5c(3bB-7Ac))\int \frac{1}{b+cx^2}}{8b^4} \\
&= -\frac{A}{3b^3x^3}-\frac{bB-3Ac}{b^4x}-\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2}-\frac{c(7bB-11Ac)x}{8b^4\left(b+cx^2\right)}-\frac{5\sqrt{c}\,(3bB-7Ac)\tan^{-1}}{8b^{9/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.05, size = 119, normalized size = 1.02

$$-\frac{A}{3b^3x^3}+\frac{-bB+3Ac}{b^4x}-\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2}-\frac{\left(7bBc-11Ac^2\right)x}{8b^4\left(b+cx^2\right)}-\frac{5\sqrt{c}\,(3bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/3*A/(b^3*x^3) + (-(b*B) + 3*A*c)/(b^4*x) - (c*(b*B - A*c)*x)/(4*b^3*(b +
 c*x^2)^2) - ((7*b*B*c - 11*A*c^2)*x)/(8*b^4*(b + c*x^2)) - (5*Sqrt[c]*(3*b
*B - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

Maple [A]
time = 0.43, size = 98, normalized size = 0.84

method	result
default	$\frac{c\left(\frac{\left(\frac{11}{8}Ac^2-\frac{7}{8}bBc\right)x^3+\frac{b(13Ac-9Bb)x}{8}}{\left(cx^2+b\right)^2}+\frac{5(7Ac-3Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{b^4}-\frac{A}{3b^3x^3}-\frac{-3Ac+Bb}{b^4x}$
risch	$\frac{\frac{5c^2(7Ac-3Bb)x^6}{8b^4}+\frac{25c(7Ac-3Bb)x^4}{24b^3}+\frac{(7Ac-3Bb)x^2}{3b^2}-\frac{A}{3b}}{x^3(cx^2+b)^2}+\frac{35\sqrt{-bc}\,\ln\left(-cx-\sqrt{-bc}\right)Ac}{16b^5}-\frac{15\sqrt{-bc}\,\ln\left(-cx-\sqrt{-bc}\right)B}{16b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*c*(((11/8*A*c^2-7/8*b*B*c)*x^3+1/8*b*(13*A*c-9*B*b)*x)/(c*x^2+b)^2+5/
8*(7*A*c-3*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))-1/3*A/b^3/x^3-(-3*A*c+
B*b)/b^4/x
```

Maxima [A]
time = 0.49, size = 128, normalized size = 1.09

$$-\frac{15\,(3\,Bbc^2-7\,Ac^3)x^6+25\,(3\,Bb^2c-7\,Abc^2)x^4+8\,Ab^3+8\,(3\,Bb^3-7\,Ab^2c)x^2}{24\,(b^4c^2x^7+2\,b^5cx^5+b^6x^3)}-\frac{5\,(3\,Bbc-7\,Ac^2)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(15*(3*B*b*c^2 - 7*A*c^3)*x^6 + 25*(3*B*b^2*c - 7*A*b*c^2)*x^4 + 8*A*
b^3 + 8*(3*B*b^3 - 7*A*b^2*c)*x^2)/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3) -
5/8*(3*B*b*c - 7*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4)
```

Fricas [A]
time = 1.28, size = 368, normalized size = 3.15

$$\left[-\frac{30\,(3\,Bbc^2-7\,Ac^3)x^6+50\,(3\,Bb^2c-7\,Abc^2)x^4+16\,Ab^3+16\,(3\,Bb^3-7\,Ab^2c)x^2+15\,\left((3\,Bbc^2-7\,Ac^3)x^7+2\,(3\,Bb^2c-7\,Abc^2)x^5+(3\,Bb^3-7\,Ab^2c)x^3\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2+2bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)}{48\,(b^4c^2x^7+2\,b^5cx^5+b^6x^3)},\ -\frac{15\,(3\,Bbc^2-7\,Ac^3)x^6+25\,(3\,Bb^2c-7\,Abc^2)x^4+8\,Ab^3+8\,(3\,Bb^3-7\,Ab^2c)x^2+15\,\left((3\,Bbc^2-7\,Ac^3)x^7+2\,(3\,Bb^2c-7\,Abc^2)x^5+(3\,Bb^3-7\,Ab^2c)x^3\right)\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)}{24\,(b^4c^2x^7+2\,b^5cx^5+b^6x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(30*(3*B*b*c^2 - 7*A*c^3)*x^6 + 50*(3*B*b^2*c - 7*A*b*c^2)*x^4 + 16*
A*b^3 + 16*(3*B*b^3 - 7*A*b^2*c)*x^2 + 15*((3*B*b*c^2 - 7*A*c^3)*x^7 + 2*(3
*B*b^2*c - 7*A*b*c^2)*x^5 + (3*B*b^3 - 7*A*b^2*c)*x^3)*sqrt(-c/b)*log((c*x^
2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^
3), -1/24*(15*(3*B*b*c^2 - 7*A*c^3)*x^6 + 25*(3*B*b^2*c - 7*A*b*c^2)*x^4 +
8*A*b^3 + 8*(3*B*b^3 - 7*A*b^2*c)*x^2 + 15*((3*B*b*c^2 - 7*A*c^3)*x^7 + 2*(
3*B*b^2*c - 7*A*b*c^2)*x^5 + (3*B*b^3 - 7*A*b^2*c)*x^3)*sqrt(c/b)*arctan(x*
sqrt(c/b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(109) = 218.
time = 0.40, size = 226, normalized size = 1.93

$$\frac{5\sqrt{-\frac{c}{b^9}}\,(-7Ac+3Bb)\log\left(-\frac{5b^5\sqrt{-\frac{c}{b^9}}\,(-7Ac+3Bb)}{-35Ac^2+15Bbc}+x\right)}{16}-\frac{5\sqrt{-\frac{c}{b^9}}\,(-7Ac+3Bb)\log\left(\frac{5b^5\sqrt{-\frac{c}{b^9}}\,(-7Ac+3Bb)}{-35Ac^2+15Bbc}+x\right)}{16}+\frac{-8Ab^3+x^6\cdot(105Ac^3-45Bbc^2)+x^4\cdot(175Abc^2-75Bb^2c)+x^2\cdot(56Ab^2c-24Bb^3)}{24b^6x^3+48b^5cx^5+24b^4c^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] 5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)*log(-5*b**5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)
/(-35*A*c**2 + 15*B*b*c) + x)/16 - 5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)*log(5*b
**5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)/(-35*A*c**2 + 15*B*b*c) + x)/16 + (-8*A*
b**3 + x**6*(105*A*c**3 - 45*B*b*c**2) + x**4*(175*A*b*c**2 - 75*B*b**2*c)
+ x**2*(56*A*b**2*c - 24*B*b**3))/(24*b**6*x**3 + 48*b**5*c*x**5 + 24*b**4*
c**2*x**7)
```

Giac [A]
time = 1.02, size = 108, normalized size = 0.92

$$-\frac{5\,(3\,Bbc-7\,Ac^2)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^4}-\frac{7\,Bbc^2x^3-11\,Ac^3x^3+9\,Bb^2cx-13\,Abc^2x}{8\,(cx^2+b)^2b^4}-\frac{3\,Bbx^2-9\,Acx^2+Ab}{3\,b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -5/8*(3*B*b*c - 7*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) - 1/8*(7*B*b
*c^2*x^3 - 11*A*c^3*x^3 + 9*B*b^2*c*x - 13*A*b*c^2*x)/((c*x^2 + b)^2*b^4) -
 1/3*(3*B*b*x^2 - 9*A*c*x^2 + A*b)/(b^4*x^3)
```

Mupad [B]
time = 0.18, size = 114, normalized size = 0.97

$$\frac{\frac{x^2\,(7\,A\,c-3\,B\,b)}{3\,b^2}-\frac{A}{3\,b}+\frac{5\,c^2\,x^6\,(7\,A\,c-3\,B\,b)}{8\,b^4}+\frac{25\,c\,x^4\,(7\,A\,c-3\,B\,b)}{24\,b^3}}{b^2\,x^3+2\,b\,c\,x^5+c^2\,x^7}+\frac{5\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(7\,A\,c-3\,B\,b)}{8\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^2*(7*A*c - 3*B*b))/(3*b^2) - A/(3*b) + (5*c^2*x^6*(7*A*c - 3*B*b))/(8*b
^4) + (25*c*x^4*(7*A*c - 3*B*b))/(24*b^3))/(b^2*x^3 + c^2*x^7 + 2*b*c*x^5)
+ (5*c^(1/2)*atan((c^(1/2)*x)/b^(1/2))*(7*A*c - 3*B*b))/(8*b^(9/2))
```

3.86 $\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=121

$$-\frac{A}{4b^3x^4}-\frac{bB-3Ac}{2b^4x^2}-\frac{c(bB-Ac)}{4b^3(b+cx^2)^2}-\frac{c(2bB-3Ac)}{2b^4(b+cx^2)}-\frac{3c(bB-2Ac)\log(x)}{b^5}+\frac{3c(bB-2Ac)\log(b+cx^2)}{2b^5}$$

```
[Out] -1/4*A/b^3/x^4+1/2*(3*A*c-B*b)/b^4/x^2-1/4*c*(-A*c+B*b)/b^3/(c*x^2+b)^2-1/2
*c*(-3*A*c+2*B*b)/b^4/(c*x^2+b)-3*c*(-2*A*c+B*b)*ln(x)/b^5+3/2*c*(-2*A*c+B*
b)*ln(c*x^2+b)/b^5
```

Rubi [A]
time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1598, 457, 78}

$$\frac{3c(bB-2Ac)\log(b+cx^2)}{2b^5}-\frac{3c\log(x)(bB-2Ac)}{b^5}-\frac{c(2bB-3Ac)}{2b^4(b+cx^2)}-\frac{bB-3Ac}{2b^4x^2}-\frac{c(bB-Ac)}{4b^3(b+cx^2)^2}-\frac{A}{4b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*A/(b^3*x^4) - (b*B - 3*A*c)/(2*b^4*x^2) - (c*(b*B - A*c))/(4*b^3*(b +
c*x^2)^2) - (c*(2*b*B - 3*A*c))/(2*b^4*(b + c*x^2)) - (3*c*(b*B - 2*A*c)*Lo
g[x])/b^5 + (3*c*(b*B - 2*A*c)*Log[b + c*x^2])/(2*b^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= \int \frac{A+Bx^2}{x^5\,(b+cx^2)^3}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3(b+cx)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{b^3x^3}+\frac{bB-3Ac}{b^4x^2}-\frac{3c(bB-2Ac)}{b^5x}+\frac{c^2(bB-Ac)}{b^3(b+cx)^3}+\frac{c^2(2bB-3Ac)}{b^4(b+cx)^2}+\frac{3\right.\right. \\
&= -\frac{A}{4b^3x^4}-\frac{bB-3Ac}{2b^4x^2}-\frac{c(bB-Ac)}{4b^3\,(b+cx^2)^2}-\frac{c(2bB-3Ac)}{2b^4\,(b+cx^2)}-\frac{3c(bB-2Ac)\log(x)}{b^5}+\frac{3c(b
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 108, normalized size = 0.89

$$\frac{-\frac{Ab^2}{x^4}-\frac{2b(bB-3Ac)}{x^2}+\frac{b^2c(-bB+Ac)}{(b+cx^2)^2}+\frac{2bc(-2bB+3Ac)}{b+cx^2}+12c(-bB+2Ac)\log(x)+6c(bB-2Ac)\log\left(b+cx^2\right)}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-((A*b^2)/x^4) - (2*b*(b*B - 3*A*c))/x^2 + (b^2*c*(-(b*B) + A*c))/(b + c*x
^2)^2 + (2*b*c*(-2*b*B + 3*A*c))/(b + c*x^2) + 12*c*(-(b*B) + 2*A*c)*Log[x]
 + 6*c*(b*B - 2*A*c)*Log[b + c*x^2])/(4*b^5)
```

Maple [A]
time = 0.39, size = 123, normalized size = 1.02

method	result	size
default	$-\frac{c^2\left(\frac{(6Ac-3Bb)\ln\left(cx^2+b\right)}{c}-\frac{b^2(Ac-Bb)}{2c\left(cx^2+b\right)^2}-\frac{b(3Ac-2Bb)}{c\left(cx^2+b\right)}\right)}{2b^5}-\frac{A}{4b^3x^4}-\frac{-3Ac+Bb}{2b^4x^2}+\frac{3c(2Ac-Bb)\ln(x)}{b^5}$	123
norman	$\frac{-\frac{Ax}{4b}+\frac{(2Ac-Bb)x^3}{2b^2}-\frac{c\left(6Ac^2-3bBc\right)x^7}{b^4}-\frac{c^2\left(18Ac^2-9bBc\right)x^9}{4b^5}}{x^5(cx^2+b)^2}+\frac{3c(2Ac-Bb)\ln(x)}{b^5}-\frac{3c(2Ac-Bb)\ln\left(cx^2+b\right)}{2b^5}$	124
risch	$\frac{\frac{3c^2(2Ac-Bb)x^6}{2b^4}+\frac{9c(2Ac-Bb)x^4}{4b^3}+\frac{(2Ac-Bb)x^2}{2b^2}-\frac{A}{4b}}{x^4(cx^2+b)^2}+\frac{6c^2\ln(x)A}{b^5}-\frac{3c\ln(x)B}{b^4}-\frac{3c^2\ln\left(cx^2+b\right)A}{b^5}+\frac{3c\ln\left(cx^2+b\right)B}{2b^4}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^5*c^2*((6*A*c-3*B*b)/c*ln(c*x^2+b)-1/2*b^2*(A*c-B*b)/c/(c*x^2+b)^2-b
*(3*A*c-2*B*b)/c/(c*x^2+b))-1/4*A/b^3/x^4-1/2*(-3*A*c+B*b)/b^4/x^2+3*c*(2*A
*c-B*b)/b^5*ln(x)
```

Maxima [A]
time = 0.29, size = 137, normalized size = 1.13

$$-\frac{6\,(Bbc^2-2\,Ac^3)x^6+9\,(Bb^2c-2\,Abc^2)x^4+Ab^3+2\,(Bb^3-2\,Ab^2c)x^2}{4\,(b^4c^2x^8+2\,b^5cx^6+b^6x^4)}+\frac{3\,(Bbc-2\,Ac^2)\log\left(cx^2+b\right)}{2\,b^5}-\frac{3\,(Bbc-2\,Ac^2)\log\left(x^2\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*(B*b*c^2 - 2*A*c^3)*x^6 + 9*(B*b^2*c - 2*A*b*c^2)*x^4 + A*b^3 + 2*(
B*b^3 - 2*A*b^2*c)*x^2)/(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4) + 3/2*(B*b*c
 - 2*A*c^2)*log(c*x^2 + b)/b^5 - 3/2*(B*b*c - 2*A*c^2)*log(x^2)/b^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(111) = 222.
time = 1.59, size = 229, normalized size = 1.89

$$-\frac{6\,(Bb^2c^2-2\,Abc^3)x^6+Ab^4+9\,(Bb^3c-2\,Ab^2c^2)x^4+2\,(Bb^4-2\,Ab^3c)x^2-6\,((Bbc^3-2\,Ac^4)x^8+2\,(Bb^2c^2-2\,Abc^3)x^6+(Bb^3c-2\,Ab^2c^2)x^4)\log\left(cx^2+b\right)+12\,((Bbc^3-2\,Ac^4)x^8+2\,(Bb^2c^2-2\,Abc^3)x^6+(Bb^3c-2\,Ab^2c^2)x^4)\log\left(x\right)}{4\,(b^5c^2x^8+2\,b^6cx^6+b^7x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + A*b^4 + 9*(B*b^3*c - 2*A*b^2*c^2)*x^4
 + 2*(B*b^4 - 2*A*b^3*c)*x^2 - 6*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 -
2*A*b*c^3)*x^6 + (B*b^3*c - 2*A*b^2*c^2)*x^4)*log(c*x^2 + b) + 12*((B*b*c^3
 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + (B*b^3*c - 2*A*b^2*c^2)*x
^4)*log(x))/(b^5*c^2*x^8 + 2*b^6*c*x^6 + b^7*x^4)
```

Sympy [A]
time = 0.65, size = 136, normalized size = 1.12

$$\frac{-Ab^3+x^6\cdot(12Ac^3-6Bbc^2)+x^4\cdot(18Abc^2-9Bb^2c)+x^2\cdot(4Ab^2c-2Bb^3)}{4b^6x^4+8b^5cx^6+4b^4c^2x^8}-\frac{3c(-2Ac+Bb)\log\left(x\right)}{b^5}+\frac{3c(-2Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-A*b**3 + x**6*(12*A*c**3 - 6*B*b*c**2) + x**4*(18*A*b*c**2 - 9*B*b**2*c)
+ x**2*(4*A*b**2*c - 2*B*b**3))/(4*b**6*x**4 + 8*b**5*c*x**6 + 4*b**4*c**2*
x**8) - 3*c*(-2*A*c + B*b)*log(x)/b**5 + 3*c*(-2*A*c + B*b)*log(b/c + x**2)
/(2*b**5)
```

Giac [A]
time = 1.08, size = 132, normalized size = 1.09

$$-\frac{3\,(Bbc-2\,Ac^2)\log\left(|x|\right)}{b^5}+\frac{3\,(Bbc^2-2\,Ac^3)\log\left(|cx^2+b|\right)}{2\,b^5c}-\frac{6\,Bbc^2x^6-12\,Ac^3x^6+9\,Bb^2cx^4-18\,Abc^2x^4+2\,Bb^3x^2-4\,Ab^2cx^2+Ab^3}{4\,(cx^4+bx^2)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -3*(B*b*c - 2*A*c^2)*log(abs(x))/b^5 + 3/2*(B*b*c^2 - 2*A*c^3)*log(abs(c*x^
2 + b))/(b^5*c) - 1/4*(6*B*b*c^2*x^6 - 12*A*c^3*x^6 + 9*B*b^2*c*x^4 - 18*A*
b*c^2*x^4 + 2*B*b^3*x^2 - 4*A*b^2*c*x^2 + A*b^3)/((c*x^4 + b*x^2)^2*b^4)
```

Mupad [B]
time = 0.17, size = 131, normalized size = 1.08

$$\frac{\frac{x^2\,\left(2\,A\,c-B\,b\right)}{2\,b^2}-\frac{A}{4\,b}+\frac{3\,c^2\,x^6\,\left(2\,A\,c-B\,b\right)}{2\,b^4}+\frac{9\,c\,x^4\,\left(2\,A\,c-B\,b\right)}{4\,b^3}}{b^2\,x^4+2\,b\,c\,x^6+c^2\,x^8}-\frac{\ln\left(c\,x^2+b\right)\,\left(6\,A\,c^2-3\,B\,b\,c\right)}{2\,b^5}+\frac{\ln\left(x\right)\,\left(6\,A\,c^2-3\,B\,b\,c\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^2*(2*A*c - B*b))/(2*b^2) - A/(4*b) + (3*c^2*x^6*(2*A*c - B*b))/(2*b^4)
+ (9*c*x^4*(2*A*c - B*b))/(4*b^3))/(b^2*x^4 + c^2*x^8 + 2*b*c*x^6) - (log(b
 + c*x^2)*(6*A*c^2 - 3*B*b*c))/(2*b^5) + (log(x)*(6*A*c^2 - 3*B*b*c))/b^5
```

3.87 $\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=140

$$-\frac{A}{5b^3x^5}-\frac{bB-3Ac}{3b^4x^3}+\frac{3c(bB-2Ac)}{b^5x}+\frac{c^2(bB-Ac)x}{4b^4\left(b+cx^2\right)^2}+\frac{c^2(11bB-15Ac)x}{8b^5\left(b+cx^2\right)}+\frac{7c^{3/2}(5bB-9Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{11/2}}$$

```
[Out] -1/5*A/b^3/x^5+1/3*(3*A*c-B*b)/b^4/x^3+3*c*(-2*A*c+B*b)/b^5/x+1/4*c^2*(-A*c
+B*b)*x/b^4/(c*x^2+b)^2+1/8*c^2*(-15*A*c+11*B*b)*x/b^5/(c*x^2+b)+7/8*c^(3/2
)*(-9*A*c+5*B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(11/2)
```

Rubi [A]
time = 0.22, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1607, 467, 1819, 1816, 211}

$$\frac{7c^{3/2}(5bB-9Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{11/2}}+\frac{c^2x(11bB-15Ac)}{8b^5\left(b+cx^2\right)}+\frac{3c(bB-2Ac)}{b^5x}+\frac{c^2x(bB-Ac)}{4b^4\left(b+cx^2\right)^2}-\frac{bB-3Ac}{3b^4x^3}-\frac{A}{5b^3x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/5*A/(b^3*x^5) - (b*B - 3*A*c)/(3*b^4*x^3) + (3*c*(b*B - 2*A*c))/(b^5*x)
+ (c^2*(b*B - A*c)*x)/(4*b^4*(b + c*x^2)^2) + (c^2*(11*b*B - 15*A*c)*x)/(8*
b^5*(b + c*x^2)) + (7*c^(3/2)*(5*b*B - 9*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/
(8*b^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^3}\,dx = \int \frac{A+Bx^2}{x^6(b+cx^2)^3}\,dx$$

$$= \frac{c^2(bB-Ac)x}{4b^4(b+cx^2)^2} - \frac{1}{4}c^2\int \frac{-\frac{4A}{bc^2}-\frac{4(bB-Ac)x^2}{b^2c^2}+\frac{4(bB-Ac)x^4}{b^3c}-\frac{3(bB-Ac)x^6}{b^4}}{x^6(b+cx^2)^2}\,dx$$

$$= \frac{c^2(bB-Ac)x}{4b^4(b+cx^2)^2} + \frac{c^2(11bB-15Ac)x}{8b^5(b+cx^2)} + \frac{c^2\int \frac{\frac{8A}{bc^2}+\frac{8(bB-2Ac)x^2}{b^2c^2}-\frac{8(2bB-3Ac)x^4}{b^3c}+\frac{(11bB-15Ac)x^6}{b^4}}{x^6(b+cx^2)}\,dx}{8b}$$

$$= \frac{c^2(bB-Ac)x}{4b^4(b+cx^2)^2} + \frac{c^2(11bB-15Ac)x}{8b^5(b+cx^2)} + \frac{c^2\int \left(\frac{8A}{b^2c^2x^6}+\frac{8(bB-3Ac)}{b^3c^2x^4}-\frac{24(bB-2Ac)}{b^4cx^2}+\frac{7(5bB-9Ac}{b^4(b+cx^2)}\right.}{8b}$$

$$= -\frac{A}{5b^3x^5} - \frac{bB-3Ac}{3b^4x^3} + \frac{3c(bB-2Ac)}{b^5x} + \frac{c^2(bB-Ac)x}{4b^4(b+cx^2)^2} + \frac{c^2(11bB-15Ac)x}{8b^5(b+cx^2)} + \frac{(7c^2(5b}{}$$

$$= -\frac{A}{5b^3x^5} - \frac{bB-3Ac}{3b^4x^3} + \frac{3c(bB-2Ac)}{b^5x} + \frac{c^2(bB-Ac)x}{4b^4(b+cx^2)^2} + \frac{c^2(11bB-15Ac)x}{8b^5(b+cx^2)} + \frac{7c^{3/2}(5}{}$$

Mathematica [A]

time = 0.05, size = 140, normalized size = 1.00

$$-\frac{A}{5b^3x^5}-\frac{bB-3Ac}{3b^4x^3}+\frac{3c(bB-2Ac)}{b^5x}+\frac{c^2(bB-Ac)x}{4b^4\left(b+cx^2\right)^2}+\frac{c^2(11bB-15Ac)x}{8b^5\left(b+cx^2\right)}+\frac{7c^{3/2}(5bB-9Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/5*A/(b^3*x^5) - (b*B - 3*A*c)/(3*b^4*x^3) + (3*c*(b*B - 2*A*c))/(b^5*x)
+ (c^2*(b*B - A*c)*x)/(4*b^4*(b + c*x^2)^2) + (c^2*(11*b*B - 15*A*c)*x)/(8*
b^5*(b + c*x^2)) + (7*c^(3/2)*(5*b*B - 9*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/
(8*b^(11/2))
```

Maple [A]
time = 0.41, size = 119, normalized size = 0.85

method	result
default	$-\frac{c^2\left(\frac{\left(\frac{15}{8}A\,c^2-\frac{11}{8}bBc\right)x^3+\frac{b(17Ac-13Bb)x}{8}}{\left(c\,x^2+b\right)^2}+\frac{7(9Ac-5Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{b^5}-\frac{A}{5b^3x^5}-\frac{-3Ac+Bb}{3b^4x^3}-\frac{3c(2Ac-Bb)}{b^5x}$
risch	$\frac{-\frac{7c^3(9Ac-5Bb)x^8}{8b^5}-\frac{35c^2(9Ac-5Bb)x^6}{24b^4}-\frac{7c(9Ac-5Bb)x^4}{15b^3}+\frac{(9Ac-5Bb)x^2}{15b^2}-\frac{A}{5b}}{x^5\left(c\,x^2+b\right)^2}+\frac{7\left(\sum_{\_R=\mathrm{RootOf}\left(b^{11}\_Z^2+81A^2c^5-90ABb\,c^4+25B^2b^2c^3\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^5*c^2*(((15/8*A*c^2-11/8*b*B*c)*x^3+1/8*b*(17*A*c-13*B*b)*x)/(c*x^2+b)
^2+7/8*(9*A*c-5*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))-1/5*A/b^3/x^5-1/3
*(-3*A*c+B*b)/b^4/x^3-3*c*(2*A*c-B*b)/b^5/x
```

Maxima [A]
time = 0.50, size = 154, normalized size = 1.10

$$\frac{105\,(5\,Bbc^3-9\,Ac^4)x^8+175\,(5\,Bb^2c^2-9\,Abc^3)x^6-24\,Ab^4+56\,(5\,Bb^3c-9\,Ab^2c^2)x^4-8\,(5\,Bb^4-9\,Ab^3c)x^2}{120\,(b^5c^2x^9+2\,b^6cx^7+b^7x^5)}+\frac{7\,(5\,Bbc^2-9\,Ac^3)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(105*(5*B*b*c^3 - 9*A*c^4)*x^8 + 175*(5*B*b^2*c^2 - 9*A*b*c^3)*x^6 -
24*A*b^4 + 56*(5*B*b^3*c - 9*A*b^2*c^2)*x^4 - 8*(5*B*b^4 - 9*A*b^3*c)*x^2)/
```

```
(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5) + 7/8*(5*B*b*c^2 - 9*A*c^3)*arctan(c*
x/sqrt(b*c))/(sqrt(b*c)*b^5)
```

Fricas [A]
time = 1.49, size = 426, normalized size = 3.04

$$\left[\frac{210\,(5\,Bbc^3-9\,Ac^4)x^8+350\,(5\,Bb^2c^2-9\,Abc^3)x^6-48\,Ab^4+112\,(5\,Bb^3c-9\,Ab^2c^2)x^4-16\,(5\,Bb^4-9\,Ab^3c)x^2-105\,((5\,Bbc^3-9\,Ac^4)x^9+2\,(5\,Bb^2c^2-9\,Abc^3)x^7+(5\,Bb^3c-9\,Ab^2c^2)x^5)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)}{240\,(b^5c^2x^9+2\,b^6cx^7+b^7x^5)}, \frac{105\,(5\,Bbc^3-9\,Ac^4)x^8+175\,(5\,Bb^2c^2-9\,Abc^3)x^6-24\,Ab^4+56\,(5\,Bb^3c-9\,Ab^2c^2)x^4-8\,(5\,Bb^4-9\,Ab^3c)x^2+105\,((5\,Bbc^3-9\,Ac^4)x^9+2\,(5\,Bb^2c^2-9\,Abc^3)x^7+(5\,Bb^3c-9\,Ab^2c^2)x^5)\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)}{120\,(b^5c^2x^9+2\,b^6cx^7+b^7x^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(210*(5*B*b*c^3 - 9*A*c^4)*x^8 + 350*(5*B*b^2*c^2 - 9*A*b*c^3)*x^6 -
 48*A*b^4 + 112*(5*B*b^3*c - 9*A*b^2*c^2)*x^4 - 16*(5*B*b^4 - 9*A*b^3*c)*x^
2 - 105*((5*B*b*c^3 - 9*A*c^4)*x^9 + 2*(5*B*b^2*c^2 - 9*A*b*c^3)*x^7 + (5*B
*b^3*c - 9*A*b^2*c^2)*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c
*x^2 + b)))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5), 1/120*(105*(5*B*b*c^3 -
9*A*c^4)*x^8 + 175*(5*B*b^2*c^2 - 9*A*b*c^3)*x^6 - 24*A*b^4 + 56*(5*B*b^3*c
 - 9*A*b^2*c^2)*x^4 - 8*(5*B*b^4 - 9*A*b^3*c)*x^2 + 105*((5*B*b*c^3 - 9*A*c
^4)*x^9 + 2*(5*B*b^2*c^2 - 9*A*b*c^3)*x^7 + (5*B*b^3*c - 9*A*b^2*c^2)*x^5)*
sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)]
```

Sympy [A]
time = 0.44, size = 260, normalized size = 1.86

$$-\frac{7\sqrt{-\frac{c^3}{b^{11}}}\,(-9Ac+5Bb)\log\left(-\frac{7b^6\sqrt{-\frac{c^3}{b^{11}}}\,(-9Ac+5Bb)}{-63Ac^3+35Bbc^2}+x\right)}{16}+\frac{7\sqrt{-\frac{c^3}{b^{11}}}\,(-9Ac+5Bb)\log\left(\frac{7b^6\sqrt{-\frac{c^3}{b^{11}}}\,(-9Ac+5Bb)}{-63Ac^3+35Bbc^2}+x\right)}{16}+\frac{-24Ab^4+x^8(-945Ac^4+525Bbc^3)+x^6(-1575Abc^3+875Bb^2c^2)+x^4(-504Ab^2c^2+280Bb^3c)+x^2\cdot(72Ab^3c-40Bb^4)}{120b^7x^5+240b^6cx^7+120b^5c^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -7*sqrt(-c**3/b**11)*(-9*A*c + 5*B*b)*log(-7*b**6*sqrt(-c**3/b**11)*(-9*A*c
 + 5*B*b)/(-63*A*c**3 + 35*B*b*c**2) + x)/16 + 7*sqrt(-c**3/b**11)*(-9*A*c
+ 5*B*b)*log(7*b**6*sqrt(-c**3/b**11)*(-9*A*c + 5*B*b)/(-63*A*c**3 + 35*B*b
*c**2) + x)/16 + (-24*A*b**4 + x**8*(-945*A*c**4 + 525*B*b*c**3) + x**6*(-1
575*A*b*c**3 + 875*B*b**2*c**2) + x**4*(-504*A*b**2*c**2 + 280*B*b**3*c) +
x**2*(72*A*b**3*c - 40*B*b**4))/(120*b**7*x**5 + 240*b**6*c*x**7 + 120*b**5
*c**2*x**9)
```

Giac [A]
time = 0.94, size = 135, normalized size = 0.96

$$\frac{7\,(5\,Bbc^2-9\,Ac^3)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^5}+\frac{11\,Bbc^3x^3-15\,Ac^4x^3+13\,Bb^2c^2x-17\,Abc^3x}{8\,(cx^2+b)^2b^5}+\frac{45\,Bbcx^4-90\,Ac^2x^4-5\,Bb^2x^2+15\,Abcx^2-3\,Ab^2}{15\,b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 7/8*(5*B*b*c^2 - 9*A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^5) + 1/8*(11*B
*b*c^3*x^3 - 15*A*c^4*x^3 + 13*B*b^2*c^2*x - 17*A*b*c^3*x)/((c*x^2 + b)^2*b
^5) + 1/15*(45*B*b*c*x^4 - 90*A*c^2*x^4 - 5*B*b^2*x^2 + 15*A*b*c*x^2 - 3*A*
b^2)/(b^5*x^5)
```

Mupad [B]
time = 0.19, size = 135, normalized size = 0.96

$$-\frac{\frac{A}{5\,b}-\frac{x^2\,\left(9\,A\,c-5\,B\,b\right)}{15\,b^2}+\frac{35\,c^2\,x^6\,\left(9\,A\,c-5\,B\,b\right)}{24\,b^4}+\frac{7\,c^3\,x^8\,\left(9\,A\,c-5\,B\,b\right)}{8\,b^5}+\frac{7\,c\,x^4\,\left(9\,A\,c-5\,B\,b\right)}{15\,b^3}}{b^2\,x^5+2\,b\,c\,x^7+c^2\,x^9}-\frac{7\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,\left(9\,A\,c-5\,B\,b\right)}{8\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - (A/(5*b) - (x^2*(9*A*c - 5*B*b))/(15*b^2) + (35*c^2*x^6*(9*A*c - 5*B*b))/
(24*b^4) + (7*c^3*x^8*(9*A*c - 5*B*b))/(8*b^5) + (7*c*x^4*(9*A*c - 5*B*b))/
(15*b^3))/(b^2*x^5 + c^2*x^9 + 2*b*c*x^7) - (7*c^(3/2)*atan((c^(1/2)*x)/b^(
1/2))*(9*A*c - 5*B*b))/(8*b^(11/2))
```

3.88 $\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=148

$$-\frac{A}{6b^3x^6}-\frac{bB-3Ac}{4b^4x^4}+\frac{3c(bB-2Ac)}{2b^5x^2}+\frac{c^2(bB-Ac)}{4b^4\left(b+cx^2\right)^2}+\frac{c^2(3bB-4Ac)}{2b^5\left(b+cx^2\right)}+\frac{2c^2(3bB-5Ac)\log(x)}{b^6}-\frac{c^2(3bB-5A$$

```
[Out] -1/6*A/b^3/x^6+1/4*(3*A*c-B*b)/b^4/x^4+3/2*c*(-2*A*c+B*b)/b^5/x^2+1/4*c^2*(
-A*c+B*b)/b^4/(c*x^2+b)^2+1/2*c^2*(-4*A*c+3*B*b)/b^5/(c*x^2+b)+2*c^2*(-5*A*
c+3*B*b)*ln(x)/b^6-c^2*(-5*A*c+3*B*b)*ln(c*x^2+b)/b^6
```

Rubi [A]
time = 0.13, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$-\frac{c^2(3bB-5Ac)\log\left(b+cx^2\right)}{b^6}+\frac{2c^2\log(x)(3bB-5Ac)}{b^6}+\frac{c^2(3bB-4Ac)}{2b^5\left(b+cx^2\right)}+\frac{3c(bB-2Ac)}{2b^5x^2}+\frac{c^2(bB-Ac)}{4b^4\left(b+cx^2\right)^2}-\frac{bB-3Ac}{4b^4x^4}-\frac{A}{6b^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -1/6*A/(b^3*x^6) - (b*B - 3*A*c)/(4*b^4*x^4) + (3*c*(b*B - 2*A*c))/(2*b^5*x
^2) + (c^2*(b*B - A*c))/(4*b^4*(b + c*x^2)^2) + (c^2*(3*b*B - 4*A*c))/(2*b^
5*(b + c*x^2)) + (2*c^2*(3*b*B - 5*A*c)*Log[x])/b^6 - (c^2*(3*b*B - 5*A*c)*
Log[b + c*x^2])/b^6
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{A+Bx^2}{x^7\left(b+cx^2\right)^3}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4(b+cx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{b^3x^4}+\frac{bB-3Ac}{b^4x^3}-\frac{3c(bB-2Ac)}{b^5x^2}+\frac{2c^2(3bB-5Ac)}{b^6x}-\frac{c^3(bB-Ac)}{b^4(b+cx)^3}\right.\right.\\ &= -\frac{A}{6b^3x^6}-\frac{bB-3Ac}{4b^4x^4}+\frac{3c(bB-2Ac)}{2b^5x^2}+\frac{c^2(bB-Ac)}{4b^4\left(b+cx^2\right)^2}+\frac{c^2(3bB-4Ac)}{2b^5\left(b+cx^2\right)}+\frac{2c^2(3bB}{}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 135, normalized size = 0.91

$$\frac{-\frac{2Ab^3}{x^6}-\frac{3b^2(bB-3Ac)}{x^4}+\frac{18bc(bB-2Ac)}{x^2}+\frac{3b^2c^2(bB-Ac)}{(b+cx^2)^2}+\frac{6bc^2(3bB-4Ac)}{b+cx^2}+24c^2(3bB-5Ac)\log(x)+12c^2(-3bB+5Ac)\log\left(b+cx^2\right)}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)^3),x]
[Out] ((-2*A*b^3)/x^6 - (3*b^2*(b*B - 3*A*c))/x^4 + (18*b*c*(b*B - 2*A*c))/x^2 +
(3*b^2*c^2*(b*B - A*c))/(b + c*x^2)^2 + (6*b*c^2*(3*b*B - 4*A*c))/(b + c*x^
2) + 24*c^2*(3*b*B - 5*A*c)*Log[x] + 12*c^2*(-3*b*B + 5*A*c)*Log[b + c*x^2]
)/(12*b^6)
```

Maple [A]
time = 0.39, size = 143, normalized size = 0.97

method	result
default	$\frac{c^3\left(\frac{(10Ac-6Bb)\ln\left(c\,x^2+b\right)}{c}-\frac{b^2(Ac-Bb)}{2c\left(c\,x^2+b\right)^2}-\frac{b(4Ac-3Bb)}{c\left(c\,x^2+b\right)}\right)}{2b^6}-\frac{A}{6b^3x^6}-\frac{-3Ac+Bb}{4b^4x^4}-\frac{3c(2Ac-Bb)}{2b^5x^2}-\frac{2c^2(5Ac-3Bb)\ln(x)}{b^6}$
norman	$\frac{-\frac{A}{6b}+\frac{(5Ac-3Bb)x^2}{12b^2}-\frac{c(5Ac-3Bb)x^4}{3b^3}+\frac{2c\left(5A\,c^3-3Bb\,c^2\right)x^8}{b^5}+\frac{c^2\left(15A\,c^3-9Bb\,c^2\right)x^{10}}{2b^6}}{x^6(c\,x^2+b)^2}+\frac{c^2(5Ac-3Bb)\ln\left(c\,x^2+b\right)}{b^6}-\frac{2c^2(5Ac-3Bb)\ln}{b^6}$
risch	$\frac{-\frac{c^3(5Ac-3Bb)x^8}{b^5}-\frac{3c^2(5Ac-3Bb)x^6}{2b^4}-\frac{c(5Ac-3Bb)x^4}{3b^3}+\frac{(5Ac-3Bb)x^2}{12b^2}-\frac{A}{6b}}{x^6(c\,x^2+b)^2}-\frac{10c^3\ln(x)A}{b^6}+\frac{6c^2\ln(x)B}{b^5}+\frac{5c^3\ln\left(-c\,x^2-b\right)A}{b^6}-\frac{3c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^6*c^3*((10*A*c-6*B*b)/c*ln(c*x^2+b)-1/2*b^2*(A*c-B*b)/c/(c*x^2+b)^2-b
*(4*A*c-3*B*b)/c/(c*x^2+b))-1/6*A/b^3/x^6-1/4*(-3*A*c+B*b)/b^4/x^4-3/2*c*(2
*A*c-B*b)/b^5/x^2-2*c^2*(5*A*c-3*B*b)/b^6*ln(x)
```

Maxima [A]
time = 0.29, size = 170, normalized size = 1.15

$$\frac{12\,(3\,Bbc^{3}-5\,Ac^{4})x^{8}+18\,(3\,Bb^{2}c^{2}-5\,Abc^{3})x^{6}-2\,Ab^{4}+4\,(3\,Bb^{3}c-5\,Ab^{2}c^{2})x^{4}-(3\,Bb^{4}-5\,Ab^{3}c)x^{2}}{12\,(b^{5}c^{2}x^{10}+2\,b^{6}cx^{8}+b^{7}x^{6})}-\frac{(3\,Bbc^{2}-5\,Ac^{3})\log\left(cx^{2}+b\right)}{b^{6}}+\frac{(3\,Bbc^{2}-5\,Ac^{3})\log\left(x^{2}\right)}{b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(3*B*b*c^3 - 5*A*c^4)*x^8 + 18*(3*B*b^2*c^2 - 5*A*b*c^3)*x^6 - 2*A
*b^4 + 4*(3*B*b^3*c - 5*A*b^2*c^2)*x^4 - (3*B*b^4 - 5*A*b^3*c)*x^2)/(b^5*c^
2*x^10 + 2*b^6*c*x^8 + b^7*x^6) - (3*B*b*c^2 - 5*A*c^3)*log(c*x^2 + b)/b^6
+ (3*B*b*c^2 - 5*A*c^3)*log(x^2)/b^6
```

Fricas [A]
time = 1.38, size = 267, normalized size = 1.80

$$\frac{12\,(3\,Bb^{2}c^{3}-5\,Abc^{4})x^{8}+18\,(3\,Bb^{3}c^{2}-5\,Ab^{2}c^{3})x^{6}-2\,Ab^{5}+4\,(3\,Bb^{4}c-5\,Ab^{3}c^{2})x^{4}-(3\,Bb^{5}-5\,Ab^{4}c)x^{2}-12\,((3\,Bbc^{4}-5\,Ac^{5})x^{10}+2\,(3\,Bb^{2}c^{3}-5\,Abc^{4})x^{8}+(3\,Bb^{3}c^{2}-5\,Ab^{2}c^{3})x^{6})\log\left(cx^{2}+b\right)+24\,((3\,Bbc^{4}-5\,Ac^{5})x^{10}+2\,(3\,Bb^{2}c^{3}-5\,Abc^{4})x^{8}+(3\,Bb^{3}c^{2}-5\,Ab^{2}c^{3})x^{6})\log\left(x\right)}{12\,(b^{6}c^{2}x^{10}+2\,b^{7}cx^{8}+b^{8}x^{6})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*(3*B*b^2*c^3 - 5*A*b*c^4)*x^8 + 18*(3*B*b^3*c^2 - 5*A*b^2*c^3)*x^6
 - 2*A*b^5 + 4*(3*B*b^4*c - 5*A*b^3*c^2)*x^4 - (3*B*b^5 - 5*A*b^4*c)*x^2 -
12*((3*B*b*c^4 - 5*A*c^5)*x^10 + 2*(3*B*b^2*c^3 - 5*A*b*c^4)*x^8 + (3*B*b^3
*c^2 - 5*A*b^2*c^3)*x^6)*log(c*x^2 + b) + 24*((3*B*b*c^4 - 5*A*c^5)*x^10 +
2*(3*B*b^2*c^3 - 5*A*b*c^4)*x^8 + (3*B*b^3*c^2 - 5*A*b^2*c^3)*x^6)*log(x))/
(b^6*c^2*x^10 + 2*b^7*c*x^8 + b^8*x^6)
```

Sympy [A]
time = 0.70, size = 165, normalized size = 1.11

$$\frac{-2Ab^{4}+x^{8}(-60Ac^{4}+36Bbc^{3})+x^{6}(-90Abc^{3}+54Bb^{2}c^{2})+x^{4}(-20Ab^{2}c^{2}+12Bb^{3}c)+x^{2}\cdot(5Ab^{3}c-3Bb^{4})}{12b^{7}x^{6}+24b^{6}cx^{8}+12b^{5}c^{2}x^{10}}+\frac{2c^{2}(-5Ac+3Bb)\log\left(x\right)}{b^{6}}-\frac{c^{2}(-5Ac+3Bb)\log\left(\frac{b}{c}+x^{2}\right)}{b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-2*A*b**4 + x**8*(-60*A*c**4 + 36*B*b*c**3) + x**6*(-90*A*b*c**3 + 54*B*b*
*2*c**2) + x**4*(-20*A*b**2*c**2 + 12*B*b**3*c) + x**2*(5*A*b**3*c - 3*B*b*
*4))/(12*b**7*x**6 + 24*b**6*c*x**8 + 12*b**5*c**2*x**10) + 2*c**2*(-5*A*c
+ 3*B*b)*log(x)/b**6 - c**2*(-5*A*c + 3*B*b)*log(b/c + x**2)/b**6
```

Giac [A]
time = 0.84, size = 201, normalized size = 1.36

$$\frac{(3\,Bbc^2-5\,Ac^3)\log\left(x^2\right)}{b^6}-\frac{(3\,Bbc^3-5\,Ac^4)\log\left(|cx^2+b|\right)}{b^6c}+\frac{18\,Bbc^4x^4-30\,Ac^5x^4+42\,Bb^2c^3x^2-68\,Abc^4x^2+25\,Bb^3c^2-39\,Ab^2c^3}{4\,(cx^2+b)^2b^6}-\frac{66\,Bbc^2x^6-110\,Ac^3x^6-18\,Bb^2cx^4+36\,Abc^2x^4+3\,Bb^3x^2-9\,Ab^2cx^2+2\,Ab^3}{12\,b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] (3*B*b*c^2 - 5*A*c^3)*log(x^2)/b^6 - (3*B*b*c^3 - 5*A*c^4)*log(abs(c*x^2 +
b))/(b^6*c) + 1/4*(18*B*b*c^4*x^4 - 30*A*c^5*x^4 + 42*B*b^2*c^3*x^2 - 68*A*
b*c^4*x^2 + 25*B*b^3*c^2 - 39*A*b^2*c^3)/((c*x^2 + b)^2*b^6) - 1/12*(66*B*b
*c^2*x^6 - 110*A*c^3*x^6 - 18*B*b^2*c*x^4 + 36*A*b*c^2*x^4 + 3*B*b^3*x^2 -
9*A*b^2*c*x^2 + 2*A*b^3)/(b^6*x^6)
```

Mupad [B]
time = 0.18, size = 155, normalized size = 1.05

$$\frac{\ln\left(c\,x^2+b\right)\,\left(5\,A\,c^3-3\,B\,b\,c^2\right)}{b^6}-\frac{\frac{A}{6\,b}-\frac{x^2\,(5\,A\,c-3\,B\,b)}{12\,b^2}+\frac{3\,c^2\,x^6\,(5\,A\,c-3\,B\,b)}{2\,b^4}+\frac{c^3\,x^8\,(5\,A\,c-3\,B\,b)}{b^5}+\frac{c\,x^4\,(5\,A\,c-3\,B\,b)}{3\,b^3}}{b^2\,x^6+2\,b\,c\,x^8+c^2\,x^{10}}-\frac{\ln\left(x\right)\,\left(10\,A\,c^3-6\,B\,b\,c^2\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(b*x^2 + c*x^4)^3),x)
```

```
[Out] (log(b + c*x^2)*(5*A*c^3 - 3*B*b*c^2))/b^6 - (A/(6*b) - (x^2*(5*A*c - 3*B*b
))/(12*b^2) + (3*c^2*x^6*(5*A*c - 3*B*b))/(2*b^4) + (c^3*x^8*(5*A*c - 3*B*b
))/b^5 + (c*x^4*(5*A*c - 3*B*b))/(3*b^3))/(b^2*x^6 + c^2*x^10 + 2*b*c*x^8)
- (log(x)*(10*A*c^3 - 6*B*b*c^2))/b^6
```

3.89 $\int x^7(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=218

$$\frac{7b^3(3bB-4Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^5}-\frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4}+\frac{7b(3bB-4Ac)x^2\left(bx^2+cx^4\right)^{3/2}}{320c^3}-$$

```
[Out] -7/384*b^2*(-4*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/c^4+7/320*b*(-4*A*c+3*B*b)*x^
2*(c*x^4+b*x^2)^(3/2)/c^3-1/40*(-4*A*c+3*B*b)*x^4*(c*x^4+b*x^2)^(3/2)/c^2+1
/12*B*x^6*(c*x^4+b*x^2)^(3/2)/c-7/1024*b^5*(-4*A*c+3*B*b)*arctanh(x^2*c^(1/
2)/(c*x^4+b*x^2)^(1/2))/c^(11/2)+7/1024*b^3*(-4*A*c+3*B*b)*(2*c*x^2+b)*(c*x
^4+b*x^2)^(1/2)/c^5
```

Rubi [A]
time = 0.25, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2059, 808, 684, 654, 626, 634, 212}

$$-\frac{7b^5(3bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{11/2}}+\frac{7b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}\,(3bB-4Ac)}{1024c^5}-\frac{7b^2\left(bx^2+cx^4\right)^{3/2}(3bB-4Ac)}{384c^4}+\frac{7bx^2\left(bx^2+cx^4\right)^{3/2}(3bB-4Ac)}{320c^3}-\frac{x^4\left(bx^2+cx^4\right)^{3/2}(3bB-4Ac)}{40c^2}+\frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (7*b^3*(3*b*B - 4*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(1024*c^5) - (7*b
^2*(3*b*B - 4*A*c)*(b*x^2 + c*x^4)^(3/2))/(384*c^4) + (7*b*(3*b*B - 4*A*c)*
x^2*(b*x^2 + c*x^4)^(3/2))/(320*c^3) - ((3*b*B - 4*A*c)*x^4*(b*x^2 + c*x^4)
^(3/2))/(40*c^2) + (B*x^6*(b*x^2 + c*x^4)^(3/2))/(12*c) - (7*b^5*(3*b*B - 4
*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(1024*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x^7 \left(A+Bx^2\right) \sqrt{bx^2+cx^4}\, dx = \frac{1}{2}\text{Subst}\left(\int x^3(A+Bx)\sqrt{bx+cx^2}\, dx, x, x^2\right)$$

$$= \frac{Bx^6(bx^2+cx^4)^{3/2}}{12c} + \frac{\left(3(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int x^3\sqrt{bx+\right.}{12c}$$

$$= -\frac{(3bB-4Ac)x^4(bx^2+cx^4)^{3/2}}{40c^2} + \frac{Bx^6(bx^2+cx^4)^{3/2}}{12c} + \frac{(7b(3bB-4Ac))\text{S}}{}$$

$$= \frac{7b(3bB-4Ac)x^2(bx^2+cx^4)^{3/2}}{320c^3} - \frac{(3bB-4Ac)x^4(bx^2+cx^4)^{3/2}}{40c^2} + \frac{Bx^6(b}{}$$

$$= -\frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4} + \frac{7b(3bB-4Ac)x^2(bx^2+cx^4)^{3/2}}{320c^3} - \frac{(3b}{}$$

$$= \frac{7b^3(3bB-4Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^5} - \frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4}$$

$$= \frac{7b^3(3bB-4Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^5} - \frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4}$$

$$= \frac{7b^3(3bB-4Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^5} - \frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4}$$

Mathematica [A]
time = 0.29, size = 187, normalized size = 0.86

$$\frac{x\left(\sqrt{c}\,x(b+cx^2)\left(315b^5B-210b^4c(2A+Bx^2)+64bc^4x^6(3A+2Bx^2)+56b^3c^2x^2(5A+3Bx^2)+256c^5x^8(6A+5Bx^2)-16b^2c^3x^4(14A+9Bx^2)\right)+105b^5(3bB-4Ac)\sqrt{b+cx^2}\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{15360c^{11/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(315*b^5*B - 210*b^4*c*(2*A + B*x^2) + 64*b*c^4*x
^6*(3*A + 2*B*x^2) + 56*b^3*c^2*x^2*(5*A + 3*B*x^2) + 256*c^5*x^8*(6*A + 5*
B*x^2) - 16*b^2*c^3*x^4*(14*A + 9*B*x^2)) + 105*b^5*(3*b*B - 4*A*c)*Sqrt[b
+ c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(15360*c^(11/2)*Sqrt[x^2*(b
+ c*x^2)])
```

Maple [A]
time = 0.41, size = 290, normalized size = 1.33

method	result

risch	$-\frac{(-1280Bc^5x^{10}-1536Ac^5x^8-128Bbc^4x^8-192Abc^4x^6+144Bb^2c^3x^6+224Ab^2c^3x^4-168Bb^3c^2x^4-280Ab^3c^2x^2+210Bb^4cx^2+4}{15360c^5}$
default	$\frac{\sqrt{x^4c+b\,x^2}\,\left(1280B(c\,x^2+b)^{\frac{3}{2}}c^{\frac{9}{2}}x^9+1536A(c\,x^2+b)^{\frac{3}{2}}c^{\frac{9}{2}}x^7-1152B(c\,x^2+b)^{\frac{3}{2}}c^{\frac{7}{2}}b\,x^7-1344A(c\,x^2+b)^{\frac{3}{2}}c^{\frac{7}{2}}b\,x^5+1008B(c\,x^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15360*(c*x^4+b*x^2)^(1/2)*(1280*B*(c*x^2+b)^(3/2)*c^(9/2)*x^9+1536*A*(c*x
^2+b)^(3/2)*c^(9/2)*x^7-1152*B*(c*x^2+b)^(3/2)*c^(7/2)*b*x^7-1344*A*(c*x^2+
b)^(3/2)*c^(7/2)*b*x^5+1008*B*(c*x^2+b)^(3/2)*c^(5/2)*b^2*x^5+1120*A*(c*x^2
+b)^(3/2)*c^(5/2)*b^2*x^3-840*B*(c*x^2+b)^(3/2)*c^(3/2)*b^3*x^3-840*A*(c*x^
2+b)^(3/2)*c^(3/2)*b^3*x+420*A*(c*x^2+b)^(1/2)*c^(3/2)*b^4*x+630*B*(c*x^2+b
)^(3/2)*c^(1/2)*b^4*x-315*B*(c*x^2+b)^(1/2)*c^(1/2)*b^5*x+420*A*ln(c^(1/2)*
x+(c*x^2+b)^(1/2))*b^5*c-315*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^6)/x/(c*x^2+
b)^(1/2)/c^(11/2)
```

<u>Maxima</u> [**A**]
time = 0.30, size = 321, normalized size = 1.47

$$\frac{1}{7680}\left(\frac{768\,(cx^4+bx^2)^{\frac{3}{2}}x^4}{c}-\frac{420\,\sqrt{cx^4+bx^2}\,b^3x^2}{c^3}-\frac{672\,(cx^4+bx^2)^{\frac{3}{2}}bx^2}{c^2}+\frac{105\,b^5\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{9}{2}}}-\frac{210\,\sqrt{cx^4+bx^2}\,b^4}{c^4}+\frac{560\,(cx^4+bx^2)^{\frac{3}{2}}b^2}{c^3}\right)A+\frac{1}{30720}\left(\frac{2560\,(cx^4+bx^2)^{\frac{3}{2}}x^6}{c}-\frac{2304\,(cx^4+bx^2)^{\frac{3}{2}}bx^4}{c^2}+\frac{1260\,\sqrt{cx^4+bx^2}\,b^4x^2}{c^4}+\frac{2016\,(cx^4+bx^2)^{\frac{3}{2}}b^2x^2}{c^3}-\frac{315\,b^6\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{11}{2}}}+\frac{630\,\sqrt{cx^4+bx^2}\,b^5}{c^5}-\frac{1680\,(cx^4+bx^2)^{\frac{3}{2}}b^3}{c^4}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7680*(768*(c*x^4 + b*x^2)^(3/2)*x^4/c - 420*sqrt(c*x^4 + b*x^2)*b^3*x^2/c
^3 - 672*(c*x^4 + b*x^2)^(3/2)*b*x^2/c^2 + 105*b^5*log(2*c*x^2 + b + 2*sqrt
(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) - 210*sqrt(c*x^4 + b*x^2)*b^4/c^4 + 560*(c
*x^4 + b*x^2)^(3/2)*b^2/c^3)*A + 1/30720*(2560*(c*x^4 + b*x^2)^(3/2)*x^6/c
- 2304*(c*x^4 + b*x^2)^(3/2)*b*x^4/c^2 + 1260*sqrt(c*x^4 + b*x^2)*b^4*x^2/c
^4 + 2016*(c*x^4 + b*x^2)^(3/2)*b^2*x^2/c^3 - 315*b^6*log(2*c*x^2 + b + 2*s
qrt(c*x^4 + b*x^2)*sqrt(c))/c^(11/2) + 630*sqrt(c*x^4 + b*x^2)*b^5/c^5 - 16
80*(c*x^4 + b*x^2)^(3/2)*b^3/c^4)*B
```

<u>Fricas</u> [**A**]
time = 1.92, size = 368, normalized size = 1.69

$$\left[-\frac{105\,(3\,Bb^6-4\,Ab^5c)\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\,(1280\,Bc^6x^{10}+128\,(Bbc^5+12\,Ac^6)x^8+315\,Bb^5c-420\,Ab^4c^2-48\,(3\,Bb^2c^4-4\,Abc^5)x^6+56\,(3\,Bb^3c^3-4\,Ab^2c^4)x^4-70\,(3\,Bb^4c^2-4\,Ab^3c^3)x^2)\sqrt{cx^4+bx^2}}{30720\,c^6},\ \frac{105\,(3\,Bb^6-4\,Ab^5c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+(1280\,Bc^6x^{10}+128\,(Bbc^5+12\,Ac^6)x^8+315\,Bb^5c-420\,Ab^4c^2-48\,(3\,Bb^2c^4-4\,Abc^5)x^6+56\,(3\,Bb^3c^3-4\,Ab^2c^4)x^4-70\,(3\,Bb^4c^2-4\,Ab^3c^3)x^2)\sqrt{cx^4+bx^2}}{15360\,c^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/30720*(105*(3*B*b^6 - 4*A*b^5*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^
4 + b*x^2)*sqrt(c)) - 2*(1280*B*c^6*x^10 + 128*(B*b*c^5 + 12*A*c^6)*x^8 + 3
15*B*b^5*c - 420*A*b^4*c^2 - 48*(3*B*b^2*c^4 - 4*A*b*c^5)*x^6 + 56*(3*B*b^3
*c^3 - 4*A*b^2*c^4)*x^4 - 70*(3*B*b^4*c^2 - 4*A*b^3*c^3)*x^2)*sqrt(c*x^4 +
b*x^2))/c^6, 1/15360*(105*(3*B*b^6 - 4*A*b^5*c)*sqrt(-c)*arctan(sqrt(c*x^4
+ b*x^2)*sqrt(-c)/(c*x^2 + b)) + (1280*B*c^6*x^10 + 128*(B*b*c^5 + 12*A*c^6
)*x^8 + 315*B*b^5*c - 420*A*b^4*c^2 - 48*(3*B*b^2*c^4 - 4*A*b*c^5)*x^6 + 56
*(3*B*b^3*c^3 - 4*A*b^2*c^4)*x^4 - 70*(3*B*b^4*c^2 - 4*A*b^3*c^3)*x^2)*sqrt
(c*x^4 + b*x^2))/c^6]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \sqrt{x^2 \left(b + cx^2\right)} \left(A + Bx^2\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**7*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [A]
time = 1.04, size = 245, normalized size = 1.12

$$\frac{1}{15360} \left(2 \left(4 \left(2 \left(8 \left(10 \, Bx^2 \mathrm{sgn}(x) + \frac{Bbc^9 \mathrm{sgn}(x) + 12 \, Ac^{10} \mathrm{sgn}(x)}{c^{10}}\right) x^2 - \frac{3 \left(3 \, Bb^2 c^8 \mathrm{sgn}(x) - 4 \, Abc^9 \mathrm{sgn}(x)\right)}{c^{10}}\right) x^2 + \frac{7 \left(3 \, Bb^3 c^7 \mathrm{sgn}(x) - 4 \, Ab^2 c^8 \mathrm{sgn}(x)\right)}{c^{10}}\right) x^2 - \frac{35 \left(3 \, Bb^4 c^6 \mathrm{sgn}(x) - 4 \, Ab^3 c^7 \mathrm{sgn}(x)\right)}{c^{10}}\right) x^2 + \frac{105 \left(3 \, Bb^5 c^5 \mathrm{sgn}(x) - 4 \, Ab^4 c^6 \mathrm{sgn}(x)\right)}{c^{10}}\right) \sqrt{cx^2 + b} \, x + \frac{7 \left(3 \, Bb^6 \mathrm{sgn}(x) - 4 \, Ab^5 c \mathrm{sgn}(x)\right) \log\left(\left|-\sqrt{c} \, x + \sqrt{cx^2 + b}\right|\right)}{1024 \, c^{\frac{11}{2}}} - \frac{7 \left(3 \, Bb^6 \log\left(|b|\right) - 4 \, Ab^5 c \log\left(|b|\right)\right) \mathrm{sgn}(x)}{2048 \, c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*B*x^2*sgn(x) + (B*b*c^9*sgn(x) + 12*A*c^10*sgn(x))/
c^10)*x^2 - 3*(3*B*b^2*c^8*sgn(x) - 4*A*b*c^9*sgn(x))/c^10)*x^2 + 7*(3*B*b^
3*c^7*sgn(x) - 4*A*b^2*c^8*sgn(x))/c^10)*x^2 - 35*(3*B*b^4*c^6*sgn(x) - 4*A
*b^3*c^7*sgn(x))/c^10)*x^2 + 105*(3*B*b^5*c^5*sgn(x) - 4*A*b^4*c^6*sgn(x))/
c^10)*sqrt(c*x^2 + b)*x + 7/1024*(3*B*b^6*sgn(x) - 4*A*b^5*c*sgn(x))*log(ab
s(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(11/2) - 7/2048*(3*B*b^6*log(abs(b)) - 4
*A*b^5*c*log(abs(b)))*sgn(x)/c^(11/2)
```

Mupad [B]
time = 1.47, size = 289, normalized size = 1.33

$$\frac{A\,x^4\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{10\,c}+\frac{B\,x^6\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{12\,c}-\frac{3\,B\,b\,\left(\frac{7\,b\,\left(\frac{5\,b\,\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,\left|x\right|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{8\,c}-\frac{x^2\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{4\,c}\right)}{10\,c}+\frac{x^4\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{5\,c}\right)}{8\,c}+\frac{7\,A\,b\,\left(\frac{5\,b\,\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,\left|x\right|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{8\,c}-\frac{x^2\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{4\,c}\right)}{20\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (A*x^4*(b*x^2 + c*x^4)^(3/2))/(10*c) + (B*x^6*(b*x^2 + c*x^4)^(3/2))/(12*c)
 - (3*B*b*((7*b*((5*b*((b^3*log(b + 2*c*x^2 + 2*c^(1/2)*abs(x)*(b + c*x^2)^
(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - 3*b^2 + 2*b*c*x^
2))/(24*c^2)))/(8*c) - (x^2*(b*x^2 + c*x^4)^(3/2))/(4*c)))/(10*c) + (x^4*(b
*x^2 + c*x^4)^(3/2))/(5*c)))/(8*c) + (7*A*b*((5*b*((b^3*log(b + 2*c*x^2 + 2
*c^(1/2)*abs(x)*(b + c*x^2)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(
8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(8*c) - (x^2*(b*x^2 + c*x^4)^(3/
2))/(4*c)))/(20*c)
```

3.90 $\int x^5(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=181

$$-\frac{b^2(7bB-10Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^4}+\frac{b(7bB-10Ac)\left(bx^2+cx^4\right)^{3/2}}{96c^3}-\frac{(7bB-10Ac)x^2\left(bx^2+cx^4\right)^{3/2}}{80c^2}+$$

```
[Out] 1/96*b*(-10*A*c+7*B*b)*(c*x^4+b*x^2)^(3/2)/c^3-1/80*(-10*A*c+7*B*b)*x^2*(c*
x^4+b*x^2)^(3/2)/c^2+1/10*B*x^4*(c*x^4+b*x^2)^(3/2)/c+1/256*b^4*(-10*A*c+7*
B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(9/2)-1/256*b^2*(-10*A*c+7*
B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^4
```

Rubi [A]
time = 0.22, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2059, 808, 684, 654, 626, 634, 212}

$$\frac{b^4(7bB-10Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{9/2}}-\frac{b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}\,(7bB-10Ac)}{256c^4}+\frac{b\left(bx^2+cx^4\right)^{3/2}(7bB-10Ac)}{96c^3}-\frac{x^2\left(bx^2+cx^4\right)^{3/2}(7bB-10Ac)}{80c^2}+\frac{Bx^4\left(bx^2+cx^4\right)^{3/2}}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -1/256*(b^2*(7*b*B - 10*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/c^4 + (b*(7
*b*B - 10*A*c)*(b*x^2 + c*x^4)^(3/2))/(96*c^3) - ((7*b*B - 10*A*c)*x^2*(b*x
^2 + c*x^4)^(3/2))/(80*c^2) + (B*x^4*(b*x^2 + c*x^4)^(3/2))/(10*c) + (b^4*(
7*b*B - 10*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(256*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x^5 \left(A+Bx^2\right) \sqrt{bx^2+cx^4}\, dx = \frac{1}{2}\text{Subst}\left(\int x^2(A+Bx)\sqrt{bx+cx^2}\, dx, x, x^2\right)$$

$$= \frac{Bx^4(bx^2+cx^4)^{3/2}}{10c} + \frac{\left(2(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int x^2\sqrt{bx+\right.}{10c}$$

$$= -\frac{(7bB-10Ac)x^2(bx^2+cx^4)^{3/2}}{80c^2} + \frac{Bx^4(bx^2+cx^4)^{3/2}}{10c} + \frac{(b(7bB-10Ac)}{}$$

$$= \frac{b(7bB-10Ac)\left(bx^2+cx^4\right)^{3/2}}{96c^3} - \frac{(7bB-10Ac)x^2(bx^2+cx^4)^{3/2}}{80c^2} + \frac{Bx^4(bx}{}$$

$$= -\frac{b^2(7bB-10Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^4} + \frac{b(7bB-10Ac)\left(bx^2+cx^4\right)^{3/}}{96c^3}$$

$$= -\frac{b^2(7bB-10Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^4} + \frac{b(7bB-10Ac)\left(bx^2+cx^4\right)^{3/}}{96c^3}$$

$$= -\frac{b^2(7bB-10Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^4} + \frac{b(7bB-10Ac)\left(bx^2+cx^4\right)^{3/}}{96c^3}$$

Mathematica [A]
time = 0.25, size = 168, normalized size = 0.93

$$\frac{x\left(-\sqrt{c}\,x(b+cx^2)\left(105b^4B-16bc^3x^4(5A+3Bx^2)-96c^4x^6(5A+4Bx^2)-10b^3c(15A+7Bx^2)+4b^2c^2x^2(25A+14Bx^2)\right)-15b^4(7bB-10Ac)\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{3840c^{9/2}\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(-(Sqrt[c]*x*(b + c*x^2)*(105*b^4*B - 16*b*c^3*x^4*(5*A + 3*B*x^2) - 96*
c^4*x^6*(5*A + 4*B*x^2) - 10*b^3*c*(15*A + 7*B*x^2) + 4*b^2*c^2*x^2*(25*A +
 14*B*x^2))) - 15*b^4*(7*b*B - 10*A*c)*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + S
qrt[b + c*x^2]]))/(3840*c^(9/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.38, size = 248, normalized size = 1.37

method	result
risch	$\frac{\left(384B\,c^4x^8+480A\,c^4x^6+48Bb\,c^3x^6+80Ab\,c^3x^4-56B\,b^2c^2x^4-100A\,b^2c^2x^2+70B\,b^3c\,x^2+150A\,b^3c-105B\,b^4\right)\sqrt{x^2\left(c\,x^2+b\right)}}{3840c^4}+$

default	$\frac{\sqrt{x^4c+b\,x^2}\,\left(384B(c\,x^2+b)^{\frac{3}{2}}c^{\frac{7}{2}}x^7+480A(c\,x^2+b)^{\frac{3}{2}}c^{\frac{7}{2}}x^5-336B(c\,x^2+b)^{\frac{3}{2}}c^{\frac{5}{2}}b\,x^5-400A(c\,x^2+b)^{\frac{3}{2}}c^{\frac{5}{2}}b\,x^3+280B(c\,x^2+b)^{\frac{3}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3840*(c*x^4+b*x^2)^(1/2)*(384*B*(c*x^2+b)^(3/2)*c^(7/2)*x^7+480*A*(c*x^2+
b)^(3/2)*c^(7/2)*x^5-336*B*(c*x^2+b)^(3/2)*c^(5/2)*b*x^5-400*A*(c*x^2+b)^(3
/2)*c^(5/2)*b*x^3+280*B*(c*x^2+b)^(3/2)*c^(3/2)*b^2*x^3+300*A*(c*x^2+b)^(3/
2)*c^(3/2)*b^2*x-210*B*(c*x^2+b)^(3/2)*c^(1/2)*b^3*x-150*A*(c*x^2+b)^(1/2)*
c^(3/2)*b^3*x+105*B*(c*x^2+b)^(1/2)*c^(1/2)*b^4*x-150*A*ln(c^(1/2)*x+(c*x^2
+b)^(1/2))*b^4*c+105*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^5)/x/(c*x^2+b)^(1/2)
/c^(9/2)
```

Maxima [A]
time = 0.28, size = 273, normalized size = 1.51

$$\frac{1}{768}\left(\frac{60\sqrt{cx^4+bx^2}\,b^2x^2}{c^2}+\frac{96\,(cx^4+bx^2)^{\frac{3}{2}}x^2}{c}-\frac{15\,b^4\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{7}{2}}}+\frac{30\,\sqrt{cx^4+bx^2}\,b^3}{c^3}-\frac{80\,(cx^4+bx^2)^{\frac{3}{2}}b}{c^2}\right)A+\frac{1}{7680}\left(\frac{768\,(cx^4+bx^2)^{\frac{3}{2}}x^4}{c}-\frac{420\,\sqrt{cx^4+bx^2}\,b^3x^2}{c^3}-\frac{672\,(cx^4+bx^2)^{\frac{3}{2}}bx^2}{c^2}+\frac{105\,b^5\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{9}{2}}}-\frac{210\,\sqrt{cx^4+bx^2}\,b^4}{c^4}+\frac{560\,(cx^4+bx^2)^{\frac{3}{2}}b^2}{c^3}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/768*(60*sqrt(c*x^4 + b*x^2)*b^2*x^2/c^2 + 96*(c*x^4 + b*x^2)^(3/2)*x^2/c
- 15*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*sqrt
(c*x^4 + b*x^2)*b^3/c^3 - 80*(c*x^4 + b*x^2)^(3/2)*b/c^2)*A + 1/7680*(768*(
c*x^4 + b*x^2)^(3/2)*x^4/c - 420*sqrt(c*x^4 + b*x^2)*b^3*x^2/c^3 - 672*(c*x
^4 + b*x^2)^(3/2)*b*x^2/c^2 + 105*b^5*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^
2)*sqrt(c))/c^(9/2) - 210*sqrt(c*x^4 + b*x^2)*b^4/c^4 + 560*(c*x^4 + b*x^2)
^(3/2)*b^2/c^3)*B
```

Fricas [A]
time = 2.72, size = 321, normalized size = 1.77

$$\left[-\frac{15\,(7\,Bb^5-10\,Ab^4c)\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\,(384\,Bc^5x^8+48\,(Bbc^4+10\,Ac^5)x^6-105\,Bb^4c+150\,Ab^3c^2-8\,(7\,Bb^2c^3-10\,Abc^4)x^4+10\,(7\,Bb^3c^2-10\,Ab^2c^3)x^2)\sqrt{cx^4+bx^2}}{7680\,c^5},\ -\frac{15\,(7\,Bb^5-10\,Ab^4c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)-(384\,Bc^5x^8+48\,(Bbc^4+10\,Ac^5)x^6-105\,Bb^4c+150\,Ab^3c^2-8\,(7\,Bb^2c^3-10\,Abc^4)x^4+10\,(7\,Bb^3c^2-10\,Ab^2c^3)x^2)\sqrt{cx^4+bx^2}}{3840\,c^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(7*B*b^5 - 10*A*b^4*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4
 + b*x^2)*sqrt(c)) - 2*(384*B*c^5*x^8 + 48*(B*b*c^4 + 10*A*c^5)*x^6 - 105*B
*b^4*c + 150*A*b^3*c^2 - 8*(7*B*b^2*c^3 - 10*A*b*c^4)*x^4 + 10*(7*B*b^3*c^2
 - 10*A*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^5, -1/3840*(15*(7*B*b^5 - 10*A
*b^4*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (384*B*
c^5*x^8 + 48*(B*b*c^4 + 10*A*c^5)*x^6 - 105*B*b^4*c + 150*A*b^3*c^2 - 8*(7*
```

```
B*b^2*c^3 - 10*A*b*c^4)*x^4 + 10*(7*B*b^3*c^2 - 10*A*b^2*c^3)*x^2)*sqrt(c*x
^4 + b*x^2))/c^5]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [A]
time = 0.77, size = 211, normalized size = 1.17

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,Bx^2\mathrm{sgn}(x)+\frac{Bbc^7\mathrm{sgn}(x)+10\,Ac^8\mathrm{sgn}(x)}{c^8}\right)x^2-\frac{7\,Bb^2c^6\mathrm{sgn}(x)-10\,Abc^7\mathrm{sgn}(x)}{c^8}\right)x^2+\frac{5\left(7\,Bb^3c^5\mathrm{sgn}(x)-10\,Ab^2c^6\mathrm{sgn}(x)\right)}{c^8}\right)x^2-\frac{15\left(7\,Bb^4c^4\mathrm{sgn}(x)-10\,Ab^3c^5\mathrm{sgn}(x)\right)}{c^8}\right)\sqrt{cx^2+b}\,x-\frac{\left(7\,Bb^5\mathrm{sgn}(x)-10\,Ab^4c\mathrm{sgn}(x)\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{256\,c^{\frac{9}{2}}}+\frac{\left(7\,Bb^5\log\left(|b|\right)-10\,Ab^4c\log\left(|b|\right)\right)\mathrm{sgn}(x)}{512\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*B*x^2*sgn(x) + (B*b*c^7*sgn(x) + 10*A*c^8*sgn(x))/c^8)*x
^2 - (7*B*b^2*c^6*sgn(x) - 10*A*b*c^7*sgn(x))/c^8)*x^2 + 5*(7*B*b^3*c^5*sgn
(x) - 10*A*b^2*c^6*sgn(x))/c^8)*x^2 - 15*(7*B*b^4*c^4*sgn(x) - 10*A*b^3*c^5
*sgn(x))/c^8)*sqrt(c*x^2 + b)*x - 1/256*(7*B*b^5*sgn(x) - 10*A*b^4*c*sgn(x)
)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(9/2) + 1/512*(7*B*b^5*log(abs(b
)) - 10*A*b^4*c*log(abs(b)))*sgn(x)/c^(9/2)
```

Mupad [B]
time = 0.89, size = 233, normalized size = 1.29

$$\frac{A\,x^2\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{8\,c}-\frac{5\,A\,b\,\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,\left|x\right|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{16\,c}+\frac{B\,x^4\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{10\,c}+\frac{7\,B\,b\,\left(\frac{5\,b\,\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,\left|x\right|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{8\,c}-\frac{x^2\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{4\,c}\right)}{20\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (A*x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*A*b*((b^3*log(b + 2*c*x^2 + 2*c^(1
/2)*abs(x)*(b + c*x^2)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2
*x^4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(16*c) + (B*x^4*(b*x^2 + c*x^4)^(3/2)
)/(10*c) + (7*B*b*((5*b*((b^3*log(b + 2*c*x^2 + 2*c^(1/2)*abs(x)*(b + c*x^2
)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - 3*b^2 + 2*b*c*
x^2))/(24*c^2)))/(8*c) - (x^2*(b*x^2 + c*x^4)^(3/2))/(4*c)))/(20*c)
```

3.91 $\int x^3(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=125

$$\frac{b(5bB-8Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{\left(5bB-8Ac-6Bcx^2\right)\left(bx^2+cx^4\right)^{3/2}}{48c^2}-\frac{b^3(5bB-8Ac)\tanh^{-1}\left(\frac{\cdots}{\sqrt{bx\cdots}}\right.}{128c^{7/2}}$$

```
[Out] -1/48*(-6*B*c*x^2-8*A*c+5*B*b)*(c*x^4+b*x^2)^(3/2)/c^2-1/128*b^3*(-8*A*c+5*
B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)+1/128*b*(-8*A*c+5*B*b
)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^3
```

Rubi [A]

time = 0.13, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.192$, Rules used = {2059, 793, 626, 634, 212}

$$-\frac{b^3(5bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}+\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}\,(5bB-8Ac)}{128c^3}-\frac{\left(bx^2+cx^4\right)^{3/2}\left(-8Ac+5bB-6Bcx^2\right)}{48c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (b*(5*b*B - 8*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^3) - ((5*b*B -
 8*A*c - 6*B*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(48*c^2) - (b^3*(5*b*B - 8*A*c)*
ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x^3(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int x(A+Bx)\sqrt{bx+cx^2}\,dx,x,x^2\right)$$

$$= -\frac{(5bB-8Ac-6Bcx^2)(bx^2+cx^4)^{3/2}}{48c^2} + \frac{(b(5bB-8Ac))\text{Subst}\left(\int \sqrt{bx+\right.}{32c^2}$$

$$= \frac{b(5bB-8Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^3} - \frac{(5bB-8Ac-6Bcx^2)(bx^2+cx}{48c^2}$$

$$= \frac{b(5bB-8Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^3} - \frac{(5bB-8Ac-6Bcx^2)(bx^2+cx}{48c^2}$$

$$= \frac{b(5bB-8Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^3} - \frac{(5bB-8Ac-6Bcx^2)(bx^2+cx}{48c^2}$$

Mathematica [A]
time = 0.20, size = 145, normalized size = 1.16

$$\frac{x\left(\sqrt{c}\,x(b+cx^2)\left(15b^3B+8bc^2x^2(2A+Bx^2)+16c^3x^4(4A+3Bx^2)-2b^2c(12A+5Bx^2)\right)+3b^3(5bB-8Ac)\sqrt{b+cx^2}\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{384c^{7/2}\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(15*b^3*B + 8*b*c^2*x^2*(2*A + B*x^2) + 16*c^3*x^
4*(4*A + 3*B*x^2) - 2*b^2*c*(12*A + 5*B*x^2)) + 3*b^3*(5*b*B - 8*A*c)*Sqrt[
b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(384*c^(7/2)*Sqrt[x^2*(b +
 c*x^2)])
```

Maple [A]
time = 0.38, size = 206, normalized size = 1.65

method	result
risch	$-\frac{\left(-48B\,c^{3}x^{6}-64A\,c^{3}x^{4}-8Bb\,c^{2}x^{4}-16Ab\,c^{2}x^{2}+10B\,b^{2}c\,x^{2}+24A\,b^{2}c-15B\,b^{3}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{384c^{3}}+\left(\frac{b^{3}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}\ldots}\right.}{16c^{\frac{5}{2}}}\ldots\right.$
default	$\frac{\sqrt{x^{4}c+b\,x^{2}}\,\left(48B\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{5}{2}}x^{5}+64A\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{5}{2}}x^{3}-40B\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}b\,x^{3}-48A\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}bx+24A\sqrt{c\,x^{2}+b}\,c^{\frac{3}{2}}b\ldots\right.}{384x\sqrt{c\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*(c*x^4+b*x^2)^(1/2)*(48*B*(c*x^2+b)^(3/2)*c^(5/2)*x^5+64*A*(c*x^2+b)^
(3/2)*c^(5/2)*x^3-40*B*(c*x^2+b)^(3/2)*c^(3/2)*b*x^3-48*A*(c*x^2+b)^(3/2)*c
^(3/2)*b*x+24*A*(c*x^2+b)^(1/2)*c^(3/2)*b^2*x+30*B*(c*x^2+b)^(3/2)*c^(1/2)*
b^2*x-15*B*(c*x^2+b)^(1/2)*c^(1/2)*b^3*x+24*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2))
*b^3*c-15*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^4)/x/(c*x^2+b)^(1/2)/c^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(109) = 218.
time = 0.29, size = 225, normalized size = 1.80

$$-\frac{1}{96}\left(\frac{12\sqrt{cx^{4}+bx^{2}}\,bx^{2}}{c}-\frac{3\,b^{3}\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{c^{\frac{5}{2}}}+\frac{6\,\sqrt{cx^{4}+bx^{2}}\,b^{2}}{c^{2}}-\frac{16\,(cx^{4}+bx^{2})^{\frac{3}{2}}}{c}\right)A+\frac{1}{768}\left(\frac{60\,\sqrt{cx^{4}+bx^{2}}\,b^{2}x^{2}}{c^{2}}+\frac{96\,(cx^{4}+bx^{2})^{\frac{3}{2}}x^{2}}{c}-\frac{15\,b^{4}\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{c^{\frac{7}{2}}}+\frac{30\,\sqrt{cx^{4}+bx^{2}}\,b^{3}}{c^{3}}-\frac{80\,(cx^{4}+bx^{2})^{\frac{3}{2}}b}{c^{2}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(12*sqrt(c*x^4 + b*x^2)*b*x^2/c - 3*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^
4 + b*x^2)*sqrt(c))/c^(5/2) + 6*sqrt(c*x^4 + b*x^2)*b^2/c^2 - 16*(c*x^4 + b
*x^2)^(3/2)/c)*A + 1/768*(60*sqrt(c*x^4 + b*x^2)*b^2*x^2/c^2 + 96*(c*x^4 +
b*x^2)^(3/2)*x^2/c - 15*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)
)/c^(7/2) + 30*sqrt(c*x^4 + b*x^2)*b^3/c^3 - 80*(c*x^4 + b*x^2)^(3/2)*b/c^2
)*B
```

Fricas [A]
time = 1.82, size = 272, normalized size = 2.18

$$\left[-\frac{3\,(5\,Bb^{4}-8\,Ab^{3}c)\sqrt{c}\,\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)-2\,(48\,Bc^{4}x^{6}+15\,Bb^{3}c-24\,Ab^{2}c^{2}+8\,(Bbc^{3}+8\,Ac^{4})x^{4}-2\,(5\,Bb^{2}c^{2}-8\,Abc^{3})x^{2})\sqrt{cx^{4}+bx^{2}}}{768\,c^{4}},\ \frac{3\,(5\,Bb^{4}-8\,Ab^{3}c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)+(48\,Bc^{4}x^{6}+15\,Bb^{3}c-24\,Ab^{2}c^{2}+8\,(Bbc^{3}+8\,Ac^{4})x^{4}-2\,(5\,Bb^{2}c^{2}-8\,Abc^{3})x^{2})\sqrt{cx^{4}+bx^{2}}}{384\,c^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(5*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 +
b*x^2)*sqrt(c)) - 2*(48*B*c^4*x^6 + 15*B*b^3*c - 24*A*b^2*c^2 + 8*(B*b*c^3
+ 8*A*c^4)*x^4 - 2*(5*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4,
 1/384*(3*(5*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c
)/(c*x^2 + b)) + (48*B*c^4*x^6 + 15*B*b^3*c - 24*A*b^2*c^2 + 8*(B*b*c^3 + 8
*A*c^4)*x^4 - 2*(5*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{x^2\,(b+cx^2)}\,(A+Bx^2)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [A]
time = 1.18, size = 177, normalized size = 1.42

$$\frac{1}{384}\left(2\left(4\left(6\,Bx^2\mathrm{sgn}(x)+\frac{Bbc^5\mathrm{sgn}(x)+8\,Ac^6\mathrm{sgn}(x)}{c^6}\right)x^2-\frac{5\,Bb^2c^4\mathrm{sgn}(x)-8\,Abc^5\mathrm{sgn}(x)}{c^6}\right)x^2+\frac{3\,(5\,Bb^3c^3\mathrm{sgn}(x)-8\,Ab^2c^4\mathrm{sgn}(x))}{c^6}\right)\sqrt{cx^2+b}\,x+\frac{(5\,Bb^4\mathrm{sgn}(x)-8\,Ab^3c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{128\,c^{\frac{7}{2}}}-\frac{(5\,Bb^4\log(|b|)-8\,Ab^3c\log(|b|))\mathrm{sgn}(x)}{256\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*x^2*sgn(x) + (B*b*c^5*sgn(x) + 8*A*c^6*sgn(x))/c^6)*x^2 -
(5*B*b^2*c^4*sgn(x) - 8*A*b*c^5*sgn(x))/c^6)*x^2 + 3*(5*B*b^3*c^3*sgn(x) -
8*A*b^2*c^4*sgn(x))/c^6)*sqrt(c*x^2 + b)*x + 1/128*(5*B*b^4*sgn(x) - 8*A*b^
3*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(7/2) - 1/256*(5*B*b^4
*log(abs(b)) - 8*A*b^3*c*log(abs(b)))*sgn(x)/c^(7/2)
```

Mupad [B]
time = 0.74, size = 177, normalized size = 1.42

$$\frac{B\,x^2\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{8\,c}-\frac{5\,B\,b\,\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,\left|x\right|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{16\,c}+\frac{A\,b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,\left|x\right|\,\sqrt{c\,x^2+b}\right)}{32\,c^{5/2}}+\frac{A\,\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{48\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (B*x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*B*b*((b^3*log(b + 2*c*x^2 + 2*c^(1
/2)*abs(x)*(b + c*x^2)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2
*x^4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(16*c) + (A*b^3*log(b + 2*c*x^2 + 2*c
^(1/2)*abs(x)*(b + c*x^2)^(1/2)))/(32*c^(5/2)) + (A*(b*x^2 + c*x^4)^(1/2)*(
8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(48*c^2)
```

3.92 $\int x(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=107

$$-\frac{(bB-2Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{16c^2}+\frac{B(bx^2+cx^4)^{3/2}}{6c}+\frac{b^2(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}$$

```
[Out] 1/6*B*(c*x^4+b*x^2)^(3/2)/c+1/16*b^2*(-2*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^
4+b*x^2)^(1/2))/c^(5/2)-1/16*(-2*A*c+B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c
^2
```

Rubi [A]
time = 0.10, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2059, 654, 626, 634, 212}

$$\frac{b^2(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{(b+2cx^2)\sqrt{bx^2+cx^4}\,(bB-2Ac)}{16c^2}+\frac{B(bx^2+cx^4)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -1/16*((b*B - 2*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/c^2 + (B*(b*x^2 + c
*x^4)^(3/2))/(6*c) + (b^2*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
c*x^4]])/(16*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\frac{1}{2}\text{Subst}\left(\int(A+Bx)\sqrt{bx+cx^2}\,dx,x,x^2\right)$$

$$=\frac{B(bx^2+cx^4)^{3/2}}{6c}+\frac{(-bB+2Ac)\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{4c}$$

$$=-\frac{(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{B(bx^2+cx^4)^{3/2}}{6c}+\frac{(b^2(bB-2Ac)}{}$$

$$=-\frac{(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{B(bx^2+cx^4)^{3/2}}{6c}+\frac{(b^2(bB-2Ac)}{}$$

$$=-\frac{(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{B(bx^2+cx^4)^{3/2}}{6c}+\frac{b^2(bB-2Ac)\,\mathrm{t}}{}$$

Mathematica [A]
time = 0.15, size = 123, normalized size = 1.15

$$\frac{x\left(\sqrt{c}\,x(b+cx^2)\left(-3b^2B+2bc(3A+Bx^2)+4c^2x^2(3A+2Bx^2)\right)-3b^2(bB-2Ac)\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{48c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(-3*b^2*B + 2*b*c*(3*A + B*x^2) + 4*c^2*x^2*(3*A
+ 2*B*x^2)) - 3*b^2*(b*B - 2*A*c)*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b
 + c*x^2]]))/(48*c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.39, size = 164, normalized size = 1.53

method	result
risch	$\frac{(8Bc^2x^4+12Ac^2x^2+2bBx^2c+6Abc-3b^2B)\sqrt{x^2(cx^2+b)}}{48c^2}+\frac{\left(-\frac{b^2\ln\left(\sqrt{c}\,x+\sqrt{cx^2+b}\right)A}{8c^{\frac{3}{2}}}+\frac{b^3\ln\left(\sqrt{c}\,x+\sqrt{cx^2}\right.}{16c^{\frac{5}{2}}}\right.}{x\sqrt{cx^2+b}}$
default	$\frac{\sqrt{x^4c+bx^2}\left(8B(cx^2+b)^{\frac{3}{2}}c^{\frac{3}{2}}x^3+12A(cx^2+b)^{\frac{3}{2}}c^{\frac{3}{2}}x-6B(cx^2+b)^{\frac{3}{2}}\sqrt{c}\,bx-6A\sqrt{cx^2+b}\,c^{\frac{3}{2}}bx+3B\sqrt{cx^2+b}\,\sqrt{c}\right.}{48x\sqrt{cx^2+b}\,c^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(1/2)*(8*B*(c*x^2+b)^(3/2)*c^(3/2)*x^3+12*A*(c*x^2+b)^(3
/2)*c^(3/2)*x-6*B*(c*x^2+b)^(3/2)*c^(1/2)*b*x-6*A*(c*x^2+b)^(1/2)*c^(3/2)*b
*x+3*B*(c*x^2+b)^(1/2)*c^(1/2)*b^2*x-6*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2*
c+3*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^3)/x/(c*x^2+b)^(1/2)/c^(5/2)
```

Maxima [A]
time = 0.29, size = 177, normalized size = 1.65

$$\frac{1}{16}\left(4\sqrt{cx^4+bx^2}\,x^2-\frac{b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{3}{2}}}+\frac{2\,\sqrt{cx^4+bx^2}\,b}{c}\right)A-\frac{1}{96}\left(\frac{12\,\sqrt{cx^4+bx^2}\,bx^2}{c}-\frac{3\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{5}{2}}}+\frac{6\,\sqrt{cx^4+bx^2}\,b^2}{c^2}-\frac{16\,(cx^4+bx^2)^{\frac{3}{2}}}{c}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*sqrt(c*x^4 + b*x^2)*x^2 - b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^
2)*sqrt(c))/c^(3/2) + 2*sqrt(c*x^4 + b*x^2)*b/c)*A - 1/96*(12*sqrt(c*x^4 +
b*x^2)*b*x^2/c - 3*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(
5/2) + 6*sqrt(c*x^4 + b*x^2)*b^2/c^2 - 16*(c*x^4 + b*x^2)^(3/2)/c)*B
```

Fricas [A]
time = 1.86, size = 223, normalized size = 2.08

$$\left[-\frac{3\,(Bb^3-2\,Ab^2c)\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\,(8\,Bc^3x^4-3\,Bb^2c+6\,Abc^2+2\,(Bbc^2+6\,Ac^3)x^2)\sqrt{cx^4+bx^2}}{96\,c^3},\,-\frac{3\,(Bb^3-2\,Ab^2c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)-(8\,Bc^3x^4-3\,Bb^2c+6\,Abc^2+2\,(Bbc^2+6\,Ac^3)x^2)\sqrt{cx^4+bx^2}}{48\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(B*b^3 - 2*A*b^2*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x
^2)*sqrt(c)) - 2*(8*B*c^3*x^4 - 3*B*b^2*c + 6*A*b*c^2 + 2*(B*b*c^2 + 6*A*c^
3)*x^2)*sqrt(c*x^4 + b*x^2))/c^3, -1/48*(3*(B*b^3 - 2*A*b^2*c)*sqrt(-c)*arc
```

```
tan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (8*B*c^3*x^4 - 3*B*b^2*c +
6*A*b*c^2 + 2*(B*b*c^2 + 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [A]
time = 0.90, size = 140, normalized size = 1.31

$$\frac{1}{48}\left(2\left(4\,Bx^2\mathrm{sgn}(x)+\frac{Bbc^3\mathrm{sgn}(x)+6\,Ac^4\mathrm{sgn}(x)}{c^4}\right)x^2-\frac{3\,(Bb^2c^2\mathrm{sgn}(x)-2\,Abc^3\mathrm{sgn}(x))}{c^4}\right)\sqrt{cx^2+b}\,x-\frac{(Bb^3\mathrm{sgn}(x)-2\,Ab^2c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{16\,c^{\frac{5}{2}}}+\frac{(Bb^3\log\left(|b|\right)-2\,Ab^2c\log\left(|b|\right))\mathrm{sgn}(x)}{32\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*x^2*sgn(x) + (B*b*c^3*sgn(x) + 6*A*c^4*sgn(x))/c^4)*x^2 - 3*(B
*b^2*c^2*sgn(x) - 2*A*b*c^3*sgn(x))/c^4)*sqrt(c*x^2 + b)*x - 1/16*(B*b^3*sg
n(x) - 2*A*b^2*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(5/2) + 1
/32*(B*b^3*log(abs(b)) - 2*A*b^2*c*log(abs(b)))*sgn(x)/c^(5/2)
```

Mupad [B]
time = 0.75, size = 140, normalized size = 1.31

$$\frac{A\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2}}{2}+\frac{B\,b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,\left|x\right|\,\sqrt{c\,x^2+b}\right)}{32\,c^{5/2}}-\frac{A\,b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{16\,c^{3/2}}+\frac{B\,\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{48\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (A*(b/(4*c) + x^2/2)*(b*x^2 + c*x^4)^(1/2))/2 + (B*b^3*log(b + 2*c*x^2 + 2*
c^(1/2)*abs(x)*(b + c*x^2)^(1/2)))/(32*c^(5/2)) - (A*b^2*log((b/2 + c*x^2)/
c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(16*c^(3/2)) + (B*(b*x^2 + c*x^4)^(1/2)*(
8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(48*c^2)
```

3.93 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx$

Optimal. Leaf size=100

$$-\frac{(bB-4Ac)\sqrt{bx^2+cx^4}}{8c}+\frac{B(bx^2+cx^4)^{3/2}}{4cx^2}-\frac{b(bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

```
[Out] 1/4*B*(c*x^4+b*x^2)^(3/2)/c/x^2-1/8*b*(-4*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x
^4+b*x^2)^(1/2))/c^(3/2)-1/8*(-4*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A]
time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2059, 808, 678, 634, 212}

$$-\frac{b(bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}-\frac{\sqrt{bx^2+cx^4}\,(bB-4Ac)}{8c}+\frac{B(bx^2+cx^4)^{3/2}}{4cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x,x]
```

```
[Out] -1/8*((b*B - 4*A*c)*Sqrt[b*x^2 + c*x^4])/c + (B*(b*x^2 + c*x^4)^(3/2))/(4*c
*x^2) - (b*(b*B - 4*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(
3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x}\,dx, x, x^2\right)\\
&= \frac{B(bx^2+cx^4)^{3/2}}{4cx^2} + \frac{\left(bB-Ac+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx, x, \ldots\right.}{4c}\\
&= -\frac{(bB-4Ac)\sqrt{bx^2+cx^4}}{8c} + \frac{B(bx^2+cx^4)^{3/2}}{4cx^2} - \frac{(b(bB-4Ac))\text{Subst}\left(\int \frac{\ldots}{\sqrt{\ldots}}\right.}{16c}\\
&= -\frac{(bB-4Ac)\sqrt{bx^2+cx^4}}{8c} + \frac{B(bx^2+cx^4)^{3/2}}{4cx^2} - \frac{(b(bB-4Ac))\text{Subst}\left(\int \frac{1}{1-\ldots}\right.}{8c}\\
&= -\frac{(bB-4Ac)\sqrt{bx^2+cx^4}}{8c} + \frac{B(bx^2+cx^4)^{3/2}}{4cx^2} - \frac{b(bB-4Ac)\tanh^{-1}\left(\frac{\ldots}{\sqrt{bx\ldots}}\right.}{8c^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 97, normalized size = 0.97

$$
\frac{x\left(\sqrt{c}\,x(b+cx^2)\,(bB+4Ac+2Bcx^2)+b(bB-4Ac)\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{8c^{3/2}\sqrt{x^2\,(b+cx^2)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x,x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(b*B + 4*A*c + 2*B*c*x^2) + b*(b*B - 4*A*c)*Sqrt[
b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(8*c^(3/2)*Sqrt[x^2*(b + c
*x^2)])
```

Maple [A]
time = 0.43, size = 124, normalized size = 1.24

method	result
risch	$\frac{\left(2Bc\,x^2+4Ac+Bb\right)\sqrt{x^2\left(c\,x^2+b\right)}}{8c}+\frac{\left(\frac{b\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)A}{2\sqrt{c}}-\frac{b^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)B}{8c^{\frac{3}{2}}}\right)\sqrt{x^2\left(c\,x^2+\right.}$
default	$\frac{\sqrt{x^4c+b\,x^2}\,\left(2B\sqrt{c}\,\left(c\,x^2+b\right)^{\frac{3}{2}}x+4A\,c^{\frac{3}{2}}\sqrt{c\,x^2+b}\,x-B\sqrt{c}\,\sqrt{c\,x^2+b}\,bx+4A\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)bc-B\right.}{8c^{\frac{3}{2}}\sqrt{c\,x^2+b}\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(1/2)*(2*B*c^(1/2)*(c*x^2+b)^(3/2)*x+4*A*c^(3/2)*(c*x^2+b
)^(1/2)*x-B*c^(1/2)*(c*x^2+b)^(1/2)*b*x+4*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b
*c-B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2)/c^(3/2)/(c*x^2+b)^(1/2)/x
```

Maxima [A]
time = 0.29, size = 128, normalized size = 1.28

$$\frac{1}{4}\left(\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{\sqrt{c}}+2\,\sqrt{cx^4+bx^2}\right)A+\frac{1}{16}\left(4\,\sqrt{cx^4+bx^2}\,x^2-\frac{b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{3}{2}}}+\frac{2\,\sqrt{cx^4+bx^2}\,b}{c}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*(b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 2*sqrt(c*
x^4 + b*x^2))*A + 1/16*(4*sqrt(c*x^4 + b*x^2)*x^2 - b^2*log(2*c*x^2 + b + 2
*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) + 2*sqrt(c*x^4 + b*x^2)*b/c)*B
```

Fricas [A]
time = 2.04, size = 172, normalized size = 1.72

$$\left[-\frac{\left(Bb^2-4\,Abc\right)\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\left(2\,Bc^2x^2+Bbc+4\,Ac^2\right)\sqrt{cx^4+bx^2}}{16\,c^2},\ \frac{\left(Bb^2-4\,Abc\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\left(2\,Bc^2x^2+Bbc+4\,Ac^2\right)\sqrt{cx^4+bx^2}}{8\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/16*((B*b^2 - 4*A*b*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*
sqrt(c)) - 2*(2*B*c^2*x^2 + B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^2, 1/8*
((B*b^2 - 4*A*b*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)
) + (2*B*c^2*x^2 + B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\,\left(A+Bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x, x)
```

Giac [A]
time = 1.13, size = 103, normalized size = 1.03

$$\frac{1}{8}\left(2\,Bx^2\mathrm{sgn}(x)+\frac{Bbc\mathrm{sgn}(x)+4\,Ac^2\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+b}\,x+\frac{\left(Bb^2\mathrm{sgn}(x)-4\,Abc\mathrm{sgn}(x)\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{8\,c^{\frac{3}{2}}}-\frac{\left(Bb^2\log\left(|b|\right)-4\,Abc\log\left(|b|\right)\right)\mathrm{sgn}(x)}{16\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*B*x^2*sgn(x) + (B*b*c*sgn(x) + 4*A*c^2*sgn(x))/c^2)*sqrt(c*x^2 + b)*
x + 1/8*(B*b^2*sgn(x) - 4*A*b*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b
)))/c^(3/2) - 1/16*(B*b^2*log(abs(b)) - 4*A*b*c*log(abs(b)))*sgn(x)/c^(3/2)
```

Mupad [B]
time = 0.61, size = 117, normalized size = 1.17

$$\frac{A\,\sqrt{c\,x^4+b\,x^2}}{2}+\frac{B\,\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\,\sqrt{c\,x^4+b\,x^2}}{2}+\frac{A\,b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,\sqrt{c}}-\frac{B\,b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{16\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x,x)
```

```
[Out] (A*(b*x^2 + c*x^4)^(1/2))/2 + (B*(b/(4*c) + x^2/2)*(b*x^2 + c*x^4)^(1/2))/2
 + (A*b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(4*c^(1/2)) - (
B*b^2*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(16*c^(3/2))
```

3.94 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx$

Optimal. Leaf size=97

$$\frac{(bB+2Ac)\sqrt{bx^2+cx^4}}{2b}-\frac{A(bx^2+cx^4)^{3/2}}{bx^4}+\frac{(bB+2Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] -A*(c*x^4+b*x^2)^(3/2)/b/x^4+1/2*(2*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x
^2)^(1/2))/c^(1/2)+1/2*(2*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/b
```

Rubi [A]
time = 0.14, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2059, 806, 678, 634, 212}

$$\frac{\sqrt{bx^2+cx^4}\,(2Ac+bB)}{2b}+\frac{(2Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}-\frac{A(bx^2+cx^4)^{3/2}}{bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^3,x]
```

```
[Out] ((b*B + 2*A*c)*Sqrt[b*x^2 + c*x^4])/(2*b) - (A*(b*x^2 + c*x^4)^(3/2))/(b*x^
4) + ((b*B + 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{bx^4} + \frac{\left(-2(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+c}}{x}\right.}{b}$$

$$= \frac{(bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{A(bx^2+cx^4)^{3/2}}{bx^4} + \frac{1}{4}(bB+2Ac)\text{Subst}\left(\int \frac{}{\sqrt{bx}}\right.$$

$$= \frac{(bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{A(bx^2+cx^4)^{3/2}}{bx^4} + \frac{1}{2}(bB+2Ac)\text{Subst}\left(\int \frac{1}{1-c}\right.$$

$$= \frac{(bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{A(bx^2+cx^4)^{3/2}}{bx^4} + \frac{(bB+2Ac)\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{bx^2+}}\right.}{2\sqrt{c}}$$

Mathematica [A]
time = 0.12, size = 90, normalized size = 0.93

$$\frac{\sqrt{c}\,(-2A+Bx^2)\,(b+cx^2) - (bB+2Ac)x\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)}{2\sqrt{c}\,\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^3,x]
```

```
[Out] (Sqrt[c]*(-2*A + B*x^2)*(b + c*x^2) - (b*B + 2*A*c)*x*Sqrt[b + c*x^2]*Log[-
(Sqrt[c]*x) + Sqrt[b + c*x^2]])/(2*Sqrt[c]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.38, size = 132, normalized size = 1.36

method	result
risch	$-\frac{(-B x^2+2A)\sqrt{x^2(c x^2+b)}}{2x^2}+\frac{\left(\ln\left(\sqrt{c}\,x+\sqrt{c x^2+b}\right)\sqrt{c}\,A+\frac{\ln\left(\sqrt{c}\,x+\sqrt{c x^2+b}\right)Bb}{2\sqrt{c}}\right)\sqrt{x^2(c x^2+}}{x\sqrt{c x^2+b}}$
default	$-\frac{\sqrt{x^4c+b x^2}\left(-2A\sqrt{c x^2+b}\,c^{\frac{3}{2}}x^2-B\sqrt{c x^2+b}\,\sqrt{c}\,b x^2+2A(c x^2+b)^{\frac{3}{2}}\sqrt{c}-2A\ln\left(\sqrt{c}\,x+\sqrt{c x^2+b}\right)b\right.}{2x^2\sqrt{c x^2+b}\,b\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x^4+b*x^2)^(1/2)*(-2*A*(c*x^2+b)^(1/2)*c^(3/2)*x^2-B*(c*x^2+b)^(1/2
)*c^(1/2)*b*x^2+2*A*(c*x^2+b)^(3/2)*c^(1/2)-2*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2
))*b*c*x-B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2*x)/x^2/(c*x^2+b)^(1/2)/b/c^(1/
2)
```

Maxima [A]
time = 0.28, size = 105, normalized size = 1.08

$$\frac{1}{2}\left(\sqrt{c}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-\frac{2\,\sqrt{cx^4+bx^2}}{x^2}\right)A+\frac{1}{4}\left(\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{\sqrt{c}}+2\,\sqrt{cx^4+bx^2}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*sqrt(c*x^
4 + b*x^2)/x^2)*A + 1/4*(b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))
/sqrt(c) + 2*sqrt(c*x^4 + b*x^2))*B
```

Fricas [A]
time = 1.80, size = 161, normalized size = 1.66

$$\left[\frac{(Bb+2\,Ac)\sqrt{c}\,x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\,(Bcx^2-2\,Ac)}{4\,cx^2},-\frac{(Bb+2\,Ac)\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)-\sqrt{cx^4+bx^2}\,(Bcx^2-2\,Ac)}{2\,cx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*((B*b + 2*A*c)*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sq
rt(c)) + 2*sqrt(c*x^4 + b*x^2)*(B*c*x^2 - 2*A*c))/(c*x^2), -1/2*((B*b + 2*A
*c)*sqrt(-c)*x^2*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*
x^4 + b*x^2)*(B*c*x^2 - 2*A*c))/(c*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**3, x)
```

Giac [A]
time = 1.69, size = 92, normalized size = 0.95

$$\frac{1}{2}\sqrt{cx^2+b}\,Bx\mathrm{sgn}(x)+\frac{2\,Ab\sqrt{c}\,\mathrm{sgn}(x)}{\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b}-\frac{\left(Bb\sqrt{c}\,\mathrm{sgn}(x)+2\,Ac^{\frac{3}{2}}\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + b)*B*x*sgn(x) + 2*A*b*sqrt(c)*sgn(x)/((sqrt(c)*x - sqrt(c*
x^2 + b))^2 - b) - 1/4*(B*b*sqrt(c)*sgn(x) + 2*A*c^(3/2)*sgn(x))*log((sqrt(
c)*x - sqrt(c*x^2 + b))^2)/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^3,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^3, x)
```

3.95 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx$

Optimal. Leaf size=80

$$-\frac{B\sqrt{bx^2+cx^4}}{x^2}-\frac{A(bx^2+cx^4)^{3/2}}{3bx^6}+B\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] -1/3*A*(c*x^4+b*x^2)^(3/2)/b/x^6+B*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))
*c^(1/2)-B*(c*x^4+b*x^2)^(1/2)/x^2
```

Rubi [A]

time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2059, 806, 676, 634, 212}

$$-\frac{A(bx^2+cx^4)^{3/2}}{3bx^6}-\frac{B\sqrt{bx^2+cx^4}}{x^2}+B\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^5,x]
```

```
[Out] -((B*Sqrt[b*x^2 + c*x^4])/x^2) - (A*(b*x^2 + c*x^4)^(3/2))/(3*b*x^6) + B*Sq
rt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 676

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Dist[c*(p/(e^2*(m + p + 1))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^3}\,dx, x, x^2\right)\\ &= -\frac{A(bx^2+cx^4)^{3/2}}{3bx^6}+\frac{1}{2}B\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx, x, x^2\right)\\ &= -\frac{B\sqrt{bx^2+cx^4}}{x^2}-\frac{A(bx^2+cx^4)^{3/2}}{3bx^6}+\frac{1}{2}(Bc)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\ &= -\frac{B\sqrt{bx^2+cx^4}}{x^2}-\frac{A(bx^2+cx^4)^{3/2}}{3bx^6}+(Bc)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\ &= -\frac{B\sqrt{bx^2+cx^4}}{x^2}-\frac{A(bx^2+cx^4)^{3/2}}{3bx^6}+B\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)\end{aligned}$$

Mathematica [A]
time = 0.10, size = 99, normalized size = 1.24

$$-\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b+cx^2}\,(3bBx^2+A(b+cx^2))+3bB\sqrt{c}\,x^3\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{3bx^4\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^5,x]
```

```
[Out] -1/3*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b + c*x^2]*(3*b*B*x^2 + A*(b + c*x^2)) +
3*b*B*Sqrt[c]*x^3*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(b*x^4*Sqrt[b + c*x
^2])
```

Maple [A]
time = 0.39, size = 109, normalized size = 1.36

method	result
risch	$-\frac{(A c x^2+3 b B x^2+A b)\sqrt{x^2(c x^2+b)}}{3 x^4 b}+\frac{B\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{c x^2+b}\right)\sqrt{x^2(c x^2+b)}}{x\sqrt{c x^2+b}}$
default	$-\frac{\sqrt{x^4 c+b x^2}\left(-3B\sqrt{c x^2+b}\,c^{\frac{3}{2}} x^4+3B(c x^2+b)^{\frac{3}{2}}\sqrt{c}\,x^2-3B\ln\left(\sqrt{c}\,x+\sqrt{c x^2+b}\right) b c x^3+A(c x^2+b)^{\frac{3}{2}}\sqrt{c}\right)}{3x^4\sqrt{c x^2+b}\,b\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*x^4+b*x^2)^(1/2)*(-3*B*(c*x^2+b)^(1/2)*c^(3/2)*x^4+3*B*(c*x^2+b)^(3
/2)*c^(1/2)*x^2-3*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b*c*x^3+A*(c*x^2+b)^(3/2)
*c^(1/2))/x^4/(c*x^2+b)^(1/2)/b/c^(1/2)
```

Maxima [A]
time = 0.28, size = 96, normalized size = 1.20

$$\frac{1}{2}\left(\sqrt{c}\,\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-\frac{2\,\sqrt{cx^4+bx^2}}{x^2}\right)B-\frac{1}{3}\,A\left(\frac{\sqrt{cx^4+bx^2}\,c}{bx^2}+\frac{\sqrt{cx^4+bx^2}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*sqrt(c*x^
4 + b*x^2)/x^2)*B - 1/3*A*(sqrt(c*x^4 + b*x^2)*c/(b*x^2) + sqrt(c*x^4 + b*x
^2)/x^4)
```

Fricas [A]
time = 1.21, size = 160, normalized size = 2.00

$$\left[\frac{3\,Bb\sqrt{c}\,x^4\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}\,((3\,Bb+Ac)x^2+Ab)}{6\,bx^4},\,-\frac{3\,Bb\sqrt{-c}\,x^4\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}\,((3\,Bb+Ac)x^2+Ab)}{3\,bx^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/6*(3*B*b*sqrt(c)*x^4*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) -
 2*sqrt(c*x^4 + b*x^2)*((3*B*b + A*c)*x^2 + A*b))/(b*x^4), -1/3*(3*B*b*sqrt
```

```
(-c)*x^4*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*
x^2)*((3*B*b + A*c)*x^2 + A*b))/(b*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\,(b+cx^2)}\,(A+Bx^2)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(68) = 136.
time = 1.29, size = 163, normalized size = 2.04

$$-\frac{1}{2}\,B\sqrt{c}\,\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2\right)\operatorname{sgn}(x)+\frac{2\left(3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4Bb\sqrt{c}\,\operatorname{sgn}(x)+3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4Ac^{\frac{3}{2}}\operatorname{sgn}(x)-6\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2Bb^2\sqrt{c}\,\operatorname{sgn}(x)+3\,Bb^3\sqrt{c}\,\operatorname{sgn}(x)+Ab^2c^{\frac{3}{2}}\operatorname{sgn}(x)\right)}{3\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/2*B*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 2/3*(3*(sqrt(c
)*x - sqrt(c*x^2 + b))^4*B*b*sqrt(c)*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + b
))^4*A*c^(3/2)*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^2*sqrt(c)*sgn
(x) + 3*B*b^3*sqrt(c)*sgn(x) + A*b^2*c^(3/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x
^2 + b))^2 - b)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^5,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^5, x)
```

3.96 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx$

Optimal. Leaf size=61

$$-\frac{A(bx^2+cx^4)^{3/2}}{5bx^8}-\frac{(5bB-2Ac)\,(bx^2+cx^4)^{3/2}}{15b^2x^6}$$

```
[Out] -1/5*A*(c*x^4+b*x^2)^(3/2)/b/x^8-1/15*(-2*A*c+5*B*b)*(c*x^4+b*x^2)^(3/2)/b^
2/x^6
```

Rubi [A]
time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2059, 806, 664}

$$-\frac{(bx^2+cx^4)^{3/2}\,(5bB-2Ac)}{15b^2x^6}-\frac{A(bx^2+cx^4)^{3/2}}{5bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^7,x]
```

```
[Out] -1/5*(A*(b*x^2 + c*x^4)^(3/2))/(b*x^8) - ((5*b*B - 2*A*c)*(b*x^2 + c*x^4)^(
3/2))/(15*b^2*x^6)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
```

```
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{5bx^8} + \frac{\left(-4(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^3}\right.}{5b}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{5bx^8} - \frac{(5bB-2Ac)(bx^2+cx^4)^{3/2}}{15b^2x^6}$$

Mathematica [A]
time = 0.10, size = 44, normalized size = 0.72

$$-\frac{(x^2(b+cx^2))^{3/2}(3Ab+5bBx^2-2Acx^2)}{15b^2x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^7,x]
[Out] -1/15*((x^2*(b + c*x^2))^(3/2)*(3*A*b + 5*b*B*x^2 - 2*A*c*x^2))/(b^2*x^8)
```

Maple [A]
time = 0.39, size = 48, normalized size = 0.79

method	result	size
gosper	$-\frac{(cx^2+b)(-2Acx^2+5bBx^2+3Ab)\sqrt{x^4c+bx^2}}{15b^2x^6}$	48
default	$-\frac{(cx^2+b)(-2Acx^2+5bBx^2+3Ab)\sqrt{x^4c+bx^2}}{15b^2x^6}$	48
trager	$-\frac{(-2Ac^2x^4+5x^4bBc+Abcx^2+5b^2Bx^2+3b^2A)\sqrt{x^4c+bx^2}}{15b^2x^6}$	62
risch	$-\frac{\sqrt{x^2(cx^2+b)}\,(-2Ac^2x^4+5x^4bBc+Abcx^2+5b^2Bx^2+3b^2A)}{15x^6b^2}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(c*x^2+b)*(-2*A*c*x^2+5*B*b*x^2+3*A*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(53) = 106$.
time = 0.30, size = 111, normalized size = 1.82

$$-\frac{1}{3}B\left(\frac{\sqrt{cx^4+bx^2}\,c}{bx^2}+\frac{\sqrt{cx^4+bx^2}}{x^4}\right)+\frac{1}{15}A\left(\frac{2\sqrt{cx^4+bx^2}\,c^2}{b^2x^2}-\frac{\sqrt{cx^4+bx^2}\,c}{bx^4}-\frac{3\sqrt{cx^4+bx^2}}{x^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/3*B*(sqrt(c*x^4 + b*x^2)*c/(b*x^2) + sqrt(c*x^4 + b*x^2)/x^4) + 1/15*A*(
2*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^2) - sqrt(c*x^4 + b*x^2)*c/(b*x^4) - 3*sqr
t(c*x^4 + b*x^2)/x^6)
```

Fricas [A]
time = 2.78, size = 59, normalized size = 0.97

$$-\frac{((5\,Bbc-2\,Ac^2)x^4+3\,Ab^2+(5\,Bb^2+Abc)x^2)\sqrt{cx^4+bx^2}}{15\,b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/15*((5*B*b*c - 2*A*c^2)*x^4 + 3*A*b^2 + (5*B*b^2 + A*b*c)*x^2)*sqrt(c*x^
4 + b*x^2)/(b^2*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\,(b+cx^2)}\,(A+Bx^2)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**7, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(53) = 106$.
time = 1.08, size = 250, normalized size = 4.10

$$\frac{2\left(15\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^8Bc^{\frac{3}{2}}\mathrm{sgn}(x)-30\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^6Bbc^{\frac{3}{2}}\mathrm{sgn}(x)+30\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^6Ac^{\frac{5}{2}}\mathrm{sgn}(x)+20\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4Bb^2c^{\frac{3}{2}}\mathrm{sgn}(x)+10\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4Abc^{\frac{5}{2}}\mathrm{sgn}(x)-10\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2Bb^3c^{\frac{3}{2}}\mathrm{sgn}(x)+10\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2Ab^2c^{\frac{5}{2}}\mathrm{sgn}(x)+5\,Bb^4c^{\frac{3}{2}}\mathrm{sgn}(x)-2\,Ab^3c^{\frac{5}{2}}\mathrm{sgn}(x)\right)}{15\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*c^(3/2)*sgn(x) - 30*(sqrt(c)*x -
 sqrt(c*x^2 + b))^6*B*b*c^(3/2)*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 + b))^6
*A*c^(5/2)*sgn(x) + 20*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^2*c^(3/2)*sgn(x)
 + 10*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b*c^(5/2)*sgn(x) - 10*(sqrt(c)*x -
sqrt(c*x^2 + b))^2*B*b^3*c^(3/2)*sgn(x) + 10*(sqrt(c)*x - sqrt(c*x^2 + b))^
2*A*b^2*c^(5/2)*sgn(x) + 5*B*b^4*c^(3/2)*sgn(x) - 2*A*b^3*c^(5/2)*sgn(x))/(
(sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5
```

Mupad [B]
time = 0.47, size = 113, normalized size = 1.85

$$\frac{(A\,c^2+B\,b\,c)\,\sqrt{c\,x^4+b\,x^2}}{5\,b^2\,x^2}-\frac{(5\,B\,b^2+A\,c\,b)\,\sqrt{c\,x^4+b\,x^2}}{15\,b^2\,x^4}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{5\,x^6}-\frac{c\,(A\,c+8\,B\,b)\,\sqrt{c\,x^4+b\,x^2}}{15\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^7,x)
```

```
[Out] ((A*c^2 + B*b*c)*(b*x^2 + c*x^4)^(1/2))/(5*b^2*x^2) - ((5*B*b^2 + A*b*c)*(b
*x^2 + c*x^4)^(1/2))/(15*b^2*x^4) - (A*(b*x^2 + c*x^4)^(1/2))/(5*x^6) - (c*
(A*c + 8*B*b)*(b*x^2 + c*x^4)^(1/2))/(15*b^2*x^2)
```

3.97 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx$

Optimal. Leaf size=96

$$-\frac{A(bx^2+cx^4)^{3/2}}{7bx^{10}}-\frac{(7bB-4Ac)\,(bx^2+cx^4)^{3/2}}{35b^2x^8}+\frac{2c(7bB-4Ac)\,(bx^2+cx^4)^{3/2}}{105b^3x^6}$$

```
[Out] -1/7*A*(c*x^4+b*x^2)^(3/2)/b/x^10-1/35*(-4*A*c+7*B*b)*(c*x^4+b*x^2)^(3/2)/b
^2/x^8+2/105*c*(-4*A*c+7*B*b)*(c*x^4+b*x^2)^(3/2)/b^3/x^6
```

Rubi [A]
time = 0.14, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2059, 806, 672, 664}

$$\frac{2c(bx^2+cx^4)^{3/2}\,(7bB-4Ac)}{105b^3x^6}-\frac{(bx^2+cx^4)^{3/2}\,(7bB-4Ac)}{35b^2x^8}-\frac{A(bx^2+cx^4)^{3/2}}{7bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^9,x]
```

```
[Out] -1/7*(A*(b*x^2 + c*x^4)^(3/2))/(b*x^10) - ((7*b*B - 4*A*c)*(b*x^2 + c*x^4)^
(3/2))/(35*b^2*x^8) + (2*c*(7*b*B - 4*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*b^3*
x^6)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
```

```
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{7bx^{10}} + \frac{\left(-5(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx}}{x^4}\right.}{7b}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{7bx^{10}} - \frac{(7bB-4Ac)\,(bx^2+cx^4)^{3/2}}{35b^2x^8} - \frac{(c(7bB-4Ac))\text{Subst}\left(\right.}{35}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{7bx^{10}} - \frac{(7bB-4Ac)\,(bx^2+cx^4)^{3/2}}{35b^2x^8} + \frac{2c(7bB-4Ac)\,(bx^2+c}{105b^3x^6}$$

Mathematica [A]
time = 0.12, size = 66, normalized size = 0.69

$$\frac{(x^2(b+cx^2))^{3/2}\,(7bBx^2(-3b+2cx^2)+A(-15b^2+12bcx^2-8c^2x^4))}{105b^3x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^9,x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(7*b*B*x^2*(-3*b + 2*c*x^2) + A*(-15*b^2 + 12*b*c*
x^2 - 8*c^2*x^4)))/(105*b^3*x^10)
```

Maple [A]
time = 0.38, size = 70, normalized size = 0.73

method	result	size
gosper	$-\frac{(c\,x^{2}+b)\,(8A\,c^{2}x^{4}-14x^{4}bBc-12Abc\,x^{2}+21b^{2}B\,x^{2}+15b^{2}A)\,\sqrt{x^{4}c+b\,x^{2}}}{105b^{3}x^{8}}$	70
default	$-\frac{(c\,x^{2}+b)\,(8A\,c^{2}x^{4}-14x^{4}bBc-12Abc\,x^{2}+21b^{2}B\,x^{2}+15b^{2}A)\,\sqrt{x^{4}c+b\,x^{2}}}{105b^{3}x^{8}}$	70
trager	$-\frac{(8A\,c^{3}x^{6}-14x^{6}Bb\,c^{2}-4Ab\,c^{2}x^{4}+7x^{4}B\,b^{2}c+3A\,b^{2}c\,x^{2}+21x^{2}B\,b^{3}+15A\,b^{3})\,\sqrt{x^{4}c+b\,x^{2}}}{105b^{3}x^{8}}$	87
risch	$-\frac{\sqrt{x^{2}\,(c\,x^{2}+b)}\,(8A\,c^{3}x^{6}-14x^{6}Bb\,c^{2}-4Ab\,c^{2}x^{4}+7x^{4}B\,b^{2}c+3A\,b^{2}c\,x^{2}+21x^{2}B\,b^{3}+15A\,b^{3})}{105x^{8}b^{3}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(c*x^2+b)*(8*A*c^2*x^4-14*B*b*c*x^4-12*A*b*c*x^2+21*B*b^2*x^2+15*A*b
^2)*(c*x^4+b*x^2)^(1/2)/b^3/x^8
```

Maxima [A]
time = 0.28, size = 161, normalized size = 1.68

$$\frac{1}{15}\,B\left(\frac{2\,\sqrt{cx^{4}+bx^{2}}\,c^{2}}{b^{2}x^{2}}-\frac{\sqrt{cx^{4}+bx^{2}}\,c}{bx^{4}}-\frac{3\,\sqrt{cx^{4}+bx^{2}}}{x^{6}}\right)-\frac{1}{105}\,A\left(\frac{8\,\sqrt{cx^{4}+bx^{2}}\,c^{3}}{b^{3}x^{2}}-\frac{4\,\sqrt{cx^{4}+bx^{2}}\,c^{2}}{b^{2}x^{4}}+\frac{3\,\sqrt{cx^{4}+bx^{2}}\,c}{bx^{6}}+\frac{15\,\sqrt{cx^{4}+bx^{2}}}{x^{8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/15*B*(2*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^2) - sqrt(c*x^4 + b*x^2)*c/(b*x^4)
 - 3*sqrt(c*x^4 + b*x^2)/x^6) - 1/105*A*(8*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^2
) - 4*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^4) + 3*sqrt(c*x^4 + b*x^2)*c/(b*x^6) +
 15*sqrt(c*x^4 + b*x^2)/x^8)
```

Fricas [A]
time = 1.82, size = 85, normalized size = 0.89

$$\frac{(2\,(7\,Bbc^{2}-4\,Ac^{3})x^{6}-(7\,Bb^{2}c-4\,Abc^{2})x^{4}-15\,Ab^{3}-3\,(7\,Bb^{3}+Ab^{2}c)x^{2})\sqrt{cx^{4}+bx^{2}}}{105\,b^{3}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(7*B*b*c^2 - 4*A*c^3)*x^6 - (7*B*b^2*c - 4*A*b*c^2)*x^4 - 15*A*b^3
 - 3*(7*B*b^3 + A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^3*x^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^{2}\,(b+cx^{2})}\,(A+Bx^{2})}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**9,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**9, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(84) = 168.
time = 1.41, size = 310, normalized size = 3.23

$$\frac{4\left(105\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bc^{\frac{5}{2}}\mathrm{sgn}(x)-175\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Bbc^{\frac{5}{2}}\mathrm{sgn}(x)+280\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Ac^{\frac{7}{2}}\mathrm{sgn}(x)+70\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}Bb^2c^{\frac{5}{2}}\mathrm{sgn}(x)+140\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}Abc^{\frac{7}{2}}\mathrm{sgn}(x)-42\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}Bb^3c^{\frac{5}{2}}\mathrm{sgn}(x)+84\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}Ab^2c^{\frac{7}{2}}\mathrm{sgn}(x)+49\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}Bb^4c^{\frac{5}{2}}\mathrm{sgn}(x)-28\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}Ab^3c^{\frac{7}{2}}\mathrm{sgn}(x)-7Bb^5c^{\frac{5}{2}}\mathrm{sgn}(x)+4Ab^4c^{\frac{7}{2}}\mathrm{sgn}(x)\right)}{105\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 4/105*(105*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*c^(5/2)*sgn(x) - 175*(sqrt(c)
*x - sqrt(c*x^2 + b))^8*B*b*c^(5/2)*sgn(x) + 280*(sqrt(c)*x - sqrt(c*x^2 +
b))^8*A*c^(7/2)*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^2*c^(5/2)*s
gn(x) + 140*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b*c^(7/2)*sgn(x) - 42*(sqrt(c
)*x - sqrt(c*x^2 + b))^4*B*b^3*c^(5/2)*sgn(x) + 84*(sqrt(c)*x - sqrt(c*x^2
+ b))^4*A*b^2*c^(7/2)*sgn(x) + 49*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^4*c^(
5/2)*sgn(x) - 28*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^3*c^(7/2)*sgn(x) - 7*B
*b^5*c^(5/2)*sgn(x) + 4*A*b^4*c^(7/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b)
)^2 - b)^7
```

Mupad [B]
time = 0.68, size = 160, normalized size = 1.67

$$\frac{4\,A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^4}-\frac{B\,\sqrt{c\,x^4+b\,x^2}}{5\,x^6}-\frac{A\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^6}-\frac{B\,c\,\sqrt{c\,x^4+b\,x^2}}{15\,b\,x^4}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{7\,x^8}-\frac{8\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{105\,b^3\,x^2}+\frac{2\,B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{15\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^9,x)
```

```
[Out] (4*A*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^4) - (B*(b*x^2 + c*x^4)^(1/2))/(
5*x^6) - (A*c*(b*x^2 + c*x^4)^(1/2))/(35*b*x^6) - (B*c*(b*x^2 + c*x^4)^(1/2
))/(15*b*x^4) - (A*(b*x^2 + c*x^4)^(1/2))/(7*x^8) - (8*A*c^3*(b*x^2 + c*x^4
)^(1/2))/(105*b^3*x^2) + (2*B*c^2*(b*x^2 + c*x^4)^(1/2))/(15*b^2*x^2)
```

3.98 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx$

Optimal. Leaf size=133

$$-\frac{A(bx^2+cx^4)^{3/2}}{9bx^{12}}-\frac{(3bB-2Ac)\,(bx^2+cx^4)^{3/2}}{21b^2x^{10}}+\frac{4c(3bB-2Ac)\,(bx^2+cx^4)^{3/2}}{105b^3x^8}-\frac{8c^2(3bB-2Ac)\,(bx^2+cx^4)^{3/2}}{315b^4x^6}$$

```
[Out] -1/9*A*(c*x^4+b*x^2)^(3/2)/b/x^12-1/21*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b
^2/x^10+4/105*c*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b^3/x^8-8/315*c^2*(-2*A*
c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b^4/x^6
```

Rubi [A]

time = 0.16, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2059, 806, 672, 664}

$$-\frac{8c^2(bx^2+cx^4)^{3/2}\,(3bB-2Ac)}{315b^4x^6}+\frac{4c(bx^2+cx^4)^{3/2}\,(3bB-2Ac)}{105b^3x^8}-\frac{(bx^2+cx^4)^{3/2}\,(3bB-2Ac)}{21b^2x^{10}}-\frac{A(bx^2+cx^4)^{3/2}}{9bx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^11,x]
```

```
[Out] -1/9*(A*(b*x^2 + c*x^4)^(3/2))/(b*x^12) - ((3*b*B - 2*A*c)*(b*x^2 + c*x^4)^
(3/2))/(21*b^2*x^10) + (4*c*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*b^3
*x^8) - (8*c^2*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(3/2))/(315*b^4*x^6)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
```

```
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{9bx^{12}} + \frac{\left(-6(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx}}{x^5}\right.}{9b}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{9bx^{12}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{3/2}}{21b^2x^{10}} - \frac{(2c(3bB-2Ac))\text{Subst}\left(\right.}{21}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{9bx^{12}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{3/2}}{21b^2x^{10}} + \frac{4c(3bB-2Ac)(bx^2+c}{105b^3x^8}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{9bx^{12}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{3/2}}{21b^2x^{10}} + \frac{4c(3bB-2Ac)(bx^2+c}{105b^3x^8}$$

Mathematica [A]
time = 0.15, size = 88, normalized size = 0.66

$$-\frac{(x^2(b+cx^2))^{3/2}(3bBx^2(15b^2-12bcx^2+8c^2x^4)+A(35b^3-30b^2cx^2+24bc^2x^4-16c^3x^6))}{315b^4x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^11,x]
```

```
[Out] -1/315*((x^2*(b + c*x^2))^(3/2)*(3*b*B*x^2*(15*b^2 - 12*b*c*x^2 + 8*c^2*x^4
) + A*(35*b^3 - 30*b^2*c*x^2 + 24*b*c^2*x^4 - 16*c^3*x^6)))/(b^4*x^12)
```

Maple [A]
time = 0.38, size = 94, normalized size = 0.71

method	result	s
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-16 A\,c^{3} x^{6}+24 x^{6} B b\,c^{2}+24 A b\,c^{2} x^{4}-36 x^{4} B\,b^{2} c-30 A\,b^{2} c\,x^{2}+45 x^{2} B\,b^{3}+35 A\,b^{3}\right)\sqrt{x^{4} c+b\,x^{2}}}{315 b^{4} x^{10}}$	9
default	$-\frac{\left(c\,x^{2}+b\right)\left(-16 A\,c^{3} x^{6}+24 x^{6} B b\,c^{2}+24 A b\,c^{2} x^{4}-36 x^{4} B\,b^{2} c-30 A\,b^{2} c\,x^{2}+45 x^{2} B\,b^{3}+35 A\,b^{3}\right)\sqrt{x^{4} c+b\,x^{2}}}{315 b^{4} x^{10}}$	9
trager	$-\frac{\left(-16 A\,c^{4} x^{8}+24 B b\,c^{3} x^{8}+8 A b\,c^{3} x^{6}-12 B\,b^{2} c^{2} x^{6}-6 A\,b^{2} c^{2} x^{4}+9 B\,b^{3} c\,x^{4}+5 A\,b^{3} c\,x^{2}+45 B\,b^{4} x^{2}+35 A\,b^{4}\right)\sqrt{x^{4} c+b\,x^{2}}}{315 b^{4} x^{10}}$	
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(-16 A\,c^{4} x^{8}+24 B b\,c^{3} x^{8}+8 A b\,c^{3} x^{6}-12 B\,b^{2} c^{2} x^{6}-6 A\,b^{2} c^{2} x^{4}+9 B\,b^{3} c\,x^{4}+5 A\,b^{3} c\,x^{2}+45 B\,b^{4} x^{2}+35 A\,b^{4}\right)}{315 x^{10} b^{4}}$	

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(c*x^2+b)*(-16*A*c^3*x^6+24*B*b*c^2*x^6+24*A*b*c^2*x^4-36*B*b^2*c*x^
4-30*A*b^2*c*x^2+45*B*b^3*x^2+35*A*b^3)*(c*x^4+b*x^2)^(1/2)/b^4/x^10
```

Maxima [A]
time = 0.29, size = 209, normalized size = 1.57

$$-\frac{1}{105}\,B\left(\frac{8\,\sqrt{cx^4+bx^2}\,c^3}{b^3x^2}-\frac{4\,\sqrt{cx^4+bx^2}\,c^2}{b^2x^4}+\frac{3\,\sqrt{cx^4+bx^2}\,c}{bx^6}+\frac{15\,\sqrt{cx^4+bx^2}}{x^8}\right)+\frac{1}{315}\,A\left(\frac{16\,\sqrt{cx^4+bx^2}\,c^4}{b^4x^2}-\frac{8\,\sqrt{cx^4+bx^2}\,c^3}{b^3x^4}+\frac{6\,\sqrt{cx^4+bx^2}\,c^2}{b^2x^6}-\frac{5\,\sqrt{cx^4+bx^2}\,c}{bx^8}-\frac{35\,\sqrt{cx^4+bx^2}}{x^{10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/105*B*(8*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^2) - 4*sqrt(c*x^4 + b*x^2)*c^2/(
b^2*x^4) + 3*sqrt(c*x^4 + b*x^2)*c/(b*x^6) + 15*sqrt(c*x^4 + b*x^2)/x^8) +
1/315*A*(16*sqrt(c*x^4 + b*x^2)*c^4/(b^4*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^3/(
b^3*x^4) + 6*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^6) - 5*sqrt(c*x^4 + b*x^2)*c/(b
*x^8) - 35*sqrt(c*x^4 + b*x^2)/x^10)
```

Fricas [A]
time = 1.29, size = 109, normalized size = 0.82

$$-\frac{\left(8\,\left(3\,Bbc^{3}-2\,Ac^{4}\right)x^{8}-4\,\left(3\,Bb^{2}c^{2}-2\,Abc^{3}\right)x^{6}+35\,Ab^{4}+3\,\left(3\,Bb^{3}c-2\,Ab^{2}c^{2}\right)x^{4}+5\,\left(9\,Bb^{4}+Ab^{3}c\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}}}{315\,b^{4}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/315*(8*(3*B*b*c^3 - 2*A*c^4)*x^8 - 4*(3*B*b^2*c^2 - 2*A*b*c^3)*x^6 + 35*
A*b^4 + 3*(3*B*b^3*c - 2*A*b^2*c^2)*x^4 + 5*(9*B*b^4 + A*b^3*c)*x^2)*sqrt(c
*x^4 + b*x^2)/(b^4*x^10)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**11,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**11, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(117) = 234.
time = 1.74, size = 370, normalized size = 2.78

$$\frac{16\left(210\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bc^{\frac{7}{2}}\mathrm{sgn}(x)-315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bbc^{\frac{7}{2}}\mathrm{sgn}(x)+630\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Ac^{\frac{9}{2}}\mathrm{sgn}(x)+63\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Bb^2c^{\frac{7}{2}}\mathrm{sgn}(x)+378\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Abc^{\frac{9}{2}}\mathrm{sgn}(x)-42\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}Bb^3c^{\frac{7}{2}}\mathrm{sgn}(x)+168\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}Ab^2c^{\frac{9}{2}}\mathrm{sgn}(x)+108\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}Bb^4c^{\frac{7}{2}}\mathrm{sgn}(x)-72\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}Ab^3c^{\frac{9}{2}}\mathrm{sgn}(x)-27\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}Bb^5c^{\frac{7}{2}}\mathrm{sgn}(x)+18\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}Ab^4c^{\frac{9}{2}}\mathrm{sgn}(x)+3Bb^6c^{\frac{7}{2}}\mathrm{sgn}(x)-2Ab^5c^{\frac{9}{2}}\mathrm{sgn}(x)\right)}{315\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*c^(7/2)*sgn(x) - 315*(sqrt(c
)*x - sqrt(c*x^2 + b))^10*B*b*c^(7/2)*sgn(x) + 630*(sqrt(c)*x - sqrt(c*x^2
+ b))^10*A*c^(9/2)*sgn(x) + 63*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^2*c^(7/2
)*sgn(x) + 378*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b*c^(9/2)*sgn(x) - 42*(sqr
t(c)*x - sqrt(c*x^2 + b))^6*B*b^3*c^(7/2)*sgn(x) + 168*(sqrt(c)*x - sqrt(c*
x^2 + b))^6*A*b^2*c^(9/2)*sgn(x) + 108*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^
4*c^(7/2)*sgn(x) - 72*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^3*c^(9/2)*sgn(x)
- 27*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^5*c^(7/2)*sgn(x) + 18*(sqrt(c)*x -
 sqrt(c*x^2 + b))^2*A*b^4*c^(9/2)*sgn(x) + 3*B*b^6*c^(7/2)*sgn(x) - 2*A*b^5
*c^(9/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^9
```

Mupad [B]
time = 1.04, size = 210, normalized size = 1.58

$$\frac{2\,A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^6}-\frac{B\,\sqrt{c\,x^4+b\,x^2}}{7\,x^8}-\frac{A\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,b\,x^8}-\frac{B\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^6}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}}-\frac{8\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^4}+\frac{16\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^4\,x^2}+\frac{4\,B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^4}-\frac{8\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{105\,b^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^11,x)
```

```
[Out] (2*A*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^6) - (B*(b*x^2 + c*x^4)^(1/2))/(
7*x^8) - (A*c*(b*x^2 + c*x^4)^(1/2))/(63*b*x^8) - (B*c*(b*x^2 + c*x^4)^(1/2
))/(35*b*x^6) - (A*(b*x^2 + c*x^4)^(1/2))/(9*x^10) - (8*A*c^3*(b*x^2 + c*x^
4)^(1/2))/(315*b^3*x^4) + (16*A*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^2) +
(4*B*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^4) - (8*B*c^3*(b*x^2 + c*x^4)^(1
/2))/(105*b^3*x^2)
```

3.99 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx$

Optimal. Leaf size=170

$$-\frac{A(bx^2+cx^4)^{3/2}}{11bx^{14}}-\frac{(11bB-8Ac)\,(bx^2+cx^4)^{3/2}}{99b^2x^{12}}+\frac{2c(11bB-8Ac)\,(bx^2+cx^4)^{3/2}}{231b^3x^{10}}-\frac{8c^2(11bB-8Ac)\,(bx^2+}{1155b^4x^8}$$

```
[Out] -1/11*A*(c*x^4+b*x^2)^(3/2)/b/x^14-1/99*(-8*A*c+11*B*b)*(c*x^4+b*x^2)^(3/2)
/b^2/x^12+2/231*c*(-8*A*c+11*B*b)*(c*x^4+b*x^2)^(3/2)/b^3/x^10-8/1155*c^2*(
-8*A*c+11*B*b)*(c*x^4+b*x^2)^(3/2)/b^4/x^8+16/3465*c^3*(-8*A*c+11*B*b)*(c*x
^4+b*x^2)^(3/2)/b^5/x^6
```

Rubi [A]
time = 0.20, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2059, 806, 672, 664}

$$\frac{16c^3(bx^2+cx^4)^{3/2}(11bB-8Ac)}{3465b^5x^6}-\frac{8c^2(bx^2+cx^4)^{3/2}(11bB-8Ac)}{1155b^4x^8}+\frac{2c(bx^2+cx^4)^{3/2}(11bB-8Ac)}{231b^3x^{10}}-\frac{(bx^2+cx^4)^{3/2}(11bB-8Ac)}{99b^2x^{12}}-\frac{A(bx^2+cx^4)^{3/2}}{11bx^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^13,x]
```

```
[Out] -1/11*(A*(b*x^2 + c*x^4)^(3/2))/(b*x^14) - ((11*b*B - 8*A*c)*(b*x^2 + c*x^4
)^(3/2))/(99*b^2*x^12) + (2*c*(11*b*B - 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(231*
b^3*x^10) - (8*c^2*(11*b*B - 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(1155*b^4*x^8) +
 (16*c^3*(11*b*B - 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(3465*b^5*x^6)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^7}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{11bx^{14}} + \frac{\left(-7(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+c}}{x^6}\right.}{11b}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{11bx^{14}} - \frac{(11bB-8Ac)(bx^2+cx^4)^{3/2}}{99b^2x^{12}} - \frac{(c(11bB-8Ac))\text{Subst}}{}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{11bx^{14}} - \frac{(11bB-8Ac)(bx^2+cx^4)^{3/2}}{99b^2x^{12}} + \frac{2c(11bB-8Ac)(bx^2+}{231b^3x^{10}}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{11bx^{14}} - \frac{(11bB-8Ac)(bx^2+cx^4)^{3/2}}{99b^2x^{12}} + \frac{2c(11bB-8Ac)(bx^2+}{231b^3x^{10}}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{11bx^{14}} - \frac{(11bB-8Ac)(bx^2+cx^4)^{3/2}}{99b^2x^{12}} + \frac{2c(11bB-8Ac)(bx^2+}{231b^3x^{10}}$$

Mathematica [A]
time = 0.16, size = 110, normalized size = 0.65

$$\frac{(x^2(b+cx^2))^{3/2}\left(11bBx^2(-35b^3+30b^2cx^2-24bc^2x^4+16c^3x^6)+A(-315b^4+280b^3cx^2-240b^2c^2x^4+192bc^3x^6-128c^4x^8)\right)}{3465b^5x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^13,x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(11*b*B*x^2*(-35*b^3 + 30*b^2*c*x^2 - 24*b*c^2*x^4
 + 16*c^3*x^6) + A*(-315*b^4 + 280*b^3*c*x^2 - 240*b^2*c^2*x^4 + 192*b*c^3*
x^6 - 128*c^4*x^8)))/(3465*b^5*x^14)
```

Maple [A]
time = 0.40, size = 118, normalized size = 0.69

method	result
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(128A\,c^{4}x^{8}-176Bb\,c^{3}x^{8}-192Ab\,c^{3}x^{6}+264B\,b^{2}c^{2}x^{6}+240A\,b^{2}c^{2}x^{4}-330B\,b^{3}c\,x^{4}-280A\,b^{3}c\,x^{2}+385B\,b^{4}x^{2}+315A\,b^{4}\right)\sqrt{}}{3465b^{5}x^{12}}$
default	$-\frac{\left(c\,x^{2}+b\right)\left(128A\,c^{4}x^{8}-176Bb\,c^{3}x^{8}-192Ab\,c^{3}x^{6}+264B\,b^{2}c^{2}x^{6}+240A\,b^{2}c^{2}x^{4}-330B\,b^{3}c\,x^{4}-280A\,b^{3}c\,x^{2}+385B\,b^{4}x^{2}+315A\,b^{4}\right)\sqrt{}}{3465b^{5}x^{12}}$
trager	$-\frac{\left(128A\,c^{5}x^{10}-176Bb\,c^{4}x^{10}-64Ab\,c^{4}x^{8}+88B\,b^{2}c^{3}x^{8}+48A\,b^{2}c^{3}x^{6}-66B\,b^{3}c^{2}x^{6}-40A\,b^{3}c^{2}x^{4}+55B\,b^{4}c\,x^{4}+35A\,b^{4}c\,x^{2}+385b^{5}B\,x^{2}\right.}{3465b^{5}x^{12}}$
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(128A\,c^{5}x^{10}-176Bb\,c^{4}x^{10}-64Ab\,c^{4}x^{8}+88B\,b^{2}c^{3}x^{8}+48A\,b^{2}c^{3}x^{6}-66B\,b^{3}c^{2}x^{6}-40A\,b^{3}c^{2}x^{4}+55B\,b^{4}c\,x^{4}+3\right.}{3465x^{12}b^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3465*(c*x^2+b)*(128*A*c^4*x^8-176*B*b*c^3*x^8-192*A*b*c^3*x^6+264*B*b^2*
c^2*x^6+240*A*b^2*c^2*x^4-330*B*b^3*c*x^4-280*A*b^3*c*x^2+385*B*b^4*x^2+315
*A*b^4)*(c*x^4+b*x^2)^(1/2)/b^5/x^12
```

Maxima [A]
time = 0.30, size = 257, normalized size = 1.51

$$\frac{1}{315}\,B\left(\frac{16\,\sqrt{cx^4+bx^2}\,c^4}{b^4x^2}-\frac{8\,\sqrt{cx^4+bx^2}\,c^3}{b^3x^4}+\frac{6\,\sqrt{cx^4+bx^2}\,c^2}{b^2x^6}-\frac{5\,\sqrt{cx^4+bx^2}\,c}{bx^8}-\frac{35\,\sqrt{cx^4+bx^2}}{x^{10}}\right)-\frac{1}{3465}\,A\left(\frac{128\,\sqrt{cx^4+bx^2}\,c^5}{b^5x^2}-\frac{64\,\sqrt{cx^4+bx^2}\,c^4}{b^4x^4}+\frac{48\,\sqrt{cx^4+bx^2}\,c^3}{b^3x^6}-\frac{40\,\sqrt{cx^4+bx^2}\,c^2}{b^2x^8}+\frac{35\,\sqrt{cx^4+bx^2}\,c}{bx^{10}}+\frac{315\,\sqrt{cx^4+bx^2}}{x^{12}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="maxima")
```

```
[Out] 1/315*B*(16*sqrt(c*x^4 + b*x^2)*c^4/(b^4*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^3/(
b^3*x^4) + 6*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^6) - 5*sqrt(c*x^4 + b*x^2)*c/(b
*x^8) - 35*sqrt(c*x^4 + b*x^2)/x^10) - 1/3465*A*(128*sqrt(c*x^4 + b*x^2)*c^
5/(b^5*x^2) - 64*sqrt(c*x^4 + b*x^2)*c^4/(b^4*x^4) + 48*sqrt(c*x^4 + b*x^2)
*c^3/(b^3*x^6) - 40*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^8) + 35*sqrt(c*x^4 + b*x
^2)*c/(b*x^10) + 315*sqrt(c*x^4 + b*x^2)/x^12)
```

Fricas [A]
time = 1.74, size = 133, normalized size = 0.78

$$\frac{\left(16\,(11\,Bbc^4-8\,Ac^5)x^{10}-8\,(11\,Bb^2c^3-8\,Abc^4)x^8+6\,(11\,Bb^3c^2-8\,Ab^2c^3)x^6-315\,Ab^5-5\,(11\,Bb^4c-8\,Ab^3c^2)x^4-35\,(11\,Bb^5+Ab^4c)x^2\right)\sqrt{cx^4+bx^2}}{3465\,b^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="fricas")
```

```
[Out] 1/3465*(16*(11*B*b*c^4 - 8*A*c^5)*x^10 - 8*(11*B*b^2*c^3 - 8*A*b*c^4)*x^8 +
 6*(11*B*b^3*c^2 - 8*A*b^2*c^3)*x^6 - 315*A*b^5 - 5*(11*B*b^4*c - 8*A*b^3*c
^2)*x^4 - 35*(11*B*b^5 + A*b^4*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^5*x^12)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\,(b+cx^2)}\,(A+Bx^2)}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**13,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**13, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(150) = 300.
time = 2.20, size = 430, normalized size = 2.53

$$\frac{32\left(3465\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Bc^{\frac{9}{2}}\mathrm{sgn}(x)-4851\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bbc^{\frac{9}{2}}\mathrm{sgn}(x)+11088\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Ac^{\frac{11}{2}}\mathrm{sgn}(x)+231\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bb^2c^{\frac{9}{2}}\mathrm{sgn}(x)+7392\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Abc^{\frac{11}{2}}\mathrm{sgn}(x)-165\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Bb^3c^{\frac{9}{2}}\mathrm{sgn}(x)+2640\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Ab^2c^{\frac{11}{2}}\mathrm{sgn}(x)+1815\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}Bb^4c^{\frac{9}{2}}\mathrm{sgn}(x)-1320\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}Ab^3c^{\frac{11}{2}}\mathrm{sgn}(x)-605\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}Bb^5c^{\frac{9}{2}}\mathrm{sgn}(x)+440\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}Ab^4c^{\frac{11}{2}}\mathrm{sgn}(x)+121\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}Bb^6c^{\frac{9}{2}}\mathrm{sgn}(x)-88\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}Ab^5c^{\frac{11}{2}}\mathrm{sgn}(x)-11\,Bb^7c^{\frac{9}{2}}\mathrm{sgn}(x)+8\,Ab^6c^{\frac{11}{2}}\mathrm{sgn}(x)\right)}{3465\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] 32/3465*(3465*(sqrt(c)*x - sqrt(c*x^2 + b))^14*B*c^(9/2)*sgn(x) - 4851*(sqr
t(c)*x - sqrt(c*x^2 + b))^12*B*b*c^(9/2)*sgn(x) + 11088*(sqrt(c)*x - sqrt(c
*x^2 + b))^12*A*c^(11/2)*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^
2*c^(9/2)*sgn(x) + 7392*(sqrt(c)*x - sqrt(c*x^2 + b))^10*A*b*c^(11/2)*sgn(x
) - 165*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^3*c^(9/2)*sgn(x) + 2640*(sqrt(c
)*x - sqrt(c*x^2 + b))^8*A*b^2*c^(11/2)*sgn(x) + 1815*(sqrt(c)*x - sqrt(c*x
^2 + b))^6*B*b^4*c^(9/2)*sgn(x) - 1320*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^
3*c^(11/2)*sgn(x) - 605*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^5*c^(9/2)*sgn(x
) + 440*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^4*c^(11/2)*sgn(x) + 121*(sqrt(c
)*x - sqrt(c*x^2 + b))^2*B*b^6*c^(9/2)*sgn(x) - 88*(sqrt(c)*x - sqrt(c*x^2
+ b))^2*A*b^5*c^(11/2)*sgn(x) - 11*B*b^7*c^(9/2)*sgn(x) + 8*A*b^6*c^(11/2)*
sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^11
```

Mupad [B]
time = 1.41, size = 260, normalized size = 1.53

$$\frac{8\,A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{693\,b^2\,x^8}-\frac{B\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}}-\frac{A\,c\,\sqrt{c\,x^4+b\,x^2}}{99\,b\,x^{10}}-\frac{B\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,b\,x^8}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}}-\frac{16\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^3\,x^6}+\frac{64\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{3465\,b^4\,x^4}-\frac{128\,A\,c^5\,\sqrt{c\,x^4+b\,x^2}}{3465\,b^5\,x^2}+\frac{2\,B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^6}-\frac{8\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^4}+\frac{16\,B\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^13,x)
```

```
[Out] (8*A*c^2*(b*x^2 + c*x^4)^(1/2))/(693*b^2*x^8) - (B*(b*x^2 + c*x^4)^(1/2))/(
9*x^10) - (A*c*(b*x^2 + c*x^4)^(1/2))/(99*b*x^10) - (B*c*(b*x^2 + c*x^4)^(1
/2))/(63*b*x^8) - (A*(b*x^2 + c*x^4)^(1/2))/(11*x^12) - (16*A*c^3*(b*x^2 +
c*x^4)^(1/2))/(1155*b^3*x^6) + (64*A*c^4*(b*x^2 + c*x^4)^(1/2))/(3465*b^4*x
^4) - (128*A*c^5*(b*x^2 + c*x^4)^(1/2))/(3465*b^5*x^2) + (2*B*c^2*(b*x^2 +
c*x^4)^(1/2))/(105*b^2*x^6) - (8*B*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^4)
 + (16*B*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^2)
```

3.100 $\int x^4(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=131

$$-\frac{8b^2(2bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{315c^4x^3}+\frac{4b(2bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{105c^3x}-\frac{(2bB-3Ac)x(bx^2+cx^4)^{3/2}}{21c^2}+\frac{Bx^3(bx^2+c$$

```
[Out] -8/315*b^2*(-3*A*c+2*B*b)*(c*x^4+b*x^2)^(3/2)/c^4/x^3+4/105*b*(-3*A*c+2*B*b
)*(c*x^4+b*x^2)^(3/2)/c^3/x-1/21*(-3*A*c+2*B*b)*x*(c*x^4+b*x^2)^(3/2)/c^2+1
/9*B*x^3*(c*x^4+b*x^2)^(3/2)/c
```

Rubi [A]
time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2064, 2041, 2025}

$$-\frac{8b^2\left(bx^2+cx^4\right)^{3/2}(2bB-3Ac)}{315c^4x^3}+\frac{4b\left(bx^2+cx^4\right)^{3/2}(2bB-3Ac)}{105c^3x}-\frac{x\left(bx^2+cx^4\right)^{3/2}(2bB-3Ac)}{21c^2}+\frac{Bx^3\left(bx^2+cx^4\right)^{3/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-8*b^2*(2*b*B - 3*A*c)*(b*x^2 + c*x^4)^(3/2))/(315*c^4*x^3) + (4*b*(2*b*B
- 3*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x) - ((2*b*B - 3*A*c)*x*(b*x^2 + c
*x^4)^(3/2))/(21*c^2) + (B*x^3*(b*x^2 + c*x^4)^(3/2))/(9*c)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
```

```
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^4\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\frac{Bx^3\left(bx^2+cx^4\right)^{3/2}}{9c}-\frac{(6bB-9Ac)\int x^4\sqrt{bx^2+cx^4}\,dx}{9c}$$
$$=-\frac{(2bB-3Ac)x\left(bx^2+cx^4\right)^{3/2}}{21c^2}+\frac{Bx^3\left(bx^2+cx^4\right)^{3/2}}{9c}+\frac{(4b(2bB-3Ac))}{2}$$
$$=\frac{4b(2bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{105c^3x}-\frac{(2bB-3Ac)x\left(bx^2+cx^4\right)^{3/2}}{21c^2}+\frac{Bx^3(bx}{}$$
$$=-\frac{8b^2(2bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{315c^4x^3}+\frac{4b(2bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{105c^3x}-\frac{(2b}{}$$

Mathematica [A]
time = 0.06, size = 82, normalized size = 0.63

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(-16b^3B+24b^2c\left(A+Bx^2\right)-6bc^2x^2\left(6A+5Bx^2\right)+5c^3x^4\left(9A+7Bx^2\right)\right)}{315c^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
[Out] ((x^2*(b + c*x^2))^(3/2)*(-16*b^3*B + 24*b^2*c*(A + B*x^2) - 6*b*c^2*x^2*(6
*A + 5*B*x^2) + 5*c^3*x^4*(9*A + 7*B*x^2)))/(315*c^4*x^3)
```

Maple [A]
time = 0.41, size = 91, normalized size = 0.69

method	result	size
gosper	$\frac{\left(c\,x^2+b\right)\left(35B\,c^3x^6+45A\,c^3x^4-30Bb\,c^2x^4-36Ab\,c^2x^2+24B\,b^2c\,x^2+24A\,b^2c-16B\,b^3\right)\sqrt{x^4c+b\,x^2}}{315c^4x}$	91
default	$\frac{\left(c\,x^2+b\right)\left(35B\,c^3x^6+45A\,c^3x^4-30Bb\,c^2x^4-36Ab\,c^2x^2+24B\,b^2c\,x^2+24A\,b^2c-16B\,b^3\right)\sqrt{x^4c+b\,x^2}}{315c^4x}$	91
trager	$\frac{\left(35B\,c^4x^8+45A\,c^4x^6+5Bb\,c^3x^6+9Ab\,c^3x^4-6B\,b^2c^2x^4-12A\,b^2c^2x^2+8B\,b^3c\,x^2+24A\,b^3c-16B\,b^4\right)\sqrt{x^4c+b\,x^2}}{315c^4x}$	108
risch	$\frac{\sqrt{x^2\left(c\,x^2+b\right)}\,\left(35B\,c^4x^8+45A\,c^4x^6+5Bb\,c^3x^6+9Ab\,c^3x^4-6B\,b^2c^2x^4-12A\,b^2c^2x^2+8B\,b^3c\,x^2+24A\,b^3c-16B\,b^4\right)}{315x\,c^4}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/315*(c*x^2+b)*(35*B*c^3*x^6+45*A*c^3*x^4-30*B*b*c^2*x^4-36*A*b*c^2*x^2+24
*B*b^2*c*x^2+24*A*b^2*c-16*B*b^3)*(c*x^4+b*x^2)^(1/2)/c^4/x
```

Maxima [A]
time = 0.29, size = 106, normalized size = 0.81

$$\frac{\left(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3\right)\sqrt{cx^2+b}\,A}{105\,c^3}+\frac{\left(35\,c^4x^8+5\,bc^3x^6-6\,b^2c^2x^4+8\,b^3cx^2-16\,b^4\right)\sqrt{cx^2+b}\,B}{315\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^2 + b)*A/c^
3 + 1/315*(35*c^4*x^8 + 5*b*c^3*x^6 - 6*b^2*c^2*x^4 + 8*b^3*c*x^2 - 16*b^4)
*sqrt(c*x^2 + b)*B/c^4
```

Fricas [A]
time = 1.34, size = 106, normalized size = 0.81

$$\frac{\left(35\,Bc^4x^8+5\,(Bbc^3+9\,Ac^4)x^6-16\,Bb^4+24\,Ab^3c-3\,(2\,Bb^2c^2-3\,Abc^3)x^4+4\,(2\,Bb^3c-3\,Ab^2c^2)x^2\right)\sqrt{cx^4+bx^2}}{315\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*B*c^4*x^8 + 5*(B*b*c^3 + 9*A*c^4)*x^6 - 16*B*b^4 + 24*A*b^3*c - 3
*(2*B*b^2*c^2 - 3*A*b*c^3)*x^4 + 4*(2*B*b^3*c - 3*A*b^2*c^2)*x^2)*sqrt(c*x^
4 + b*x^2)/(c^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [A]
time = 0.63, size = 140, normalized size = 1.07

$$\frac{8\left(2\,Bb^{\frac{9}{2}}-3\,Ab^{\frac{7}{2}}c\right)\mathrm{sgn}(x)}{315\,c^4}+\frac{35\,(cx^2+b)^{\frac{9}{2}}B\mathrm{sgn}(x)-135\,(cx^2+b)^{\frac{7}{2}}Bb\mathrm{sgn}(x)+189\,(cx^2+b)^{\frac{5}{2}}Bb^2\mathrm{sgn}(x)-105\,(cx^2+b)^{\frac{3}{2}}Bb^3\mathrm{sgn}(x)+45\,(cx^2+b)^{\frac{7}{2}}Ac\mathrm{sgn}(x)-126\,(cx^2+b)^{\frac{5}{2}}Abc\mathrm{sgn}(x)+105\,(cx^2+b)^{\frac{3}{2}}Ab^2c\mathrm{sgn}(x)}{315\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 8/315*(2*B*b^(9/2) - 3*A*b^(7/2)*c)*sgn(x)/c^4 + 1/315*(35*(c*x^2 + b)^(9/2
)*B*sgn(x) - 135*(c*x^2 + b)^(7/2)*B*b*sgn(x) + 189*(c*x^2 + b)^(5/2)*B*b^2
*sgn(x) - 105*(c*x^2 + b)^(3/2)*B*b^3*sgn(x) + 45*(c*x^2 + b)^(7/2)*A*c*sgn
(x) - 126*(c*x^2 + b)^(5/2)*A*b*c*sgn(x) + 105*(c*x^2 + b)^(3/2)*A*b^2*c*sg
n(x))/c^4
```

Mupad [B]
time = 0.25, size = 103, normalized size = 0.79

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{B\,x^8}{9}-\frac{16\,B\,b^4-24\,A\,b^3\,c}{315\,c^4}+\frac{x^6\,\left(45\,A\,c^4+5\,B\,b\,c^3\right)}{315\,c^4}-\frac{4\,b^2\,x^2\,\left(3\,A\,c-2\,B\,b\right)}{315\,c^3}+\frac{b\,x^4\,\left(3\,A\,c-2\,B\,b\right)}{105\,c^2}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((B*x^8)/9 - (16*B*b^4 - 24*A*b^3*c)/(315*c^4) + (x^
6*(45*A*c^4 + 5*B*b*c^3))/(315*c^4) - (4*b^2*x^2*(3*A*c - 2*B*b))/(315*c^3)
 + (b*x^4*(3*A*c - 2*B*b))/(105*c^2)))/x
```

3.101 $\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=94

$$\frac{2b(4bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{105c^3x^3}-\frac{(4bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{35c^2x}+\frac{Bx\left(bx^2+cx^4\right)^{3/2}}{7c}$$

```
[Out] 2/105*b*(-7*A*c+4*B*b)*(c*x^4+b*x^2)^(3/2)/c^3/x^3-1/35*(-7*A*c+4*B*b)*(c*x
^4+b*x^2)^(3/2)/c^2/x+1/7*B*x*(c*x^4+b*x^2)^(3/2)/c
```

Rubi [A]
time = 0.11, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2064, 2041, 2025}

$$\frac{2b\left(bx^2+cx^4\right)^{3/2}(4bB-7Ac)}{105c^3x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}(4bB-7Ac)}{35c^2x}+\frac{Bx\left(bx^2+cx^4\right)^{3/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*b*(4*b*B - 7*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x^3) - ((4*b*B - 7*A*c
)*(b*x^2 + c*x^4)^(3/2))/(35*c^2*x) + (B*x*(b*x^2 + c*x^4)^(3/2))/(7*c)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
```

```
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx &= \frac{Bx(bx^2+cx^4)^{3/2}}{7c}-\frac{(4bB-7Ac)\int x^2\sqrt{bx^2+cx^4}\,dx}{7c}\\ &=-\frac{(4bB-7Ac)(bx^2+cx^4)^{3/2}}{35c^2x}+\frac{Bx(bx^2+cx^4)^{3/2}}{7c}+\frac{(2b(4bB-7Ac))\int}{35c}\\ &=\frac{2b(4bB-7Ac)(bx^2+cx^4)^{3/2}}{105c^3x^3}-\frac{(4bB-7Ac)(bx^2+cx^4)^{3/2}}{35c^2x}+\frac{Bx(bx^2}{7}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 64, normalized size = 0.68

$$\frac{(x^2(b+cx^2))^{3/2}(8b^2B+3c^2x^2(7A+5Bx^2)-2bc(7A+6Bx^2))}{105c^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(8*b^2*B + 3*c^2*x^2*(7*A + 5*B*x^2) - 2*b*c*(7*A
+ 6*B*x^2)))/(105*c^3*x^3)
```

Maple [A]
time = 0.41, size = 67, normalized size = 0.71

method	result	size
gosper	$-\frac{(c\,x^2+b)(-15B\,c^2x^4-21A\,c^2x^2+12bB\,x^2c+14Abc-8b^2B)\sqrt{x^4c+b\,x^2}}{105c^3x}$	67
default	$-\frac{(c\,x^2+b)(-15B\,c^2x^4-21A\,c^2x^2+12bB\,x^2c+14Abc-8b^2B)\sqrt{x^4c+b\,x^2}}{105c^3x}$	67
trager	$-\frac{(-15B\,c^3x^6-21A\,c^3x^4-3Bb\,c^2x^4-7Ab\,c^2x^2+4B\,b^2c\,x^2+14A\,b^2c-8B\,b^3)\sqrt{x^4c+b\,x^2}}{105c^3x}$	84
risch	$-\frac{\sqrt{x^2\,(c\,x^2+b)}\,(-15B\,c^3x^6-21A\,c^3x^4-3Bb\,c^2x^4-7Ab\,c^2x^2+4B\,b^2c\,x^2+14A\,b^2c-8B\,b^3)}{105x\,c^3}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(c*x^2+b)*(-15*B*c^2*x^4-21*A*c^2*x^2+12*B*b*c*x^2+14*A*b*c-8*B*b^2)
*(c*x^4+b*x^2)^(1/2)/c^3/x
```

Maxima [A]
time = 0.28, size = 83, normalized size = 0.88

$$\frac{\left(3\,c^{2}x^{4}+bcx^{2}-2\,b^{2}\right)\sqrt{cx^{2}+b}\,A}{15\,c^{2}}+\frac{\left(15\,c^{3}x^{6}+3\,bc^{2}x^{4}-4\,b^{2}cx^{2}+8\,b^{3}\right)\sqrt{cx^{2}+b}\,B}{105\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^2 + b)*A/c^2 + 1/105*(15*c^3*x^
6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^2 + b)*B/c^3
```

Fricas [A]
time = 1.50, size = 82, normalized size = 0.87

$$\frac{\left(15\,Bc^{3}x^{6}+3\,\left(Bbc^{2}+7\,Ac^{3}\right)x^{4}+8\,Bb^{3}-14\,Ab^{2}c-\left(4\,Bb^{2}c-7\,Abc^{2}\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}}}{105\,c^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*c^3*x^6 + 3*(B*b*c^2 + 7*A*c^3)*x^4 + 8*B*b^3 - 14*A*b^2*c - (4
*B*b^2*c - 7*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{x^{2}\left(b+cx^{2}\right)}\,\left(A+Bx^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [A]
time = 0.58, size = 105, normalized size = 1.12

$$-\frac{2\,\left(4\,Bb^{\frac{7}{2}}-7\,Ab^{\frac{5}{2}}c\right)\mathrm{sgn}(x)}{105\,c^{3}}+\frac{15\,(cx^{2}+b)^{\frac{7}{2}}B\mathrm{sgn}(x)-42\,(cx^{2}+b)^{\frac{5}{2}}Bb\mathrm{sgn}(x)+35\,(cx^{2}+b)^{\frac{3}{2}}Bb^{2}\mathrm{sgn}(x)+21\,(cx^{2}+b)^{\frac{5}{2}}Ac\mathrm{sgn}(x)-35\,(cx^{2}+b)^{\frac{3}{2}}Abc\mathrm{sgn}(x)}{105\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/105*(4*B*b^(7/2) - 7*A*b^(5/2)*c)*sgn(x)/c^3 + 1/105*(15*(c*x^2 + b)^(7/
2)*B*sgn(x) - 42*(c*x^2 + b)^(5/2)*B*b*sgn(x) + 35*(c*x^2 + b)^(3/2)*B*b^2*
sgn(x) + 21*(c*x^2 + b)^(5/2)*A*c*sgn(x) - 35*(c*x^2 + b)^(3/2)*A*b*c*sgn(x
))/c^3
```

Mupad [B]
time = 0.19, size = 83, normalized size = 0.88

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{B\,x^6}{7}+\frac{8\,B\,b^3-14\,A\,b^2\,c}{105\,c^3}+\frac{x^4\,\left(21\,A\,c^3+3\,B\,b\,c^2\right)}{105\,c^3}+\frac{b\,x^2\,\left(7\,A\,c-4\,B\,b\right)}{105\,c^2}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((B*x^6)/7 + (8*B*b^3 - 14*A*b^2*c)/(105*c^3) + (x^4
*(21*A*c^3 + 3*B*b*c^2))/(105*c^3) + (b*x^2*(7*A*c - 4*B*b))/(105*c^2)))/x
```

3.102 $\int (A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=61

$$-\frac{(2bB-5Ac)\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{5cx}$$

```
[Out] -1/15*(-5*A*c+2*B*b)*(c*x^4+b*x^2)^(3/2)/c^2/x^3+1/5*B*(c*x^4+b*x^2)^(3/2)/
c/x
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1159, 2025}

$$\frac{B\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{\left(bx^2+cx^4\right)^{3/2}(2bB-5Ac)}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -1/15*((2*b*B - 5*A*c)*(b*x^2 + c*x^4)^(3/2))/(c^2*x^3) + (B*(b*x^2 + c*x^4
)^(3/2))/(5*c*x)
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[e*((b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 3)*x)), x] - Dist[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3)), Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
c*d*(4*p + 3), 0]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx &= \frac{B\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{(2bB-5Ac)\int \sqrt{bx^2+cx^4}\,dx}{5c}\\ &= -\frac{(2bB-5Ac)\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{5cx}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 41, normalized size = 0.67

$$\frac{(x^2(b+cx^2))^{3/2}(-2bB+5Ac+3Bcx^2)}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(-2*b*B + 5*A*c + 3*B*c*x^2))/(15*c^2*x^3)
```

Maple [A]
time = 0.38, size = 45, normalized size = 0.74

method	result	size
gosper	$\frac{(c\,x^2+b)(3Bc\,x^2+5Ac-2Bb)\sqrt{x^4c+b\,x^2}}{15c^2x}$	45
default	$\frac{(c\,x^2+b)(3Bc\,x^2+5Ac-2Bb)\sqrt{x^4c+b\,x^2}}{15c^2x}$	45
trager	$\frac{(3B\,c^2x^4+5A\,c^2x^2+bB\,x^2c+5Abc-2b^2B)\sqrt{x^4c+b\,x^2}}{15c^2x}$	59
risch	$\frac{\sqrt{x^2\,(c\,x^2+b)}\,(3B\,c^2x^4+5A\,c^2x^2+bB\,x^2c+5Abc-2b^2B)}{15x\,c^2}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(c*x^2+b)*(3*B*c*x^2+5*A*c-2*B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x
```

Maxima [A]
time = 0.29, size = 51, normalized size = 0.84

$$\frac{(cx^2+b)^{\frac{3}{2}}A}{3\,c}+\frac{(3\,c^2x^4+bcx^2-2\,b^2)\sqrt{cx^2+b}\,B}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^2 + b)^(3/2)*A/c + 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^2 +
 b)*B/c^2
```

Fricas [A]
time = 1.89, size = 57, normalized size = 0.93

$$\frac{(3\,Bc^2x^4-2\,Bb^2+5\,Abc+(Bbc+5\,Ac^2)x^2)\sqrt{cx^4+bx^2}}{15\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*c^2*x^4 - 2*B*b^2 + 5*A*b*c + (B*b*c + 5*A*c^2)*x^2)*sqrt(c*x^4 +
 b*x^2)/(c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [A]
time = 0.48, size = 72, normalized size = 1.18

$$\frac{\left(2\,Bb^{\frac{5}{2}}-5\,Ab^{\frac{3}{2}}c\right)\mathrm{sgn}(x)}{15\,c^2}+\frac{3\,(cx^2+b)^{\frac{5}{2}}B\mathrm{sgn}(x)-5\,(cx^2+b)^{\frac{3}{2}}Bb\mathrm{sgn}(x)+5\,(cx^2+b)^{\frac{3}{2}}Ac\mathrm{sgn}(x)}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(2*B*b^(5/2) - 5*A*b^(3/2)*c)*sgn(x)/c^2 + 1/15*(3*(c*x^2 + b)^(5/2)*B
*sgn(x) - 5*(c*x^2 + b)^(3/2)*B*b*sgn(x) + 5*(c*x^2 + b)^(3/2)*A*c*sgn(x))/
c^2
```

Mupad [B]
time = 0.15, size = 60, normalized size = 0.98

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{B\,x^4}{5}-\frac{2\,B\,b^2-5\,A\,b\,c}{15\,c^2}+\frac{x^2\,\left(5\,A\,c^2+B\,b\,c\right)}{15\,c^2}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((B*x^4)/5 - (2*B*b^2 - 5*A*b*c)/(15*c^2) + (x^2*(5*
A*c^2 + B*b*c))/(15*c^2)))/x
```

3.103 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}\,dx$

Optimal. Leaf size=78

$$\frac{A\sqrt{bx^2+cx^4}}{x}+\frac{B(bx^2+cx^4)^{3/2}}{3cx^3}-A\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] 1/3*B*(c*x^4+b*x^2)^(3/2)/c/x^3-A*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))*b^
(1/2)+A*(c*x^4+b*x^2)^(1/2)/x
```

Rubi [A]
time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2064, 2046, 2033, 212}

$$\frac{A\sqrt{bx^2+cx^4}}{x}-A\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)+\frac{B(bx^2+cx^4)^{3/2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^2,x]
```

```
[Out] (A*Sqrt[b*x^2 + c*x^4])/x + (B*(b*x^2 + c*x^4)^(3/2))/(3*c*x^3) - A*Sqrt[b]
*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}\,dx &= \frac{B(bx^2+cx^4)^{3/2}}{3cx^3}+A\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx\\ &= \frac{A\sqrt{bx^2+cx^4}}{x}+\frac{B(bx^2+cx^4)^{3/2}}{3cx^3}+(Ab)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\ &= \frac{A\sqrt{bx^2+cx^4}}{x}+\frac{B(bx^2+cx^4)^{3/2}}{3cx^3}-(Ab)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+c}}\right.\\ &= \frac{A\sqrt{bx^2+cx^4}}{x}+\frac{B(bx^2+cx^4)^{3/2}}{3cx^3}-A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)\end{aligned}$$

Mathematica [A]
time = 0.08, size = 84, normalized size = 1.08

$$\frac{x\left((b+cx^2)\,(bB+3Ac+Bcx^2)-3A\sqrt{b}\,c\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{3c\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^2,x]
```

```
[Out] (x*((b + c*x^2)*(b*B + 3*A*c + B*c*x^2) - 3*A*Sqrt[b]*c*Sqrt[b + c*x^2]*Arc
Tanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(3*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.39, size = 85, normalized size = 1.09

method	result	size
default	$-\frac{\sqrt{x^{4}c+b\,x^{2}}\,\left(3A\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)\sqrt{b}\,c-B\left(c\,x^{2}+b\right)^{\frac{3}{2}}-3A\sqrt{c\,x^{2}+b}\,c\right)}{3x\sqrt{c\,x^{2}+b}\,c}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*x^4+b*x^2)^(1/2)*(3*A*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b^(1/2)*c
-B*(c*x^2+b)^(3/2)-3*A*(c*x^2+b)^(1/2)*c)/x/(c*x^2+b)^(1/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] A*integrate(sqrt(c*x^2 + b)/x, x) + 1/3*(c*x^2 + b)^(3/2)*B/c
```

Fricas [A]
time = 2.44, size = 159, normalized size = 2.04

$$\left[\frac{3\,A\sqrt{b}\,cx\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\,(Bcx^2+Bb+3\,Ac)}{6\,cx},\ \frac{3\,A\sqrt{-b}\,cx\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\,(Bcx^2+Bb+3\,Ac)}{3\,cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*A*sqrt(b)*c*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/
x^3) + 2*sqrt(c*x^4 + b*x^2)*(B*c*x^2 + B*b + 3*A*c))/(c*x), 1/3*(3*A*sqrt(
-b)*c*x*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b
*x^2)*(B*c*x^2 + B*b + 3*A*c))/(c*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\,(b+cx^2)}\,(A+Bx^2)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**2, x)
```

Giac [A]
time = 0.59, size = 116, normalized size = 1.49

$$\frac{Ab\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}}-\frac{\left(3\,Abc\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+B\sqrt{-b}\,b^{\frac{3}{2}}+3\,A\sqrt{-b}\,\sqrt{b}\,c\right)\mathrm{sgn}(x)}{3\,\sqrt{-b}\,c}+\frac{(cx^2+b)^{\frac{3}{2}}Bc^2\mathrm{sgn}(x)+3\,\sqrt{cx^2+b}\,Ac^3\mathrm{sgn}(x)}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] A*b*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - 1/3*(3*A*b*c*arctan(
sqrt(b)/sqrt(-b)) + B*sqrt(-b)*b^(3/2) + 3*A*sqrt(-b)*sqrt(b)*c)*sgn(x)/(sq
rt(-b)*c) + 1/3*((c*x^2 + b)^(3/2)*B*c^2*sgn(x) + 3*sqrt(c*x^2 + b)*A*c^3*s
gn(x))/c^3
```

Mupad [B]
time = 0.50, size = 99, normalized size = 1.27

$$\frac{A\sqrt{c\,x^4+b\,x^2}}{x}+\frac{B\,\left(c\,x^2+b\right)\,\sqrt{c\,x^4+b\,x^2}}{3\,c\,x}+\frac{A\,\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,1{}\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c\,x^4+b\,x^2}\,1{}\mathrm{i}}{\sqrt{c}\,x^2\,\sqrt{\frac{b}{c\,x^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^2,x)
```

```
[Out] (A*(b*x^2 + c*x^4)^(1/2))/x + (B*(b + c*x^2)*(b*x^2 + c*x^4)^(1/2))/(3*c*x)
 + (A*b^(1/2)*asin((b^(1/2)*1i)/(c^(1/2)*x))*(b*x^2 + c*x^4)^(1/2)*1i)/(c^(
1/2)*x^2*(b/(c*x^2) + 1)^(1/2))
```

3.104 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx$

Optimal. Leaf size=100

$$\frac{(2bB+Ac)\sqrt{bx^2+cx^4}}{2bx}-\frac{A(bx^2+cx^4)^{3/2}}{2bx^5}-\frac{(2bB+Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}$$

```
[Out] -1/2*A*(c*x^4+b*x^2)^(3/2)/b/x^5-1/2*(A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b
*x^2)^(1/2))/b^(1/2)+1/2*(A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b/x
```

Rubi [A]
time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2063, 2046, 2033, 212}

$$\frac{\sqrt{bx^2+cx^4}\,(Ac+2bB)}{2bx}-\frac{(Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}-\frac{A(bx^2+cx^4)^{3/2}}{2bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^4,x]
```

```
[Out] ((2*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(2*b*x) - (A*(b*x^2 + c*x^4)^(3/2))/(2*
b*x^5) - ((2*b*B + A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*Sqrt[b
])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
```

```
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx = -\frac{A(bx^2+cx^4)^{3/2}}{2bx^5} - \frac{(-2bB-Ac)\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx}{2b}$$

$$= \frac{(2bB+Ac)\sqrt{bx^2+cx^4}}{2bx} - \frac{A(bx^2+cx^4)^{3/2}}{2bx^5} - \frac{1}{2}(-2bB-Ac)\int \frac{1}{\sqrt{bx^2+cx}$$

$$= \frac{(2bB+Ac)\sqrt{bx^2+cx^4}}{2bx} - \frac{A(bx^2+cx^4)^{3/2}}{2bx^5} - \frac{1}{2}(2bB+Ac)\text{Subst}\left(\int \frac{1}{1-b}\right.$$

$$= \frac{(2bB+Ac)\sqrt{bx^2+cx^4}}{2bx} - \frac{A(bx^2+cx^4)^{3/2}}{2bx^5} - \frac{(2bB+Ac)\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{bx^2+}}\right.}{2\sqrt{b}}$$

Mathematica [A]
time = 0.10, size = 94, normalized size = 0.94

$$-\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}(A-2Bx^2)\sqrt{b+cx^2}+(2bB+Ac)x^2\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{2\sqrt{b}\,x^3\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^4,x]
```

```
[Out] -1/2*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*(A - 2*B*x^2)*Sqrt[b + c*x^2] + (2*b*B
 + A*c)*x^2*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(Sqrt[b]*x^3*Sqrt[b + c*x^2]
)
```

Maple [A]
time = 0.39, size = 135, normalized size = 1.35

method	result
risch	$-\frac{A\sqrt{x^2(cx^2+b)}}{2x^3}+\frac{\left(B\sqrt{cx^2+b}-\frac{\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{cx^2+b}}{x}\right)Ac}{2\sqrt{b}}-\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{cx^2+b}}{x}\right)B\right)\sqrt{x}}{x\sqrt{cx^2+b}}$
default	$-\frac{\sqrt{x^4c+bx^2}\left(A\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{cx^2+b}}{x}\right)cx^2+2Bb^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{cx^2+b}}{x}\right)x^2-A\sqrt{cx^2+b}\,cx^2-\right.}{2x^3\sqrt{cx^2+b}\,b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x^4+b*x^2)^(1/2)*(A*b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c*x
^2+2*B*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^2-A*(c*x^2+b)^(1/2)*c*
x^2-2*B*(c*x^2+b)^(1/2)*b*x^2+A*(c*x^2+b)^(3/2))/x^3/(c*x^2+b)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^4, x)
```

Fricas [A]
time = 1.39, size = 169, normalized size = 1.69

$$\left[\frac{(2\,Bb+Ac)\sqrt{b}\,x^3\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\,(2\,Bbx^2-Ab)}{4\,bx^3},\ \frac{(2\,Bb+Ac)\sqrt{-b}\,x^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\,(2\,Bbx^2-Ab)}{2\,bx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/4*((2*B*b + A*c)*sqrt(b)*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)
*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*B*b*x^2 - A*b))/(b*x^3), 1/2*((2*
B*b + A*c)*sqrt(-b)*x^3*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x))
+ sqrt(c*x^4 + b*x^2)*(2*B*b*x^2 - A*b))/(b*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\,(b+cx^2)}\,(A+Bx^2)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**4, x)
```

Giac [A]
time = 0.54, size = 76, normalized size = 0.76

$$\frac{2\sqrt{cx^2+b}\,Bc\mathrm{sgn}(x) + \frac{(2\,Bbc\mathrm{sgn}(x)+Ac^2\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{\sqrt{cx^2+b}\,Ac\mathrm{sgn}(x)}{x^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(c*x^2 + b)*B*c*sgn(x) + (2*B*b*c*sgn(x) + A*c^2*sgn(x))*arctan(
sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) - sqrt(c*x^2 + b)*A*c*sgn(x)/x^2)/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^4,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^4, x)
```

3.105 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx$

Optimal. Leaf size=103

$$-\frac{(4bB-Ac)\sqrt{bx^2+cx^4}}{8bx^3}-\frac{A(bx^2+cx^4)^{3/2}}{4bx^7}-\frac{c(4bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}$$

```
[Out] -1/4*A*(c*x^4+b*x^2)^(3/2)/b/x^7-1/8*c*(-A*c+4*B*b)*arctanh(x*b^(1/2)/(c*x^
4+b*x^2)^(1/2))/b^(3/2)-1/8*(-A*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^3
```

Rubi [A]
time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2063, 2045, 2033, 212}

$$-\frac{c(4bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}-\frac{\sqrt{bx^2+cx^4}\,(4bB-Ac)}{8bx^3}-\frac{A(bx^2+cx^4)^{3/2}}{4bx^7}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^6,x]
```

```
[Out] -1/8*((4*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(b*x^3) - (A*(b*x^2 + c*x^4)^(3/2)
)/(4*b*x^7) - (c*(4*b*B - A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8
*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
```

```
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx = -\frac{A(bx^2+cx^4)^{3/2}}{4bx^7} - \frac{(-4bB+Ac)\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx}{4b}$$

$$= -\frac{(4bB-Ac)\sqrt{bx^2+cx^4}}{8bx^3} - \frac{A(bx^2+cx^4)^{3/2}}{4bx^7} + \frac{(c(4bB-Ac))\int \frac{1}{\sqrt{bx^2+c}}$$

$$= -\frac{(4bB-Ac)\sqrt{bx^2+cx^4}}{8bx^3} - \frac{A(bx^2+cx^4)^{3/2}}{4bx^7} - \frac{(c(4bB-Ac))\text{Subst}\left(\int \frac{1}{1-b}\right.}{8b}$$

$$= -\frac{(4bB-Ac)\sqrt{bx^2+cx^4}}{8bx^3} - \frac{A(bx^2+cx^4)^{3/2}}{4bx^7} - \frac{c(4bB-Ac)\tanh^{-1}\left(\frac{}{\sqrt{bx}}\right.}{8b^{3/2}}$$

Mathematica [A]
time = 0.13, size = 106, normalized size = 1.03

$$-\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}\sqrt{b+cx^2}\,(2Ab+4bBx^2+Acx^2)+c(4bB-Ac)x^4\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{8b^{3/2}x^5\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^6,x]
```

```
[Out] -1/8*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(2*A*b + 4*b*B*x^2 + A
*c*x^2) + c*(4*b*B - A*c)*x^4*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(b^(3/2)*x
^5*Sqrt[b + c*x^2])
```

Maple [A]
time = 0.40, size = 175, normalized size = 1.70

method	result
risch	$-\frac{\left(A c \,x^{2}+4 b B \,x^{2}+2 A b \right) \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{8 x^{5} b}+\frac{\left(\frac{c^{2} \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c \,x^{2}+b}}{x}\right) A}{8 b^{\frac{3}{2}}}-\frac{c \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c \,x^{2}+b}}{x}\right) B}{2 \sqrt{b}}\right) \sqrt{x^{2}}}{x \sqrt{c \,x^{2}+b}}$
default	$-\frac{\sqrt{x^{4} c +b \,x^{2}}\, \left(-A \sqrt{b}\, \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c \,x^{2}+b}}{x}\right) c^{2} x^{4}+4 B \,b^{\frac{3}{2}} \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c \,x^{2}+b}}{x}\right) c \,x^{4}+A \sqrt{c \,x^{2}+b}\, c}{8 x^{5} \sqrt{c \,x^{2}+b}\, b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(c*x^4+b*x^2)^(1/2)*(-A*b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c^
2*x^4+4*B*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c*x^4+A*(c*x^2+b)^(1/
2)*c^2*x^4-4*B*(c*x^2+b)^(1/2)*b*c*x^4-A*(c*x^2+b)^(3/2)*c*x^2+4*B*(c*x^2+b
)^(3/2)*b*x^2+2*A*(c*x^2+b)^(3/2)*b)/x^5/(c*x^2+b)^(1/2)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^6, x)
```

Fricas [A]
time = 1.62, size = 198, normalized size = 1.92

$$\left[-\frac{(4\,Bbc - Ac^2)\sqrt{b}\,x^5\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right) + 2\,\sqrt{cx^4+bx^2}\,(2\,Ab^2 + (4\,Bb^2 + Abc)x^2)}{16\,b^2x^5}, \frac{(4\,Bbc - Ac^2)\sqrt{-b}\,x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right) - \sqrt{cx^4+bx^2}\,(2\,Ab^2 + (4\,Bb^2 + Abc)x^2)}{8\,b^2x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [-1/16*((4*B*b*c - A*c^2)*sqrt(b)*x^5*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 +
b*x^2)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^2 + A*b*c)*x
^2))/(b^2*x^5), 1/8*((4*B*b*c - A*c^2)*sqrt(-b)*x^5*arctan(sqrt(c*x^4 + b*x
^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^2 + A*b
*c)*x^2))/(b^2*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 (b + cx^2)}\,(A + Bx^2)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**6, x)
```

Giac [A]
time = 0.56, size = 132, normalized size = 1.28

$$\frac{\frac{(4\,Bbc^2\mathrm{sgn}(x) - Ac^3\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b} - \frac{4\,(cx^2+b)^{\frac{3}{2}}Bbc^2\mathrm{sgn}(x) - 4\,\sqrt{cx^2+b}\,Bb^2c^2\mathrm{sgn}(x) + (cx^2+b)^{\frac{3}{2}}Ac^3\mathrm{sgn}(x) + \sqrt{cx^2+b}\,Abc^3\mathrm{sgn}(x)}{bc^2x^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/8*((4*B*b*c^2*sgn(x) - A*c^3*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sq
rt(-b)*b) - (4*(c*x^2 + b)^(3/2)*B*b*c^2*sgn(x) - 4*sqrt(c*x^2 + b)*B*b^2*c
^2*sgn(x) + (c*x^2 + b)^(3/2)*A*c^3*sgn(x) + sqrt(c*x^2 + b)*A*b*c^3*sgn(x)
)/(b*c^2*x^4))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^2 + A)\,\sqrt{c\,x^4 + b\,x^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^6,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^6, x)
```

3.106 $\int x^5(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=223

$$\frac{b^4(9bB-14Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{2048c^5}-\frac{b^2(9bB-14Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{768c^4}+\frac{b(9bB-14Ac)\left(bx^2+\right.}{240c^3}$$

```
[Out] -1/768*b^2*(-14*A*c+9*B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(3/2)/c^4+1/240*b*(-14
*A*c+9*B*b)*(c*x^4+b*x^2)^(5/2)/c^3-1/168*(-14*A*c+9*B*b)*x^2*(c*x^4+b*x^2)
^(5/2)/c^2+1/14*B*x^4*(c*x^4+b*x^2)^(5/2)/c-1/2048*b^6*(-14*A*c+9*B*b)*arct
anh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(11/2)+1/2048*b^4*(-14*A*c+9*B*b)*(2
*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^5
```

Rubi [A]
time = 0.26, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.269$, Rules used = {2059, 808, 684, 654, 626, 634, 212}

$$-\frac{b^6(9bB-14Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2048c^{11/2}}+\frac{b^4\left(b+2cx^2\right)\sqrt{bx^2+cx^4}\,(9bB-14Ac)}{2048c^5}-\frac{b^2\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}(9bB-14Ac)}{768c^4}+\frac{b\left(bx^2+cx^4\right)^{5/2}(9bB-14Ac)}{240c^3}-\frac{x^2\left(bx^2+cx^4\right)^{5/2}(9bB-14Ac)}{168c^2}+\frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{14c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (b^4*(9*b*B - 14*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(2048*c^5) - (b^2*
(9*b*B - 14*A*c)*(b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(768*c^4) + (b*(9*b*B
 - 14*A*c)*(b*x^2 + c*x^4)^(5/2))/(240*c^3) - ((9*b*B - 14*A*c)*x^2*(b*x^2
+ c*x^4)^(5/2))/(168*c^2) + (B*x^4*(b*x^2 + c*x^4)^(5/2))/(14*c) - (b^6*(9*
b*B - 14*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2048*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x^5 (A+Bx^2) \left(bx^2+cx^4\right)^{3/2} \, dx = \frac{1}{2}\text{Subst}\left(\int x^2(A+Bx) \left(bx+cx^2\right)^{3/2} \, dx, x, x^2\right)$$

$$= \frac{Bx^4(bx^2+cx^4)^{5/2}}{14c} + \frac{\left(2(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int x^2(bx\right.}{14c}$$

$$= -\frac{(9bB-14Ac)x^2(bx^2+cx^4)^{5/2}}{168c^2} + \frac{Bx^4(bx^2+cx^4)^{5/2}}{14c} + \frac{(b(9bB-14\right.}{}$$

$$= \frac{b(9bB-14Ac)\left(bx^2+cx^4\right)^{5/2}}{240c^3} - \frac{(9bB-14Ac)x^2(bx^2+cx^4)^{5/2}}{168c^2} + \frac{Bx}{}$$

$$= -\frac{b^2(9bB-14Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{768c^4} + \frac{b(9bB-14Ac)\left(bx^2+c\right.}{240c^3}$$

$$= \frac{b^4(9bB-14Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{2048c^5} - \frac{b^2(9bB-14Ac)\left(b+2cx^2\right)}{768c^4}$$

$$= \frac{b^4(9bB-14Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{2048c^5} - \frac{b^2(9bB-14Ac)\left(b+2cx^2\right)}{768c^4}$$

$$= \frac{b^4(9bB-14Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{2048c^5} - \frac{b^2(9bB-14Ac)\left(b+2cx^2\right)}{768c^4}$$

Mathematica [A]
time = 0.35, size = 209, normalized size = 0.94

$$\frac{x\left(\sqrt{c}\,x(b+cx^2)\left(945b^6B-210b^5c(7A+3Bx^2)+96b^2c^4x^6(7A+4Bx^2)+2560c^6x^{10}(7A+6Bx^2)+28b^4c^2x^2(35A+18Bx^2)-16b^3c^3x^4(49A+27Bx^2)+256bc^5x^8(91A+75Bx^2)\right)+105b^6(9bB-14Ac)\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{215040c^{11/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(945*b^6*B - 210*b^5*c*(7*A + 3*B*x^2) + 96*b^2*c
^4*x^6*(7*A + 4*B*x^2) + 2560*c^6*x^10*(7*A + 6*B*x^2) + 28*b^4*c^2*x^2*(35
*A + 18*B*x^2) - 16*b^3*c^3*x^4*(49*A + 27*B*x^2) + 256*b*c^5*x^8*(91*A + 7
5*B*x^2)) + 105*b^6*(9*b*B - 14*A*c)*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqr
t[b + c*x^2]]))/(215040*c^(11/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.44, size = 328, normalized size = 1.47

method	result

risch	$-\frac{\left(-15360 B\,c^{6} x^{12}-17920 A\,c^{6} x^{10}-19200 B b\,c^{5} x^{10}-23296 A b\,c^{5} x^{8}-384 B\,b^{2} c^{4} x^{8}-672 A\,b^{2} c^{4} x^{6}+432 B\,b^{3} c^{3} x^{6}+784 A\,b^{3} c^{3} x^{4}-504 B\,b^{4}\right.}{215040 c^{5}}$
default	$\frac{\left(x^{4} c+b\,x^{2}\right)^{\frac{3}{2}} \left(15360 B \left(c\,x^{2}+b\right)^{\frac{5}{2}} c^{\frac{9}{2}} x^{9}+17920 A \left(c\,x^{2}+b\right)^{\frac{5}{2}} c^{\frac{9}{2}} x^{7}-11520 B \left(c\,x^{2}+b\right)^{\frac{5}{2}} c^{\frac{7}{2}} b\,x^{7}-12544 A \left(c\,x^{2}+b\right)^{\frac{5}{2}} c^{\frac{7}{2}} b\,x^{5}+8064 B \left(c\,x^{2}+b\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/215040*(c*x^4+b*x^2)^(3/2)*(15360*B*(c*x^2+b)^(5/2)*c^(9/2)*x^9+17920*A*(
c*x^2+b)^(5/2)*c^(9/2)*x^7-11520*B*(c*x^2+b)^(5/2)*c^(7/2)*b*x^7-12544*A*(c
*x^2+b)^(5/2)*c^(7/2)*b*x^5+8064*B*(c*x^2+b)^(5/2)*c^(5/2)*b^2*x^5+7840*A*(
c*x^2+b)^(5/2)*c^(5/2)*b^2*x^3-5040*B*(c*x^2+b)^(5/2)*c^(3/2)*b^3*x^3-3920*
A*(c*x^2+b)^(5/2)*c^(3/2)*b^3*x+980*A*(c*x^2+b)^(3/2)*c^(3/2)*b^4*x+2520*B*
(c*x^2+b)^(5/2)*c^(1/2)*b^4*x+1470*A*(c*x^2+b)^(1/2)*c^(3/2)*b^5*x-630*B*(c
*x^2+b)^(3/2)*c^(1/2)*b^5*x-945*B*(c*x^2+b)^(1/2)*c^(1/2)*b^6*x+1470*A*ln(c
^(1/2)*x+(c*x^2+b)^(1/2))*b^6*c-945*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^7)/x^
3/(c*x^2+b)^(3/2)/c^(11/2)
```

Maxima [A]
time = 0.29, size = 363, normalized size = 1.63

$$-\frac{1}{30720} \left(\frac{420 \sqrt{cx^4+bx^2}\, b^4 x^2}{c^3} - \frac{1120 (cx^4+bx^2)^{\frac{3}{2}} b^2 x^2}{c^2} - \frac{2560 (cx^4+bx^2)^{\frac{5}{2}} x^2}{c} - \frac{105 b^6 \log\left(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{9}{2}}} + \frac{210 \sqrt{cx^4+bx^2}\, b^5}{c^4} - \frac{560 (cx^4+bx^2)^{\frac{3}{2}} b^3}{c^3} + \frac{1792 (cx^4+bx^2)^{\frac{5}{2}} b}{c^2}\right) A + \frac{1}{143360} \left(\frac{10240 (cx^4+bx^2)^{\frac{5}{2}} x^4}{c} + \frac{1260 \sqrt{cx^4+bx^2}\, b^5 x^2}{c^4} - \frac{3360 (cx^4+bx^2)^{\frac{3}{2}} b^3 x^2}{c^3} - \frac{7680 (cx^4+bx^2)^{\frac{5}{2}} b x^2}{c^2} - \frac{315 b^7 \log\left(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{11}{2}}} + \frac{630 \sqrt{cx^4+bx^2}\, b^6}{c^5} - \frac{1680 (cx^4+bx^2)^{\frac{3}{2}} b^4}{c^4} + \frac{5376 (cx^4+bx^2)^{\frac{5}{2}} b^2}{c^3}\right) B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/30720*(420*sqrt(c*x^4 + b*x^2)*b^4*x^2/c^3 - 1120*(c*x^4 + b*x^2)^(3/2)*
b^2*x^2/c^2 - 2560*(c*x^4 + b*x^2)^(5/2)*x^2/c - 105*b^6*log(2*c*x^2 + b +
2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) + 210*sqrt(c*x^4 + b*x^2)*b^5/c^4 -
560*(c*x^4 + b*x^2)^(3/2)*b^3/c^3 + 1792*(c*x^4 + b*x^2)^(5/2)*b/c^2)*A + 1
/143360*(10240*(c*x^4 + b*x^2)^(5/2)*x^4/c + 1260*sqrt(c*x^4 + b*x^2)*b^5*x
^2/c^4 - 3360*(c*x^4 + b*x^2)^(3/2)*b^3*x^2/c^3 - 7680*(c*x^4 + b*x^2)^(5/2
)*b*x^2/c^2 - 315*b^7*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(1
1/2) + 630*sqrt(c*x^4 + b*x^2)*b^6/c^5 - 1680*(c*x^4 + b*x^2)^(3/2)*b^4/c^4
 + 5376*(c*x^4 + b*x^2)^(5/2)*b^2/c^3)*B
```

Fricas [A]
time = 1.54, size = 418, normalized size = 1.87

$$\left[-\frac{105 (9 Bb^7 - 14 Ab^6 c)\sqrt{c} \log\left(-2cx^2 - b - 2\sqrt{cx^4+bx^2}\sqrt{c}\right) - 2 (15360 Bc^7 x^{12} + 1280 (15 Bbc^6 + 14 Ac^7) x^{10} + 128 (3 Bb^2 c^5 + 182 Abc^6) x^8 + 945 Bb^6 c - 1470 Ab^5 c^2 - 48 (9 Bb^3 c^4 - 14 Ab^2 c^5) x^6 + 56 (9 Bb^4 c^3 - 14 Ab^3 c^4) x^4 - 70 (9 Bb^5 c^2 - 14 Ab^4 c^3) x^2)\sqrt{cx^4+bx^2}}{430080 c^6}, \frac{105 (9 Bb^7 - 14 Ab^6 c)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right) + (15360 Bc^7 x^{12} + 1280 (15 Bbc^6 + 14 Ac^7) x^{10} + 128 (3 Bb^2 c^5 + 182 Abc^6) x^8 + 945 Bb^6 c - 1470 Ab^5 c^2 - 48 (9 Bb^3 c^4 - 14 Ab^2 c^5) x^6 + 56 (9 Bb^4 c^3 - 14 Ab^3 c^4) x^4 - 70 (9 Bb^5 c^2 - 14 Ab^4 c^3) x^2)\sqrt{cx^4+bx^2}}{215040 c^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/430080*(105*(9*B*b^7 - 14*A*b^6*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*
x^4 + b*x^2)*sqrt(c)) - 2*(15360*B*c^7*x^12 + 1280*(15*B*b*c^6 + 14*A*c^7)*
x^10 + 128*(3*B*b^2*c^5 + 182*A*b*c^6)*x^8 + 945*B*b^6*c - 1470*A*b^5*c^2 -
 48*(9*B*b^3*c^4 - 14*A*b^2*c^5)*x^6 + 56*(9*B*b^4*c^3 - 14*A*b^3*c^4)*x^4
- 70*(9*B*b^5*c^2 - 14*A*b^4*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^6, 1/215040*(
105*(9*B*b^7 - 14*A*b^6*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*
x^2 + b)) + (15360*B*c^7*x^12 + 1280*(15*B*b*c^6 + 14*A*c^7)*x^10 + 128*(3*
B*b^2*c^5 + 182*A*b*c^6)*x^8 + 945*B*b^6*c - 1470*A*b^5*c^2 - 48*(9*B*b^3*c
^4 - 14*A*b^2*c^5)*x^6 + 56*(9*B*b^4*c^3 - 14*A*b^3*c^4)*x^4 - 70*(9*B*b^5*
c^2 - 14*A*b^4*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^6]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}} \left(A + Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**5*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

Giac [A]
time = 0.57, size = 280, normalized size = 1.26

$$\frac{1}{215040}\left(2\left(4\left(2\left(8\left(10\left(12\,Bcx^2\mathrm{sgn}(x) + \frac{15\,Bbc^{12}\mathrm{sgn}(x) + 14\,Ac^{13}\mathrm{sgn}(x)}{c^{12}}\right)x^2 + \frac{3\,Bb^2c^{11}\mathrm{sgn}(x) + 182\,Abc^{12}\mathrm{sgn}(x)}{c^{12}}\right)x^2 - \frac{3\left(9\,Bb^3c^{10}\mathrm{sgn}(x) - 14\,Ab^2c^{11}\mathrm{sgn}(x)\right)}{c^{12}}\right)x^2 + \frac{7\left(9\,Bb^4c^9\mathrm{sgn}(x) - 14\,Ab^3c^{10}\mathrm{sgn}(x)\right)}{c^{12}}\right)x^2 - \frac{35\left(9\,Bb^5c^8\mathrm{sgn}(x) - 14\,Ab^4c^9\mathrm{sgn}(x)\right)}{c^{12}}\right)x^2 + \frac{105\left(9\,Bb^6c^7\mathrm{sgn}(x) - 14\,Ab^5c^8\mathrm{sgn}(x)\right)}{c^{12}}\right)\sqrt{cx^2+b}\,x + \frac{\left(9\,Bb^7\mathrm{sgn}(x) - 14\,Ab^6c\mathrm{sgn}(x)\right)\log\left(\left|-\sqrt{c}\,x + \sqrt{cx^2+b}\right|\right)}{2048\,c^{\frac{11}{2}}} - \frac{\left(9\,Bb^7\log\left(|b|\right) - 14\,Ab^6c\log\left(|b|\right)\right)\mathrm{sgn}(x)}{4096\,c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/215040*(2*(4*(2*(8*(10*(12*B*c*x^2*sgn(x) + (15*B*b*c^12*sgn(x) + 14*A*c^
13*sgn(x))/c^12)*x^2 + (3*B*b^2*c^11*sgn(x) + 182*A*b*c^12*sgn(x))/c^12)*x^
2 - 3*(9*B*b^3*c^10*sgn(x) - 14*A*b^2*c^11*sgn(x))/c^12)*x^2 + 7*(9*B*b^4*c
^9*sgn(x) - 14*A*b^3*c^10*sgn(x))/c^12)*x^2 - 35*(9*B*b^5*c^8*sgn(x) - 14*A
*b^4*c^9*sgn(x))/c^12)*x^2 + 105*(9*B*b^6*c^7*sgn(x) - 14*A*b^5*c^8*sgn(x))
/c^12)*sqrt(c*x^2 + b)*x + 1/2048*(9*B*b^7*sgn(x) - 14*A*b^6*c*sgn(x))*log(
abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(11/2) - 1/4096*(9*B*b^7*log(abs(b)) -
 14*A*b^6*c*log(abs(b)))*sgn(x)/c^(11/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.107 $\int x^3(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=167

$$-\frac{b^3(7bB-12Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^4}+\frac{b(7bB-12Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{384c^3}-\frac{(7bB-12Ac-10Bcx}{120c^2}$$

```
[Out] 1/384*b*(-12*A*c+7*B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(3/2)/c^3-1/120*(-10*B*c*
x^2-12*A*c+7*B*b)*(c*x^4+b*x^2)^(5/2)/c^2+1/1024*b^5*(-12*A*c+7*B*b)*arctan
h(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(9/2)-1/1024*b^3*(-12*A*c+7*B*b)*(2*c*
x^2+b)*(c*x^4+b*x^2)^(1/2)/c^4
```

Rubi [A]
time = 0.17, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2059, 793, 626, 634, 212}

$$\frac{b^5(7bB-12Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{9/2}}-\frac{b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}\,(7bB-12Ac)}{1024c^4}+\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}(7bB-12Ac)}{384c^3}-\frac{\left(bx^2+cx^4\right)^{5/2}\left(-12Ac+7bB-10Bcx^2\right)}{120c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -1/1024*(b^3*(7*b*B - 12*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/c^4 + (b*(
7*b*B - 12*A*c)*(b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(384*c^3) - ((7*b*B -
12*A*c - 10*B*c*x^2)*(b*x^2 + c*x^4)^(5/2))/(120*c^2) + (b^5*(7*b*B - 12*A*
c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(1024*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x^3\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)$$

$$=-\frac{\left(7bB-12Ac-10Bcx^2\right)\left(bx^2+cx^4\right)^{5/2}}{120c^2}+\frac{(b(7bB-12Ac))\text{Subst}\left(\int \right.}{48}$$

$$=\frac{b(7bB-12Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{384c^3}-\frac{\left(7bB-12Ac-10Bcx^2\right)(}{120c^2}$$

$$=-\frac{b^3(7bB-12Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^4}+\frac{b(7bB-12Ac)\left(b+2cx^2\right.}{384c^3}$$

$$=-\frac{b^3(7bB-12Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^4}+\frac{b(7bB-12Ac)\left(b+2cx^2\right.}{384c^3}$$

$$=-\frac{b^3(7bB-12Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^4}+\frac{b(7bB-12Ac)\left(b+2cx^2\right.}{384c^3}$$

Mathematica [A]

time = 0.30, size = 191, normalized size = 1.14

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}\,x\sqrt{b+cx^2}\left(-105b^5B+48b^2c^3x^4\left(2A+Bx^2\right)+256c^5x^8\left(6A+5Bx^2\right)-8b^3c^2x^2\left(15A+7Bx^2\right)+10b^4c\left(18A+7Bx^2\right)+64bc^4x^6\left(33A+26Bx^2\right)\right)-15b^5(7bB-12Ac)\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{15360c^{9/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(-105*b^5*B + 48*b^2*c^3*x^4*
(2*A + B*x^2) + 256*c^5*x^8*(6*A + 5*B*x^2) - 8*b^3*c^2*x^2*(15*A + 7*B*x^2
) + 10*b^4*c*(18*A + 7*B*x^2) + 64*b*c^4*x^6*(33*A + 26*B*x^2)) - 15*b^5*(7
*b*B - 12*A*c)*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(15360*c^(9/2)*Sqrt[x^
2*(b + c*x^2)])
```

Maple [A]
time = 0.39, size = 286, normalized size = 1.71

method	result
risch	$\frac{(1280B\,c^5x^{10}+1536A\,c^5x^8+1664Bb\,c^4x^8+2112Ab\,c^4x^6+48B\,b^2c^3x^6+96A\,b^2c^3x^4-56B\,b^3c^2x^4-120A\,b^3c^2x^2+70B\,b^4c\,x^2+180A\,b^4c-}{15360c^4}$
default	$(x^4c+b\,x^2)^{\frac{3}{2}}\left(1280B(c\,x^2+b)^{\frac{5}{2}}c^{\frac{7}{2}}x^7+1536A(c\,x^2+b)^{\frac{5}{2}}c^{\frac{7}{2}}x^5-896B(c\,x^2+b)^{\frac{5}{2}}c^{\frac{5}{2}}b\,x^5-960A(c\,x^2+b)^{\frac{5}{2}}c^{\frac{5}{2}}b\,x^3+560B(c\,x^2+b)^{\frac{5}{2}}c^{\frac{3}{2}}b^2\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15360*(c*x^4+b*x^2)^(3/2)*(1280*B*(c*x^2+b)^(5/2)*c^(7/2)*x^7+1536*A*(c*x
^2+b)^(5/2)*c^(7/2)*x^5-896*B*(c*x^2+b)^(5/2)*c^(5/2)*b*x^5-960*A*(c*x^2+b)
^(5/2)*c^(5/2)*b*x^3+560*B*(c*x^2+b)^(5/2)*c^(3/2)*b^2*x^3+480*A*(c*x^2+b)^
(5/2)*c^(3/2)*b^2*x-280*B*(c*x^2+b)^(5/2)*c^(1/2)*b^3*x-120*A*(c*x^2+b)^(3/
2)*c^(3/2)*b^3*x+70*B*(c*x^2+b)^(3/2)*c^(1/2)*b^4*x-180*A*(c*x^2+b)^(1/2)*c
^(3/2)*b^4*x+105*B*(c*x^2+b)^(1/2)*c^(1/2)*b^5*x-180*A*ln(c^(1/2)*x+(c*x^2+
b)^(1/2))*b^5*c+105*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^6)/x^3/(c*x^2+b)^(3/2
)/c^(9/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(147) = 294$.
time = 0.28, size = 315, normalized size = 1.89

$$\frac{1}{2560}\left(\frac{60\sqrt{cx^4+bx^2}\,b^3x^2}{c^2}-\frac{160\,(cx^4+bx^2)^{\frac{3}{2}}bx^2}{c}-\frac{15\,b^5\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{7}{2}}}+\frac{30\sqrt{cx^4+bx^2}\,b^4}{c^3}-\frac{80\,(cx^4+bx^2)^{\frac{3}{2}}b^2}{c^2}+\frac{256\,(cx^4+bx^2)^{\frac{5}{2}}}{c}\right)A-\frac{1}{30720}\left(\frac{420\sqrt{cx^4+bx^2}\,b^4x^2}{c^3}-\frac{1120\,(cx^4+bx^2)^{\frac{3}{2}}b^2x^2}{c^2}-\frac{2560\,(cx^4+bx^2)^{\frac{5}{2}}x^2}{c}-\frac{105\,b^6\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{9}{2}}}+\frac{210\sqrt{cx^4+bx^2}\,b^5}{c^4}-\frac{560\,(cx^4+bx^2)^{\frac{3}{2}}b^3}{c^3}+\frac{1792\,(cx^4+bx^2)^{\frac{5}{2}}b}{c^2}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2560*(60*sqrt(c*x^4 + b*x^2)*b^3*x^2/c^2 - 160*(c*x^4 + b*x^2)^(3/2)*b*x^
2/c - 15*b^5*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*
sqrt(c*x^4 + b*x^2)*b^4/c^3 - 80*(c*x^4 + b*x^2)^(3/2)*b^2/c^2 + 256*(c*x^4
 + b*x^2)^(5/2)/c)*A - 1/30720*(420*sqrt(c*x^4 + b*x^2)*b^4*x^2/c^3 - 1120*
(c*x^4 + b*x^2)^(3/2)*b^2*x^2/c^2 - 2560*(c*x^4 + b*x^2)^(5/2)*x^2/c - 105*
```

b^6*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) + 210*sqrt(c*x
^4 + b*x^2)*b^5/c^4 - 560*(c*x^4 + b*x^2)^(3/2)*b^3/c^3 + 1792*(c*x^4 + b*x
^2)^(5/2)*b/c^2)*B

Fricas [A]
time = 2.23, size = 369, normalized size = 2.21

$$\left[-\frac{15\,(7\,Bb^{6}-12\,Ab^{5}c)\sqrt{c}\,\log\left(-2\,cx^{2}-b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)-2\,(1280\,Bc^{6}x^{10}+128\,(13\,Bbc^{5}+12\,Ac^{6})x^{8}-105\,Bb^{5}c+180\,Ab^{4}c^{2}+48\,(Bb^{2}c^{4}+44\,Abc^{5})x^{6}-8\,(7\,Bb^{3}c^{3}-12\,Ab^{2}c^{4})x^{4}+10\,(7\,Bb^{4}c^{2}-12\,Ab^{3}c^{3})x^{2})\sqrt{cx^{4}+bx^{2}}}{30720\,c^{5}},\,-\frac{15\,(7\,Bb^{6}-12\,Ab^{5}c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)-(1280\,Bc^{6}x^{10}+128\,(13\,Bbc^{5}+12\,Ac^{6})x^{8}-105\,Bb^{5}c+180\,Ab^{4}c^{2}+48\,(Bb^{2}c^{4}+44\,Abc^{5})x^{6}-8\,(7\,Bb^{3}c^{3}-12\,Ab^{2}c^{4})x^{4}+10\,(7\,Bb^{4}c^{2}-12\,Ab^{3}c^{3})x^{2})\sqrt{cx^{4}+bx^{2}}}{15360\,c^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")

[Out] [-1/30720*(15*(7*B*b^6 - 12*A*b^5*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^
4 + b*x^2)*sqrt(c)) - 2*(1280*B*c^6*x^10 + 128*(13*B*b*c^5 + 12*A*c^6)*x^8
- 105*B*b^5*c + 180*A*b^4*c^2 + 48*(B*b^2*c^4 + 44*A*b*c^5)*x^6 - 8*(7*B*b^
3*c^3 - 12*A*b^2*c^4)*x^4 + 10*(7*B*b^4*c^2 - 12*A*b^3*c^3)*x^2)*sqrt(c*x^4
 + b*x^2))/c^5, -1/15360*(15*(7*B*b^6 - 12*A*b^5*c)*sqrt(-c)*arctan(sqrt(c*
x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (1280*B*c^6*x^10 + 128*(13*B*b*c^5 + 1
2*A*c^6)*x^8 - 105*B*b^5*c + 180*A*b^4*c^2 + 48*(B*b^2*c^4 + 44*A*b*c^5)*x^
6 - 8*(7*B*b^3*c^3 - 12*A*b^2*c^4)*x^4 + 10*(7*B*b^4*c^2 - 12*A*b^3*c^3)*x^
2)*sqrt(c*x^4 + b*x^2))/c^5]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\left(A+Bx^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)

[Out] Integral(x**3*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)

Giac [A]
time = 0.87, size = 246, normalized size = 1.47

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,Bcx^{2}\mathrm{sgn}(x)+\frac{13\,Bbc^{10}\mathrm{sgn}(x)+12\,Ac^{11}\mathrm{sgn}(x)}{c^{10}}\right)x^{2}+\frac{3\,(Bb^{2}c^{9}\mathrm{sgn}(x)+44\,Abc^{10}\mathrm{sgn}(x))}{c^{10}}\right)x^{2}-\frac{7\,Bb^{3}c^{8}\mathrm{sgn}(x)-12\,Ab^{2}c^{9}\mathrm{sgn}(x)}{c^{10}}\right)x^{2}+\frac{5\,(7\,Bb^{4}c^{7}\mathrm{sgn}(x)-12\,Ab^{3}c^{8}\mathrm{sgn}(x))}{c^{10}}\right)x^{2}-\frac{15\,(7\,Bb^{5}c^{6}\mathrm{sgn}(x)-12\,Ab^{4}c^{7}\mathrm{sgn}(x))}{c^{10}}\right)\sqrt{cx^{2}+b}\,x-\frac{(7\,Bb^{6}\mathrm{sgn}(x)-12\,Ab^{5}c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^{2}+b}\right|\right)}{1024\,c^{\frac{9}{2}}}+\frac{(7\,Bb^{6}\log\left(|b|\right)-12\,Ab^{5}c\log\left(|b|\right))\mathrm{sgn}(x)}{2048\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")

[Out] 1/15360*(2*(4*(2*(8*(10*B*c*x^2*sgn(x) + (13*B*b*c^10*sgn(x) + 12*A*c^11*sg
n(x))/c^10)*x^2 + 3*(B*b^2*c^9*sgn(x) + 44*A*b*c^10*sgn(x))/c^10)*x^2 - (7*
B*b^3*c^8*sgn(x) - 12*A*b^2*c^9*sgn(x))/c^10)*x^2 + 5*(7*B*b^4*c^7*sgn(x) -
 12*A*b^3*c^8*sgn(x))/c^10)*x^2 - 15*(7*B*b^5*c^6*sgn(x) - 12*A*b^4*c^7*sgn
(x))/c^10)*sqrt(c*x^2 + b)*x - 1/1024*(7*B*b^6*sgn(x) - 12*A*b^5*c*sgn(x))*

```
log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(9/2) + 1/2048*(7*B*b^6*log(abs(b)
) - 12*A*b^5*c*log(abs(b)))*sgn(x)/c^(9/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.108 $\int x(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=148

$$\frac{3b^2(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{(bB-2Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{3b^4(bB-}{}$$

```
[Out] -1/32*(-2*A*c+B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(3/2)/c^2+1/10*B*(c*x^4+b*x^2)
^(5/2)/c-3/256*b^4*(-2*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^
(7/2)+3/256*b^2*(-2*A*c+B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^3
```

Rubi [A]
time = 0.13, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2059, 654, 626, 634, 212}

$$-\frac{3b^4(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}+\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}\,(bB-2Ac)}{256c^3}-\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}(bB-2Ac)}{32c^2}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b^2*(b*B - 2*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(256*c^3) - ((b*B -
 2*A*c)*(b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(32*c^2) + (B*(b*x^2 + c*x^4)^
(5/2))/(10*c) - (3*b^4*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x
^4]])/(256*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{2}\text{Subst}\left(\int (A+Bx)\left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)$$

$$=\frac{B\left(bx^2+cx^4\right)^{5/2}}{10c}+\frac{(-bB+2Ac)\text{Subst}\left(\int \left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)}{4c}$$

$$=-\frac{(bB-2Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{10c}+\frac{(3b^2(bB-$$

$$=\frac{3b^2(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{(bB-2Ac)\left(b+2cx^2\right)(bx^2+c}{32c^2}$$

$$=\frac{3b^2(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{(bB-2Ac)\left(b+2cx^2\right)(bx^2+c}{32c^2}$$

$$=\frac{3b^2(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{(bB-2Ac)\left(b+2cx^2\right)(bx^2+c}{32c^2}$$

Mathematica [A]
time = 0.22, size = 165, normalized size = 1.11

$$\frac{x\left(\sqrt{c}\,x(b+cx^2)\left(15b^4B-10b^3c(3A+Bx^2)+4b^2c^2x^2(5A+2Bx^2)+32c^4x^6(5A+4Bx^2)+16bc^3x^4(15A+11Bx^2)\right)+15b^4(bB-2Ac)\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{1280c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(15*b^4*B - 10*b^3*c*(3*A + B*x^2) + 4*b^2*c^2*x^
2*(5*A + 2*B*x^2) + 32*c^4*x^6*(5*A + 4*B*x^2) + 16*b*c^3*x^4*(15*A + 11*B*
x^2)) + 15*b^4*(b*B - 2*A*c)*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b + c*
x^2]]))/(1280*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.38, size = 244, normalized size = 1.65

method	result
risch	$-\frac{\left(-128 B\,c^{4} x^{8}-160 A\,c^{4} x^{6}-176 B b\,c^{3} x^{6}-240 A b\,c^{3} x^{4}-8 B\,b^{2} c^{2} x^{4}-20 A\,b^{2} c^{2} x^{2}+10 B\,b^{3} c\,x^{2}+30 A\,b^{3} c-15 B\,b^{4}\right) \sqrt{x^{2} \left(c\,x^{2}+b\right)}}{1280 c^{3}}$
default	$\frac{\left(x^{4} c+b\,x^{2}\right)^{\frac{3}{2}} \left(128 B \left(c\,x^{2}+b\right)^{\frac{5}{2}} c^{\frac{5}{2}} x^{5}+160 A \left(c\,x^{2}+b\right)^{\frac{5}{2}} c^{\frac{5}{2}} x^{3}-80 B \left(c\,x^{2}+b\right)^{\frac{5}{2}} c^{\frac{3}{2}} b\,x^{3}-80 A \left(c\,x^{2}+b\right)^{\frac{5}{2}} c^{\frac{3}{2}} b x+20 A \left(c\,x^{2}+b\right)^{\frac{3}{2}} c^{\frac{3}{2}} b^{2} x+40\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1280*(c*x^4+b*x^2)^(3/2)*(128*B*(c*x^2+b)^(5/2)*c^(5/2)*x^5+160*A*(c*x^2+
b)^(5/2)*c^(5/2)*x^3-80*B*(c*x^2+b)^(5/2)*c^(3/2)*b*x^3-80*A*(c*x^2+b)^(5/2
)*c^(3/2)*b*x+20*A*(c*x^2+b)^(3/2)*c^(3/2)*b^2*x+40*B*(c*x^2+b)^(5/2)*c^(1/
2)*b^2*x+30*A*(c*x^2+b)^(1/2)*c^(3/2)*b^3*x-10*B*(c*x^2+b)^(3/2)*c^(1/2)*b^
3*x-15*B*(c*x^2+b)^(1/2)*c^(1/2)*b^4*x+30*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b
^4*c-15*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^5)/x^3/(c*x^2+b)^(3/2)/c^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(128) = 256$.
time = 0.30, size = 267, normalized size = 1.80

$$\frac{1}{256}\left(32\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}x^{2}-\frac{12\sqrt{cx^{4}+bx^{2}}\,b^{2}x^{2}}{c}+\frac{3\,b^{4}\log\left(2\,cx^{2}+b+2\sqrt{cx^{4}+bx^{2}}\sqrt{c}\right)}{c^{\frac{5}{2}}}-\frac{6\sqrt{cx^{4}+bx^{2}}\,b^{3}}{c^{2}}+\frac{16\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}b}{c}\right)A+\frac{1}{2560}\left(\frac{60\sqrt{cx^{4}+bx^{2}}\,b^{3}x^{2}}{c^{2}}-\frac{160\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}bx^{2}}{c}-\frac{15\,b^{5}\log\left(2\,cx^{2}+b+2\sqrt{cx^{4}+bx^{2}}\sqrt{c}\right)}{c^{\frac{7}{2}}}+\frac{30\sqrt{cx^{4}+bx^{2}}\,b^{4}}{c^{3}}-\frac{80\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}b^{2}}{c^{2}}+\frac{256\left(cx^{4}+bx^{2}\right)^{\frac{5}{2}}}{c}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/256*(32*(c*x^4 + b*x^2)^(3/2)*x^2 - 12*sqrt(c*x^4 + b*x^2)*b^2*x^2/c + 3*
b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2) - 6*sqrt(c*x^4
 + b*x^2)*b^3/c^2 + 16*(c*x^4 + b*x^2)^(3/2)*b/c)*A + 1/2560*(60*sqrt(c*x^4
 + b*x^2)*b^3*x^2/c^2 - 160*(c*x^4 + b*x^2)^(3/2)*b*x^2/c - 15*b^5*log(2*c*
x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*sqrt(c*x^4 + b*x^2)*b
^4/c^3 - 80*(c*x^4 + b*x^2)^(3/2)*b^2/c^2 + 256*(c*x^4 + b*x^2)^(5/2)/c)*B
```

Fricas [A]
time = 1.24, size = 316, normalized size = 2.14

$$\left[-\frac{15\left(Bb^{5}-2\,Ab^{4}c\right)\sqrt{c}\,\log\left(-2\,cx^{2}-b-2\sqrt{cx^{4}+bx^{2}}\sqrt{c}\right)-2\left(128\,Bc^{5}x^{8}+16\left(11\,Bbc^{4}+10\,Ac^{5}\right)x^{6}+15\,Bb^{4}c-30\,Ab^{3}c^{2}+8\left(Bb^{2}c^{3}+30\,Abc^{4}\right)x^{4}-10\left(Bb^{3}c^{2}-2\,Ab^{2}c^{3}\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}}}{2560\,c^{4}},\ \frac{15\left(Bb^{5}-2\,Ab^{4}c\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\sqrt{-c}}{cx^{2}+b}\right)+\left(128\,Bc^{5}x^{8}+16\left(11\,Bbc^{4}+10\,Ac^{5}\right)x^{6}+15\,Bb^{4}c-30\,Ab^{3}c^{2}+8\left(Bb^{2}c^{3}+30\,Abc^{4}\right)x^{4}-10\left(Bb^{3}c^{2}-2\,Ab^{2}c^{3}\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}}}{1280\,c^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(B*b^5 - 2*A*b^4*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 +
b*x^2)*sqrt(c)) - 2*(128*B*c^5*x^8 + 16*(11*B*b*c^4 + 10*A*c^5)*x^6 + 15*B*
b^4*c - 30*A*b^3*c^2 + 8*(B*b^2*c^3 + 30*A*b*c^4)*x^4 - 10*(B*b^3*c^2 - 2*A
*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4, 1/1280*(15*(B*b^5 - 2*A*b^4*c)*sqr
t(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (128*B*c^5*x^8 + 1
6*(11*B*b*c^4 + 10*A*c^5)*x^6 + 15*B*b^4*c - 30*A*b^3*c^2 + 8*(B*b^2*c^3 +
30*A*b*c^4)*x^4 - 10*(B*b^3*c^2 - 2*A*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^
4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

Giac [A]
time = 0.65, size = 207, normalized size = 1.40

$$\frac{1}{1280}\left(2\left(4\left(2\left(8\,Bcx^2\mathrm{sgn}(x)+\frac{11\,Bbc^8\mathrm{sgn}(x)+10\,Ac^9\mathrm{sgn}(x)}{c^8}\right)x^2+\frac{Bb^2c^7\mathrm{sgn}(x)+30\,Abc^8\mathrm{sgn}(x)}{c^8}\right)x^2-\frac{5\left(Bb^3c^6\mathrm{sgn}(x)-2\,Ab^2c^7\mathrm{sgn}(x)\right)}{c^8}\right)x^2+\frac{15\left(Bb^4c^5\mathrm{sgn}(x)-2\,Ab^3c^6\mathrm{sgn}(x)\right)}{c^8}\right)\sqrt{cx^2+b}\,x+\frac{3\left(Bb^5\mathrm{sgn}(x)-2\,Ab^4c\mathrm{sgn}(x)\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{256\,c^{\frac{7}{2}}}-\frac{3\left(Bb^5\log\left(|b|\right)-2\,Ab^4c\log\left(|b|\right)\right)\mathrm{sgn}(x)}{512\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1280*(2*(4*(2*(8*B*c*x^2*sgn(x) + (11*B*b*c^8*sgn(x) + 10*A*c^9*sgn(x))/c
^8)*x^2 + (B*b^2*c^7*sgn(x) + 30*A*b*c^8*sgn(x))/c^8)*x^2 - 5*(B*b^3*c^6*sg
n(x) - 2*A*b^2*c^7*sgn(x))/c^8)*x^2 + 15*(B*b^4*c^5*sgn(x) - 2*A*b^3*c^6*sg
n(x))/c^8)*sqrt(c*x^2 + b)*x + 3/256*(B*b^5*sgn(x) - 2*A*b^4*c*sgn(x))*log(
abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(7/2) - 3/512*(B*b^5*log(abs(b)) - 2*A
*b^4*c*log(abs(b)))*sgn(x)/c^(7/2)
```

Mupad [B]
time = 1.01, size = 236, normalized size = 1.59

$$\frac{B\,{\left(c\,x^4+b\,x^2\right)}^{5/2}}{10\,c}+\frac{A\,{\left(c\,x^4+b\,x^2\right)}^{3/2}\,\left(c\,x^2+\frac{b}{2}\right)}{8\,c}-\frac{3\,A\,b^2\,\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\,\sqrt{c\,x^4+b\,x^2}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{8\,c^{3/2}}\right)}{32\,c}-\frac{B\,b\,\left(\frac{x^2\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{4}-\frac{3\,b^2\,\left(\frac{\left(2\,c\,x^2+b\right)\,\sqrt{c\,x^4+b\,x^2}}{4\,c}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{8\,c^{3/2}}\right)}{16\,c}+\frac{b\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{8\,c}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (B*(b*x^2 + c*x^4)^(5/2))/(10*c) + (A*(b*x^2 + c*x^4)^(3/2)*(b/2 + c*x^2))/
(8*c) - (3*A*b^2*((b/(4*c) + x^2/2)*(b*x^2 + c*x^4)^(1/2) - (b^2*log((b/2 +
 c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(8*c^(3/2))))/(32*c) - (B*b*((x^2
*(b*x^2 + c*x^4)^(3/2))/4 - (3*b^2*(((b + 2*c*x^2)*(b*x^2 + c*x^4)^(1/2))/(
4*c) - (b^2*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(8*c^(3/2))
))/(16*c) + (b*(b*x^2 + c*x^4)^(3/2))/(8*c)))/(4*c)
```

3.109 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x} \, dx$

Optimal. Leaf size=144

$$-\frac{b(3bB-8Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^2}-\frac{(3bB-8Ac)(bx^2+cx^4)^{3/2}}{48c}+\frac{B(bx^2+cx^4)^{5/2}}{8cx^2}+\frac{b^3(3bB-8Ac)\tanh}{128}$$

```
[Out] -1/48*(-8*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/c+1/8*B*(c*x^4+b*x^2)^(5/2)/c/x^2+
1/128*b^3*(-8*A*c+3*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(5/2)-1
/128*b*(-8*A*c+3*B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^2
```

Rubi [A]
time = 0.17, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2059, 808, 678, 626, 634, 212}

$$\frac{b^3(3bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}-\frac{b(b+2cx^2)\sqrt{bx^2+cx^4}\,(3bB-8Ac)}{128c^2}-\frac{(bx^2+cx^4)^{3/2}(3bB-8Ac)}{48c}+\frac{B(bx^2+cx^4)^{5/2}}{8cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x,x]
```

```
[Out] -1/128*(b*(3*b*B - 8*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/c^2 - ((3*b*B
- 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(48*c) + (B*(b*x^2 + c*x^4)^(5/2))/(8*c*x^2
) + (b^3*(3*b*B - 8*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c
^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x}\,dx, x, x^2\right)$$

$$= \frac{B(bx^2+cx^4)^{5/2}}{8cx^2} + \frac{\left(bB-Ac+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x}\,dx, x,\right.}{8c}$$

$$= -\frac{(3bB-8Ac)(bx^2+cx^4)^{3/2}}{48c} + \frac{B(bx^2+cx^4)^{5/2}}{8cx^2} - \frac{(b(3bB-8Ac))\text{Subst}(}{3}$$

$$= -\frac{b(3bB-8Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^2} - \frac{(3bB-8Ac)(bx^2+cx^4)^{3/2}}{48c} +$$

$$= -\frac{b(3bB-8Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^2} - \frac{(3bB-8Ac)(bx^2+cx^4)^{3/2}}{48c} +$$

$$= -\frac{b(3bB-8Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^2} - \frac{(3bB-8Ac)(bx^2+cx^4)^{3/2}}{48c} +$$

Mathematica [A]
time = 0.20, size = 149, normalized size = 1.03

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}\,x\sqrt{b+cx^2}\,(-9b^3B+6b^2c(4A+Bx^2)+16c^3x^4(4A+3Bx^2)+8bc^2x^2(14A+9Bx^2))+3b^3(-3bB+8Ac)\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{384c^{5/2}\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x,x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(-9*b^3*B + 6*b^2*c*(4*A + B*
x^2) + 16*c^3*x^4*(4*A + 3*B*x^2) + 8*b*c^2*x^2*(14*A + 9*B*x^2)) + 3*b^3*(
-3*b*B + 8*A*c)*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(384*c^(5/2)*Sqrt[x^2
*(b + c*x^2)])
```

Maple [A]
time = 0.37, size = 202, normalized size = 1.40

method	result
risch	$\frac{(48Bc^3x^6+64Ac^3x^4+72Bbc^2x^4+112Abc^2x^2+6Bb^2cx^2+24Ab^2c-9Bb^3)\sqrt{x^2(cx^2+b)}}{384c^2} + \frac{\left(-\frac{b^3\ln\left(\sqrt{c}\,x+\sqrt{cx^2+b}\right.}{16c^{\frac{3}{2}}}\right.}{}$

default	$\frac{(x^4c+b\,x^2)^{\frac{3}{2}}\left(48B(c\,x^2+b)^{\frac{5}{2}}c^{\frac{3}{2}}x^3+64A(c\,x^2+b)^{\frac{5}{2}}c^{\frac{3}{2}}x-24B(c\,x^2+b)^{\frac{5}{2}}\sqrt{c}\,bx-16A(c\,x^2+b)^{\frac{3}{2}}c^{\frac{3}{2}}bx+6B(c\,x^2+b)^{\frac{3}{2}}\sqrt{c}\,b^2x-24}{384x^3(c\,x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*(c*x^4+b*x^2)^(3/2)*(48*B*(c*x^2+b)^(5/2)*c^(3/2)*x^3+64*A*(c*x^2+b)^
(5/2)*c^(3/2)*x-24*B*(c*x^2+b)^(5/2)*c^(1/2)*b*x-16*A*(c*x^2+b)^(3/2)*c^(3/
2)*b*x+6*B*(c*x^2+b)^(3/2)*c^(1/2)*b^2*x-24*A*(c*x^2+b)^(1/2)*c^(3/2)*b^2*x
+9*B*(c*x^2+b)^(1/2)*c^(1/2)*b^3*x-24*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^3*c
+9*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^4)/x^3/(c*x^2+b)^(3/2)/c^(5/2)
```

Maxima [A]
time = 0.28, size = 216, normalized size = 1.50

$$\frac{1}{96}\left(12\sqrt{cx^4+bx^2}\,bx^2-\frac{3\,b^3\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}}+16\,(cx^4+bx^2)^{\frac{3}{2}}+\frac{6\sqrt{cx^4+bx^2}\,b^2}{c}\right)A+\frac{1}{256}\left(32\,(cx^4+bx^2)^{\frac{3}{2}}x^2-\frac{12\sqrt{cx^4+bx^2}\,b^2x^2}{c}+\frac{3\,b^4\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}}-\frac{6\sqrt{cx^4+bx^2}\,b^3}{c^2}+\frac{16\,(cx^4+bx^2)^{\frac{3}{2}}b}{c}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/96*(12*sqrt(c*x^4 + b*x^2)*b*x^2 - 3*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 +
 b*x^2)*sqrt(c))/c^(3/2) + 16*(c*x^4 + b*x^2)^(3/2) + 6*sqrt(c*x^4 + b*x^2)
*b^2/c)*A + 1/256*(32*(c*x^4 + b*x^2)^(3/2)*x^2 - 12*sqrt(c*x^4 + b*x^2)*b^
2*x^2/c + 3*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2) -
6*sqrt(c*x^4 + b*x^2)*b^3/c^2 + 16*(c*x^4 + b*x^2)^(3/2)*b/c)*B
```

Fricas [A]
time = 1.78, size = 275, normalized size = 1.91

$$\left[-\frac{3\,(3\,Bb^4-8\,Ab^3c)\sqrt{c}\,\log\left(-2\,cx^2-b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(48\,Bc^4x^6-9\,Bb^3c+24\,Ab^2c^2+8\,(9\,Bbc^3+8\,Ac^4)x^4+2\,(3\,Bb^2c^2+56\,Abc^3)x^2)\sqrt{cx^4+bx^2}}{768\,c^3},\ -\frac{3\,(3\,Bb^4-8\,Ab^3c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(48\,Bc^4x^6-9\,Bb^3c+24\,Ab^2c^2+8\,(9\,Bbc^3+8\,Ac^4)x^4+2\,(3\,Bb^2c^2+56\,Abc^3)x^2)\sqrt{cx^4+bx^2}}{384\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(3*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 +
b*x^2)*sqrt(c)) - 2*(48*B*c^4*x^6 - 9*B*b^3*c + 24*A*b^2*c^2 + 8*(9*B*b*c^3
 + 8*A*c^4)*x^4 + 2*(3*B*b^2*c^2 + 56*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^
3, -1/384*(3*(3*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt
(-c)/(c*x^2 + b)) - (48*B*c^4*x^6 - 9*B*b^3*c + 24*A*b^2*c^2 + 8*(9*B*b*c^3
 + 8*A*c^4)*x^4 + 2*(3*B*b^2*c^2 + 56*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^
3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x, x)
```

Giac [A]
time = 0.57, size = 178, normalized size = 1.24

$$\frac{1}{384}\left(2\left(4\left(6\,Bcx^{2}\mathrm{sgn}(x)+\frac{9\,Bbc^{6}\mathrm{sgn}(x)+8\,Ac^{7}\mathrm{sgn}(x)}{c^{6}}\right)x^{2}+\frac{3\,Bb^{2}c^{5}\mathrm{sgn}(x)+56\,Abc^{6}\mathrm{sgn}(x)}{c^{6}}\right)x^{2}-\frac{3\,(3\,Bb^{3}c^{4}\mathrm{sgn}(x)-8\,Ab^{2}c^{5}\mathrm{sgn}(x))}{c^{6}}\right)\sqrt{cx^{2}+b}\,x-\frac{(3\,Bb^{4}\mathrm{sgn}(x)-8\,Ab^{3}c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^{2}+b}\right|\right)}{128\,c^{\frac{5}{2}}}+\frac{(3\,Bb^{4}\log\left(|b|\right)-8\,Ab^{3}c\log\left(|b|\right))\mathrm{sgn}(x)}{256\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*c*x^2*sgn(x) + (9*B*b*c^6*sgn(x) + 8*A*c^7*sgn(x))/c^6)*x^
2 + (3*B*b^2*c^5*sgn(x) + 56*A*b*c^6*sgn(x))/c^6)*x^2 - 3*(3*B*b^3*c^4*sgn(
x) - 8*A*b^2*c^5*sgn(x))/c^6)*sqrt(c*x^2 + b)*x - 1/128*(3*B*b^4*sgn(x) - 8
*A*b^3*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(5/2) + 1/256*(3*
B*b^4*log(abs(b)) - 8*A*b^3*c*log(abs(b)))*sgn(x)/c^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x, x)
```

3.110 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=137

$$\frac{(bB-6Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{16c}+\frac{(bB-6Ac)(bx^2+cx^4)^{3/2}}{6b}+\frac{A(bx^2+cx^4)^{5/2}}{bx^4}-\frac{b^2(bB-6Ac)\tanh^{-1}\Big(}{16c^{3/2}}$$

```
[Out] 1/6*(-6*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/b+A*(c*x^4+b*x^2)^(5/2)/b/x^4-1/16*b^2
*(-6*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)+1/16*(-6*A*c
+B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A]
time = 0.19, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2059, 806, 678, 626, 634, 212}

$$-\frac{b^2(bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}+\frac{(bx^2+cx^4)^{3/2}(bB-6Ac)}{6b}+\frac{(b+2cx^2)\sqrt{bx^2+cx^4}\,(bB-6Ac)}{16c}+\frac{A(bx^2+cx^4)^{5/2}}{bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3,x]
```

```
[Out] ((b*B - 6*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c) + ((b*B - 6*A*c)*(
b*x^2 + c*x^4)^(3/2))/(6*b) + (A*(b*x^2 + c*x^4)^(5/2))/(b*x^4) - (b^2*(b*B
 - 6*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^2}\,dx, x, x^2\right)$$

$$= \frac{A(bx^2+cx^4)^{5/2}}{bx^4} - \frac{\left(-2(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^3}{x}\right.}{b}$$

$$= \frac{(bB-6Ac)(bx^2+cx^4)^{3/2}}{6b} + \frac{A(bx^2+cx^4)^{5/2}}{bx^4} - \frac{1}{4}(-bB+6Ac)\text{Subst}\left(\int\right.$$

$$= \frac{(bB-6Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{16c} + \frac{(bB-6Ac)(bx^2+cx^4)^{3/2}}{6b} + \frac{A(bx}{}$$

$$= \frac{(bB-6Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{16c} + \frac{(bB-6Ac)(bx^2+cx^4)^{3/2}}{6b} + \frac{A(bx}{}$$

$$= \frac{(bB-6Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{16c} + \frac{(bB-6Ac)(bx^2+cx^4)^{3/2}}{6b} + \frac{A(bx}{}$$

Mathematica [A]
time = 0.18, size = 124, normalized size = 0.91

$$\frac{x\left(\sqrt{c}\,x(b+cx^2)\left(3b^2B+4c^2x^2(3A+2Bx^2)+2bc(15A+7Bx^2)\right)+3b^2(bB-6Ac)\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{48c^{3/2}\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3,x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(3*b^2*B + 4*c^2*x^2*(3*A + 2*B*x^2) + 2*b*c*(15*
A + 7*B*x^2)) + 3*b^2*(b*B - 6*A*c)*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt
[b + c*x^2]]))/(48*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.38, size = 162, normalized size = 1.18

method	result
risch	$\frac{(8Bc^2x^4+12Ac^2x^2+14bBx^2c+30Abc+3b^2B)\sqrt{x^2(cx^2+b)}}{48c} + \frac{\left(\frac{3b^2\ln\left(\sqrt{c}\,x+\sqrt{cx^2+b}\right)A}{8\sqrt{c}} - \frac{b^3\ln\left(\sqrt{c}\,x+\sqrt{cx^2}\right.}{16c^{\frac{3}{2}}}\right.}{x\sqrt{cx^2+b}}$

default	$\frac{(x^4c+b\,x^2)^{\frac{3}{2}}\left(8B\sqrt{c}\,(c\,x^2+b)^{\frac{5}{2}}x+12A(c\,x^2+b)^{\frac{3}{2}}c^{\frac{3}{2}}x-2B(c\,x^2+b)^{\frac{3}{2}}\sqrt{c}\,bx+18A\sqrt{c\,x^2+b}\,c^{\frac{3}{2}}bx-3B\sqrt{c\,x^2+b}\,\sqrt{c}\,b^2\right.}{48x^3(c\,x^2+b)^{\frac{3}{2}}c^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(3/2)*(8*B*c^(1/2)*(c*x^2+b)^(5/2)*x+12*A*(c*x^2+b)^(3/2
)*c^(3/2)*x-2*B*(c*x^2+b)^(3/2)*c^(1/2)*b*x+18*A*(c*x^2+b)^(1/2)*c^(3/2)*b*
x-3*B*(c*x^2+b)^(1/2)*c^(1/2)*b^2*x+18*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2*
c-3*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^3)/x^3/(c*x^2+b)^(3/2)/c^(3/2)
```

Maxima [A]
time = 0.29, size = 168, normalized size = 1.23

$$\frac{1}{16}\left(\frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{\sqrt{c}}+6\,\sqrt{cx^4+bx^2}\,b+\frac{4\,(cx^4+bx^2)^{\frac{3}{2}}}{x^2}\right)A+\frac{1}{96}\left(12\,\sqrt{cx^4+bx^2}\,bx^2-\frac{3\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{3}{2}}}+16\,(cx^4+bx^2)^{\frac{3}{2}}+\frac{6\,\sqrt{cx^4+bx^2}\,b^2}{c}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 6*sq
rt(c*x^4 + b*x^2)*b + 4*(c*x^4 + b*x^2)^(3/2)/x^2)*A + 1/96*(12*sqrt(c*x^4
+ b*x^2)*b*x^2 - 3*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(
3/2) + 16*(c*x^4 + b*x^2)^(3/2) + 6*sqrt(c*x^4 + b*x^2)*b^2/c)*B
```

Fricas [A]
time = 1.11, size = 224, normalized size = 1.64

$$\left[-\frac{3\,(Bb^3-6\,Ab^2c)\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\,(8\,Bc^3x^4+3\,Bb^2c+30\,Abc^2+2\,(7\,Bbc^2+6\,Ac^3)x^2)\sqrt{cx^4+bx^2}}{96\,c^2},\ \frac{3\,(Bb^3-6\,Ab^2c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+(8\,Bc^3x^4+3\,Bb^2c+30\,Abc^2+2\,(7\,Bbc^2+6\,Ac^3)x^2)\sqrt{cx^4+bx^2}}{48\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(B*b^3 - 6*A*b^2*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x
^2)*sqrt(c)) - 2*(8*B*c^3*x^4 + 3*B*b^2*c + 30*A*b*c^2 + 2*(7*B*b*c^2 + 6*A
*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^2, 1/48*(3*(B*b^3 - 6*A*b^2*c)*sqrt(-c)*a
rctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*B*c^3*x^4 + 3*B*b^2*c
+ 30*A*b*c^2 + 2*(7*B*b*c^2 + 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**3, x)
```

Giac [A]
time = 0.61, size = 142, normalized size = 1.04

$$\frac{1}{48}\left(2\left(4\,Bcx^2\mathrm{sgn}(x)+\frac{7\,Bbc^4\mathrm{sgn}(x)+6\,Ac^5\mathrm{sgn}(x)}{c^4}\right)x^2+\frac{3\,(Bb^2c^3\mathrm{sgn}(x)+10\,Abc^4\mathrm{sgn}(x))}{c^4}\right)\sqrt{cx^2+b}\,x+\frac{(Bb^3\mathrm{sgn}(x)-6\,Ab^2c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{16\,c^{\frac{3}{2}}}-\frac{(Bb^3\log(|b|)-6\,Ab^2c\log(|b|))\mathrm{sgn}(x)}{32\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*c*x^2*sgn(x) + (7*B*b*c^4*sgn(x) + 6*A*c^5*sgn(x))/c^4)*x^2 +
3*(B*b^2*c^3*sgn(x) + 10*A*b*c^4*sgn(x))/c^4)*sqrt(c*x^2 + b)*x + 1/16*(B*b
^3*sgn(x) - 6*A*b^2*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(3/2
) - 1/32*(B*b^3*log(abs(b)) - 6*A*b^2*c*log(abs(b)))*sgn(x)/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3, x)
```

3.111 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=128

$$\frac{3}{8}(bB+4Ac)\sqrt{bx^2+cx^4}+\frac{(bB+4Ac)\,(bx^2+cx^4)^{3/2}}{4bx^2}-\frac{A(bx^2+cx^4)^{5/2}}{bx^6}+\frac{3b(bB+4Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}$$

```
[Out] 1/4*(4*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/b/x^2-A*(c*x^4+b*x^2)^(5/2)/b/x^6+3/8*b
*(4*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(1/2)+3/8*(4*A*c+B*
b)*(c*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2059, 806, 678, 634, 212}

$$\frac{(bx^2+cx^4)^{3/2}(4Ac+bB)}{4bx^2}+\frac{3}{8}\sqrt{bx^2+cx^4}\,(4Ac+bB)+\frac{3b(4Ac+bB)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}-\frac{A(bx^2+cx^4)^{5/2}}{bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5,x]
```

```
[Out] (3*(b*B + 4*A*c)*Sqrt[b*x^2 + c*x^4])/8 + ((b*B + 4*A*c)*(b*x^2 + c*x^4)^(3
/2))/(4*b*x^2) - (A*(b*x^2 + c*x^4)^(5/2))/(b*x^6) + (3*b*(b*B + 4*A*c)*Arc
Tanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
```

```
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{bx^6} + \frac{\left(-3(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2\ldots}{x^2}\right.}{b}$$

$$= \frac{(bB+4Ac)\left(bx^2+cx^4\right)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/2}}{bx^6} + \frac{1}{8}(3(bB+4Ac))\text{Subst}\Bigg(\ldots$$

$$= \frac{3}{8}(bB+4Ac)\sqrt{bx^2+cx^4} + \frac{(bB+4Ac)\left(bx^2+cx^4\right)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/\ldots}}{bx^6}$$

$$= \frac{3}{8}(bB+4Ac)\sqrt{bx^2+cx^4} + \frac{(bB+4Ac)\left(bx^2+cx^4\right)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/\ldots}}{bx^6}$$

$$= \frac{3}{8}(bB+4Ac)\sqrt{bx^2+cx^4} + \frac{(bB+4Ac)\left(bx^2+cx^4\right)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/\ldots}}{bx^6}$$

Mathematica [A]

time = 0.19, size = 108, normalized size = 0.84

$$\frac{\sqrt{c}\,(b+cx^2)\,(-8Ab+5bBx^2+4Acx^2+2Bcx^4)-3b(bB+4Ac)x\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)}{8\sqrt{c}\,\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[c]*(b + c*x^2)*(-8*A*b + 5*b*B*x^2 + 4*A*c*x^2 + 2*B*c*x^4) - 3*b*(b*
B + 4*A*c)*x*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]])/(8*Sqrt[c
]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.37, size = 174, normalized size = 1.36

method	result
risch	$-\frac{(-2Bc\,x^4-4Ac\,x^2-5bB\,x^2+8Ab)\sqrt{x^2\,(c\,x^2+b)}}{8x^2}+\frac{\left(\frac{3b\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)\sqrt{c}\,A}{2}+\frac{3b^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right.}{8\sqrt{c}}\right.}{x\sqrt{c\,x^2+b}}$
default	$-\frac{(x^4c+b\,x^2)^{\frac{3}{2}}\left(-8A(c\,x^2+b)^{\frac{3}{2}}c^{\frac{3}{2}}x^2-2B(c\,x^2+b)^{\frac{3}{2}}\sqrt{c}\,b\,x^2+8A(c\,x^2+b)^{\frac{5}{2}}\sqrt{c}-12A\sqrt{c\,x^2+b}\,c^{\frac{3}{2}}b\,x^2-3B\sqrt{c\,x^2+b}\,\right.}{8x^4(c\,x^2+b)^{\frac{3}{2}}b\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(c*x^4+b*x^2)^(3/2)*(-8*A*(c*x^2+b)^(3/2)*c^(3/2)*x^2-2*B*(c*x^2+b)^(3
/2)*c^(1/2)*b*x^2+8*A*(c*x^2+b)^(5/2)*c^(1/2)-12*A*(c*x^2+b)^(1/2)*c^(3/2)*
b*x^2-3*B*(c*x^2+b)^(1/2)*c^(1/2)*b^2*x^2-12*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2)
)*b^2*c*x-3*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^3*x)/x^4/(c*x^2+b)^(3/2)/b/c^
(1/2)
```

Maxima [A]
time = 0.30, size = 148, normalized size = 1.16

$$\frac{1}{4}\left(3\,b\sqrt{c}\,\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-\frac{6\,\sqrt{cx^4+bx^2}\,b}{x^2}+\frac{2\,(cx^4+bx^2)^{\frac{3}{2}}}{x^4}\right)A+\frac{1}{16}\left(\frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{\sqrt{c}}+6\,\sqrt{cx^4+bx^2}\,b+\frac{4\,(cx^4+bx^2)^{\frac{3}{2}}}{x^2}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(3*b*sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 6*sqrt(
c*x^4 + b*x^2)*b/x^2 + 2*(c*x^4 + b*x^2)^(3/2)/x^4)*A + 1/16*(3*b^2*log(2*c
*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 6*sqrt(c*x^4 + b*x^2)*b
 + 4*(c*x^4 + b*x^2)^(3/2)/x^2)*B
```

Fricas [A]
time = 2.27, size = 209, normalized size = 1.63

$$\left[\frac{3\,(Bb^2+4\,Abc)\sqrt{c}\,x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,(2\,Bc^2x^4-8\,Abc+(5\,Bbc+4\,Ac^2)x^2)\sqrt{cx^4+bx^2}}{16\,cx^2}, -\frac{3\,(Bb^2+4\,Abc)\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)-(2\,Bc^2x^4-8\,Abc+(5\,Bbc+4\,Ac^2)x^2)\sqrt{cx^4+bx^2}}{8\,cx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(B*b^2 + 4*A*b*c)*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*
x^2)*sqrt(c)) + 2*(2*B*c^2*x^4 - 8*A*b*c + (5*B*b*c + 4*A*c^2)*x^2)*sqrt(c*
x^4 + b*x^2))/(c*x^2), -1/8*(3*(B*b^2 + 4*A*b*c)*sqrt(-c)*x^2*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (2*B*c^2*x^4 - 8*A*b*c + (5*B*b*c + 4
*A*c^2)*x^2)*sqrt(c*x^4 + b*x^2))/(c*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**5, x)
```

Giac [A]
time = 0.61, size = 126, normalized size = 0.98

$$\frac{2\,Ab^2\sqrt{c}\,\mathrm{sgn}(x)}{\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b}+\frac{1}{8}\left(2\,Bcx^2\mathrm{sgn}(x)+\frac{5\,Bbc^2\mathrm{sgn}(x)+4\,Ac^3\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+b}\,x-\frac{3\left(Bb^2\sqrt{c}\,\mathrm{sgn}(x)+4\,Abc^{\frac{3}{2}}\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2\right)}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 2*A*b^2*sqrt(c)*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b) + 1/8*(2*B*c*x
^2*sgn(x) + (5*B*b*c^2*sgn(x) + 4*A*c^3*sgn(x))/c^2)*sqrt(c*x^2 + b)*x - 3/
16*(B*b^2*sqrt(c)*sgn(x) + 4*A*b*c^(3/2)*sgn(x))*log((sqrt(c)*x - sqrt(c*x^
2 + b))^2)/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5, x)
```

3.112 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=136

$$\frac{c(3bB+2Ac)\sqrt{bx^2+cx^4}}{2b}-\frac{(3bB+2Ac)\left(bx^2+cx^4\right)^{3/2}}{3bx^4}-\frac{A(bx^2+cx^4)^{5/2}}{3bx^8}+\frac{1}{2}\sqrt{c}\,(3bB+2Ac)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{bx^2}}\right.$$

```
[Out] -1/3*(2*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^4-1/3*A*(c*x^4+b*x^2)^(5/2)/b/x^
8+1/2*(2*A*c+3*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))*c^(1/2)+1/2*c*
(2*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/b
```

Rubi [A]
time = 0.18, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2059, 806, 676, 678, 634, 212}

$$-\frac{(bx^2+cx^4)^{3/2}(2Ac+3bB)}{3bx^4}+\frac{c\sqrt{bx^2+cx^4}\,(2Ac+3bB)}{2b}+\frac{1}{2}\sqrt{c}\,(2Ac+3bB)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{A(bx^2+cx^4)^{5/2}}{3bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7,x]
```

```
[Out] (c*(3*b*B + 2*A*c)*Sqrt[b*x^2 + c*x^4])/(2*b) - ((3*b*B + 2*A*c)*(b*x^2 + c
*x^4)^(3/2))/(3*b*x^4) - (A*(b*x^2 + c*x^4)^(5/2))/(3*b*x^8) + (Sqrt[c]*(3*
b*B + 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 676

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Dist[c*(p/(e^2*(m + p + 1))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{3bx^8} + \frac{\left(-4(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3}}{x^3}\right.}{3b}$$

$$= -\frac{(3bB+2Ac)(bx^2+cx^4)^{3/2}}{3bx^4} - \frac{A(bx^2+cx^4)^{5/2}}{3bx^8} + \frac{\left(c\left(-4(-bB+Ac)+\frac{5}{2}\right.\right.}{}$$

$$= \frac{c(3bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{(3bB+2Ac)(bx^2+cx^4)^{3/2}}{3bx^4} - \frac{A(bx^2+cx^4)^{5}}{3bx^8}$$

$$= \frac{c(3bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{(3bB+2Ac)(bx^2+cx^4)^{3/2}}{3bx^4} - \frac{A(bx^2+cx^4)^{5}}{3bx^8}$$

$$= \frac{c(3bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{(3bB+2Ac)(bx^2+cx^4)^{3/2}}{3bx^4} - \frac{A(bx^2+cx^4)^{5}}{3bx^8}$$

Mathematica [A]
time = 0.20, size = 112, normalized size = 0.82

$$-\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b+cx^2}\,(6bBx^2-3Bcx^4+2A(b+4cx^2))+3\sqrt{c}\,(3bB+2Ac)x^3\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{6x^4\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7,x]
```

```
[Out] -1/6*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b + c*x^2]*(6*b*B*x^2 - 3*B*c*x^4 + 2*A*(
b + 4*c*x^2)) + 3*Sqrt[c]*(3*b*B + 2*A*c)*x^3*Log[-(Sqrt[c]*x) + Sqrt[b + c
*x^2]]))/(x^4*Sqrt[b + c*x^2])
```

Maple [A]
time = 0.38, size = 219, normalized size = 1.61

method	result
risch	$-\frac{(-3Bc\,x^4+8Ac\,x^2+6bB\,x^2+2Ab)\sqrt{x^2(c\,x^2+b)}}{6x^4}+\frac{\left(A\,c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)+\frac{3B\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+\right.}{2}\right.}{x\sqrt{c\,x^2+b}}$
default	$-\frac{(x^4c+b\,x^2)^{\frac{3}{2}}\left(-4A(c\,x^2+b)^{\frac{3}{2}}c^{\frac{5}{2}}x^4-6B(c\,x^2+b)^{\frac{3}{2}}c^{\frac{3}{2}}b\,x^4+4A(c\,x^2+b)^{\frac{5}{2}}c^{\frac{3}{2}}x^2-6A\sqrt{c\,x^2+b}\,c^{\frac{5}{2}}b\,x^4+6B(c\,x^2+b)^{\frac{5}{2}}\sqrt{c}\,b\,x^2-\right.}{6x^6(c\,x^2+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(c*x^4+b*x^2)^(3/2)*(-4*A*(c*x^2+b)^(3/2)*c^(5/2)*x^4-6*B*(c*x^2+b)^(3
/2)*c^(3/2)*b*x^4+4*A*(c*x^2+b)^(5/2)*c^(3/2)*x^2-6*A*(c*x^2+b)^(1/2)*c^(5/
2)*b*x^4+6*B*(c*x^2+b)^(5/2)*c^(1/2)*b*x^2-9*B*(c*x^2+b)^(1/2)*c^(3/2)*b^2*
x^4-6*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2*c^2*x^3-9*B*ln(c^(1/2)*x+(c*x^2+b
)^(1/2))*b^3*c*x^3+2*A*(c*x^2+b)^(5/2)*c^(1/2)*b)/x^6/(c*x^2+b)^(3/2)/b^2/c
^(1/2)
```

Maxima [A]
time = 0.30, size = 167, normalized size = 1.23

$$\frac{1}{6}\left(3\,c^{\frac{3}{2}}\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)-\frac{7\,\sqrt{cx^{4}+bx^{2}}\,c}{x^{2}}-\frac{\sqrt{cx^{4}+bx^{2}}\,b}{x^{4}}-\frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{x^{6}}\right)A+\frac{1}{4}\left(3\,b\sqrt{c}\,\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)-\frac{6\,\sqrt{cx^{4}+bx^{2}}\,b}{x^{2}}+\frac{2\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{x^{4}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*(3*c^(3/2)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 7*sqrt(c*
x^4 + b*x^2)*c/x^2 - sqrt(c*x^4 + b*x^2)*b/x^4 - (c*x^4 + b*x^2)^(3/2)/x^6)
*A + 1/4*(3*b*sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 6*
sqrt(c*x^4 + b*x^2)*b/x^2 + 2*(c*x^4 + b*x^2)^(3/2)/x^4)*B
```

Fricas [A]
time = 1.07, size = 189, normalized size = 1.39

$$\left[\frac{3\,(3\,Bb+2\,Ac)\sqrt{c}\,x^{4}\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)+2\,(3\,Bcx^{4}-2\,(3\,Bb+4\,Ac)x^{2}-2\,Ab)\sqrt{cx^{4}+bx^{2}}}{12\,x^{4}},\,-\frac{3\,(3\,Bb+2\,Ac)\sqrt{-c}\,x^{4}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)-(3\,Bcx^{4}-2\,(3\,Bb+4\,Ac)x^{2}-2\,Ab)\sqrt{cx^{4}+bx^{2}}}{6\,x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(3*B*b + 2*A*c)*sqrt(c)*x^4*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^
2)*sqrt(c)) + 2*(3*B*c*x^4 - 2*(3*B*b + 4*A*c)*x^2 - 2*A*b)*sqrt(c*x^4 + b*
x^2))/x^4, -1/6*(3*(3*B*b + 2*A*c)*sqrt(-c)*x^4*arctan(sqrt(c*x^4 + b*x^2)*
sqrt(-c)/(c*x^2 + b)) - (3*B*c*x^4 - 2*(3*B*b + 4*A*c)*x^2 - 2*A*b)*sqrt(c*
x^4 + b*x^2))/x^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^{2}(b+cx^{2})\right)^{\frac{3}{2}}\left(A+Bx^{2}\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**7,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**7, x)
```

Giac [A]
time = 0.87, size = 225, normalized size = 1.65

$$\frac{1}{2}\sqrt{cx^2+b}\,Bcx\,\mathrm{sgn}(x)-\frac{1}{4}\left(3\,Bb\sqrt{c}\,\mathrm{sgn}(x)+2\,Ac^{\frac{3}{2}}\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2\right)+\frac{2\left(3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4Bb^2\sqrt{c}\,\mathrm{sgn}(x)+6\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4Abc^{\frac{3}{2}}\mathrm{sgn}(x)-6\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2Bb^3\sqrt{c}\,\mathrm{sgn}(x)-6\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2Ab^2c^{\frac{3}{2}}\mathrm{sgn}(x)+3\,Bb^4\sqrt{c}\,\mathrm{sgn}(x)+4\,Ab^3c^{\frac{3}{2}}\mathrm{sgn}(x)\right)}{3\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + b)*B*c*x*sgn(x) - 1/4*(3*B*b*sqrt(c)*sgn(x) + 2*A*c^(3/2)*
sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + b))^2) + 2/3*(3*(sqrt(c)*x - sqrt(c*x
^2 + b))^4*B*b^2*sqrt(c)*sgn(x) + 6*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b*c^(
3/2)*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^3*sqrt(c)*sgn(x) - 6*(s
qrt(c)*x - sqrt(c*x^2 + b))^2*A*b^2*c^(3/2)*sgn(x) + 3*B*b^4*sqrt(c)*sgn(x)
 + 4*A*b^3*c^(3/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7, x)
```

3.113 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=104

$$-\frac{Bc\sqrt{bx^2+cx^4}}{x^2}-\frac{B(bx^2+cx^4)^{3/2}}{3x^6}-\frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}}+Bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] -1/3*B*(c*x^4+b*x^2)^(3/2)/x^6-1/5*A*(c*x^4+b*x^2)^(5/2)/b/x^10+B*c^(3/2)*a
rctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))-B*c*(c*x^4+b*x^2)^(1/2)/x^2
```

Rubi [A]
time = 0.16, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2059, 806, 676, 634, 212}

$$-\frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}}+Bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{Bc\sqrt{bx^2+cx^4}}{x^2}-\frac{B(bx^2+cx^4)^{3/2}}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9,x]
```

```
[Out] -((B*c*Sqrt[b*x^2 + c*x^4])/x^2) - (B*(b*x^2 + c*x^4)^(3/2))/(3*x^6) - (A*(
b*x^2 + c*x^4)^(5/2))/(5*b*x^10) + B*c^(3/2)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x
^2 + c*x^4]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 676

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Dist[c*(p/(e^2*(m + p + 1))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^5}\,dx,x,x^2\right)\\
&= -\frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}}+\frac{1}{2}B\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{B(bx^2+cx^4)^{3/2}}{3x^6}-\frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}}+\frac{1}{2}(Bc)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx,x\right.\\
&= -\frac{Bc\sqrt{bx^2+cx^4}}{x^2}-\frac{B(bx^2+cx^4)^{3/2}}{3x^6}-\frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}}+\frac{1}{2}(Bc^2)\,\text{Subst}\left(\right.\\
&= -\frac{Bc\sqrt{bx^2+cx^4}}{x^2}-\frac{B(bx^2+cx^4)^{3/2}}{3x^6}-\frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}}+(Bc^2)\,\text{Subst}\left(\int\right.\\
&= -\frac{Bc\sqrt{bx^2+cx^4}}{x^2}-\frac{B(bx^2+cx^4)^{3/2}}{3x^6}-\frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}}+Bc^{3/2}\tanh^{-1}\left(\right.
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 110, normalized size = 1.06

$$-\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b+cx^2}\left(3A(b+cx^2)^2+5bBx^2(b+4cx^2)\right)+15bBc^{3/2}x^5\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{15bx^6\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9,x]
```

```
[Out] -1/15*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b + c*x^2]*(3*A*(b + c*x^2)^2 + 5*b*B*x^
2*(b + 4*c*x^2)) + 15*b*B*c^(3/2)*x^5*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))
/(b*x^6*Sqrt[b + c*x^2])
```

Maple [**A**]
time = 0.38, size = 153, normalized size = 1.47

method	result
risch	$-\frac{\left(3A\,c^{2}x^{4}+20x^{4}bBc+6Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{15x^{6}b}+\frac{B\,c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{x\sqrt{c\,x^{2}+b}}$
default	$-\frac{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(-10B\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{5}{2}}x^{6}+10B\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{3}{2}}x^{4}-15B\sqrt{c\,x^{2}+b}\,c^{\frac{5}{2}}b\,x^{6}-15B\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)b^{2}c^{2}x^{5}\right.}{15x^{8}\left(c\,x^{2}+b\right)^{\frac{3}{2}}b^{2}\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(c*x^4+b*x^2)^(3/2)*(-10*B*(c*x^2+b)^(3/2)*c^(5/2)*x^6+10*B*(c*x^2+b)
^(5/2)*c^(3/2)*x^4-15*B*(c*x^2+b)^(1/2)*c^(5/2)*b*x^6-15*B*ln(c^(1/2)*x+(c*
x^2+b)^(1/2))*b^2*c^2*x^5+5*B*(c*x^2+b)^(5/2)*c^(1/2)*b*x^2+3*A*(c*x^2+b)^(
5/2)*c^(1/2)*b)/x^8/(c*x^2+b)^(3/2)/b^2/c^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(88) = 176.
time = 0.28, size = 177, normalized size = 1.70

$$\frac{1}{6}\left(3\,c^{\frac{3}{2}}\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)-\frac{7\,\sqrt{cx^{4}+bx^{2}}\,c}{x^{2}}-\frac{\sqrt{cx^{4}+bx^{2}}\,b}{x^{4}}-\frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{x^{6}}\right)B-\frac{1}{10}\,A\left(\frac{2\,\sqrt{cx^{4}+bx^{2}}\,c^{2}}{bx^{2}}-\frac{\sqrt{cx^{4}+bx^{2}}\,c}{x^{4}}-\frac{3\,\sqrt{cx^{4}+bx^{2}}\,b}{x^{6}}+\frac{5\,\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{x^{8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/6*(3*c^(3/2)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 7*sqrt(c*
x^4 + b*x^2)*c/x^2 - sqrt(c*x^4 + b*x^2)*b/x^4 - (c*x^4 + b*x^2)^(3/2)/x^6)
*B - 1/10*A*(2*sqrt(c*x^4 + b*x^2)*c^2/(b*x^2) - sqrt(c*x^4 + b*x^2)*c/x^4
- 3*sqrt(c*x^4 + b*x^2)*b/x^6 + 5*(c*x^4 + b*x^2)^(3/2)/x^8)
```

Fricas [**A**]
time = 1.86, size = 207, normalized size = 1.99

$$\left[\frac{15\,Bbc^{\frac{3}{2}}x^{6}\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)-2\left(\left(20\,Bbc+3\,Ac^{2}\right)x^{4}+3\,Ab^{2}+\left(5\,Bb^{2}+6\,Abc\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}}}{30\,bx^{6}},-\frac{15\,Bb\sqrt{-c}\,cx^{6}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)+\left(\left(20\,Bbc+3\,Ac^{2}\right)x^{4}+3\,Ab^{2}+\left(5\,Bb^{2}+6\,Abc\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}}}{15\,bx^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/30*(15*B*b*c^(3/2)*x^6*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))
 - 2*((20*B*b*c + 3*A*c^2)*x^4 + 3*A*b^2 + (5*B*b^2 + 6*A*b*c)*x^2)*sqrt(c*
x^4 + b*x^2))/(b*x^6), -1/15*(15*B*b*sqrt(-c)*c*x^6*arctan(sqrt(c*x^4 + b*x
^2)*sqrt(-c)/(c*x^2 + b)) + ((20*B*b*c + 3*A*c^2)*x^4 + 3*A*b^2 + (5*B*b^2
+ 6*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}(b+cx^{2})\right)^{\frac{3}{2}}\left(A+Bx^{2}\right)}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**9,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**9, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(88) = 176.
time = 1.20, size = 254, normalized size = 2.44

$$-\frac{1}{2}Bc^{\frac{3}{2}}\log\left(\left(\sqrt{c}\,x-\sqrt{cx^{2}+b}\right)^{2}\right)\mathrm{sgn}(x)+\frac{2\left(30\left(\sqrt{c}\,x-\sqrt{cx^{2}+b}\right)^{8}Bbc^{\frac{3}{2}}\mathrm{sgn}(x)+15\left(\sqrt{c}\,x-\sqrt{cx^{2}+b}\right)^{8}Ac^{\frac{5}{2}}\mathrm{sgn}(x)-90\left(\sqrt{c}\,x-\sqrt{cx^{2}+b}\right)^{6}Bb^{2}c^{\frac{3}{2}}\mathrm{sgn}(x)+110\left(\sqrt{c}\,x-\sqrt{cx^{2}+b}\right)^{4}Bb^{3}c^{\frac{3}{2}}\mathrm{sgn}(x)+30\left(\sqrt{c}\,x-\sqrt{cx^{2}+b}\right)^{4}Ab^{2}c^{\frac{5}{2}}\mathrm{sgn}(x)-70\left(\sqrt{c}\,x-\sqrt{cx^{2}+b}\right)^{2}Bb^{4}c^{\frac{3}{2}}\mathrm{sgn}(x)+20\,Bb^{5}c^{\frac{3}{2}}\mathrm{sgn}(x)+3\,Ab^{4}c^{\frac{5}{2}}\mathrm{sgn}(x)\right)}{15\left(\left(\sqrt{c}\,x-\sqrt{cx^{2}+b}\right)^{2}-b\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/2*B*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 2/15*(30*(sqrt
(c)*x - sqrt(c*x^2 + b))^8*B*b*c^(3/2)*sgn(x) + 15*(sqrt(c)*x - sqrt(c*x^2
+ b))^8*A*c^(5/2)*sgn(x) - 90*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^2*c^(3/2)
*sgn(x) + 110*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^3*c^(3/2)*sgn(x) + 30*(sq
rt(c)*x - sqrt(c*x^2 + b))^4*A*b^2*c^(5/2)*sgn(x) - 70*(sqrt(c)*x - sqrt(c*
x^2 + b))^2*B*b^4*c^(3/2)*sgn(x) + 20*B*b^5*c^(3/2)*sgn(x) + 3*A*b^4*c^(5/2
)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^{2}+A\right)\,{\left(c\,x^{4}+b\,x^{2}\right)}^{3/2}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9, x)
```

3.114 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}} dx$

Optimal. Leaf size=61

$$-\frac{A(bx^2+cx^4)^{5/2}}{7bx^{12}}-\frac{(7bB-2Ac)(bx^2+cx^4)^{5/2}}{35b^2x^{10}}$$

```
[Out] -1/7*A*(c*x^4+b*x^2)^(5/2)/b/x^12-1/35*(-2*A*c+7*B*b)*(c*x^4+b*x^2)^(5/2)/b
^2/x^10
```

Rubi [A]
time = 0.11, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2059, 806, 664}

$$-\frac{(bx^2+cx^4)^{5/2}(7bB-2Ac)}{35b^2x^{10}}-\frac{A(bx^2+cx^4)^{5/2}}{7bx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^11,x]
```

```
[Out] -1/7*(A*(b*x^2 + c*x^4)^(5/2))/(b*x^12) - ((7*b*B - 2*A*c)*(b*x^2 + c*x^4)^
(5/2))/(35*b^2*x^10)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
```

```
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{7bx^{12}} + \frac{\left(-6(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3}}{x^5}\right.}{7b}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{7bx^{12}} - \frac{(7bB-2Ac)(bx^2+cx^4)^{5/2}}{35b^2x^{10}}$$

Mathematica [A]
time = 0.16, size = 44, normalized size = 0.72

$$\frac{(x^2(b+cx^2))^{5/2}(-5Ab-7bBx^2+2Acx^2)}{35b^2x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^11,x]
[Out] ((x^2*(b + c*x^2))^(5/2)*(-5*A*b - 7*b*B*x^2 + 2*A*c*x^2))/(35*b^2*x^12)
```

Maple [A]
time = 0.50, size = 48, normalized size = 0.79

method	result	size
gosper	$-\frac{(c\,x^2+b)(-2Ac\,x^2+7bB\,x^2+5Ab)(x^4c+b\,x^2)^{\frac{3}{2}}}{35b^2x^{10}}$	48
default	$-\frac{(c\,x^2+b)(-2Ac\,x^2+7bB\,x^2+5Ab)(x^4c+b\,x^2)^{\frac{3}{2}}}{35b^2x^{10}}$	48
trager	$-\frac{(-2A\,c^3x^6+7x^6Bb\,c^2+Ab\,c^2x^4+14x^4B\,b^2c+8A\,b^2c\,x^2+7x^2B\,b^3+5A\,b^3)\sqrt{x^4c+b\,x^2}}{35b^2x^8}$	86
risch	$-\frac{\sqrt{x^2\,(c\,x^2+b)}\,(-2A\,c^3x^6+7x^6Bb\,c^2+Ab\,c^2x^4+14x^4B\,b^2c+8A\,b^2c\,x^2+7x^2B\,b^3+5A\,b^3)}{35x^8b^2}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*(c*x^2+b)*(-2*A*c*x^2+7*B*b*x^2+5*A*b)*(c*x^4+b*x^2)^(3/2)/b^2/x^10
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(53) = 106.
time = 0.29, size = 193, normalized size = 3.16

$$-\frac{1}{10}\,B\left(\frac{2\,\sqrt{cx^4+bx^2}\,c^2}{bx^2}-\frac{\sqrt{cx^4+bx^2}\,c}{x^4}-\frac{3\,\sqrt{cx^4+bx^2}\,b}{x^6}+\frac{5\,(cx^4+bx^2)^{\frac{3}{2}}}{x^8}\right)+\frac{1}{140}\,A\left(\frac{8\,\sqrt{cx^4+bx^2}\,c^3}{b^2x^2}-\frac{4\,\sqrt{cx^4+bx^2}\,c^2}{bx^4}+\frac{3\,\sqrt{cx^4+bx^2}\,c}{x^6}+\frac{15\,\sqrt{cx^4+bx^2}\,b}{x^8}-\frac{35\,(cx^4+bx^2)^{\frac{3}{2}}}{x^{10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/10*B*(2*sqrt(c*x^4 + b*x^2)*c^2/(b*x^2) - sqrt(c*x^4 + b*x^2)*c/x^4 - 3*
sqrt(c*x^4 + b*x^2)*b/x^6 + 5*(c*x^4 + b*x^2)^(3/2)/x^8) + 1/140*A*(8*sqrt(
c*x^4 + b*x^2)*c^3/(b^2*x^2) - 4*sqrt(c*x^4 + b*x^2)*c^2/(b*x^4) + 3*sqrt(c
*x^4 + b*x^2)*c/x^6 + 15*sqrt(c*x^4 + b*x^2)*b/x^8 - 35*(c*x^4 + b*x^2)^(3/
2)/x^10)
```

Fricas [A]
time = 1.13, size = 82, normalized size = 1.34

$$-\frac{\left((7\,Bbc^2-2\,Ac^3)x^6+(14\,Bb^2c+Abc^2)x^4+5\,Ab^3+(7\,Bb^3+8\,Ab^2c)x^2\right)\sqrt{cx^4+bx^2}}{35\,b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/35*((7*B*b*c^2 - 2*A*c^3)*x^6 + (14*B*b^2*c + A*b*c^2)*x^4 + 5*A*b^3 + (
7*B*b^3 + 8*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^2*x^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**11,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**11, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(53) = 106.
time = 1.87, size = 370, normalized size = 6.07

$$\frac{2\left(35\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{12}Bc^{\frac{5}{2}}\mathrm{sgn}(x)-70\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}Bbc^{\frac{5}{2}}\mathrm{sgn}(x)+70\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}Ac^{\frac{7}{2}}\mathrm{sgn}(x)+105\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{8}Bb^2c^{\frac{5}{2}}\mathrm{sgn}(x)+70\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{8}Abc^{\frac{7}{2}}\mathrm{sgn}(x)-140\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{6}Bb^3c^{\frac{5}{2}}\mathrm{sgn}(x)+140\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{6}Ab^2c^{\frac{7}{2}}\mathrm{sgn}(x)+77\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{4}Bb^4c^{\frac{5}{2}}\mathrm{sgn}(x)+28\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{4}Ab^3c^{\frac{7}{2}}\mathrm{sgn}(x)-14\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{2}Bb^5c^{\frac{5}{2}}\mathrm{sgn}(x)+14\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{2}Ab^4c^{\frac{7}{2}}\mathrm{sgn}(x)+7\,Bb^6c^{\frac{5}{2}}\mathrm{sgn}(x)-2\,Ab^5c^{\frac{7}{2}}\mathrm{sgn}(x)\right)}{35\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] 2/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*c^(5/2)*sgn(x) - 70*(sqrt(c)*x
- sqrt(c*x^2 + b))^10*B*b*c^(5/2)*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))
^10*A*c^(7/2)*sgn(x) + 105*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^2*c^(5/2)*sg
n(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b*c^(7/2)*sgn(x) - 140*(sqrt(c)
*x - sqrt(c*x^2 + b))^6*B*b^3*c^(5/2)*sgn(x) + 140*(sqrt(c)*x - sqrt(c*x^2
+ b))^6*A*b^2*c^(7/2)*sgn(x) + 77*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^4*c^(
5/2)*sgn(x) + 28*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^3*c^(7/2)*sgn(x) - 14*
(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^5*c^(5/2)*sgn(x) + 14*(sqrt(c)*x - sqrt
(c*x^2 + b))^2*A*b^4*c^(7/2)*sgn(x) + 7*B*b^6*c^(5/2)*sgn(x) - 2*A*b^5*c^(7
/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7
```

Mupad [**B**]
time = 1.03, size = 156, normalized size = 2.56

$$\frac{2\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{35\,b^2\,x^2}-\frac{8\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,x^6}-\frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{5\,x^6}-\frac{2\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{5\,x^4}-\frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^4}-\frac{A\,b\,\sqrt{c\,x^4+b\,x^2}}{7\,x^8}-\frac{B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{5\,b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^11,x)
```

```
[Out] (2*A*c^3*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^2) - (8*A*c*(b*x^2 + c*x^4)^(1/2)
)/(35*x^6) - (B*b*(b*x^2 + c*x^4)^(1/2))/(5*x^6) - (2*B*c*(b*x^2 + c*x^4)^(
1/2))/(5*x^4) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b*x^4) - (A*b*(b*x^2 + c*
x^4)^(1/2))/(7*x^8) - (B*c^2*(b*x^2 + c*x^4)^(1/2))/(5*b*x^2)
```

3.115 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=96

$$-\frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}}-\frac{(9bB-4Ac)(bx^2+cx^4)^{5/2}}{63b^2x^{12}}+\frac{2c(9bB-4Ac)(bx^2+cx^4)^{5/2}}{315b^3x^{10}}$$

```
[Out] -1/9*A*(c*x^4+b*x^2)^(5/2)/b/x^14-1/63*(-4*A*c+9*B*b)*(c*x^4+b*x^2)^(5/2)/b
^2/x^12+2/315*c*(-4*A*c+9*B*b)*(c*x^4+b*x^2)^(5/2)/b^3/x^10
```

Rubi [A]
time = 0.15, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2059, 806, 672, 664}

$$\frac{2c(bx^2+cx^4)^{5/2}(9bB-4Ac)}{315b^3x^{10}}-\frac{(bx^2+cx^4)^{5/2}(9bB-4Ac)}{63b^2x^{12}}-\frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^13,x]
```

```
[Out] -1/9*(A*(b*x^2 + c*x^4)^(5/2))/(b*x^14) - ((9*b*B - 4*A*c)*(b*x^2 + c*x^4)^
(5/2))/(63*b^2*x^12) + (2*c*(9*b*B - 4*A*c)*(b*x^2 + c*x^4)^(5/2))/(315*b^3
*x^10)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
```

```
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^7}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}} + \frac{\left(-7(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3}}{x^6}\right.}{9b}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}} - \frac{(9bB-4Ac)(bx^2+cx^4)^{5/2}}{63b^2x^{12}} - \frac{(c(9bB-4Ac))\text{Subst}\left(\right.}{63}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}} - \frac{(9bB-4Ac)(bx^2+cx^4)^{5/2}}{63b^2x^{12}} + \frac{2c(9bB-4Ac)(bx^2+}{315b^3x^{10}}$$

Mathematica [**A**]
time = 0.18, size = 66, normalized size = 0.69

$$\frac{(x^2(b+cx^2))^{5/2}\left(9bBx^2(-5b+2cx^2)+A(-35b^2+20bcx^2-8c^2x^4)\right)}{315b^3x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^13,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(9*b*B*x^2*(-5*b + 2*c*x^2) + A*(-35*b^2 + 20*b*c*
x^2 - 8*c^2*x^4)))/(315*b^3*x^14)
```

Maple [**A**]
time = 0.38, size = 70, normalized size = 0.73

method	result	si
gosper	$-\frac{(c\,x^2+b)\left(8A\,c^2x^4-18x^4bBc-20Abc\,x^2+45b^2B\,x^2+35b^2A\right)\left(x^4c+b\,x^2\right)^{\frac{3}{2}}}{315b^3x^{12}}$	70
default	$-\frac{(c\,x^2+b)\left(8A\,c^2x^4-18x^4bBc-20Abc\,x^2+45b^2B\,x^2+35b^2A\right)\left(x^4c+b\,x^2\right)^{\frac{3}{2}}}{315b^3x^{12}}$	70
trager	$-\frac{\left(8A\,c^4x^8-18Bb\,c^3x^8-4Ab\,c^3x^6+9B\,b^2c^2x^6+3A\,b^2c^2x^4+72B\,b^3c\,x^4+50A\,b^3c\,x^2+45B\,b^4x^2+35A\,b^4\right)\sqrt{x^4c+b\,x^2}}{315b^3x^{10}}$	1
risch	$-\frac{\sqrt{x^2\,(c\,x^2+b)}\,\left(8A\,c^4x^8-18Bb\,c^3x^8-4Ab\,c^3x^6+9B\,b^2c^2x^6+3A\,b^2c^2x^4+72B\,b^3c\,x^4+50A\,b^3c\,x^2+45B\,b^4x^2+35A\,b^4\right)}{315x^{10}b^3}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(c*x^2+b)*(8*A*c^2*x^4-18*B*b*c*x^4-20*A*b*c*x^2+45*B*b^2*x^2+35*A*b
^2)*(c*x^4+b*x^2)^(3/2)/b^3/x^12
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(84) = 168$.
time = 0.29, size = 241, normalized size = 2.51

$$\frac{1}{140}\,B\left(\frac{8\,\sqrt{cx^4+bx^2}\,c^3}{b^2x^2}-\frac{4\,\sqrt{cx^4+bx^2}\,c^2}{bx^4}+\frac{3\,\sqrt{cx^4+bx^2}\,c}{x^6}+\frac{15\,\sqrt{cx^4+bx^2}\,b}{x^8}-\frac{35\,(cx^4+bx^2)^{\frac{3}{2}}}{x^{10}}\right)-\frac{1}{630}\,A\left(\frac{16\,\sqrt{cx^4+bx^2}\,c^4}{b^3x^2}-\frac{8\,\sqrt{cx^4+bx^2}\,c^3}{b^2x^4}+\frac{6\,\sqrt{cx^4+bx^2}\,c^2}{bx^6}-\frac{5\,\sqrt{cx^4+bx^2}\,c}{x^8}-\frac{35\,\sqrt{cx^4+bx^2}\,b}{x^{10}}+\frac{105\,(cx^4+bx^2)^{\frac{3}{2}}}{x^{12}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] 1/140*B*(8*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^2) - 4*sqrt(c*x^4 + b*x^2)*c^2/(b
*x^4) + 3*sqrt(c*x^4 + b*x^2)*c/x^6 + 15*sqrt(c*x^4 + b*x^2)*b/x^8 - 35*(c*
x^4 + b*x^2)^(3/2)/x^10) - 1/630*A*(16*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^2) -
8*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^4) + 6*sqrt(c*x^4 + b*x^2)*c^2/(b*x^6) - 5
*sqrt(c*x^4 + b*x^2)*c/x^8 - 35*sqrt(c*x^4 + b*x^2)*b/x^10 + 105*(c*x^4 + b
*x^2)^(3/2)/x^12)
```

Fricas [A]
time = 1.75, size = 109, normalized size = 1.14

$$\frac{\left(2\,(9\,Bbc^3-4\,Ac^4)x^8-(9\,Bb^2c^2-4\,Abc^3)x^6-35\,Ab^4-3\,(24\,Bb^3c+Ab^2c^2)x^4-5\,(9\,Bb^4+10\,Ab^3c)x^2\right)\sqrt{cx^4+bx^2}}{315\,b^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] 1/315*(2*(9*B*b*c^3 - 4*A*c^4)*x^8 - (9*B*b^2*c^2 - 4*A*b*c^3)*x^6 - 35*A*b
^4 - 3*(24*B*b^3*c + A*b^2*c^2)*x^4 - 5*(9*B*b^4 + 10*A*b^3*c)*x^2)*sqrt(c*
x^4 + b*x^2)/(b^3*x^10)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**13,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**13, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(84) = 168.
time = 2.77, size = 430, normalized size = 4.48

$$\frac{4\left(315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Bc^{\frac{7}{2}}\mathrm{sgn}(x)-315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bbc^{\frac{7}{2}}\mathrm{sgn}(x)+840\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Ac^{\frac{9}{2}}\mathrm{sgn}(x)+315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bb^2c^{\frac{7}{2}}\mathrm{sgn}(x)+1260\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Abc^{\frac{9}{2}}\mathrm{sgn}(x)-819\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Bb^3c^{\frac{7}{2}}\mathrm{sgn}(x)+1764\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Ab^2c^{\frac{9}{2}}\mathrm{sgn}(x)+441\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}Bb^4c^{\frac{7}{2}}\mathrm{sgn}(x)+504\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}Ab^3c^{\frac{9}{2}}\mathrm{sgn}(x)-9\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}Bb^5c^{\frac{7}{2}}\mathrm{sgn}(x)+144\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}Ab^4c^{\frac{9}{2}}\mathrm{sgn}(x)+81\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}Bb^6c^{\frac{7}{2}}\mathrm{sgn}(x)-36\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}Ab^5c^{\frac{9}{2}}\mathrm{sgn}(x)-9Bb^7c^{\frac{7}{2}}\mathrm{sgn}(x)+4Ab^6c^{\frac{9}{2}}\mathrm{sgn}(x)\right)}{315\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(c)*x - sqrt(c*x^2 + b))^14*B*c^(7/2)*sgn(x) - 315*(sqrt(c)
*x - sqrt(c*x^2 + b))^12*B*b*c^(7/2)*sgn(x) + 840*(sqrt(c)*x - sqrt(c*x^2 +
 b))^12*A*c^(9/2)*sgn(x) + 315*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^2*c^(7/
2)*sgn(x) + 1260*(sqrt(c)*x - sqrt(c*x^2 + b))^10*A*b*c^(9/2)*sgn(x) - 819*
(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^3*c^(7/2)*sgn(x) + 1764*(sqrt(c)*x - sq
rt(c*x^2 + b))^8*A*b^2*c^(9/2)*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 + b))^6
*B*b^4*c^(7/2)*sgn(x) + 504*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^3*c^(9/2)*s
gn(x) - 9*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^5*c^(7/2)*sgn(x) + 144*(sqrt(
c)*x - sqrt(c*x^2 + b))^4*A*b^4*c^(9/2)*sgn(x) + 81*(sqrt(c)*x - sqrt(c*x^2
 + b))^2*B*b^6*c^(7/2)*sgn(x) - 36*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^5*c^
(9/2)*sgn(x) - 9*B*b^7*c^(7/2)*sgn(x) + 4*A*b^6*c^(9/2)*sgn(x))/((sqrt(c)*x
 - sqrt(c*x^2 + b))^2 - b)^9
```

Mupad [B]
time = 1.44, size = 206, normalized size = 2.15

$$\frac{4\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^2\,x^4}-\frac{10\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,x^8}-\frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{7\,x^8}-\frac{8\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,x^6}-\frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b\,x^6}-\frac{A\,b\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}}-\frac{8\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^2}-\frac{B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^4}+\frac{2\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{35\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^13,x)
```

```
[Out] (4*A*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^2*x^4) - (10*A*c*(b*x^2 + c*x^4)^(1/
2))/(63*x^8) - (B*b*(b*x^2 + c*x^4)^(1/2))/(7*x^8) - (8*B*c*(b*x^2 + c*x^4)
^(1/2))/(35*x^6) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b*x^6) - (A*b*(b*x^2
+ c*x^4)^(1/2))/(9*x^10) - (8*A*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^2) -
(B*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b*x^4) + (2*B*c^3*(b*x^2 + c*x^4)^(1/2))/
(35*b^2*x^2)
```

3.116 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15}} \, dx$

Optimal. Leaf size=133

$$-\frac{A(bx^2+cx^4)^{5/2}}{11bx^{16}}-\frac{(11bB-6Ac)(bx^2+cx^4)^{5/2}}{99b^2x^{14}}+\frac{4c(11bB-6Ac)(bx^2+cx^4)^{5/2}}{693b^3x^{12}}-\frac{8c^2(11bB-6Ac)(bx^2+}{3465b^4x^{10}}$$

```
[Out] -1/11*A*(c*x^4+b*x^2)^(5/2)/b/x^16-1/99*(-6*A*c+11*B*b)*(c*x^4+b*x^2)^(5/2)
/b^2/x^14+4/693*c*(-6*A*c+11*B*b)*(c*x^4+b*x^2)^(5/2)/b^3/x^12-8/3465*c^2*(
-6*A*c+11*B*b)*(c*x^4+b*x^2)^(5/2)/b^4/x^10
```

Rubi [A]
time = 0.19, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2059, 806, 672, 664}

$$-\frac{8c^2(bx^2+cx^4)^{5/2}(11bB-6Ac)}{3465b^4x^{10}}+\frac{4c(bx^2+cx^4)^{5/2}(11bB-6Ac)}{693b^3x^{12}}-\frac{(bx^2+cx^4)^{5/2}(11bB-6Ac)}{99b^2x^{14}}-\frac{A(bx^2+cx^4)^{5/2}}{11bx^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^15,x]
```

```
[Out] -1/11*(A*(b*x^2 + c*x^4)^(5/2))/(b*x^16) - ((11*b*B - 6*A*c)*(b*x^2 + c*x^4
)^(5/2))/(99*b^2*x^14) + (4*c*(11*b*B - 6*A*c)*(b*x^2 + c*x^4)^(5/2))/(693*
b^3*x^12) - (8*c^2*(11*b*B - 6*A*c)*(b*x^2 + c*x^4)^(5/2))/(3465*b^4*x^10)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
```

```
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^8}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{11bx^{16}} + \frac{\left(-8(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3}}{x^7}\right.}{11b}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{11bx^{16}} - \frac{(11bB-6Ac)(bx^2+cx^4)^{5/2}}{99b^2x^{14}} - \frac{(2c(11bB-6Ac))\text{Sul}}{}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{11bx^{16}} - \frac{(11bB-6Ac)(bx^2+cx^4)^{5/2}}{99b^2x^{14}} + \frac{4c(11bB-6Ac)(bx^2}{693b^3x^{12}}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{11bx^{16}} - \frac{(11bB-6Ac)(bx^2+cx^4)^{5/2}}{99b^2x^{14}} + \frac{4c(11bB-6Ac)(bx^2}{693b^3x^{12}}$$

Mathematica [A]
time = 0.23, size = 89, normalized size = 0.67

$$-\frac{(x^2(b+cx^2))^{5/2}\,(11bBx^2(35b^2-20bcx^2+8c^2x^4)+3A(105b^3-70b^2cx^2+40bc^2x^4-16c^3x^6))}{3465b^4x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^15,x]
```

```
[Out] -1/3465*((x^2*(b + c*x^2))^(5/2)*(11*b*B*x^2*(35*b^2 - 20*b*c*x^2 + 8*c^2*x
^4) + 3*A*(105*b^3 - 70*b^2*c*x^2 + 40*b*c^2*x^4 - 16*c^3*x^6)))/(b^4*x^16)
```

Maple [A]
time = 0.40, size = 94, normalized size = 0.71

method	result
gosper	$-\frac{(c\,x^2+b)\,(-48A\,c^3x^6+88x^6Bb\,c^2+120Ab\,c^2x^4-220x^4B\,b^2c-210A\,b^2c\,x^2+385x^2B\,b^3+315A\,b^3)\,(x^4c+b\,x^2)^{\frac{3}{2}}}{3465b^4x^{14}}$
default	$-\frac{(c\,x^2+b)\,(-48A\,c^3x^6+88x^6Bb\,c^2+120Ab\,c^2x^4-220x^4B\,b^2c-210A\,b^2c\,x^2+385x^2B\,b^3+315A\,b^3)\,(x^4c+b\,x^2)^{\frac{3}{2}}}{3465b^4x^{14}}$
trager	$-\frac{(-48A\,c^5x^{10}+88Bb\,c^4x^{10}+24Ab\,c^4x^8-44B\,b^2c^3x^8-18A\,b^2c^3x^6+33B\,b^3c^2x^6+15A\,b^3c^2x^4+550B\,b^4c\,x^4+420A\,b^4c\,x^2+385b^5B\,x}{3465b^4x^{12}}$
risch	$-\frac{\sqrt{x^2\,(c\,x^2+b)}\,(-48A\,c^5x^{10}+88Bb\,c^4x^{10}+24Ab\,c^4x^8-44B\,b^2c^3x^8-18A\,b^2c^3x^6+33B\,b^3c^2x^6+15A\,b^3c^2x^4+550B\,b^4c\,x^4+4}{3465x^{12}b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3465*(c*x^2+b)*(-48*A*c^3*x^6+88*B*b*c^2*x^6+120*A*b*c^2*x^4-220*B*b^2*c
*x^4-210*A*b^2*c*x^2+385*B*b^3*x^2+315*A*b^3)*(c*x^4+b*x^2)^(3/2)/b^4/x^14
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(117) = 234$.
time = 0.30, size = 289, normalized size = 2.17

$$-\frac{1}{630}\,B\left(\frac{16\,\sqrt{cx^4+bx^2}\,c^4}{b^3x^2}-\frac{8\,\sqrt{cx^4+bx^2}\,c^3}{b^2x^4}+\frac{6\,\sqrt{cx^4+bx^2}\,c^2}{bx^6}-\frac{5\,\sqrt{cx^4+bx^2}\,c}{x^8}-\frac{35\,\sqrt{cx^4+bx^2}\,b}{x^{10}}+\frac{105\,(cx^4+bx^2)^{\frac{3}{2}}}{x^{12}}\right)+\frac{1}{9240}\,A\left(\frac{128\,\sqrt{cx^4+bx^2}\,c^5}{b^4x^2}-\frac{64\,\sqrt{cx^4+bx^2}\,c^4}{b^3x^4}+\frac{48\,\sqrt{cx^4+bx^2}\,c^3}{b^2x^6}-\frac{40\,\sqrt{cx^4+bx^2}\,c^2}{bx^8}+\frac{35\,\sqrt{cx^4+bx^2}\,c}{x^{10}}+\frac{315\,\sqrt{cx^4+bx^2}\,b}{x^{12}}-\frac{1155\,(cx^4+bx^2)^{\frac{3}{2}}}{x^{14}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] -1/630*B*(16*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^3/
(b^2*x^4) + 6*sqrt(c*x^4 + b*x^2)*c^2/(b*x^6) - 5*sqrt(c*x^4 + b*x^2)*c/x^8
 - 35*sqrt(c*x^4 + b*x^2)*b/x^10 + 105*(c*x^4 + b*x^2)^(3/2)/x^12) + 1/9240
*A*(128*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^2) - 64*sqrt(c*x^4 + b*x^2)*c^4/(b^3
*x^4) + 48*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^6) - 40*sqrt(c*x^4 + b*x^2)*c^2/(
b*x^8) + 35*sqrt(c*x^4 + b*x^2)*c/x^10 + 315*sqrt(c*x^4 + b*x^2)*b/x^12 - 1
155*(c*x^4 + b*x^2)^(3/2)/x^14)
```

Fricas [A]
time = 1.50, size = 134, normalized size = 1.01

$$-\frac{(8\,(11\,Bbc^4-6\,Ac^5)x^{10}-4\,(11\,Bb^2c^3-6\,Abc^4)x^8+3\,(11\,Bb^3c^2-6\,Ab^2c^3)x^6+315\,Ab^5+5\,(110\,Bb^4c+3\,Ab^3c^2)x^4+35\,(11\,Bb^5+12\,Ab^4c)x^2)\sqrt{cx^4+bx^2}}{3465\,b^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/3465*(8*(11*B*b*c^4 - 6*A*c^5)*x^10 - 4*(11*B*b^2*c^3 - 6*A*b*c^4)*x^8 +
 3*(11*B*b^3*c^2 - 6*A*b^2*c^3)*x^6 + 315*A*b^5 + 5*(110*B*b^4*c + 3*A*b^3*
c^2)*x^4 + 35*(11*B*b^5 + 12*A*b^4*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^4*x^12)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**15,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**15, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(117) = 234.
time = 2.56, size = 490, normalized size = 3.68

$$\frac{16\left(2310\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{16}Bc^{\frac{9}{2}}\operatorname{sgn}(x)-1155\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Bbc^{\frac{9}{2}}\operatorname{sgn}(x)+6930\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Ac^{\frac{11}{2}}\operatorname{sgn}(x)+231\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bb^2c^{\frac{9}{2}}\operatorname{sgn}(x)+12474\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Abc^{\frac{11}{2}}\operatorname{sgn}(x)-4851\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bb^3c^{\frac{9}{2}}\operatorname{sgn}(x)+15246\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Ab^2c^{\frac{11}{2}}\operatorname{sgn}(x)+2475\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Bb^4c^{\frac{9}{2}}\operatorname{sgn}(x)+4950\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Ab^3c^{\frac{11}{2}}\operatorname{sgn}(x)+495\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}Bb^5c^{\frac{9}{2}}\operatorname{sgn}(x)+990\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}Ab^4c^{\frac{11}{2}}\operatorname{sgn}(x)+605\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}Bb^6c^{\frac{9}{2}}\operatorname{sgn}(x)-330\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}Ab^5c^{\frac{11}{2}}\operatorname{sgn}(x)-121\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}Bb^7c^{\frac{9}{2}}\operatorname{sgn}(x)+66\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}Ab^6c^{\frac{11}{2}}\operatorname{sgn}(x)+11\,Bb^8c^{\frac{9}{2}}\operatorname{sgn}(x)-6\,Ab^7c^{\frac{11}{2}}\operatorname{sgn}(x)\right)}{3465\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] 16/3465*(2310*(sqrt(c)*x - sqrt(c*x^2 + b))^16*B*c^(9/2)*sgn(x) - 1155*(sqr
t(c)*x - sqrt(c*x^2 + b))^14*B*b*c^(9/2)*sgn(x) + 6930*(sqrt(c)*x - sqrt(c*
x^2 + b))^14*A*c^(11/2)*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*b^2
*c^(9/2)*sgn(x) + 12474*(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*b*c^(11/2)*sgn(x
) - 4851*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^3*c^(9/2)*sgn(x) + 15246*(sqr
t(c)*x - sqrt(c*x^2 + b))^10*A*b^2*c^(11/2)*sgn(x) + 2475*(sqrt(c)*x - sqrt
(c*x^2 + b))^8*B*b^4*c^(9/2)*sgn(x) + 4950*(sqrt(c)*x - sqrt(c*x^2 + b))^8*
A*b^3*c^(11/2)*sgn(x) + 495*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^5*c^(9/2)*s
gn(x) + 990*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^4*c^(11/2)*sgn(x) + 605*(sq
rt(c)*x - sqrt(c*x^2 + b))^4*B*b^6*c^(9/2)*sgn(x) - 330*(sqrt(c)*x - sqrt(c
*x^2 + b))^4*A*b^5*c^(11/2)*sgn(x) - 121*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*
b^7*c^(9/2)*sgn(x) + 66*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^6*c^(11/2)*sgn(
x) + 11*B*b^8*c^(9/2)*sgn(x) - 6*A*b^7*c^(11/2)*sgn(x))/((sqrt(c)*x - sqrt(
c*x^2 + b))^2 - b)^11
```

Mupad [B]
time = 1.92, size = 256, normalized size = 1.92

$$\frac{2\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{385\,b^2\,x^6}-\frac{4\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{33\,x^{10}}-\frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}}-\frac{10\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,x^8}-\frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{231\,b\,x^8}-\frac{A\,b\,\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}}-\frac{8\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^3\,x^4}+\frac{16\,A\,c^5\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^4\,x^2}-\frac{B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b\,x^6}+\frac{4\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^2\,x^4}-\frac{8\,B\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^15,x)
```

```
[Out] (2*A*c^3*(b*x^2 + c*x^4)^(1/2))/(385*b^2*x^6) - (4*A*c*(b*x^2 + c*x^4)^(1/2
))/(33*x^10) - (B*b*(b*x^2 + c*x^4)^(1/2))/(9*x^10) - (10*B*c*(b*x^2 + c*x^
4)^(1/2))/(63*x^8) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(231*b*x^8) - (A*b*(b*x^
2 + c*x^4)^(1/2))/(11*x^12) - (8*A*c^4*(b*x^2 + c*x^4)^(1/2))/(1155*b^3*x^4
) + (16*A*c^5*(b*x^2 + c*x^4)^(1/2))/(1155*b^4*x^2) - (B*c^2*(b*x^2 + c*x^4
)^(1/2))/(105*b*x^6) + (4*B*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^2*x^4) - (8*B
*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^2)
```

3.117 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}} dx$

Optimal. Leaf size=170

$$-\frac{A(bx^2+cx^4)^{5/2}}{13bx^{18}}-\frac{(13bB-8Ac)(bx^2+cx^4)^{5/2}}{143b^2x^{16}}+\frac{2c(13bB-8Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{14}}-\frac{8c^2(13bB-8Ac)(bx^2+\cdots}{3003b^4x^{12}}$$

```
[Out] -1/13*A*(c*x^4+b*x^2)^(5/2)/b/x^18-1/143*(-8*A*c+13*B*b)*(c*x^4+b*x^2)^(5/2
)/b^2/x^16+2/429*c*(-8*A*c+13*B*b)*(c*x^4+b*x^2)^(5/2)/b^3/x^14-8/3003*c^2*
(-8*A*c+13*B*b)*(c*x^4+b*x^2)^(5/2)/b^4/x^12+16/15015*c^3*(-8*A*c+13*B*b)*(
c*x^4+b*x^2)^(5/2)/b^5/x^10
```

Rubi [A]
time = 0.22, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2059, 806, 672, 664}

$$\frac{16c^3(bx^2+cx^4)^{5/2}(13bB-8Ac)}{15015b^5x^{10}}-\frac{8c^2(bx^2+cx^4)^{5/2}(13bB-8Ac)}{3003b^4x^{12}}+\frac{2c(bx^2+cx^4)^{5/2}(13bB-8Ac)}{429b^3x^{14}}-\frac{(bx^2+cx^4)^{5/2}(13bB-8Ac)}{143b^2x^{16}}-\frac{A(bx^2+cx^4)^{5/2}}{13bx^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^17,x]
```

```
[Out] -1/13*(A*(b*x^2 + c*x^4)^(5/2))/(b*x^18) - ((13*b*B - 8*A*c)*(b*x^2 + c*x^4
)^(5/2))/(143*b^2*x^16) + (2*c*(13*b*B - 8*A*c)*(b*x^2 + c*x^4)^(5/2))/(429
*b^3*x^14) - (8*c^2*(13*b*B - 8*A*c)*(b*x^2 + c*x^4)^(5/2))/(3003*b^4*x^12)
 + (16*c^3*(13*b*B - 8*A*c)*(b*x^2 + c*x^4)^(5/2))/(15015*b^5*x^10)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{17}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^9}\,dx, x, x^2\right)$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} + \frac{\left(-9(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2}{x^8}\right.}{13b}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}} - \frac{(3c(13bB-8Ac))\text{Su}}{}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB-8Ac)\,(bx}{429b^3x^{14}}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB-8Ac)\,(bx}{429b^3x^{14}}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB-8Ac)\,(bx}{429b^3x^{14}}$$

Mathematica [A]
time = 0.26, size = 110, normalized size = 0.65

$$\frac{\left(x^2(b+cx^2)\right)^{5/2}\left(13bBx^2(-105b^3+70b^2cx^2-40bc^2x^4+16c^3x^6)+A(-1155b^4+840b^3cx^2-560b^2c^2x^4+320bc^3x^6-128c^4x^8)\right)}{15015b^5x^{18}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^17,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(13*b*B*x^2*(-105*b^3 + 70*b^2*c*x^2 - 40*b*c^2*x^
4 + 16*c^3*x^6) + A*(-1155*b^4 + 840*b^3*c*x^2 - 560*b^2*c^2*x^4 + 320*b*c^
3*x^6 - 128*c^4*x^8)))/(15015*b^5*x^18)
```

Maple [A]
time = 0.39, size = 118, normalized size = 0.69

method	result
gosper	$-\frac{(c\,x^2+b)\,(128A\,c^4x^8-208Bb\,c^3x^8-320Ab\,c^3x^6+520B\,b^2c^2x^6+560A\,b^2c^2x^4-910B\,b^3c\,x^4-840A\,b^3c\,x^2+1365B\,b^4x^2+1155A\,b^4)\,(x^4}{15015b^5x^{16}}$
default	$-\frac{(c\,x^2+b)\,(128A\,c^4x^8-208Bb\,c^3x^8-320Ab\,c^3x^6+520B\,b^2c^2x^6+560A\,b^2c^2x^4-910B\,b^3c\,x^4-840A\,b^3c\,x^2+1365B\,b^4x^2+1155A\,b^4)\,(x^4}{15015b^5x^{16}}$
trager	$-\frac{(128A\,c^6x^{12}-208Bb\,c^5x^{12}-64Ab\,c^5x^{10}+104B\,b^2c^4x^{10}+48A\,b^2c^4x^8-78B\,b^3c^3x^8-40A\,b^3c^3x^6+65B\,b^4c^2x^6+35A\,b^4c^2x^4+1820B\,b}{15015b^5x^{14}}$
risch	$-\frac{\sqrt{x^2\,(c\,x^2+b)}\,(128A\,c^6x^{12}-208Bb\,c^5x^{12}-64Ab\,c^5x^{10}+104B\,b^2c^4x^{10}+48A\,b^2c^4x^8-78B\,b^3c^3x^8-40A\,b^3c^3x^6+65B\,b^4c^2x^6+}{15015x^{14}b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15015*(c*x^2+b)*(128*A*c^4*x^8-208*B*b*c^3*x^8-320*A*b*c^3*x^6+520*B*b^2
*c^2*x^6+560*A*b^2*c^2*x^4-910*B*b^3*c*x^4-840*A*b^3*c*x^2+1365*B*b^4*x^2+1
155*A*b^4)*(c*x^4+b*x^2)^(3/2)/b^5/x^16
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(150) = 300$.
time = 0.29, size = 337, normalized size = 1.98

$$\frac{1}{9240}B\left(\frac{128\sqrt{cx^4+bx^2}\,c^5}{b^4x^2}-\frac{64\sqrt{cx^4+bx^2}\,c^4}{b^3x^4}+\frac{48\sqrt{cx^4+bx^2}\,c^3}{b^2x^6}-\frac{40\sqrt{cx^4+bx^2}\,c^2}{bx^8}+\frac{35\sqrt{cx^4+bx^2}\,c}{x^{10}}+\frac{315\sqrt{cx^4+bx^2}\,b}{x^{12}}-\frac{1155\,(cx^4+bx^2)^{\frac{3}{2}}}{x^{14}}\right)-\frac{1}{30030}A\left(\frac{256\sqrt{cx^4+bx^2}\,c^6}{b^5x^2}-\frac{128\sqrt{cx^4+bx^2}\,c^5}{b^4x^4}+\frac{96\sqrt{cx^4+bx^2}\,c^4}{b^3x^6}-\frac{80\sqrt{cx^4+bx^2}\,c^3}{b^2x^8}+\frac{70\sqrt{cx^4+bx^2}\,c^2}{bx^{10}}-\frac{63\sqrt{cx^4+bx^2}\,c}{x^{12}}-\frac{693\sqrt{cx^4+bx^2}\,b}{x^{14}}+\frac{3003\,(cx^4+bx^2)^{\frac{3}{2}}}{x^{16}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] 1/9240*B*(128*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^2) - 64*sqrt(c*x^4 + b*x^2)*c^
4/(b^3*x^4) + 48*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^6) - 40*sqrt(c*x^4 + b*x^2)
*c^2/(b*x^8) + 35*sqrt(c*x^4 + b*x^2)*c/x^10 + 315*sqrt(c*x^4 + b*x^2)*b/x^
12 - 1155*(c*x^4 + b*x^2)^(3/2)/x^14) - 1/30030*A*(256*sqrt(c*x^4 + b*x^2)*
c^6/(b^5*x^2) - 128*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^4) + 96*sqrt(c*x^4 + b*x
^2)*c^4/(b^3*x^6) - 80*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^8) + 70*sqrt(c*x^4 +
b*x^2)*c^2/(b*x^10) - 63*sqrt(c*x^4 + b*x^2)*c/x^12 - 693*sqrt(c*x^4 + b*x^
2)*b/x^14 + 3003*(c*x^4 + b*x^2)^(3/2)/x^16)
```

Fricas [A]
time = 1.82, size = 157, normalized size = 0.92

$$\frac{\left(16\,{\left(13\,Bbc^5-8\,Ac^6\right)}x^{12}-8\,{\left(13\,Bb^2c^4-8\,Abc^5\right)}x^{10}+6\,{\left(13\,Bb^3c^3-8\,Ab^2c^4\right)}x^8-1155\,Ab^6-5\,{\left(13\,Bb^4c^2-8\,Ab^3c^3\right)}x^6-35\,{\left(52\,Bb^5c+Ab^4c^2\right)}x^4-105\,{\left(13\,Bb^6+14\,Ab^5c\right)}x^2\right)\sqrt{cx^4+bx^2}}{15015\,b^5x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] 1/15015*(16*(13*B*b*c^5 - 8*A*c^6)*x^12 - 8*(13*B*b^2*c^4 - 8*A*b*c^5)*x^10
 + 6*(13*B*b^3*c^3 - 8*A*b^2*c^4)*x^8 - 1155*A*b^6 - 5*(13*B*b^4*c^2 - 8*A*
b^3*c^3)*x^6 - 35*(52*B*b^5*c + A*b^4*c^2)*x^4 - 105*(13*B*b^6 + 14*A*b^5*c
)*x^2)*sqrt(c*x^4 + b*x^2)/(b^5*x^14)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**17,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**17, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(150) = 300.
time = 2.74, size = 550, normalized size = 3.24

$$\frac{32\left(15015\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{18}Bc^{\frac{11}{2}}\mathrm{sgn}(x)-3003\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{16}Bbc^{\frac{11}{2}}\mathrm{sgn}(x)+48048\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{16}Ac^{\frac{13}{2}}\mathrm{sgn}(x)-6006\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Bb^2c^{\frac{11}{2}}\mathrm{sgn}(x)+96096\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Abc^{\frac{13}{2}}\mathrm{sgn}(x)-28314\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bb^3c^{\frac{11}{2}}\mathrm{sgn}(x)+109824\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Ab^2c^{\frac{13}{2}}\mathrm{sgn}(x)+13728\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bb^4c^{\frac{11}{2}}\mathrm{sgn}(x)+37752\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Ab^3c^{\frac{13}{2}}\mathrm{sgn}(x)+5720\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Bb^5c^{\frac{11}{2}}\mathrm{sgn}(x)+5720\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Ab^4c^{\frac{13}{2}}\mathrm{sgn}(x)+3718\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}Bb^6c^{\frac{11}{2}}\mathrm{sgn}(x)-2288\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}Ab^5c^{\frac{13}{2}}\mathrm{sgn}(x)-1014\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}Bb^7c^{\frac{11}{2}}\mathrm{sgn}(x)+624\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}Ab^6c^{\frac{13}{2}}\mathrm{sgn}(x)+169\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}Bb^8c^{\frac{11}{2}}\mathrm{sgn}(x)-104\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}Ab^7c^{\frac{13}{2}}\mathrm{sgn}(x)-13\,Bb^9c^{\frac{11}{2}}\mathrm{sgn}(x)+8\,Ab^8c^{\frac{13}{2}}\mathrm{sgn}(x)\right)}{15015\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] 32/15015*(15015*(sqrt(c)*x - sqrt(c*x^2 + b))^18*B*c^(11/2)*sgn(x) - 3003*(
sqrt(c)*x - sqrt(c*x^2 + b))^16*B*b*c^(11/2)*sgn(x) + 48048*(sqrt(c)*x - sq
rt(c*x^2 + b))^16*A*c^(13/2)*sgn(x) - 6006*(sqrt(c)*x - sqrt(c*x^2 + b))^14
*B*b^2*c^(11/2)*sgn(x) + 96096*(sqrt(c)*x - sqrt(c*x^2 + b))^14*A*b*c^(13/2
)*sgn(x) - 28314*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*b^3*c^(11/2)*sgn(x) + 1
09824*(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*b^2*c^(13/2)*sgn(x) + 13728*(sqrt(
c)*x - sqrt(c*x^2 + b))^10*B*b^4*c^(11/2)*sgn(x) + 37752*(sqrt(c)*x - sqrt(
c*x^2 + b))^10*A*b^3*c^(13/2)*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + b))^8
*B*b^5*c^(11/2)*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b^4*c^(13/2
)*sgn(x) + 3718*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^6*c^(11/2)*sgn(x) - 228
8*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^5*c^(13/2)*sgn(x) - 1014*(sqrt(c)*x -
 sqrt(c*x^2 + b))^4*B*b^7*c^(11/2)*sgn(x) + 624*(sqrt(c)*x - sqrt(c*x^2 + b
```

```
))^4*A*b^6*c^(13/2)*sgn(x) + 169*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^8*c^(1
1/2)*sgn(x) - 104*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^7*c^(13/2)*sgn(x) - 1
3*B*b^9*c^(11/2)*sgn(x) + 8*A*b^8*c^(13/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2
 + b))^2 - b)^13
```

Mupad [B]
time = 2.50, size = 306, normalized size = 1.80

$$\frac{8\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{3003\,b^2\,x^8}-\frac{14\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{143\,x^{12}}-\frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}}-\frac{4\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{33\,x^{10}}-\frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{429\,b\,x^{10}}-\frac{A\,b\,\sqrt{c\,x^4+b\,x^2}}{13\,x^{14}}-\frac{16\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{5005\,b^3\,x^6}+\frac{64\,A\,c^5\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^4\,x^4}-\frac{128\,A\,c^6\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^5\,x^2}-\frac{B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{231\,b\,x^8}+\frac{2\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{385\,b^2\,x^6}-\frac{8\,B\,c^4\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^3\,x^4}+\frac{16\,B\,c^5\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^17,x)
```

```
[Out] (8*A*c^3*(b*x^2 + c*x^4)^(1/2))/(3003*b^2*x^8) - (14*A*c*(b*x^2 + c*x^4)^(1
/2))/(143*x^12) - (B*b*(b*x^2 + c*x^4)^(1/2))/(11*x^12) - (4*B*c*(b*x^2 + c
*x^4)^(1/2))/(33*x^10) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(429*b*x^10) - (A*b*
(b*x^2 + c*x^4)^(1/2))/(13*x^14) - (16*A*c^4*(b*x^2 + c*x^4)^(1/2))/(5005*b
^3*x^6) + (64*A*c^5*(b*x^2 + c*x^4)^(1/2))/(15015*b^4*x^4) - (128*A*c^6*(b*
x^2 + c*x^4)^(1/2))/(15015*b^5*x^2) - (B*c^2*(b*x^2 + c*x^4)^(1/2))/(231*b*
x^8) + (2*B*c^3*(b*x^2 + c*x^4)^(1/2))/(385*b^2*x^6) - (8*B*c^4*(b*x^2 + c*
x^4)^(1/2))/(1155*b^3*x^4) + (16*B*c^5*(b*x^2 + c*x^4)^(1/2))/(1155*b^4*x^2
)
```

3.118 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}}\,dx$

Optimal. Leaf size=207

$$-\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}}-\frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}}+\frac{8c(3bB-2Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{16}}-\frac{16c^2(3bB-2Ac)(bx^2+c}{1287b^4x^{14}}$$

```
[Out] -1/15*A*(c*x^4+b*x^2)^(5/2)/b/x^20-1/39*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(5/2)/
b^2/x^18+8/429*c*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(5/2)/b^3/x^16-16/1287*c^2*(-
2*A*c+3*B*b)*(c*x^4+b*x^2)^(5/2)/b^4/x^14+64/9009*c^3*(-2*A*c+3*B*b)*(c*x^4
+b*x^2)^(5/2)/b^5/x^12-128/45045*c^4*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(5/2)/b^6
/x^10
```

Rubi [A]
time = 0.24, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2059, 806, 672, 664}

$$-\frac{128c^4(bx^2+cx^4)^{5/2}(3bB-2Ac)}{45045b^6x^{10}}+\frac{64c^3(bx^2+cx^4)^{5/2}(3bB-2Ac)}{9009b^5x^{12}}-\frac{16c^2(bx^2+cx^4)^{5/2}(3bB-2Ac)}{1287b^4x^{14}}+\frac{8c(bx^2+cx^4)^{5/2}(3bB-2Ac)}{429b^3x^{16}}-\frac{(bx^2+cx^4)^{5/2}(3bB-2Ac)}{39b^2x^{18}}-\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^19,x]
```

```
[Out] -1/15*(A*(b*x^2 + c*x^4)^(5/2))/(b*x^20) - ((3*b*B - 2*A*c)*(b*x^2 + c*x^4)
^(5/2))/(39*b^2*x^18) + (8*c*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(429*b^
3*x^16) - (16*c^2*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(1287*b^4*x^14) +
(64*c^3*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(9009*b^5*x^12) - (128*c^4*(
3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(45045*b^6*x^10)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^{10}}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} + \frac{\left(-10(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2}{x^9}\right.}{15b}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} - \frac{(4c(3bB-2Ac))\text{Subst}}{3}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+}{429b^3x^{16}}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+}{429b^3x^{16}}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+}{429b^3x^{16}}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+}{429b^3x^{16}}$$

Mathematica [A]
time = 0.32, size = 132, normalized size = 0.64

$$-\frac{(x^2(b+cx^2))^{5/2}\left(3bBx^2(1155b^4-840b^3cx^2+560b^2c^2x^4-320bc^3x^6+128c^4x^8)+A(3003b^5-2310b^4cx^2+1680b^3c^2x^4-1120b^2c^3x^6+640bc^4x^8-256c^5x^{10})\right)}{45045b^6x^{20}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^19,x]
```

```
[Out] -1/45045*((x^2*(b + c*x^2))^(5/2)*(3*b*B*x^2*(1155*b^4 - 840*b^3*c*x^2 + 56
0*b^2*c^2*x^4 - 320*b*c^3*x^6 + 128*c^4*x^8) + A*(3003*b^5 - 2310*b^4*c*x^2
 + 1680*b^3*c^2*x^4 - 1120*b^2*c^3*x^6 + 640*b*c^4*x^8 - 256*c^5*x^10)))/(b
^6*x^20)
```

Maple [A]
time = 0.69, size = 142, normalized size = 0.69

method	result
gosper	$-\frac{(cx^2+b)(-256Ac^5x^{10}+384Bbc^4x^{10}+640Abc^4x^8-960Bb^2c^3x^8-1120Ab^2c^3x^6+1680Bb^3c^2x^6+1680Ab^3c^2x^4-2520Bb^4cx^4-2}{45045x^{18}b^6}$
default	$-\frac{(cx^2+b)(-256Ac^5x^{10}+384Bbc^4x^{10}+640Abc^4x^8-960Bb^2c^3x^8-1120Ab^2c^3x^6+1680Bb^3c^2x^6+1680Ab^3c^2x^4-2520Bb^4cx^4-2}{45045x^{18}b^6}$
trager	$-\frac{(-256Ac^7x^{14}+384Bbc^6x^{14}+128Abc^6x^{12}-192Bb^2c^5x^{12}-96Ab^2c^5x^{10}+144Bb^3c^4x^{10}+80Ab^3c^4x^8-120Bb^4c^3x^8-70Ab^4c^3x^6-}{45045b^6x^{16}}$
risch	$-\frac{\sqrt{x^2(cx^2+b)}\,(-256Ac^7x^{14}+384Bbc^6x^{14}+128Abc^6x^{12}-192Bb^2c^5x^{12}-96Ab^2c^5x^{10}+144Bb^3c^4x^{10}+80Ab^3c^4x^8-120B}{45045x^{16}b^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x,method=_RETURNVERBOSE)
```

```
[Out] -1/45045*(c*x^2+b)*(-256*A*c^5*x^10+384*B*b*c^4*x^10+640*A*b*c^4*x^8-960*B*
b^2*c^3*x^8-1120*A*b^2*c^3*x^6+1680*B*b^3*c^2*x^6+1680*A*b^3*c^2*x^4-2520*B
*b^4*c*x^4-2310*A*b^4*c*x^2+3465*B*b^5*x^2+3003*A*b^5)*(c*x^4+b*x^2)^(3/2)/
x^18/b^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(183) = 366.
time = 0.29, size = 385, normalized size = 1.86

$$-\frac{1}{30030}B\left(\frac{256\sqrt{cx^4+bx^2}\,c^6}{b^5x^2}-\frac{128\sqrt{cx^4+bx^2}\,c^5}{b^4x^4}+\frac{96\sqrt{cx^4+bx^2}\,c^4}{b^3x^6}-\frac{80\sqrt{cx^4+bx^2}\,c^3}{b^2x^8}+\frac{70\sqrt{cx^4+bx^2}\,c^2}{bx^{10}}-\frac{63\sqrt{cx^4+bx^2}\,c}{x^{12}}-\frac{693\sqrt{cx^4+bx^2}\,b}{x^{14}}+\frac{3003(cx^4+bx^2)^{\frac{3}{2}}}{x^{16}}\right)+\frac{1}{180180}A\left(\frac{1024\sqrt{cx^4+bx^2}\,c^7}{b^6x^2}-\frac{512\sqrt{cx^4+bx^2}\,c^6}{b^5x^4}+\frac{384\sqrt{cx^4+bx^2}\,c^5}{b^4x^6}-\frac{320\sqrt{cx^4+bx^2}\,c^4}{b^3x^8}+\frac{280\sqrt{cx^4+bx^2}\,c^3}{b^2x^{10}}-\frac{252\sqrt{cx^4+bx^2}\,c^2}{bx^{12}}+\frac{231\sqrt{cx^4+bx^2}\,c}{x^{14}}+\frac{3003\sqrt{cx^4+bx^2}\,b}{x^{16}}-\frac{15015(cx^4+bx^2)^{\frac{3}{2}}}{x^{18}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x, algorithm="maxima")
```

```
[Out] -1/30030*B*(256*sqrt(c*x^4 + b*x^2)*c^6/(b^5*x^2) - 128*sqrt(c*x^4 + b*x^2)
*c^5/(b^4*x^4) + 96*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^6) - 80*sqrt(c*x^4 + b*x
```

^2)*c^3/(b^2*x^8) + 70*sqrt(c*x^4 + b*x^2)*c^2/(b*x^10) - 63*sqrt(c*x^4 + b*x^2)*c/x^12 - 693*sqrt(c*x^4 + b*x^2)*b/x^14 + 3003*(c*x^4 + b*x^2)^(3/2)/x^16) + 1/180180*A*(1024*sqrt(c*x^4 + b*x^2)*c^7/(b^6*x^2) - 512*sqrt(c*x^4 + b*x^2)*c^6/(b^5*x^4) + 384*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^6) - 320*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^8) + 280*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^10) - 252*sqrt(c*x^4 + b*x^2)*c^2/(b*x^12) + 231*sqrt(c*x^4 + b*x^2)*c/x^14 + 3003*sqrt(c*x^4 + b*x^2)*b/x^16 - 15015*(c*x^4 + b*x^2)^(3/2)/x^18)

Fricas [A]
time = 1.61, size = 181, normalized size = 0.87

$$-\frac{(128\,(3\,Bbc^6-2\,Ac^7)x^{14}-64\,(3\,Bb^2c^5-2\,Abc^6)x^{12}+48\,(3\,Bb^3c^4-2\,Ab^2c^5)x^{10}-40\,(3\,Bb^4c^3-2\,Ab^3c^4)x^8+3003\,Ab^7+35\,(3\,Bb^5c^2-2\,Ab^4c^3)x^6+63\,(70\,Bb^6c+Ab^5c^2)x^4+231\,(15\,Bb^7+16\,Ab^6c)x^2)\sqrt{cx^4+bx^2}}{45045\,b^6x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x, algorithm="fricas")

[Out] -1/45045*(128*(3*B*b*c^6 - 2*A*c^7)*x^14 - 64*(3*B*b^2*c^5 - 2*A*b*c^6)*x^12 + 48*(3*B*b^3*c^4 - 2*A*b^2*c^5)*x^10 - 40*(3*B*b^4*c^3 - 2*A*b^3*c^4)*x^8 + 3003*A*b^7 + 35*(3*B*b^5*c^2 - 2*A*b^4*c^3)*x^6 + 63*(70*B*b^6*c + A*b^5*c^2)*x^4 + 231*(15*B*b^7 + 16*A*b^6*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^6*x^16)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**19,x)

[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**19, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 582 vs. 2(183) = 366.
time = 5.36, size = 582, normalized size = 2.81

$$\frac{256\left(18018\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{20}Bc^{\frac{13}{2}}\mathrm{sgn}(x)+60060\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{18}Ac^{\frac{15}{2}}\mathrm{sgn}(x)-12870\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{16}Bb^2c^{\frac{13}{2}}\mathrm{sgn}(x)+128700\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{16}Abc^{\frac{15}{2}}\mathrm{sgn}(x)-32175\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Bb^3c^{\frac{13}{2}}\mathrm{sgn}(x)+141570\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Ab^2c^{\frac{15}{2}}\mathrm{sgn}(x)+15015\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bb^4c^{\frac{13}{2}}\mathrm{sgn}(x)+50050\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Ab^3c^{\frac{15}{2}}\mathrm{sgn}(x)+9009\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bb^5c^{\frac{13}{2}}\mathrm{sgn}(x)+6006\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Ab^4c^{\frac{15}{2}}\mathrm{sgn}(x)+4095\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Bb^6c^{\frac{13}{2}}\mathrm{sgn}(x)-2730\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Ab^5c^{\frac{15}{2}}\mathrm{sgn}(x)-1365\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}Bb^7c^{\frac{13}{2}}\mathrm{sgn}(x)+910\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}Ab^6c^{\frac{15}{2}}\mathrm{sgn}(x)+315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}Bb^8c^{\frac{13}{2}}\mathrm{sgn}(x)-210\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}Ab^7c^{\frac{15}{2}}\mathrm{sgn}(x)-45\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}Bb^9c^{\frac{13}{2}}\mathrm{sgn}(x)+30\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}Ab^8c^{\frac{15}{2}}\mathrm{sgn}(x)+3\,Bb^{10}c^{\frac{13}{2}}\mathrm{sgn}(x)-2\,Ab^9c^{\frac{15}{2}}\mathrm{sgn}(x)\right)}{45045\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x, algorithm="giac")

[Out] 256/45045*(18018*(sqrt(c)*x - sqrt(c*x^2 + b))^20*B*c^(13/2)*sgn(x) + 60060*(sqrt(c)*x - sqrt(c*x^2 + b))^18*A*c^(15/2)*sgn(x) - 12870*(sqrt(c)*x - sqrt(c*x^2 + b))^16*B*b^2*c^(13/2)*sgn(x) + 128700*(sqrt(c)*x - sqrt(c*x^2 + b))^16*A*b*c^(15/2)*sgn(x) - 32175*(sqrt(c)*x - sqrt(c*x^2 + b))^14*B*b^3*c

```
^(13/2)*sgn(x) + 141570*(sqrt(c)*x - sqrt(c*x^2 + b))^14*A*b^2*c^(15/2)*sgn
(x) + 15015*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*b^4*c^(13/2)*sgn(x) + 50050*
(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*b^3*c^(15/2)*sgn(x) + 9009*(sqrt(c)*x -
sqrt(c*x^2 + b))^10*B*b^5*c^(13/2)*sgn(x) + 6006*(sqrt(c)*x - sqrt(c*x^2 +
b))^10*A*b^4*c^(15/2)*sgn(x) + 4095*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^6*c
^(13/2)*sgn(x) - 2730*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b^5*c^(15/2)*sgn(x)
 - 1365*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^7*c^(13/2)*sgn(x) + 910*(sqrt(c
)*x - sqrt(c*x^2 + b))^6*A*b^6*c^(15/2)*sgn(x) + 315*(sqrt(c)*x - sqrt(c*x^
2 + b))^4*B*b^8*c^(13/2)*sgn(x) - 210*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^7
*c^(15/2)*sgn(x) - 45*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^9*c^(13/2)*sgn(x)
 + 30*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^8*c^(15/2)*sgn(x) + 3*B*b^10*c^(1
3/2)*sgn(x) - 2*A*b^9*c^(15/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b
)^15
```

Mupad [B]
time = 3.21, size = 356, normalized size = 1.72

$$\frac{2\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{1287\,b^2\,x^{10}}-\frac{16\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{195\,x^{14}}-\frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{13\,x^{14}}-\frac{14\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{143\,x^{12}}-\frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{715\,b\,x^{12}}-\frac{A\,b\,\sqrt{c\,x^4+b\,x^2}}{15\,x^{16}}-\frac{16\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{9009\,b^3\,x^8}+\frac{32\,A\,c^5\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^4\,x^6}-\frac{128\,A\,c^6\,\sqrt{c\,x^4+b\,x^2}}{45045\,b^5\,x^4}+\frac{256\,A\,c^7\,\sqrt{c\,x^4+b\,x^2}}{45045\,b^6\,x^2}-\frac{B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{429\,b\,x^{10}}+\frac{8\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{3003\,b^2\,x^8}-\frac{16\,B\,c^4\,\sqrt{c\,x^4+b\,x^2}}{5005\,b^3\,x^6}+\frac{64\,B\,c^5\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^4\,x^4}-\frac{128\,B\,c^6\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^19,x)
```

```
[Out] (2*A*c^3*(b*x^2 + c*x^4)^(1/2))/(1287*b^2*x^10) - (16*A*c*(b*x^2 + c*x^4)^(
1/2))/(195*x^14) - (B*b*(b*x^2 + c*x^4)^(1/2))/(13*x^14) - (14*B*c*(b*x^2 +
 c*x^4)^(1/2))/(143*x^12) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(715*b*x^12) - (A
*b*(b*x^2 + c*x^4)^(1/2))/(15*x^16) - (16*A*c^4*(b*x^2 + c*x^4)^(1/2))/(900
9*b^3*x^8) + (32*A*c^5*(b*x^2 + c*x^4)^(1/2))/(15015*b^4*x^6) - (128*A*c^6*
(b*x^2 + c*x^4)^(1/2))/(45045*b^5*x^4) + (256*A*c^7*(b*x^2 + c*x^4)^(1/2))/
(45045*b^6*x^2) - (B*c^2*(b*x^2 + c*x^4)^(1/2))/(429*b*x^10) + (8*B*c^3*(b*
x^2 + c*x^4)^(1/2))/(3003*b^2*x^8) - (16*B*c^4*(b*x^2 + c*x^4)^(1/2))/(5005
*b^3*x^6) + (64*B*c^5*(b*x^2 + c*x^4)^(1/2))/(15015*b^4*x^4) - (128*B*c^6*(
b*x^2 + c*x^4)^(1/2))/(15015*b^5*x^2)
```

3.119 $\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=168

$$\frac{16b^3(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5}-\frac{8b^2(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{2b(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{(8bB-$$

```
[Out] 16/15015*b^3*(-13*A*c+8*B*b)*(c*x^4+b*x^2)^(5/2)/c^5/x^5-8/3003*b^2*(-13*A*
c+8*B*b)*(c*x^4+b*x^2)^(5/2)/c^4/x^3+2/429*b*(-13*A*c+8*B*b)*(c*x^4+b*x^2)^
(5/2)/c^3/x-1/143*(-13*A*c+8*B*b)*x*(c*x^4+b*x^2)^(5/2)/c^2+1/13*B*x^3*(c*x
^4+b*x^2)^(5/2)/c
```

Rubi [A]
time = 0.20, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2064, 2041, 2027, 2039}

$$\frac{16b^3\left(bx^2+cx^4\right)^{5/2}(8bB-13Ac)}{15015c^5x^5}-\frac{8b^2\left(bx^2+cx^4\right)^{5/2}(8bB-13Ac)}{3003c^4x^3}+\frac{2b\left(bx^2+cx^4\right)^{5/2}(8bB-13Ac)}{429c^3x}-\frac{x\left(bx^2+cx^4\right)^{5/2}(8bB-13Ac)}{143c^2}+\frac{Bx^3\left(bx^2+cx^4\right)^{5/2}}{13c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (16*b^3*(8*b*B - 13*A*c)*(b*x^2 + c*x^4)^(5/2))/(15015*c^5*x^5) - (8*b^2*(8
*b*B - 13*A*c)*(b*x^2 + c*x^4)^(5/2))/(3003*c^4*x^3) + (2*b*(8*b*B - 13*A*c
)*(b*x^2 + c*x^4)^(5/2))/(429*c^3*x) - ((8*b*B - 13*A*c)*x*(b*x^2 + c*x^4)^
(5/2))/(143*c^2) + (B*x^3*(b*x^2 + c*x^4)^(5/2))/(13*c)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{Bx^3(bx^2+cx^4)^{5/2}}{13c}-\frac{(8bB-13Ac)\int x^4(bx^2+cx^4)^{3/2}\,dx}{13c}$$
$$=-\frac{(8bB-13Ac)x(bx^2+cx^4)^{5/2}}{143c^2}+\frac{Bx^3(bx^2+cx^4)^{5/2}}{13c}+\frac{(6b(8bB-13$$
$$=\frac{2b(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{(8bB-13Ac)x(bx^2+cx^4)^{5/2}}{143c^2}+\frac{Bx$$
$$=-\frac{8b^2(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{2b(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-$$
$$=\frac{16b^3(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5}-\frac{8b^2(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+$$

Mathematica [A]
time = 0.08, size = 113, normalized size = 0.67

$$\frac{x(b+cx^2)^3\left(128b^4B+105c^4x^6(13A+11Bx^2)-70bc^3x^4(13A+12Bx^2)+40b^2c^2x^2(13A+14Bx^2)-16b^3c(13A+20Bx^2)\right)}{15015c^5\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(128*b^4*B + 105*c^4*x^6*(13*A + 11*B*x^2) - 70*b*c^3*x^4*
(13*A + 12*B*x^2) + 40*b^2*c^2*x^2*(13*A + 14*B*x^2) - 16*b^3*c*(13*A + 20*
B*x^2)))/(15015*c^5*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.39, size = 115, normalized size = 0.68

method	result
gosper	$-\frac{(c x^2+b)(-1155B c^4x^8-1365A c^4x^6+840Bb c^3x^6+910Ab c^3x^4-560B b^2c^2x^4-520A b^2c^2x^2+320B b^3c x^2+208A b^3c-128B b^4)(x^4c}{15015c^5x^3}$
default	$-\frac{(c x^2+b)(-1155B c^4x^8-1365A c^4x^6+840Bb c^3x^6+910Ab c^3x^4-560B b^2c^2x^4-520A b^2c^2x^2+320B b^3c x^2+208A b^3c-128B b^4)(x^4c}{15015c^5x^3}$
trager	$-\frac{(-1155B c^6x^{12}-1365A c^6x^{10}-1470Bb c^5x^{10}-1820Ab c^5x^8-35B b^2c^4x^8-65A b^2c^4x^6+40B b^3c^3x^6+78A b^3c^3x^4-48B b^4c^2x^4-104}{15015c^5x}$
risch	$-\frac{\sqrt{x^2(c x^2+b)}\,(-1155B c^6x^{12}-1365A c^6x^{10}-1470Bb c^5x^{10}-1820Ab c^5x^8-35B b^2c^4x^8-65A b^2c^4x^6+40B b^3c^3x^6+78A b^3c^3}{15015x c^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15015*(c*x^2+b)*(-1155*B*c^4*x^8-1365*A*c^4*x^6+840*B*b*c^3*x^6+910*A*b*
c^3*x^4-560*B*b^2*c^2*x^4-520*A*b^2*c^2*x^2+320*B*b^3*c*x^2+208*A*b^3*c-128
*B*b^4)*(c*x^4+b*x^2)^(3/2)/c^5/x^3
```

Maxima [A]
time = 0.29, size = 150, normalized size = 0.89

$$\frac{(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4cx^2-16\,b^5)\sqrt{cx^2+b}\,A}{1155\,c^4}+\frac{(1155\,c^6x^{12}+1470\,bc^5x^{10}+35\,b^2c^4x^8-40\,b^3c^3x^6+48\,b^4c^2x^4-64\,b^5cx^2+128\,b^6)\sqrt{cx^2+b}\,B}{15015\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b^
4*c*x^2 - 16*b^5)*sqrt(c*x^2 + b)*A/c^4 + 1/15015*(1155*c^6*x^12 + 1470*b*c
^5*x^10 + 35*b^2*c^4*x^8 - 40*b^3*c^3*x^6 + 48*b^4*c^2*x^4 - 64*b^5*c*x^2 +
 128*b^6)*sqrt(c*x^2 + b)*B/c^5
```

Fricas [A]
time = 1.50, size = 154, normalized size = 0.92

$$\frac{(1155\,Bc^6x^{12}+105\,(14\,Bbc^5+13\,Ac^6)x^{10}+35\,(Bb^2c^4+52\,Abc^5)x^8+128\,Bb^6-208\,Ab^5c-5\,(8\,Bb^3c^3-13\,Ab^2c^4)x^6+6\,(8\,Bb^4c^2-13\,Ab^3c^3)x^4-8\,(8\,Bb^5c-13\,Ab^4c^2)x^2)\sqrt{cx^4+bx^2}}{15015\,c^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*B*c^6*x^12 + 105*(14*B*b*c^5 + 13*A*c^6)*x^10 + 35*(B*b^2*c^4
 + 52*A*b*c^5)*x^8 + 128*B*b^6 - 208*A*b^5*c - 5*(8*B*b^3*c^3 - 13*A*b^2*c^
4)*x^6 + 6*(8*B*b^4*c^2 - 13*A*b^3*c^3)*x^4 - 8*(8*B*b^5*c - 13*A*b^4*c^2)*
x^2)*sqrt(c*x^4 + b*x^2)/(c^5*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(x^2 \left(b+cx^2\right)\right)^{\frac{3}{2}} \left(A+Bx^2\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**4*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

Giac [A]
time = 0.87, size = 175, normalized size = 1.04

$$-\frac{16 \left(8 \, B b^{\frac{13}{2}} - 13 \, A b^{\frac{11}{2}} c\right) \mathrm{sgn}\left(x\right)}{15015 \, c^{5}} + \frac{1155 \, {\left(c x^{2} + b\right)}^{\frac{13}{2}} B \mathrm{sgn}\left(x\right) - 5460 \, {\left(c x^{2} + b\right)}^{\frac{11}{2}} B b \mathrm{sgn}\left(x\right) + 10010 \, {\left(c x^{2} + b\right)}^{\frac{9}{2}} B b^{2} \mathrm{sgn}\left(x\right) - 8580 \, {\left(c x^{2} + b\right)}^{\frac{7}{2}} B b^{3} \mathrm{sgn}\left(x\right) + 3003 \, {\left(c x^{2} + b\right)}^{\frac{5}{2}} B b^{4} \mathrm{sgn}\left(x\right) + 1365 \, {\left(c x^{2} + b\right)}^{\frac{11}{2}} A c \mathrm{sgn}\left(x\right) - 5005 \, {\left(c x^{2} + b\right)}^{\frac{9}{2}} A b c \mathrm{sgn}\left(x\right) + 6435 \, {\left(c x^{2} + b\right)}^{\frac{7}{2}} A b^{2} c \mathrm{sgn}\left(x\right) - 3003 \, {\left(c x^{2} + b\right)}^{\frac{5}{2}} A b^{3} c \mathrm{sgn}\left(x\right)}{15015 \, c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15015*(8*B*b^(13/2) - 13*A*b^(11/2)*c)*sgn(x)/c^5 + 1/15015*(1155*(c*x^
2 + b)^(13/2)*B*sgn(x) - 5460*(c*x^2 + b)^(11/2)*B*b*sgn(x) + 10010*(c*x^2
+ b)^(9/2)*B*b^2*sgn(x) - 8580*(c*x^2 + b)^(7/2)*B*b^3*sgn(x) + 3003*(c*x^2
 + b)^(5/2)*B*b^4*sgn(x) + 1365*(c*x^2 + b)^(11/2)*A*c*sgn(x) - 5005*(c*x^2
 + b)^(9/2)*A*b*c*sgn(x) + 6435*(c*x^2 + b)^(7/2)*A*b^2*c*sgn(x) - 3003*(c*
x^2 + b)^(5/2)*A*b^3*c*sgn(x))/c^5
```

Mupad [B]
time = 0.35, size = 143, normalized size = 0.85

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{128\,B\,b^6-208\,A\,b^5\,c}{15015\,c^5}+\frac{x^{10}\,\left(1365\,A\,c^6+1470\,B\,b\,c^5\right)}{15015\,c^5}+\frac{B\,c\,x^{12}}{13}+\frac{b^2\,x^6\,\left(13\,A\,c-8\,B\,b\right)}{3003\,c^2}-\frac{2\,b^3\,x^4\,\left(13\,A\,c-8\,B\,b\right)}{5005\,c^3}+\frac{8\,b^4\,x^2\,\left(13\,A\,c-8\,B\,b\right)}{15015\,c^4}+\frac{b\,x^8\,\left(52\,A\,c+B\,b\right)}{429\,c}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((128*B*b^6 - 208*A*b^5*c)/(15015*c^5) + (x^10*(1365
*A*c^6 + 1470*B*b*c^5))/(15015*c^5) + (B*c*x^12)/13 + (b^2*x^6*(13*A*c - 8*
B*b))/(3003*c^2) - (2*b^3*x^4*(13*A*c - 8*B*b))/(5005*c^3) + (8*b^4*x^2*(13
*A*c - 8*B*b))/(15015*c^4) + (b*x^8*(52*A*c + B*b))/(429*c)))/x
```

3.120 $\int x^2(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=131

$$-\frac{8b^2(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{3465c^4x^5}+\frac{4b(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{693c^3x^3}-\frac{(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{99c^2x}+\frac{Bx(bx^2+\ldots}{11\ldots}$$

```
[Out] -8/3465*b^2*(-11*A*c+6*B*b)*(c*x^4+b*x^2)^(5/2)/c^4/x^5+4/693*b*(-11*A*c+6*
B*b)*(c*x^4+b*x^2)^(5/2)/c^3/x^3-1/99*(-11*A*c+6*B*b)*(c*x^4+b*x^2)^(5/2)/c
^2/x+1/11*B*x*(c*x^4+b*x^2)^(5/2)/c
```

Rubi [A]
time = 0.16, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2064, 2041, 2027, 2039}

$$-\frac{8b^2\left(bx^2+cx^4\right)^{5/2}(6bB-11Ac)}{3465c^4x^5}+\frac{4b\left(bx^2+cx^4\right)^{5/2}(6bB-11Ac)}{693c^3x^3}-\frac{\left(bx^2+cx^4\right)^{5/2}(6bB-11Ac)}{99c^2x}+\frac{Bx\left(bx^2+cx^4\right)^{5/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-8*b^2*(6*b*B - 11*A*c)*(b*x^2 + c*x^4)^(5/2))/(3465*c^4*x^5) + (4*b*(6*b*
B - 11*A*c)*(b*x^2 + c*x^4)^(5/2))/(693*c^3*x^3) - ((6*b*B - 11*A*c)*(b*x^2
 + c*x^4)^(5/2))/(99*c^2*x) + (B*x*(b*x^2 + c*x^4)^(5/2))/(11*c)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
```

```
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{Bx(bx^2+cx^4)^{5/2}}{11c}-\frac{(6bB-11Ac)\int x^2(bx^2+cx^4)^{3/2}\,dx}{11c}$$
$$=-\frac{(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{99c^2x}+\frac{Bx(bx^2+cx^4)^{5/2}}{11c}+\frac{(4b(6bB-11A}{}$$
$$=\frac{4b(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{693c^3x^3}-\frac{(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{99c^2x}+\frac{Bx(}{}$$
$$=-\frac{8b^2(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{3465c^4x^5}+\frac{4b(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{693c^3x^3}-$$

Mathematica [A]
time = 0.06, size = 94, normalized size = 0.72

$$\frac{(b+cx^2)\left(x^2(b+cx^2)\right)^{3/2}\left(-48b^3B+88Ab^2c+120b^2Bcx^2-220Abc^2x^2-210bBc^2x^4+385Ac^3x^4+315Bc^3x^6\right)}{3465c^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((b + c*x^2)*(x^2*(b + c*x^2))^(3/2)*(-48*b^3*B + 88*A*b^2*c + 120*b^2*B*c*
x^2 - 220*A*b*c^2*x^2 - 210*b*B*c^2*x^4 + 385*A*c^3*x^4 + 315*B*c^3*x^6))/(
3465*c^4*x^3)
```

Maple [A]
time = 0.37, size = 91, normalized size = 0.69

method	result

gosper	$\frac{(c\,x^{2}+b)\,(315B\,c^{3}x^{6}+385A\,c^{3}x^{4}-210Bb\,c^{2}x^{4}-220Ab\,c^{2}x^{2}+120B\,b^{2}c\,x^{2}+88A\,b^{2}c-48B\,b^{3})\,(x^{4}c+b\,x^{2})^{\frac{3}{2}}}{3465c^{4}x^{3}}$
default	$\frac{(c\,x^{2}+b)\,(315B\,c^{3}x^{6}+385A\,c^{3}x^{4}-210Bb\,c^{2}x^{4}-220Ab\,c^{2}x^{2}+120B\,b^{2}c\,x^{2}+88A\,b^{2}c-48B\,b^{3})\,(x^{4}c+b\,x^{2})^{\frac{3}{2}}}{3465c^{4}x^{3}}$
trager	$\frac{(315B\,c^{5}x^{10}+385A\,c^{5}x^{8}+420Bb\,c^{4}x^{8}+550Ab\,c^{4}x^{6}+15B\,b^{2}c^{3}x^{6}+33A\,b^{2}c^{3}x^{4}-18B\,b^{3}c^{2}x^{4}-44A\,b^{3}c^{2}x^{2}+24B\,b^{4}c\,x^{2}+88A\,b^{4}c-48b^{5}B}{3465c^{4}x}$
risch	$\frac{\sqrt{x^{2}\,(c\,x^{2}+b)}\,(315B\,c^{5}x^{10}+385A\,c^{5}x^{8}+420Bb\,c^{4}x^{8}+550Ab\,c^{4}x^{6}+15B\,b^{2}c^{3}x^{6}+33A\,b^{2}c^{3}x^{4}-18B\,b^{3}c^{2}x^{4}-44A\,b^{3}c^{2}x^{2}+24B\,b^{4}}{3465x\,c^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3465*(c*x^2+b)*(315*B*c^3*x^6+385*A*c^3*x^4-210*B*b*c^2*x^4-220*A*b*c^2*x
^2+120*B*b^2*c*x^2+88*A*b^2*c-48*B*b^3)*(c*x^4+b*x^2)^(3/2)/c^4/x^3
```

Maxima [A]
time = 0.30, size = 128, normalized size = 0.98

$$\frac{\left(35\,c^{4}x^{8}+50\,bc^{3}x^{6}+3\,b^{2}c^{2}x^{4}-4\,b^{3}cx^{2}+8\,b^{4}\right)\sqrt{cx^{2}+b}\,A}{315\,c^{3}}+\frac{\left(105\,c^{5}x^{10}+140\,bc^{4}x^{8}+5\,b^{2}c^{3}x^{6}-6\,b^{3}c^{2}x^{4}+8\,b^{4}cx^{2}-16\,b^{5}\right)\sqrt{cx^{2}+b}\,B}{1155\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sqr
t(c*x^2 + b)*A/c^3 + 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 -
 6*b^3*c^2*x^4 + 8*b^4*c*x^2 - 16*b^5)*sqrt(c*x^2 + b)*B/c^4
```

Fricas [A]
time = 1.58, size = 131, normalized size = 1.00

$$\frac{\left(315\,Bc^{5}x^{10}+35\,(12\,Bbc^{4}+11\,Ac^{5})x^{8}+5\,(3\,Bb^{2}c^{3}+110\,Abc^{4})x^{6}-48\,Bb^{5}+88\,Ab^{4}c-3\,(6\,Bb^{3}c^{2}-11\,Ab^{2}c^{3})x^{4}+4\,(6\,Bb^{4}c-11\,Ab^{3}c^{2})x^{2}\right)\sqrt{cx^{4}+bx^{2}}}{3465\,c^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*B*c^5*x^10 + 35*(12*B*b*c^4 + 11*A*c^5)*x^8 + 5*(3*B*b^2*c^3 +
110*A*b*c^4)*x^6 - 48*B*b^5 + 88*A*b^4*c - 3*(6*B*b^3*c^2 - 11*A*b^2*c^3)*x
^4 + 4*(6*B*b^4*c - 11*A*b^3*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\left(A+Bx^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

Giac [A]
time = 1.06, size = 140, normalized size = 1.07

$$\frac{8\left(6\,Bb^{\frac{11}{2}}-11\,Ab^{\frac{9}{2}}c\right)\mathrm{sgn}(x)}{3465\,c^4}+\frac{315\,(cx^2+b)^{\frac{11}{2}}B\mathrm{sgn}(x)-1155\,(cx^2+b)^{\frac{9}{2}}Bb\mathrm{sgn}(x)+1485\,(cx^2+b)^{\frac{7}{2}}Bb^2\mathrm{sgn}(x)-693\,(cx^2+b)^{\frac{5}{2}}Bb^3\mathrm{sgn}(x)+385\,(cx^2+b)^{\frac{9}{2}}Ac\mathrm{sgn}(x)-990\,(cx^2+b)^{\frac{7}{2}}Abc\mathrm{sgn}(x)+693\,(cx^2+b)^{\frac{5}{2}}Ab^2c\mathrm{sgn}(x)}{3465\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 8/3465*(6*B*b^(11/2) - 11*A*b^(9/2)*c)*sgn(x)/c^4 + 1/3465*(315*(c*x^2 + b)
^(11/2)*B*sgn(x) - 1155*(c*x^2 + b)^(9/2)*B*b*sgn(x) + 1485*(c*x^2 + b)^(7/
2)*B*b^2*sgn(x) - 693*(c*x^2 + b)^(5/2)*B*b^3*sgn(x) + 385*(c*x^2 + b)^(9/2
)*A*c*sgn(x) - 990*(c*x^2 + b)^(7/2)*A*b*c*sgn(x) + 693*(c*x^2 + b)^(5/2)*A
*b^2*c*sgn(x))/c^4
```

Mupad [B]
time = 0.29, size = 124, normalized size = 0.95

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{x^8\,\left(385\,A\,c^5+420\,B\,b\,c^4\right)}{3465\,c^4}-\frac{48\,B\,b^5-88\,A\,b^4\,c}{3465\,c^4}+\frac{B\,c\,x^{10}}{11}+\frac{b^2\,x^4\,\left(11\,A\,c-6\,B\,b\right)}{1155\,c^2}-\frac{4\,b^3\,x^2\,\left(11\,A\,c-6\,B\,b\right)}{3465\,c^3}+\frac{b\,x^6\,\left(110\,A\,c+3\,B\,b\right)}{693\,c}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((x^8*(385*A*c^5 + 420*B*b*c^4))/(3465*c^4) - (48*B*
b^5 - 88*A*b^4*c)/(3465*c^4) + (B*c*x^10)/11 + (b^2*x^4*(11*A*c - 6*B*b))/(
1155*c^2) - (4*b^3*x^2*(11*A*c - 6*B*b))/(3465*c^3) + (b*x^6*(110*A*c + 3*B
*b))/(693*c)))/x
```

3.121 $\int (A+Bx^2)(bx^2+cx^4)^{3/2}\,dx$

Optimal. Leaf size=96

$$\frac{2b(4bB-9Ac)(bx^2+cx^4)^{5/2}}{315c^3x^5}-\frac{(4bB-9Ac)(bx^2+cx^4)^{5/2}}{63c^2x^3}+\frac{B(bx^2+cx^4)^{5/2}}{9cx}$$

```
[Out] 2/315*b*(-9*A*c+4*B*b)*(c*x^4+b*x^2)^(5/2)/c^3/x^5-1/63*(-9*A*c+4*B*b)*(c*x
^4+b*x^2)^(5/2)/c^2/x^3+1/9*B*(c*x^4+b*x^2)^(5/2)/c/x
```

Rubi [A]

time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1159, 2027, 2039}

$$\frac{2b(bx^2+cx^4)^{5/2}(4bB-9Ac)}{315c^3x^5}-\frac{(bx^2+cx^4)^{5/2}(4bB-9Ac)}{63c^2x^3}+\frac{B(bx^2+cx^4)^{5/2}}{9cx}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*b*(4*b*B - 9*A*c)*(b*x^2 + c*x^4)^(5/2))/(315*c^3*x^5) - ((4*b*B - 9*A*c
)*(b*x^2 + c*x^4)^(5/2))/(63*c^2*x^3) + (B*(b*x^2 + c*x^4)^(5/2))/(9*c*x)
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[e*((b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 3)*x)), x] - Dist[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3)), Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
c*d*(4*p + 3), 0]
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{B\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{(4bB-9Ac)\int\left(bx^2+cx^4\right)^{3/2}\,dx}{9c}$$

$$=-\frac{(4bB-9Ac)\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{9cx}+\frac{(2b(4bB-9Ac))\int \ldots}{63c^2}$$

$$=\frac{2b(4bB-9Ac)\left(bx^2+cx^4\right)^{5/2}}{315c^3x^5}-\frac{(4bB-9Ac)\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{B(bx^2+\ldots}{9c\ldots}$$

Mathematica [A]
time = 0.05, size = 70, normalized size = 0.73

$$\frac{\left(b+cx^2\right)\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(8b^2B-18Abc-20bBcx^2+45Ac^2x^2+35Bc^2x^4\right)}{315c^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((b + c*x^2)*(x^2*(b + c*x^2))^(3/2)*(8*b^2*B - 18*A*b*c - 20*b*B*c*x^2 + 4
5*A*c^2*x^2 + 35*B*c^2*x^4))/(315*c^3*x^3)
```

Maple [A]
time = 0.40, size = 67, normalized size = 0.70

method	result	siz…
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-35B\,c^{2}x^{4}-45A\,c^{2}x^{2}+20bB\,x^{2}c+18Abc-8b^{2}B\right)\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}}{315c^{3}x^{3}}$	67
default	$-\frac{\left(c\,x^{2}+b\right)\left(-35B\,c^{2}x^{4}-45A\,c^{2}x^{2}+20bB\,x^{2}c+18Abc-8b^{2}B\right)\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}}{315c^{3}x^{3}}$	67
trager	$-\frac{\left(-35B\,c^{4}x^{8}-45A\,c^{4}x^{6}-50Bb\,c^{3}x^{6}-72Ab\,c^{3}x^{4}-3B\,b^{2}c^{2}x^{4}-9A\,b^{2}c^{2}x^{2}+4B\,b^{3}c\,x^{2}+18A\,b^{3}c-8B\,b^{4}\right)\sqrt{x^{4}c+b\,x^{2}}}{315c^{3}x}$	10…
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(-35B\,c^{4}x^{8}-45A\,c^{4}x^{6}-50Bb\,c^{3}x^{6}-72Ab\,c^{3}x^{4}-3B\,b^{2}c^{2}x^{4}-9A\,b^{2}c^{2}x^{2}+4B\,b^{3}c\,x^{2}+18A\,b^{3}c-8B\,b^{4}\right)}{315x\,c^{3}}$	10…

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(c*x^2+b)*(-35*B*c^2*x^4-45*A*c^2*x^2+20*B*b*c*x^2+18*A*b*c-8*B*b^2)
*(c*x^4+b*x^2)^(3/2)/c^3/x^3
```

Maxima [A]
time = 0.29, size = 105, normalized size = 1.09

$$\frac{\left(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3\right)\sqrt{cx^2+b}\,A}{35\,c^2}+\frac{\left(35\,c^4x^8+50\,bc^3x^6+3\,b^2c^2x^4-4\,b^3cx^2+8\,b^4\right)\sqrt{cx^2+b}\,B}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^2 + b)*A/c^2 +
1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sqr
t(c*x^2 + b)*B/c^3
```

Fricas [A]
time = 1.59, size = 106, normalized size = 1.10

$$\frac{(35\,Bc^4x^8 + 5\,(10\,Bbc^3 + 9\,Ac^4)x^6 + 8\,Bb^4 - 18\,Ab^3c + 3\,(Bb^2c^2 + 24\,Abc^3)x^4 - (4\,Bb^3c - 9\,Ab^2c^2)x^2)\sqrt{cx^4 + bx^2}}{315\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*B*c^4*x^8 + 5*(10*B*b*c^3 + 9*A*c^4)*x^6 + 8*B*b^4 - 18*A*b^3*c +
 3*(B*b^2*c^2 + 24*A*b*c^3)*x^4 - (4*B*b^3*c - 9*A*b^2*c^2)*x^2)*sqrt(c*x^4
 + b*x^2)/(c^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{2}\left(b + cx^{2}\right)\right)^{\frac{3}{2}} \left(A + Bx^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

Giac [A]
time = 1.50, size = 105, normalized size = 1.09

$$-\frac{2\left(4\,Bb^{\frac{9}{2}} - 9\,Ab^{\frac{7}{2}}c\right)\mathrm{sgn}(x)}{315\,c^3} + \frac{35\,(cx^2+b)^{\frac{9}{2}}B\mathrm{sgn}(x) - 90\,(cx^2+b)^{\frac{7}{2}}Bb\mathrm{sgn}(x) + 63\,(cx^2+b)^{\frac{5}{2}}Bb^2\mathrm{sgn}(x) + 45\,(cx^2+b)^{\frac{7}{2}}Ac\mathrm{sgn}(x) - 63\,(cx^2+b)^{\frac{5}{2}}Abc\mathrm{sgn}(x)}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2/315*(4*B*b^(9/2) - 9*A*b^(7/2)*c)*sgn(x)/c^3 + 1/315*(35*(c*x^2 + b)^(9/
2)*B*sgn(x) - 90*(c*x^2 + b)^(7/2)*B*b*sgn(x) + 63*(c*x^2 + b)^(5/2)*B*b^2*
sgn(x) + 45*(c*x^2 + b)^(7/2)*A*c*sgn(x) - 63*(c*x^2 + b)^(5/2)*A*b*c*sgn(x
))/c^3
```

Mupad [B]
time = 0.26, size = 103, normalized size = 1.07

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{8\,B\,b^4-18\,A\,b^3\,c}{315\,c^3}+\frac{x^6\,\left(45\,A\,c^4+50\,B\,b\,c^3\right)}{315\,c^3}+\frac{B\,c\,x^8}{9}+\frac{b^2\,x^2\,\left(9\,A\,c-4\,B\,b\right)}{315\,c^2}+\frac{b\,x^4\,\left(24\,A\,c+B\,b\right)}{105\,c}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((8*B*b^4 - 18*A*b^3*c)/(315*c^3) + (x^6*(45*A*c^4 +
 50*B*b*c^3))/(315*c^3) + (B*c*x^8)/9 + (b^2*x^2*(9*A*c - 4*B*b))/(315*c^2)
 + (b*x^4*(24*A*c + B*b))/(105*c)))/x
```

3.122 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2} dx$

Optimal. Leaf size=61

$$-\frac{(2bB-7Ac)(bx^2+cx^4)^{5/2}}{35c^2x^5}+\frac{B(bx^2+cx^4)^{5/2}}{7cx^3}$$

```
[Out] -1/35*(-7*A*c+2*B*b)*(c*x^4+b*x^2)^(5/2)/c^2/x^5+1/7*B*(c*x^4+b*x^2)^(5/2)/
c/x^3
```

Rubi [A]
time = 0.11, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2064, 2039}

$$\frac{B(bx^2+cx^4)^{5/2}}{7cx^3}-\frac{(bx^2+cx^4)^{5/2}(2bB-7Ac)}{35c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^2,x]
```

```
[Out] -1/35*((2*b*B - 7*A*c)*(b*x^2 + c*x^4)^(5/2))/(c^2*x^5) + (B*(b*x^2 + c*x^4
)^(5/2))/(7*c*x^3)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2}\,dx = \frac{B(bx^2+cx^4)^{5/2}}{7cx^3} - \frac{(2bB-7Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^2}\,dx}{7c}$$
$$= -\frac{(2bB-7Ac)(bx^2+cx^4)^{5/2}}{35c^2x^5} + \frac{B(bx^2+cx^4)^{5/2}}{7cx^3}$$

Mathematica [A]
time = 0.03, size = 41, normalized size = 0.67

$$\frac{(x^2(b+cx^2))^{5/2}(-2bB+7Ac+5Bcx^2)}{35c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^2,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-2*b*B + 7*A*c + 5*B*c*x^2))/(35*c^2*x^5)
```

Maple [A]
time = 0.36, size = 45, normalized size = 0.74

method	result	size
gosper	$\frac{(c\,x^2+b)(5Bc\,x^2+7Ac-2Bb)(x^4c+b\,x^2)^{\frac{3}{2}}}{35c^2x^3}$	45
default	$\frac{(c\,x^2+b)(5Bc\,x^2+7Ac-2Bb)(x^4c+b\,x^2)^{\frac{3}{2}}}{35c^2x^3}$	45
trager	$\frac{(5B\,c^3x^6+7A\,c^3x^4+8Bb\,c^2x^4+14Ab\,c^2x^2+B\,b^2c\,x^2+7A\,b^2c-2B\,b^3)\sqrt{x^4c+b\,x^2}}{35c^2x}$	83
risch	$\frac{\sqrt{x^2\,(c\,x^2+b)}\,(5B\,c^3x^6+7A\,c^3x^4+8Bb\,c^2x^4+14Ab\,c^2x^2+B\,b^2c\,x^2+7A\,b^2c-2B\,b^3)}{35x\,c^2}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*(c*x^2+b)*(5*B*c*x^2+7*A*c-2*B*b)*(c*x^4+b*x^2)^(3/2)/c^2/x^3
```

Maxima [A]
time = 0.28, size = 80, normalized size = 1.31

$$\frac{(c^2x^4+2\,bcx^2+b^2)\sqrt{cx^2+b}\,A}{5\,c} + \frac{(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3)\sqrt{cx^2+b}\,B}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

[Out] 1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^2 + b)*A/c + 1/35*(5*c^3*x^6 + 8*b *c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^2 + b)*B/c^2

Fricas [A]
time = 1.58, size = 80, normalized size = 1.31

$$\frac{\left(5\,Bc^3x^6+\left(8\,Bbc^2+7\,Ac^3\right)x^4-2\,Bb^3+7\,Ab^2c+\left(Bb^2c+14\,Abc^2\right)x^2\right)\sqrt{cx^4+bx^2}}{35\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="fricas")

[Out] 1/35*(5*B*c^3*x^6 + (8*B*b*c^2 + 7*A*c^3)*x^4 - 2*B*b^3 + 7*A*b^2*c + (B*b^2*c + 14*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^2*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**2,x)

[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**2, x)

Giac [A]
time = 1.34, size = 72, normalized size = 1.18

$$\frac{\left(2\,Bb^{\frac{7}{2}}-7\,Ab^{\frac{5}{2}}c\right)\mathrm{sgn}(x)}{35\,c^2}+\frac{5\,(cx^2+b)^{\frac{7}{2}}B\mathrm{sgn}(x)-7\,(cx^2+b)^{\frac{5}{2}}Bb\mathrm{sgn}(x)+7\,(cx^2+b)^{\frac{5}{2}}Ac\mathrm{sgn}(x)}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="giac")

[Out] 1/35*(2*B*b^(7/2) - 7*A*b^(5/2)*c)*sgn(x)/c^2 + 1/35*(5*(c*x^2 + b)^(7/2)*B *sgn(x) - 7*(c*x^2 + b)^(5/2)*B*b*sgn(x) + 7*(c*x^2 + b)^(5/2)*A*c*sgn(x))/ c^2

Mupad [B]
time = 0.23, size = 83, normalized size = 1.36

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{x^4\,\left(7\,A\,c^3+8\,B\,b\,c^2\right)}{35\,c^2}-\frac{2\,B\,b^3-7\,A\,b^2\,c}{35\,c^2}+\frac{B\,c\,x^6}{7}+\frac{b\,x^2\,\left(14\,A\,c+B\,b\right)}{35\,c}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^2,x)

[Out] ((b*x^2 + c*x^4)^(1/2)*((x^4*(7*A*c^3 + 8*B*b*c^2))/(35*c^2) - (2*B*b^3 - 7 *A*b^2*c)/(35*c^2) + (B*c*x^6)/7 + (b*x^2*(14*A*c + B*b))/(35*c)))/x

3.123 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=102

$$\frac{Ab\sqrt{bx^2+cx^4}}{x}+\frac{A(bx^2+cx^4)^{3/2}}{3x^3}+\frac{B(bx^2+cx^4)^{5/2}}{5cx^5}-Ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] 1/3*A*(c*x^4+b*x^2)^(3/2)/x^3+1/5*B*(c*x^4+b*x^2)^(5/2)/c/x^5-A*b^(3/2)*arc
tanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))+A*b*(c*x^4+b*x^2)^(1/2)/x
```

Rubi [A]
time = 0.14, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2064, 2046, 2033, 212}

$$-Ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)+\frac{Ab\sqrt{bx^2+cx^4}}{x}+\frac{A(bx^2+cx^4)^{3/2}}{3x^3}+\frac{B(bx^2+cx^4)^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4,x]
```

```
[Out] (A*b*Sqrt[b*x^2 + c*x^4])/x + (A*(b*x^2 + c*x^4)^(3/2))/(3*x^3) + (B*(b*x^2
 + c*x^4)^(5/2))/(5*c*x^5) - A*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x
^4]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4}\,dx = \frac{B(bx^2+cx^4)^{5/2}}{5cx^5} + A\int \frac{(bx^2+cx^4)^{3/2}}{x^4}\,dx$$
$$= \frac{A(bx^2+cx^4)^{3/2}}{3x^3} + \frac{B(bx^2+cx^4)^{5/2}}{5cx^5} + (Ab)\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx$$
$$= \frac{Ab\sqrt{bx^2+cx^4}}{x} + \frac{A(bx^2+cx^4)^{3/2}}{3x^3} + \frac{B(bx^2+cx^4)^{5/2}}{5cx^5} + (Ab^2)\int \frac{1}{\sqrt{bx^2+}}$$
$$= \frac{Ab\sqrt{bx^2+cx^4}}{x} + \frac{A(bx^2+cx^4)^{3/2}}{3x^3} + \frac{B(bx^2+cx^4)^{5/2}}{5cx^5} - (Ab^2)\,\text{Subst}\Big(\int$$
$$= \frac{Ab\sqrt{bx^2+cx^4}}{x} + \frac{A(bx^2+cx^4)^{3/2}}{3x^3} + \frac{B(bx^2+cx^4)^{5/2}}{5cx^5} - Ab^{3/2}\tanh^{-1}\Big(\frac{}{\sqrt{}}$$

Mathematica [A]
time = 0.12, size = 108, normalized size = 1.06

$$\frac{x\left((b+cx^2)\left(3b^2B+c^2x^2(5A+3Bx^2)+b(20Ac+6Bcx^2)\right)-15Ab^{3/2}c\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{15c\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4,x]
```

```
[Out] (x*((b + c*x^2)*(3*b^2*B + c^2*x^2*(5*A + 3*B*x^2) + b*(20*A*c + 6*B*c*x^2)
) - 15*A*b^(3/2)*c*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(15*c
*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.36, size = 99, normalized size = 0.97

method	result	size
default	$\frac{\left(x^{4} c+b\,x^{2}\right)^{\frac{3}{2}} \left(3 B \left(c\,x^{2}+b\right)^{\frac{5}{2}}+5 A \left(c\,x^{2}+b\right)^{\frac{3}{2}} c-15 A\,b^{\frac{3}{2}} \ln\left(\frac{2 b+2 \sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right) c+15 A \sqrt{c\,x^{2}+b}\,b c\right)}{15 x^{3} \left(c\,x^{2}+b\right)^{\frac{3}{2}} c}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(c*x^4+b*x^2)^(3/2)*(3*B*(c*x^2+b)^(5/2)+5*A*(c*x^2+b)^(3/2)*c-15*A*b^
(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c+15*A*(c*x^2+b)^(1/2)*b*c)/x^3/(
c*x^2+b)^(3/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^4, x)
```

Fricas [A]
time = 1.60, size = 206, normalized size = 2.02

$$\left[\frac{15\,Ab^{\frac{3}{2}}cx\log\left(-\frac{cx^{3}+2\,bx-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{b}}{x^{3}}\right)+2\,(3\,Bc^{2}x^{4}+3\,Bb^{2}+20\,Abc+(6\,Bbc+5\,Ac^{2})x^{2})\sqrt{cx^{4}+bx^{2}}}{30\,cx}, \frac{15\,A\sqrt{-b}\,bcx\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-b}}{cx^{3}+bx}\right)+(3\,Bc^{2}x^{4}+3\,Bb^{2}+20\,Abc+(6\,Bbc+5\,Ac^{2})x^{2})\sqrt{cx^{4}+bx^{2}}}{15\,cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/30*(15*A*b^(3/2)*c*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b)
)/x^3) + 2*(3*B*c^2*x^4 + 3*B*b^2 + 20*A*b*c + (6*B*b*c + 5*A*c^2)*x^2)*sqr
t(c*x^4 + b*x^2))/(c*x), 1/15*(15*A*sqrt(-b)*b*c*x*arctan(sqrt(c*x^4 + b*x^
2)*sqrt(-b)/(c*x^3 + b*x)) + (3*B*c^2*x^4 + 3*B*b^2 + 20*A*b*c + (6*B*b*c +
 5*A*c^2)*x^2)*sqrt(c*x^4 + b*x^2))/(c*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}(b+cx^{2})\right)^{\frac{3}{2}}\left(A+Bx^{2}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**4, x)
```

Giac [A]
time = 1.52, size = 140, normalized size = 1.37

$$\frac{Ab^2 \arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} - \frac{\left(15\,Ab^2c\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + 3\,B\sqrt{-b}\,b^{\frac{5}{2}} + 20\,A\sqrt{-b}\,b^{\frac{3}{2}}c\right)\operatorname{sgn}(x)}{15\,\sqrt{-b}\,c} + \frac{3\,(cx^2+b)^{\frac{5}{2}}Bc^4\operatorname{sgn}(x) + 5\,(cx^2+b)^{\frac{3}{2}}Ac^5\operatorname{sgn}(x) + 15\,\sqrt{cx^2+b}\,Abc^5\operatorname{sgn}(x)}{15\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] A*b^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - 1/15*(15*A*b^2*c*a
rctan(sqrt(b)/sqrt(-b)) + 3*B*sqrt(-b)*b^(5/2) + 20*A*sqrt(-b)*b^(3/2)*c)*s
gn(x)/(sqrt(-b)*c) + 1/15*(3*(c*x^2 + b)^(5/2)*B*c^4*sgn(x) + 5*(c*x^2 + b)
^(3/2)*A*c^5*sgn(x) + 15*sqrt(c*x^2 + b)*A*b*c^5*sgn(x))/c^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^2+A)\,{(c\,x^4+b\,x^2)}^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4, x)
```

3.124 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=133

$$\frac{(2bB+3Ac)\sqrt{bx^2+cx^4}}{2x}+\frac{(2bB+3Ac)\left(bx^2+cx^4\right)^{3/2}}{6bx^3}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{2bx^7}-\frac{1}{2}\sqrt{b}\,(2bB+3Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] 1/6*(3*A*c+2*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^3-1/2*A*(c*x^4+b*x^2)^(5/2)/b/x^7
-1/2*(3*A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))*b^(1/2)+1/2*(3*A*
c+2*B*b)*(c*x^4+b*x^2)^(1/2)/x
```

Rubi [A]
time = 0.15, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2063, 2046, 2033, 212}

$$\frac{\sqrt{bx^2+cx^4}\,(3Ac+2bB)}{2x}-\frac{1}{2}\sqrt{b}\,(3Ac+2bB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)+\frac{\left(bx^2+cx^4\right)^{3/2}(3Ac+2bB)}{6bx^3}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{2bx^7}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6,x]
```

```
[Out] ((2*b*B + 3*A*c)*Sqrt[b*x^2 + c*x^4])/(2*x) + ((2*b*B + 3*A*c)*(b*x^2 + c*x
^4)^(3/2))/(6*b*x^3) - (A*(b*x^2 + c*x^4)^(5/2))/(2*b*x^7) - (Sqrt[b]*(2*b*
B + 3*A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^6}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{2bx^7} - \frac{(-2bB-3Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^4}\,dx}{2b}$$
$$= \frac{(2bB+3Ac)\,(bx^2+cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2+cx^4)^{5/2}}{2bx^7} - \frac{1}{2}(-2bB-3Ac)\int \frac{\sqrt{bx}}{}$$
$$= \frac{(2bB+3Ac)\sqrt{bx^2+cx^4}}{2x} + \frac{(2bB+3Ac)\,(bx^2+cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2+cx^4)^{5/}}{2bx^7}$$
$$= \frac{(2bB+3Ac)\sqrt{bx^2+cx^4}}{2x} + \frac{(2bB+3Ac)\,(bx^2+cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2+cx^4)^{5/}}{2bx^7}$$
$$= \frac{(2bB+3Ac)\sqrt{bx^2+cx^4}}{2x} + \frac{(2bB+3Ac)\,(bx^2+cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2+cx^4)^{5/}}{2bx^7}$$

Mathematica [A]
time = 0.14, size = 109, normalized size = 0.82

$$\frac{\sqrt{x^2\,(b+cx^2)}\left(\sqrt{b+cx^2}\,(-3Ab+8bBx^2+6Acx^2+2Bcx^4)-3\sqrt{b}\,(2bB+3Ac)x^2\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{6x^3\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[b + c*x^2]*(-3*A*b + 8*b*B*x^2 + 6*A*c*x^2 + 2
*B*c*x^4) - 3*Sqrt[b]*(2*b*B + 3*A*c)*x^2*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])
)/(6*x^3*Sqrt[b + c*x^2])
```

Maple [A]
time = 0.40, size = 172, normalized size = 1.29

method	result
risch	$-\frac{bA\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{2x^{3}}+\frac{\left(\frac{Bc\,x^{2}\sqrt{c\,x^{2}+b}}{3}+\frac{4bB\sqrt{c\,x^{2}+b}}{3}+Ac\sqrt{c\,x^{2}+b}-\frac{3A\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\cdots\right)}{2}\cdots\right)}{x\sqrt{c\,x^{2}+b}\cdots}$
default	$-\frac{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(-3A\left(c\,x^{2}+b\right)^{\frac{3}{2}}c\,x^{2}+9A\,b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)c\,x^{2}-2B\left(c\,x^{2}+b\right)^{\frac{3}{2}}b\,x^{2}+6B\,b^{\frac{5}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}\cdots}}{x}\cdots\right)\cdots\right)}{6x^{5}\left(c\,x^{2}+b\right)^{\frac{3}{2}}b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(c*x^4+b*x^2)^(3/2)*(-3*A*(c*x^2+b)^(3/2)*c*x^2+9*A*b^(3/2)*ln(2*(b^(1
/2)*(c*x^2+b)^(1/2)+b)/x)*c*x^2-2*B*(c*x^2+b)^(3/2)*b*x^2+6*B*b^(5/2)*ln(2*
(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^2+3*A*(c*x^2+b)^(5/2)-9*A*(c*x^2+b)^(1/2)*
b*c*x^2-6*B*(c*x^2+b)^(1/2)*b^2*x^2)/x^5/(c*x^2+b)^(3/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^6, x)
```

Fricas [A]
time = 1.71, size = 195, normalized size = 1.47

$$\left[\frac{3\,(2\,Bb+3\,Ac)\sqrt{b}\,x^{3}\log\left(-\frac{cx^{3}+2\,bx-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{b}}{x^{3}}\right)+2\,(2\,Bcx^{4}+2\,(4\,Bb+3\,Ac)x^{2}-3\,Ab)\sqrt{cx^{4}+bx^{2}}}{12\,x^{3}},\ \frac{3\,(2\,Bb+3\,Ac)\sqrt{-b}\,x^{3}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-b}}{cx^{3}+bx}\right)+(2\,Bcx^{4}+2\,(4\,Bb+3\,Ac)x^{2}-3\,Ab)\sqrt{cx^{4}+bx^{2}}}{6\,x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*B*b + 3*A*c)*sqrt(b)*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b
*x^2)*sqrt(b))/x^3) + 2*(2*B*c*x^4 + 2*(4*B*b + 3*A*c)*x^2 - 3*A*b)*sqrt(c*
x^4 + b*x^2))/x^3, 1/6*(3*(2*B*b + 3*A*c)*sqrt(-b)*x^3*arctan(sqrt(c*x^4 +
b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (2*B*c*x^4 + 2*(4*B*b + 3*A*c)*x^2 - 3*A*b
)*sqrt(c*x^4 + b*x^2))/x^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\left(A+Bx^{2}\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**6, x)
```

Giac [A]
time = 1.19, size = 115, normalized size = 0.86

$$\frac{2\,(cx^2+b)^{\frac{3}{2}}Bc\mathrm{sgn}(x)+6\,\sqrt{cx^2+b}\,Bbc\mathrm{sgn}(x)+6\,\sqrt{cx^2+b}\,Ac^2\mathrm{sgn}(x)-\frac{3\,\sqrt{cx^2+b}\,Abc\mathrm{sgn}(x)}{x^2}+\frac{3\,(2\,Bb^2c\mathrm{sgn}(x)+3\,Abc^2\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/6*(2*(c*x^2 + b)^(3/2)*B*c*sgn(x) + 6*sqrt(c*x^2 + b)*B*b*c*sgn(x) + 6*sq
rt(c*x^2 + b)*A*c^2*sgn(x) - 3*sqrt(c*x^2 + b)*A*b*c*sgn(x)/x^2 + 3*(2*B*b^
2*c*sgn(x) + 3*A*b*c^2*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6, x)
```

3.125 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=135

$$\frac{3c(4bB+Ac)\sqrt{bx^2+cx^4}}{8bx}-\frac{(4bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{8bx^5}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{4bx^9}-\frac{3c(4bB+Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+}}\right.}{8\sqrt{b}}$$

```
[Out] -1/8*(A*c+4*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^5-1/4*A*(c*x^4+b*x^2)^(5/2)/b/x^9-
3/8*c*(A*c+4*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(1/2)+3/8*c*(A*c
+4*B*b)*(c*x^4+b*x^2)^(1/2)/b/x
```

Rubi [A]

time = 0.16, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2063, 2045, 2046, 2033, 212}

$$\frac{3c\sqrt{bx^2+cx^4}\,(Ac+4bB)}{8bx}-\frac{3c(Ac+4bB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}-\frac{\left(bx^2+cx^4\right)^{3/2}(Ac+4bB)}{8bx^5}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{4bx^9}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8,x]
```

```
[Out] (3*c*(4*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(8*b*x) - ((4*b*B + A*c)*(b*x^2 + c
*x^4)^(3/2))/(8*b*x^5) - (A*(b*x^2 + c*x^4)^(5/2))/(4*b*x^9) - (3*c*(4*b*B
+ A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
```

```
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^8}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{4bx^9} - \frac{(-4bB-Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^6}\,dx}{4b}$$

$$= -\frac{(4bB+Ac)\,(bx^2+cx^4)^{3/2}}{8bx^5} - \frac{A(bx^2+cx^4)^{5/2}}{4bx^9} + \frac{(3c(4bB+Ac))\int \frac{\sqrt{bx^2}}{\ldots}}{8b}$$

$$= \frac{3c(4bB+Ac)\sqrt{bx^2+cx^4}}{8bx} - \frac{(4bB+Ac)\,(bx^2+cx^4)^{3/2}}{8bx^5} - \frac{A(bx^2+cx^4)^{5/}}{4bx^9}$$

$$= \frac{3c(4bB+Ac)\sqrt{bx^2+cx^4}}{8bx} - \frac{(4bB+Ac)\,(bx^2+cx^4)^{3/2}}{8bx^5} - \frac{A(bx^2+cx^4)^{5/}}{4bx^9}$$

$$= \frac{3c(4bB+Ac)\sqrt{bx^2+cx^4}}{8bx} - \frac{(4bB+Ac)\,(bx^2+cx^4)^{3/2}}{8bx^5} - \frac{A(bx^2+cx^4)^{5/}}{4bx^9}$$

Mathematica [A]
time = 0.15, size = 114, normalized size = 0.84

$$-\frac{\sqrt{x^2\,(b+cx^2)}\left(\sqrt{b}\,\sqrt{b+cx^2}\,(2Ab+4bBx^2+5Acx^2-8Bcx^4)+3c(4bB+Ac)x^4\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{8\sqrt{b}\,x^5\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8,x]
```

```
[Out] -1/8*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(2*A*b + 4*b*B*x^2 + 5
*A*c*x^2 - 8*B*c*x^4) + 3*c*(4*b*B + A*c)*x^4*ArcTanh[Sqrt[b + c*x^2]/Sqrt[
b]]))/(Sqrt[b]*x^5*Sqrt[b + c*x^2])
```

Maple [A]
time = 0.38, size = 213, normalized size = 1.58

method	result
risch	$-\frac{\left(5Ac\,x^{2}+4bB\,x^{2}+2Ab\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{8x^{5}}+\frac{\left(cB\sqrt{c\,x^{2}+b}-\frac{3\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)A\,c^{2}}{8\sqrt{b}}-\frac{3c\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}}{}\right.}{2}\right.}{x\sqrt{c\,x^{2}+b}}$
default	$-\frac{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(-A\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{2}x^{4}+3A\,b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)c^{2}x^{4}-4B\left(c\,x^{2}+b\right)^{\frac{3}{2}}bc\,x^{4}+12B\,b^{\frac{5}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x}}{x}\right.\right.}{8x^{7}\left(c\,x^{2}+b\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(c*x^4+b*x^2)^(3/2)*(-A*(c*x^2+b)^(3/2)*c^2*x^4+3*A*b^(3/2)*ln(2*(b^(1
/2)*(c*x^2+b)^(1/2)+b)/x)*c^2*x^4-4*B*(c*x^2+b)^(3/2)*b*c*x^4+12*B*b^(5/2)*
ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c*x^4+A*(c*x^2+b)^(5/2)*c*x^2-3*A*(c*x^
2+b)^(1/2)*b*c^2*x^4+4*B*(c*x^2+b)^(5/2)*b*x^2-12*B*(c*x^2+b)^(1/2)*b^2*c*x
^4+2*A*(c*x^2+b)^(5/2)*b)/x^7/(c*x^2+b)^(3/2)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^8, x)
```

Fricas [A]
time = 1.45, size = 217, normalized size = 1.61

$$\left[\frac{3\,(4\,Bbc+Ac^{2})\sqrt{b}\,x^{5}\log\left(-\frac{cx^{3}+2\,bx-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{b}}{x^{3}}\right)+2\,(8\,Bbcx^{4}-2\,Ab^{2}-(4\,Bb^{2}+5\,Abc)x^{2})\sqrt{cx^{4}+bx^{2}}}{16\,bx^{5}},\ \frac{3\,(4\,Bbc+Ac^{2})\sqrt{-b}\,x^{5}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-b}}{cx^{3}+bx}\right)+(8\,Bbcx^{4}-2\,Ab^{2}-(4\,Bb^{2}+5\,Abc)x^{2})\sqrt{cx^{4}+bx^{2}}}{8\,bx^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(4*B*b*c + A*c^2)*sqrt(b)*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 +
 b*x^2)*sqrt(b))/x^3) + 2*(8*B*b*c*x^4 - 2*A*b^2 - (4*B*b^2 + 5*A*b*c)*x^2)
*sqrt(c*x^4 + b*x^2))/(b*x^5), 1/8*(3*(4*B*b*c + A*c^2)*sqrt(-b)*x^5*arctan
(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (8*B*b*c*x^4 - 2*A*b^2 - (4*
B*b^2 + 5*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**8, x)
```

Giac [A]
time = 0.97, size = 145, normalized size = 1.07

$$\frac{8\sqrt{cx^2+b}\,Bc^2\mathrm{sgn}(x) + \frac{3\left(4\,Bbc^2\mathrm{sgn}(x)+Ac^3\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{4\,(cx^2+b)^{\frac{3}{2}}\,Bbc^2\mathrm{sgn}(x)-4\sqrt{cx^2+b}\,Bb^2c^2\mathrm{sgn}(x)+5\,(cx^2+b)^{\frac{3}{2}}\,Ac^3\mathrm{sgn}(x)-3\sqrt{cx^2+b}\,Abc^3\mathrm{sgn}(x)}{c^2x^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(c*x^2 + b)*B*c^2*sgn(x) + 3*(4*B*b*c^2*sgn(x) + A*c^3*sgn(x))*a
rctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) - (4*(c*x^2 + b)^(3/2)*B*b*c^2*sgn
(x) - 4*sqrt(c*x^2 + b)*B*b^2*c^2*sgn(x) + 5*(c*x^2 + b)^(3/2)*A*c^3*sgn(x)
 - 3*sqrt(c*x^2 + b)*A*b*c^3*sgn(x))/(c^2*x^4))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8, x)
```

3.126 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{10}} dx$

Optimal. Leaf size=140

$$-\frac{c(6bB-Ac)\sqrt{bx^2+cx^4}}{16bx^3}-\frac{(6bB-Ac)(bx^2+cx^4)^{3/2}}{24bx^7}-\frac{A(bx^2+cx^4)^{5/2}}{6bx^{11}}-\frac{c^2(6bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}$$

```
[Out] -1/24*(-A*c+6*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^7-1/6*A*(c*x^4+b*x^2)^(5/2)/b/x^
11-1/16*c^2*(-A*c+6*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-1/1
6*c*(-A*c+6*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^3
```

Rubi [A]
time = 0.14, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2063, 2045, 2033, 212}

$$-\frac{c^2(6bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}-\frac{(bx^2+cx^4)^{3/2}(6bB-Ac)}{24bx^7}-\frac{c\sqrt{bx^2+cx^4}(6bB-Ac)}{16bx^3}-\frac{A(bx^2+cx^4)^{5/2}}{6bx^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10,x]
```

```
[Out] -1/16*(c*(6*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(b*x^3) - ((6*b*B - A*c)*(b*x^2
 + c*x^4)^(3/2))/(24*b*x^7) - (A*(b*x^2 + c*x^4)^(5/2))/(6*b*x^11) - (c^2*(
6*b*B - A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(16*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
```

```
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{6bx^{11}} - \frac{(-6bB+Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^8}\,dx}{6b}$$

$$= -\frac{(6bB-Ac)(bx^2+cx^4)^{3/2}}{24bx^7} - \frac{A(bx^2+cx^4)^{5/2}}{6bx^{11}} + \frac{(c(6bB-Ac))\int \frac{\sqrt{bx^2}}{x}$$

$$= -\frac{c(6bB-Ac)\sqrt{bx^2+cx^4}}{16bx^3} - \frac{(6bB-Ac)(bx^2+cx^4)^{3/2}}{24bx^7} - \frac{A(bx^2+cx^4)^5}{6bx^{11}}$$

$$= -\frac{c(6bB-Ac)\sqrt{bx^2+cx^4}}{16bx^3} - \frac{(6bB-Ac)(bx^2+cx^4)^{3/2}}{24bx^7} - \frac{A(bx^2+cx^4)^5}{6bx^{11}}$$

$$= -\frac{c(6bB-Ac)\sqrt{bx^2+cx^4}}{16bx^3} - \frac{(6bB-Ac)(bx^2+cx^4)^{3/2}}{24bx^7} - \frac{A(bx^2+cx^4)^5}{6bx^{11}}$$

Mathematica [A]
time = 0.21, size = 132, normalized size = 0.94

$$\frac{\sqrt{x^2(b+cx^2)}\left(-\sqrt{b}\sqrt{b+cx^2}(6bBx^2(2b+5cx^2)+A(8b^2+14bcx^2+3c^2x^4))+3c^2(-6bB+Ac)x^6\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{48b^{3/2}x^7\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-(Sqrt[b]*Sqrt[b + c*x^2]*(6*b*B*x^2*(2*b + 5*c*x^2
) + A*(8*b^2 + 14*b*c*x^2 + 3*c^2*x^4))) + 3*c^2*(-6*b*B + A*c)*x^6*ArcTanh
[Sqrt[b + c*x^2]/Sqrt[b]]))/(48*b^(3/2)*x^7*Sqrt[b + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(120) = 240$.
time = 0.40, size = 259, normalized size = 1.85

method	result
risch	$-\frac{\left(3A\,c^{2}x^{4}+30x^{4}bBc+14Abc\,x^{2}+12b^{2}B\,x^{2}+8b^{2}A\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{48x^{7}b}+\frac{\left(\frac{c^{3}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)A}{16b^{\frac{3}{2}}}-\frac{3c^{2}\ln\left(\frac{2b+2\sqrt{}}{}\right.}{}\right.}{x\sqrt{c\,x^{2}+}}$
default	$-\frac{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(A\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{3}x^{6}-3A\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)b^{\frac{3}{2}}c^{3}x^{6}-6B\left(c\,x^{2}+b\right)^{\frac{3}{2}}b\,c^{2}x^{6}+18B\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+}}{x}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(c*x^4+b*x^2)^(3/2)*(A*(c*x^2+b)^(3/2)*c^3*x^6-3*A*ln(2*(b^(1/2)*(c*x
^2+b)^(1/2)+b)/x)*b^(3/2)*c^3*x^6-6*B*(c*x^2+b)^(3/2)*b*c^2*x^6+18*B*ln(2*(
b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b^(5/2)*c^2*x^6-A*(c*x^2+b)^(5/2)*c^2*x^4+3*A
*(c*x^2+b)^(1/2)*b*c^3*x^6+6*B*(c*x^2+b)^(5/2)*b*c*x^4-18*B*(c*x^2+b)^(1/2)
*b^2*c^2*x^6-2*A*(c*x^2+b)^(5/2)*b*c*x^2+12*B*(c*x^2+b)^(5/2)*b^2*x^2+8*A*(
c*x^2+b)^(5/2)*b^2)/x^9/(c*x^2+b)^(3/2)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^10, x)
```

Fricas [A]
time = 1.68, size = 250, normalized size = 1.79

$$\left[-\frac{3\,(6\,Bbc^{2}-Ac^{3})\sqrt{b}\,x^{7}\log\left(-\frac{cx^{3}+2\,bx+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{b}}{x^{3}}\right)+2\,(3\,(10\,Bb^{2}c+Abc^{2})x^{4}+8\,Ab^{3}+2\,(6\,Bb^{3}+7\,Ab^{2}c)x^{2})\sqrt{cx^{4}+bx^{2}}}{96\,b^{2}x^{7}},\ \frac{3\,(6\,Bbc^{2}-Ac^{3})\sqrt{-b}\,x^{7}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-b}}{cx^{3}+bx}\right)-(3\,(10\,Bb^{2}c+Abc^{2})x^{4}+8\,Ab^{3}+2\,(6\,Bb^{3}+7\,Ab^{2}c)x^{2})\sqrt{cx^{4}+bx^{2}}}{48\,b^{2}x^{7}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*B*b*c^2 - A*c^3)*sqrt(b)*x^7*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^
4 + b*x^2)*sqrt(b))/x^3) + 2*(3*(10*B*b^2*c + A*b*c^2)*x^4 + 8*A*b^3 + 2*(6
*B*b^3 + 7*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^2*x^7), 1/48*(3*(6*B*b*c^2
```

```
 - A*c^3)*sqrt(-b)*x^7*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) -
 (3*(10*B*b^2*c + A*b*c^2)*x^4 + 8*A*b^3 + 2*(6*B*b^3 + 7*A*b^2*c)*x^2)*sqr
t(c*x^4 + b*x^2))/(b^2*x^7)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}(b+cx^{2})\right)^{\frac{3}{2}}(A+Bx^{2})}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**10,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**10, x)
```

Giac [A]
time = 1.06, size = 175, normalized size = 1.25

$$\frac{\frac{3\left(6\,Bbc^{3}\mathrm{sgn}(x)-Ac^{4}\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^{2}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}-\frac{30\,(cx^{2}+b)^{\frac{5}{2}}Bbc^{3}\mathrm{sgn}(x)-48\,(cx^{2}+b)^{\frac{3}{2}}Bb^{2}c^{3}\mathrm{sgn}(x)+18\,\sqrt{cx^{2}+b}\,Bb^{3}c^{3}\mathrm{sgn}(x)+3\,(cx^{2}+b)^{\frac{5}{2}}Ac^{4}\mathrm{sgn}(x)+8\,(cx^{2}+b)^{\frac{3}{2}}Abc^{4}\mathrm{sgn}(x)-3\,\sqrt{cx^{2}+b}\,Ab^{2}c^{4}\mathrm{sgn}(x)}{bc^{3}x^{6}}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/48*(3*(6*B*b*c^3*sgn(x) - A*c^4*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b))/
(sqrt(-b)*b) - (30*(c*x^2 + b)^(5/2)*B*b*c^3*sgn(x) - 48*(c*x^2 + b)^(3/2)*
B*b^2*c^3*sgn(x) + 18*sqrt(c*x^2 + b)*B*b^3*c^3*sgn(x) + 3*(c*x^2 + b)^(5/2
)*A*c^4*sgn(x) + 8*(c*x^2 + b)^(3/2)*A*b*c^4*sgn(x) - 3*sqrt(c*x^2 + b)*A*b
^2*c^4*sgn(x))/(b*c^3*x^6))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^{2}+A\right)\,{\left(c\,x^{4}+b\,x^{2}\right)}^{3/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10, x)
```

3.127 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=177

$$-\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5}-\frac{c^2(8bB-3Ac)\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{(8bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{48bx^9}-\frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}}+$$

```
[Out] -1/48*(-3*A*c+8*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^9-1/8*A*(c*x^4+b*x^2)^(5/2)/b/
x^13+1/128*c^3*(-3*A*c+8*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2
)-1/64*c*(-3*A*c+8*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^5-1/128*c^2*(-3*A*c+8*B*b)*
(c*x^4+b*x^2)^(1/2)/b^2/x^3
```

Rubi [A]
time = 0.18, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2063, 2045, 2050, 2033, 212}

$$\frac{c^3(8bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}-\frac{c^2\sqrt{bx^2+cx^4}\,(8bB-3Ac)}{128b^2x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}(8bB-3Ac)}{48bx^9}-\frac{c\sqrt{bx^2+cx^4}\,(8bB-3Ac)}{64bx^5}-\frac{A\left(bx^2+cx^4\right)^{5/2}}{8bx^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12,x]
```

```
[Out] -1/64*(c*(8*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(b*x^5) - (c^2*(8*b*B - 3*A*c
)*Sqrt[b*x^2 + c*x^4])/(128*b^2*x^3) - ((8*b*B - 3*A*c)*(b*x^2 + c*x^4)^(3/
2))/(48*b*x^9) - (A*(b*x^2 + c*x^4)^(5/2))/(8*b*x^13) + (c^3*(8*b*B - 3*A*c
)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(128*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
```

```
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}} - \frac{(-8bB+3Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^{10}}\,dx}{8b}$$

$$= -\frac{(8bB-3Ac)(bx^2+cx^4)^{3/2}}{48bx^9} - \frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}} + \frac{(c(8bB-3Ac))\int \frac{\sqrt{bx}}{}}{16b}$$

$$= -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{(8bB-3Ac)(bx^2+cx^4)^{3/2}}{48bx^9} - \frac{A(bx^2+cx^4}{8bx^{13}}$$

$$= -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{c^2(8bB-3Ac)\sqrt{bx^2+cx^4}}{128b^2x^3} - \frac{(8bB-3Ac}{4}$$

$$= -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{c^2(8bB-3Ac)\sqrt{bx^2+cx^4}}{128b^2x^3} - \frac{(8bB-3Ac}{4}$$

$$= -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{c^2(8bB-3Ac)\sqrt{bx^2+cx^4}}{128b^2x^3} - \frac{(8bB-3Ac}{4}$$

Mathematica [A]
time = 0.27, size = 154, normalized size = 0.87

$$-\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}\sqrt{b+cx^2}\,(8bBx^2(8b^2+14bcx^2+3c^2x^4)+A(48b^3+72b^2cx^2+6bc^2x^4-9c^3x^6))+3c^3(-8bB+3Ac)x^8\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{384b^{5/2}x^9\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12,x]
```

```
[Out] -1/384*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(8*b*B*x^2*(8*b^2 +
14*b*c*x^2 + 3*c^2*x^4) + A*(48*b^3 + 72*b^2*c*x^2 + 6*b*c^2*x^4 - 9*c^3*x^
6)) + 3*c^3*(-8*b*B + 3*A*c)*x^8*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(b^(5/2
)*x^9*Sqrt[b + c*x^2])
```

Maple [A]
time = 0.39, size = 302, normalized size = 1.71

method	result
risch	$-\frac{(-9Ac^3x^6+24x^6Bbc^2+6Abc^2x^4+112x^4Bb^2c+72Ab^2cx^2+64x^2Bb^3+48Ab^3)\sqrt{x^2(cx^2+b)}}{384x^9b^2}+\frac{\left(-\frac{3c^4\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{\cdots}}{\cdots}\right.}{128b\cdots}\right.}{\cdots}$
default	$-\frac{(x^4c+bx^2)^{\frac{3}{2}}\left(-3A(cx^2+b)^{\frac{3}{2}}c^4x^8+9Ab^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{cx^2+b}}{x}\right)c^4x^8+8B(cx^2+b)^{\frac{3}{2}}bc^3x^8-24Bb^{\frac{5}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{\cdots}}{x}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/384*(c*x^4+b*x^2)^(3/2)*(-3*A*(c*x^2+b)^(3/2)*c^4*x^8+9*A*b^(3/2)*ln(2*(
b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c^4*x^8+8*B*(c*x^2+b)^(3/2)*b*c^3*x^8-24*B*b^
(5/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c^3*x^8+3*A*(c*x^2+b)^(5/2)*c^3*x
^6-9*A*(c*x^2+b)^(1/2)*b*c^4*x^8-8*B*(c*x^2+b)^(5/2)*b*c^2*x^6+24*B*(c*x^2+
b)^(1/2)*b^2*c^3*x^8+6*A*(c*x^2+b)^(5/2)*b*c^2*x^4-16*B*(c*x^2+b)^(5/2)*b^2
*c*x^4-24*A*(c*x^2+b)^(5/2)*b^2*c*x^2+64*B*(c*x^2+b)^(5/2)*b^3*x^2+48*A*(c*
x^2+b)^(5/2)*b^3)/x^11/(c*x^2+b)^(3/2)/b^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="maxima")
```

[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^12, x)

Fricas [A]
time = 1.47, size = 299, normalized size = 1.69

$$\left[-\frac{3\,(8\,Bbc^3-3\,Ac^4)\sqrt{b}\,x^9\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,(3\,(8\,Bb^2c^2-3\,Abc^3)x^6+48\,Ab^4+2\,(56\,Bb^3c+3\,Ab^2c^2)x^4+8\,(8\,Bb^4+9\,Ab^3c)x^2)\sqrt{cx^4+bx^2}}{768\,b^3x^9},\ -\frac{3\,(8\,Bbc^3-3\,Ac^4)\sqrt{-b}\,x^9\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+(3\,(8\,Bb^2c^2-3\,Abc^3)x^6+48\,Ab^4+2\,(56\,Bb^3c+3\,Ab^2c^2)x^4+8\,(8\,Bb^4+9\,Ab^3c)x^2)\sqrt{cx^4+bx^2}}{384\,b^3x^9}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(8*B*b*c^3 - 3*A*c^4)*sqrt(b)*x^9*log(-(c*x^3 + 2*b*x - 2*sqrt(c
*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(3*(8*B*b^2*c^2 - 3*A*b*c^3)*x^6 + 48*A*b^4
 + 2*(56*B*b^3*c + 3*A*b^2*c^2)*x^4 + 8*(8*B*b^4 + 9*A*b^3*c)*x^2)*sqrt(c*x
^4 + b*x^2))/(b^3*x^9), -1/384*(3*(8*B*b*c^3 - 3*A*c^4)*sqrt(-b)*x^9*arctan
(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (3*(8*B*b^2*c^2 - 3*A*b*c^3)
*x^6 + 48*A*b^4 + 2*(56*B*b^3*c + 3*A*b^2*c^2)*x^4 + 8*(8*B*b^4 + 9*A*b^3*c
)*x^2)*sqrt(c*x^4 + b*x^2))/(b^3*x^9)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**12,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**12, x)
```

Giac [A]
time = 1.25, size = 214, normalized size = 1.21

$$-\frac{\frac{3\,(8\,Bbc^4\mathrm{sgn}(x)-3\,Ac^5\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^2}+\frac{24\,(cx^2+b)^{\frac{7}{2}}Bbc^4\mathrm{sgn}(x)+40\,(cx^2+b)^{\frac{5}{2}}Bb^2c^4\mathrm{sgn}(x)-88\,(cx^2+b)^{\frac{3}{2}}Bb^3c^4\mathrm{sgn}(x)+24\,\sqrt{cx^2+b}\,Bb^4c^4\mathrm{sgn}(x)-9\,(cx^2+b)^{\frac{7}{2}}Ac^5\mathrm{sgn}(x)+33\,(cx^2+b)^{\frac{5}{2}}Abc^5\mathrm{sgn}(x)+33\,(cx^2+b)^{\frac{3}{2}}Ab^2c^5\mathrm{sgn}(x)-9\,\sqrt{cx^2+b}\,Ab^3c^5\mathrm{sgn}(x)}{b^2c^4x^8}}{384\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] -1/384*(3*(8*B*b*c^4*sgn(x) - 3*A*c^5*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-
b))/(sqrt(-b)*b^2) + (24*(c*x^2 + b)^(7/2)*B*b*c^4*sgn(x) + 40*(c*x^2 + b)^
(5/2)*B*b^2*c^4*sgn(x) - 88*(c*x^2 + b)^(3/2)*B*b^3*c^4*sgn(x) + 24*sqrt(c*
x^2 + b)*B*b^4*c^4*sgn(x) - 9*(c*x^2 + b)^(7/2)*A*c^5*sgn(x) + 33*(c*x^2 +
b)^(5/2)*A*b*c^5*sgn(x) + 33*(c*x^2 + b)^(3/2)*A*b^2*c^5*sgn(x) - 9*sqrt(c*
x^2 + b)*A*b^3*c^5*sgn(x))/(b^2*c^4*x^8))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{12}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12,x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12, x)
```

3.128 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}}\,dx$

Optimal. Leaf size=214

$$-\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7}-\frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5}+\frac{3c^3(2bB-Ac)\sqrt{bx^2+cx^4}}{256b^3x^3}-\frac{(2bB-Ac)(bx^2+cx}{16bx^{11}}$$

```
[Out] -1/16*(-A*c+2*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^11-1/10*A*(c*x^4+b*x^2)^(5/2)/b/
x^15-3/256*c^4*(-A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(7/2)-
1/32*c*(-A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^7-1/128*c^2*(-A*c+2*B*b)*(c*x^4
+b*x^2)^(1/2)/b^2/x^5+3/256*c^3*(-A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^3
```

Rubi [A]

time = 0.23, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2063, 2045, 2050, 2033, 212}

$$-\frac{3c^4(2bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}+\frac{3c^3\sqrt{bx^2+cx^4}\,(2bB-Ac)}{256b^3x^3}-\frac{c^2\sqrt{bx^2+cx^4}\,(2bB-Ac)}{128b^2x^5}-\frac{(bx^2+cx^4)^{3/2}(2bB-Ac)}{16bx^{11}}-\frac{c\sqrt{bx^2+cx^4}\,(2bB-Ac)}{32bx^7}-\frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14,x]
```

```
[Out] -1/32*(c*(2*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(b*x^7) - (c^2*(2*b*B - A*c)*Sq
rt[b*x^2 + c*x^4])/(128*b^2*x^5) + (3*c^3*(2*b*B - A*c)*Sqrt[b*x^2 + c*x^4]
)/(256*b^3*x^3) - ((2*b*B - A*c)*(b*x^2 + c*x^4)^(3/2))/(16*b*x^11) - (A*(b
*x^2 + c*x^4)^(5/2))/(10*b*x^15) - (3*c^4*(2*b*B - A*c)*ArcTanh[(Sqrt[b]*x)
/Sqrt[b*x^2 + c*x^4]])/(256*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
```

```
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} - \frac{(-10bB+5Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^{12}}dx}{10b}$$

$$= -\frac{(2bB-Ac)(bx^2+cx^4)^{3/2}}{16bx^{11}} - \frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} + \frac{(3c(2bB-Ac))\int \frac{\sqrt{bx^2}}{}}{16b}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{(2bB-Ac)(bx^2+cx^4)^{3/2}}{16bx^{11}} - \frac{A(bx^2+cx^4)^{5}}{10bx^{15}}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} - \frac{(2bB-Ac)(b}{16bx}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} + \frac{3c^3(2bB-Ac}{256b}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} + \frac{3c^3(2bB-Ac}{256b}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} + \frac{3c^3(2bB-Ac}{256b}$$

Mathematica [A]
time = 0.33, size = 172, normalized size = 0.80

$$\frac{-\sqrt{b}\,(b+cx^2)\left(10bBx^2(16b^3+24b^2cx^2+2bc^2x^4-3c^3x^6)+A(128b^4+176b^3cx^2+8b^2c^2x^4-10bc^3x^6+15c^4x^8)\right)+15c^4(-2bB+Ac)x^{10}\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{1280b^{7/2}x^9\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14,x]
```

```
[Out] (-(Sqrt[b]*(b + c*x^2)*(10*b*B*x^2*(16*b^3 + 24*b^2*c*x^2 + 2*b*c^2*x^4 - 3
*c^3*x^6) + A*(128*b^4 + 176*b^3*c*x^2 + 8*b^2*c^2*x^4 - 10*b*c^3*x^6 + 15*
c^4*x^8))) + 15*c^4*(-2*b*B + A*c)*x^10*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*
x^2]/Sqrt[b]])/(1280*b^(7/2)*x^9*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.40, size = 344, normalized size = 1.61

method	result

risch	$-\frac{(15 A c^4 x^8-30 B b c^3 x^8-10 A b c^3 x^6+20 B b^2 c^2 x^6+8 A b^2 c^2 x^4+240 B b^3 c x^4+176 A b^3 c x^2+160 B b^4 x^2+128 A b^4) \sqrt{x^2 (c x^2+b}}{1280 x^{11} b^3}$
default	$-\frac{(x^4 c+b x^2)^{\frac{3}{2}} \left(5 A (c x^2+b)^{\frac{3}{2}} c^5 x^{10}-15 A b^{\frac{3}{2}} \ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c x^2+b}}{x}\right) c^5 x^{10}-10 B (c x^2+b)^{\frac{3}{2}} b c^4 x^{10}+30 B b^{\frac{5}{2}} \ln\left(\frac{2b+2\sqrt{b}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1280*(c*x^4+b*x^2)^(3/2)*(5*A*(c*x^2+b)^(3/2)*c^5*x^10-15*A*b^(3/2)*ln(2
*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c^5*x^10-10*B*(c*x^2+b)^(3/2)*b*c^4*x^10+30
*B*b^(5/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c^4*x^10-5*A*(c*x^2+b)^(5/2)
*c^4*x^8+15*A*(c*x^2+b)^(1/2)*b*c^5*x^10+10*B*(c*x^2+b)^(5/2)*b*c^3*x^8-30*
B*(c*x^2+b)^(1/2)*b^2*c^4*x^10-10*A*(c*x^2+b)^(5/2)*b*c^3*x^6+20*B*(c*x^2+b
)^(5/2)*b^2*c^2*x^6+40*A*(c*x^2+b)^(5/2)*b^2*c^2*x^4-80*B*(c*x^2+b)^(5/2)*b
^3*c*x^4-80*A*(c*x^2+b)^(5/2)*b^3*c*x^2+160*B*(c*x^2+b)^(5/2)*b^4*x^2+128*A
*(c*x^2+b)^(5/2)*b^4)/x^13/(c*x^2+b)^(3/2)/b^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^14, x)
```

Fricas [A]
time = 1.44, size = 345, normalized size = 1.61

$$\left[-\frac{15\,(2\,Bbc^4-Ac^5)\sqrt{b}\,x^{11}\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\,(15\,(2\,Bb^2c^3-Abc^4)x^8-10\,(2\,Bb^3c^2-Ab^2c^3)x^6-128\,Ab^5-8\,(30\,Bb^4c+Ab^3c^2)x^4-16\,(10\,Bb^5+11\,Ab^4c)x^2)\sqrt{cx^4+bx^2}}{2560\,b^4x^{11}},\ \frac{15\,(2\,Bbc^4-Ac^5)\sqrt{-b}\,x^{11}\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+(15\,(2\,Bb^2c^3-Abc^4)x^8-10\,(2\,Bb^3c^2-Ab^2c^3)x^6-128\,Ab^5-8\,(30\,Bb^4c+Ab^3c^2)x^4-16\,(10\,Bb^5+11\,Ab^4c)x^2)\sqrt{cx^4+bx^2}}{1280\,b^4x^{11}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(2*B*b*c^4 - A*c^5)*sqrt(b)*x^11*log(-(c*x^3 + 2*b*x + 2*sqrt(
c*x^4 + b*x^2)*sqrt(b))/x^3) - 2*(15*(2*B*b^2*c^3 - A*b*c^4)*x^8 - 10*(2*B*
b^3*c^2 - A*b^2*c^3)*x^6 - 128*A*b^5 - 8*(30*B*b^4*c + A*b^3*c^2)*x^4 - 16*
(10*B*b^5 + 11*A*b^4*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^4*x^11), 1/1280*(15*(2
*B*b*c^4 - A*c^5)*sqrt(-b)*x^11*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3
```

```
+ b*x)) + (15*(2*B*b^2*c^3 - A*b*c^4)*x^8 - 10*(2*B*b^3*c^2 - A*b^2*c^3)*x^
6 - 128*A*b^5 - 8*(30*B*b^4*c + A*b^3*c^2)*x^4 - 16*(10*B*b^5 + 11*A*b^4*c)
*x^2)*sqrt(c*x^4 + b*x^2))/(b^4*x^11)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**14,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**14, x)
```

Giac [A]
time = 0.94, size = 234, normalized size = 1.09

$$\frac{\frac{15\,(2\,Bbc^5\mathrm{sgn}(x)-Ac^6\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^3}+\frac{30\,(cx^2+b)^{\frac{9}{2}}Bbc^5\mathrm{sgn}(x)-140\,(cx^2+b)^{\frac{7}{2}}Bb^2c^5\mathrm{sgn}(x)+140\,(cx^2+b)^{\frac{3}{2}}Bb^4c^5\mathrm{sgn}(x)-30\,\sqrt{cx^2+b}\,Bb^5c^5\mathrm{sgn}(x)-15\,(cx^2+b)^{\frac{9}{2}}Ac^6\mathrm{sgn}(x)+70\,(cx^2+b)^{\frac{7}{2}}Abc^6\mathrm{sgn}(x)-128\,(cx^2+b)^{\frac{5}{2}}Ab^2c^6\mathrm{sgn}(x)-70\,(cx^2+b)^{\frac{3}{2}}Ab^3c^6\mathrm{sgn}(x)+15\,\sqrt{cx^2+b}\,Ab^4c^6\mathrm{sgn}(x)}{b^3c^5x^{10}}}{1280\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] 1/1280*(15*(2*B*b*c^5*sgn(x) - A*c^6*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b
))/(sqrt(-b)*b^3) + (30*(c*x^2 + b)^(9/2)*B*b*c^5*sgn(x) - 140*(c*x^2 + b)^
(7/2)*B*b^2*c^5*sgn(x) + 140*(c*x^2 + b)^(3/2)*B*b^4*c^5*sgn(x) - 30*sqrt(c
*x^2 + b)*B*b^5*c^5*sgn(x) - 15*(c*x^2 + b)^(9/2)*A*c^6*sgn(x) + 70*(c*x^2
+ b)^(7/2)*A*b*c^6*sgn(x) - 128*(c*x^2 + b)^(5/2)*A*b^2*c^6*sgn(x) - 70*(c*
x^2 + b)^(3/2)*A*b^3*c^6*sgn(x) + 15*sqrt(c*x^2 + b)*A*b^4*c^6*sgn(x))/(b^3
*c^5*x^10))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14, x)
```

3.129 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}\,dx$

Optimal. Leaf size=251

$$-\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9}-\frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7}+\frac{c^3(12bB-7Ac)\sqrt{bx^2+cx^4}}{1536b^3x^5}-\frac{c^4(12bB-7A$$

```
[Out] -1/120*(-7*A*c+12*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^13-1/12*A*(c*x^4+b*x^2)^(5/2
)/b/x^17+1/1024*c^5*(-7*A*c+12*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/
b^(9/2)-1/320*c*(-7*A*c+12*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^9-1/1920*c^2*(-7*A*
c+12*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^7+1/1536*c^3*(-7*A*c+12*B*b)*(c*x^4+b*x
^2)^(1/2)/b^3/x^5-1/1024*c^4*(-7*A*c+12*B*b)*(c*x^4+b*x^2)^(1/2)/b^4/x^3
```

Rubi [A]
time = 0.26, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2063, 2045, 2050, 2033, 212}

$$\frac{c^5(12bB-7Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{1024b^{9/2}}-\frac{c^4\sqrt{bx^2+cx^4}\,(12bB-7Ac)}{1024b^4x^3}+\frac{c^3\sqrt{bx^2+cx^4}\,(12bB-7Ac)}{1536b^3x^5}-\frac{c^2\sqrt{bx^2+cx^4}\,(12bB-7Ac)}{1920b^2x^7}-\frac{(bx^2+cx^4)^{3/2}(12bB-7Ac)}{120bx^{13}}-\frac{c\sqrt{bx^2+cx^4}\,(12bB-7Ac)}{320bx^9}-\frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16,x]
```

```
[Out] -1/320*(c*(12*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(b*x^9) - (c^2*(12*b*B - 7*
A*c)*Sqrt[b*x^2 + c*x^4])/(1920*b^2*x^7) + (c^3*(12*b*B - 7*A*c)*Sqrt[b*x^2
 + c*x^4])/(1536*b^3*x^5) - (c^4*(12*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(102
4*b^4*x^3) - ((12*b*B - 7*A*c)*(b*x^2 + c*x^4)^(3/2))/(120*b*x^13) - (A*(b*
x^2 + c*x^4)^(5/2))/(12*b*x^17) + (c^5*(12*b*B - 7*A*c)*ArcTanh[(Sqrt[b]*x)
/Sqrt[b*x^2 + c*x^4]])/(1024*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}} - \frac{(-12bB+7Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx}{12b}$$

$$= -\frac{(12bB-7Ac)(bx^2+cx^4)^{3/2}}{120bx^{13}} - \frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}} + \frac{(c(12bB-7Ac))\int}{40b}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{(12bB-7Ac)(bx^2+cx^4)^{3/2}}{120bx^{13}} - \frac{A(bx^2+}{12b}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} - \frac{(12bB-}{}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB}{}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB}{}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB}{}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB}{}$$

Mathematica [A]
time = 0.36, size = 198, normalized size = 0.79

$$-\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}\sqrt{b+cx^2}\,(12bBx^2(128b^4+176b^3cx^2+8b^2c^2x^4-10bc^3x^6+15c^4x^8)+A(1280b^5+1664b^4cx^2+48b^3c^2x^4-56b^2c^3x^6+70bc^4x^8-105c^5x^{10}))+15c^5(-12bB+7Ac)x^{12}\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{15360b^{9/2}x^{13}\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16,x]
```

```
[Out] -1/15360*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(12*b*B*x^2*(128*b
^4 + 176*b^3*c*x^2 + 8*b^2*c^2*x^4 - 10*b*c^3*x^6 + 15*c^4*x^8) + A*(1280*b
^5 + 1664*b^4*c*x^2 + 48*b^3*c^2*x^4 - 56*b^2*c^3*x^6 + 70*b*c^4*x^8 - 105*
c^5*x^10)) + 15*c^5*(-12*b*B + 7*A*c)*x^12*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]
))/(b^(9/2)*x^13*Sqrt[b + c*x^2])
```

Maple [A]
time = 0.40, size = 386, normalized size = 1.54

method	result

risch	$-\frac{\left(-105 A\,c^{5} x^{10}+180 B b\,c^{4} x^{10}+70 A b\,c^{4} x^{8}-120 B\,b^{2} c^{3} x^{8}-56 A\,b^{2} c^{3} x^{6}+96 B\,b^{3} c^{2} x^{6}+48 A\,b^{3} c^{2} x^{4}+2112 B\,b^{4} c\,x^{4}+1664 A\,b^{4} c\,x^{2}+1536 b}{15360 x^{13} b^{4}}$
default	$-\frac{\left(x^{4} c+b\,x^{2}\right)^{\frac{3}{2}} \left(-35 A \left(c\,x^{2}+b\right)^{\frac{3}{2}} c^{6} x^{12}+105 A\,b^{\frac{3}{2}} \ln\left(\frac{2 b+2 \sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right) c^{6} x^{12}+60 B \left(c\,x^{2}+b\right)^{\frac{3}{2}} b\,c^{5} x^{12}-180 B\,b^{\frac{5}{2}} \ln\left(\frac{2 b+2 \sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x,method=_RETURNVERBOSE)

[Out] -1/15360*(c*x^4+b*x^2)^(3/2)*(-35*A*(c*x^2+b)^(3/2)*c^6*x^12+105*A*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c^6*x^12+60*B*(c*x^2+b)^(3/2)*b*c^5*x^12-180*B*b^(5/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c^5*x^12+35*A*(c*x^2+b)^(5/2)*c^5*x^10-105*A*(c*x^2+b)^(1/2)*b*c^6*x^12-60*B*(c*x^2+b)^(5/2)*b*c^4*x^10+180*B*(c*x^2+b)^(1/2)*b^2*c^5*x^12+70*A*(c*x^2+b)^(5/2)*b*c^4*x^8-120*B*(c*x^2+b)^(5/2)*b^2*c^3*x^8-280*A*(c*x^2+b)^(5/2)*b^2*c^3*x^6+480*B*(c*x^2+b)^(5/2)*b^3*c^2*x^6+560*A*(c*x^2+b)^(5/2)*b^3*c^2*x^4-960*B*(c*x^2+b)^(5/2)*b^4*c*x^4-896*A*(c*x^2+b)^(5/2)*b^4*c*x^2+1536*B*(c*x^2+b)^(5/2)*b^5*x^2+1280*A*(c*x^2+b)^(5/2)*b^5)/x^15/(c*x^2+b)^(3/2)/b^6

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^16, x)

Fricas [A]
time = 3.39, size = 393, normalized size = 1.57

$$\left[-\frac{15\,(12\,Bbc^{5}-7\,Ac^{6})\sqrt{b}\,x^{13}\log\left(-\frac{cx^{3}+2\,bx-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{b}}{x^{3}}\right)+2\,(15\,(12\,Bb^{2}c^{4}-7\,Abc^{5})x^{10}-10\,(12\,Bb^{3}c^{3}-7\,Ab^{2}c^{4})x^{8}+1280\,Ab^{6}+8\,(12\,Bb^{4}c^{2}-7\,Ab^{3}c^{3})x^{6}+48\,(44\,Bb^{5}c+Ab^{4}c^{2})x^{4}+128\,(12\,Bb^{6}+13\,Ab^{5}c)x^{2})\sqrt{cx^{4}+bx^{2}}}{30720\,b^{5}x^{13}},\ -\frac{15\,(12\,Bbc^{5}-7\,Ac^{6})\sqrt{-b}\,x^{13}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-b}}{cx^{3}+bx}\right)+(15\,(12\,Bb^{2}c^{4}-7\,Abc^{5})x^{10}-10\,(12\,Bb^{3}c^{3}-7\,Ab^{2}c^{4})x^{8}+1280\,Ab^{6}+8\,(12\,Bb^{4}c^{2}-7\,Ab^{3}c^{3})x^{6}+48\,(44\,Bb^{5}c+Ab^{4}c^{2})x^{4}+128\,(12\,Bb^{6}+13\,Ab^{5}c)x^{2})\sqrt{cx^{4}+bx^{2}}}{15360\,b^{5}x^{13}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x, algorithm="fricas")

[Out] [-1/30720*(15*(12*B*b*c^5 - 7*A*c^6)*sqrt(b)*x^13*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(15*(12*B*b^2*c^4 - 7*A*b*c^5)*x^10 - 10*(12*B*b^3*c^3 - 7*A*b^2*c^4)*x^8 + 1280*A*b^6 + 8*(12*B*b^4*c^2 - 7*A*b^3*c^3)*x^6 + 48*(44*B*b^5*c + A*b^4*c^2)*x^4 + 128*(12*B*b^6 + 13*A*b^5*c)*

```
x^2)*sqrt(c*x^4 + b*x^2))/(b^5*x^13), -1/15360*(15*(12*B*b*c^5 - 7*A*c^6)*s
qrt(-b)*x^13*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (15*(12*B
*b^2*c^4 - 7*A*b*c^5)*x^10 - 10*(12*B*b^3*c^3 - 7*A*b^2*c^4)*x^8 + 1280*A*b
^6 + 8*(12*B*b^4*c^2 - 7*A*b^3*c^3)*x^6 + 48*(44*B*b^5*c + A*b^4*c^2)*x^4 +
 128*(12*B*b^6 + 13*A*b^5*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^5*x^13)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}(b+cx^{2})\right)^{\frac{3}{2}}\left(A+Bx^{2}\right)}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**16,x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**16, x)
```

Giac [A]
time = 1.21, size = 294, normalized size = 1.17

$$-\frac{\frac{15\,(12\,Bbc^{6}\mathrm{sgn}(x)-7\,Ac^{7}\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^{2}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^{4}}+\frac{180\,(cx^{2}+b)^{\frac{11}{2}}Bbc^{6}\mathrm{sgn}(x)-1020\,(cx^{2}+b)^{\frac{9}{2}}Bb^{2}c^{6}\mathrm{sgn}(x)+2376\,(cx^{2}+b)^{\frac{7}{2}}Bb^{3}c^{6}\mathrm{sgn}(x)-696\,(cx^{2}+b)^{\frac{5}{2}}Bb^{4}c^{6}\mathrm{sgn}(x)-1020\,(cx^{2}+b)^{\frac{3}{2}}Bb^{5}c^{6}\mathrm{sgn}(x)+180\,\sqrt{cx^{2}+b}\,Bb^{6}c^{6}\mathrm{sgn}(x)-105\,(cx^{2}+b)^{\frac{11}{2}}Ac^{7}\mathrm{sgn}(x)+595\,(cx^{2}+b)^{\frac{9}{2}}Abc^{7}\mathrm{sgn}(x)-1386\,(cx^{2}+b)^{\frac{7}{2}}Ab^{2}c^{7}\mathrm{sgn}(x)+1686\,(cx^{2}+b)^{\frac{5}{2}}Ab^{3}c^{7}\mathrm{sgn}(x)+595\,(cx^{2}+b)^{\frac{3}{2}}Ab^{4}c^{7}\mathrm{sgn}(x)-105\,\sqrt{cx^{2}+b}\,Ab^{5}c^{7}\mathrm{sgn}(x)}{b^{4}c^{6}x^{12}}}{15360\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x, algorithm="giac")
[Out] -1/15360*(15*(12*B*b*c^6*sgn(x) - 7*A*c^7*sgn(x))*arctan(sqrt(c*x^2 + b)/sq
rt(-b))/(sqrt(-b)*b^4) + (180*(c*x^2 + b)^(11/2)*B*b*c^6*sgn(x) - 1020*(c*x
^2 + b)^(9/2)*B*b^2*c^6*sgn(x) + 2376*(c*x^2 + b)^(7/2)*B*b^3*c^6*sgn(x) -
696*(c*x^2 + b)^(5/2)*B*b^4*c^6*sgn(x) - 1020*(c*x^2 + b)^(3/2)*B*b^5*c^6*s
gn(x) + 180*sqrt(c*x^2 + b)*B*b^6*c^6*sgn(x) - 105*(c*x^2 + b)^(11/2)*A*c^7
*sgn(x) + 595*(c*x^2 + b)^(9/2)*A*b*c^7*sgn(x) - 1386*(c*x^2 + b)^(7/2)*A*b
^2*c^7*sgn(x) + 1686*(c*x^2 + b)^(5/2)*A*b^3*c^7*sgn(x) + 595*(c*x^2 + b)^(
3/2)*A*b^4*c^7*sgn(x) - 105*sqrt(c*x^2 + b)*A*b^5*c^7*sgn(x))/(b^4*c^6*x^12
))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{16}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16,x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16, x)
```

3.130 $\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=176

$$-\frac{5b^2(7bB-8Ac)\sqrt{bx^2+cx^4}}{128c^4}+\frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3}-\frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2}+\frac{Bx^6\sqrt{bx^2+cx^4}}{8c}$$

```
[Out] 5/128*b^3*(-8*A*c+7*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(9/2)-5
/128*b^2*(-8*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/c^4+5/192*b*(-8*A*c+7*B*b)*x^2*
(c*x^4+b*x^2)^(1/2)/c^3-1/48*(-8*A*c+7*B*b)*x^4*(c*x^4+b*x^2)^(1/2)/c^2+1/8
*B*x^6*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A]

time = 0.21, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2059, 808, 684, 654, 634, 212}

$$\frac{5b^3(7bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{9/2}}-\frac{5b^2\sqrt{bx^2+cx^4}\,(7bB-8Ac)}{128c^4}+\frac{5bx^2\sqrt{bx^2+cx^4}\,(7bB-8Ac)}{192c^3}-\frac{x^4\sqrt{bx^2+cx^4}\,(7bB-8Ac)}{48c^2}+\frac{Bx^6\sqrt{bx^2+cx^4}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-5*b^2*(7*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(128*c^4) + (5*b*(7*b*B - 8*A*
c)*x^2*Sqrt[b*x^2 + c*x^4])/(192*c^3) - ((7*b*B - 8*A*c)*x^4*Sqrt[b*x^2 + c
*x^4])/(48*c^2) + (B*x^6*Sqrt[b*x^2 + c*x^4])/(8*c) + (5*b^3*(7*b*B - 8*A*c
)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{Bx^6\sqrt{bx^2+cx^4}}{8c} + \frac{\left(3(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{8c}$$

$$= -\frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2} + \frac{Bx^6\sqrt{bx^2+cx^4}}{8c} + \frac{(5b(7bB-8Ac))\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+}}\right.}{96c^2}$$

$$= \frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2} + \frac{Bx^6\sqrt{bx^2+cx^4}}{8c} - \frac{(5}{}$$

$$= -\frac{5b^2(7bB-8Ac)\sqrt{bx^2+cx^4}}{128c^4} + \frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx}}{48c^2}$$

$$= -\frac{5b^2(7bB-8Ac)\sqrt{bx^2+cx^4}}{128c^4} + \frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx}}{48c^2}$$

$$= -\frac{5b^2(7bB-8Ac)\sqrt{bx^2+cx^4}}{128c^4} + \frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx}}{48c^2}$$

Mathematica [A]
time = 0.18, size = 147, normalized size = 0.84

$$\frac{x\left(-\sqrt{c}\,x(b+cx^2)\left(105b^3B-16c^3x^4(4A+3Bx^2)+8bc^2x^2(10A+7Bx^2)-10b^2c(12A+7Bx^2)\right)-15b^3(7bB-8Ac)\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{384c^{9/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(-(Sqrt[c]*x*(b + c*x^2)*(105*b^3*B - 16*c^3*x^4*(4*A + 3*B*x^2) + 8*b*c
^2*x^2*(10*A + 7*B*x^2) - 10*b^2*c*(12*A + 7*B*x^2))) - 15*b^3*(7*b*B - 8*A
*c)*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(384*c^(9/2)*Sqrt
[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.39, size = 211, normalized size = 1.20

method	result

risch	$\frac{x^2\left(48B\,c^3x^6+64A\,c^3x^4-56Bb\,c^2x^4-80Ab\,c^2x^2+70B\,b^2c\,x^2+120A\,b^2c-105B\,b^3\right)\left(c\,x^2+b\right)}{384c^4\sqrt{x^2\left(c\,x^2+b\right)}}+\frac{\left(-\frac{5b^3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)A}{16c^{\frac{7}{2}}}\right.}{\sqrt{}}$
default	$\frac{x\sqrt{c\,x^2+b}\,\left(48B\sqrt{c\,x^2+b}\,c^{\frac{9}{2}}x^7+64A\sqrt{c\,x^2+b}\,c^{\frac{9}{2}}x^5-56B\sqrt{c\,x^2+b}\,c^{\frac{7}{2}}b\,x^5-80A\sqrt{c\,x^2+b}\,c^{\frac{7}{2}}b\,x^3+70B\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*x*(c*x^2+b)^(1/2)*(48*B*(c*x^2+b)^(1/2)*c^(9/2)*x^7+64*A*(c*x^2+b)^(1
/2)*c^(9/2)*x^5-56*B*(c*x^2+b)^(1/2)*c^(7/2)*b*x^5-80*A*(c*x^2+b)^(1/2)*c^(
7/2)*b*x^3+70*B*(c*x^2+b)^(1/2)*c^(5/2)*b^2*x^3+120*A*(c*x^2+b)^(1/2)*c^(5/
2)*b^2*x-105*B*(c*x^2+b)^(1/2)*c^(3/2)*b^3*x-120*A*ln(c^(1/2)*x+(c*x^2+b)^(
1/2))*b^3*c^2+105*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^4*c)/(c*x^4+b*x^2)^(1/2
)/c^(11/2)
```

Maxima [A]

time = 0.29, size = 231, normalized size = 1.31

$$\frac{1}{96}\left(\frac{16\sqrt{cx^4+bx^2}\,x^4}{c}-\frac{20\sqrt{cx^4+bx^2}\,bx^2}{c^2}-\frac{15\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{7}{2}}}+\frac{30\,\sqrt{cx^4+bx^2}\,b^2}{c^3}\right)A+\frac{1}{768}\left(\frac{96\,\sqrt{cx^4+bx^2}\,x^6}{c}-\frac{112\,\sqrt{cx^4+bx^2}\,bx^4}{c^2}+\frac{140\,\sqrt{cx^4+bx^2}\,b^2x^2}{c^3}+\frac{105\,b^4\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{c^{\frac{9}{2}}}-\frac{210\,\sqrt{cx^4+bx^2}\,b^3}{c^4}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(16*sqrt(c*x^4 + b*x^2)*x^4/c - 20*sqrt(c*x^4 + b*x^2)*b*x^2/c^2 - 15*
b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*sqrt(c*x^
4 + b*x^2)*b^2/c^3)*A + 1/768*(96*sqrt(c*x^4 + b*x^2)*x^6/c - 112*sqrt(c*x^
4 + b*x^2)*b*x^4/c^2 + 140*sqrt(c*x^4 + b*x^2)*b^2*x^2/c^3 + 105*b^4*log(2*
c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) - 210*sqrt(c*x^4 + b*x^2
)*b^3/c^4)*B
```

Fricas [A]

time = 2.42, size = 275, normalized size = 1.56

$$\left[-\frac{15\,(7\,Bb^4-8\,Ab^3c)\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\,(48\,Bc^4x^6-105\,Bb^3c+120\,Ab^2c^2-8\,(7\,Bbc^3-8\,Ac^4)x^4+10\,(7\,Bb^2c^2-8\,Abc^3)x^2)\sqrt{cx^4+bx^2}}{768\,c^5},\ -\frac{15\,(7\,Bb^4-8\,Ab^3c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)-(48\,Bc^4x^6-105\,Bb^3c+120\,Ab^2c^2-8\,(7\,Bbc^3-8\,Ac^4)x^4+10\,(7\,Bb^2c^2-8\,Abc^3)x^2)\sqrt{cx^4+bx^2}}{384\,c^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(7*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 +
 b*x^2)*sqrt(c)) - 2*(48*B*c^4*x^6 - 105*B*b^3*c + 120*A*b^2*c^2 - 8*(7*B*b
*c^3 - 8*A*c^4)*x^4 + 10*(7*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2)
```

```
)/c^5, -1/384*(15*(7*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)
*sqrt(-c)/(c*x^2 + b)) - (48*B*c^4*x^6 - 105*B*b^3*c + 120*A*b^2*c^2 - 8*(7
*B*b*c^3 - 8*A*c^4)*x^4 + 10*(7*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*
x^2))/c^5]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7(A+Bx^2)}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**7*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 0.69, size = 184, normalized size = 1.05

$$\frac{1}{384}\left(2\left(4\,x^2\left(\frac{6\,Bx^2}{c\mathrm{sgn}\,(x)}-\frac{7\,Bbc^5\mathrm{sgn}(x)-8\,Ac^6\mathrm{sgn}(x)}{c^7}\right)+\frac{5\,(7\,Bb^2c^4\mathrm{sgn}(x)-8\,Abc^5\mathrm{sgn}(x))}{c^7}\right)x^2-\frac{15\,(7\,Bb^3c^3\mathrm{sgn}(x)-8\,Ab^2c^4\mathrm{sgn}(x))}{c^7}\right)\sqrt{cx^2+b}\,x+\frac{5\,(7\,Bb^4\log\left(|b|\right)-8\,Ab^3c\log\left(|b|\right))\mathrm{sgn}(x)}{256\,c^{\frac{9}{2}}}-\frac{5\,(7\,Bb^4-8\,Ab^3c)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{128\,c^{\frac{9}{2}}\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*x^2*(6*B*x^2/(c*sgn(x)) - (7*B*b*c^5*sgn(x) - 8*A*c^6*sgn(x))/c
^7) + 5*(7*B*b^2*c^4*sgn(x) - 8*A*b*c^5*sgn(x))/c^7)*x^2 - 15*(7*B*b^3*c^3*
sgn(x) - 8*A*b^2*c^4*sgn(x))/c^7)*sqrt(c*x^2 + b)*x + 5/256*(7*B*b^4*log(ab
s(b)) - 8*A*b^3*c*log(abs(b)))*sgn(x)/c^(9/2) - 5/128*(7*B*b^4 - 8*A*b^3*c)
*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(9/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.131 $\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=139

$$\frac{b(5bB-6Ac)\sqrt{bx^2+cx^4}}{16c^3}-\frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2}+\frac{Bx^4\sqrt{bx^2+cx^4}}{6c}-\frac{b^2(5bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{\cdots}}{\sqrt{bx^2\cdots}}\right.}{16c^{7/2}}$$

```
[Out] -1/16*b^2*(-6*A*c+5*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)+1
/16*b*(-6*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/c^3-1/24*(-6*A*c+5*B*b)*x^2*(c*x^4
+b*x^2)^(1/2)/c^2+1/6*B*x^4*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A]

time = 0.18, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2059, 808, 684, 654, 634, 212}

$$-\frac{b^2(5bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}+\frac{b\sqrt{bx^2+cx^4}\,(5bB-6Ac)}{16c^3}-\frac{x^2\sqrt{bx^2+cx^4}\,(5bB-6Ac)}{24c^2}+\frac{Bx^4\sqrt{bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (b*(5*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/(16*c^3) - ((5*b*B - 6*A*c)*x^2*Sqr
t[b*x^2 + c*x^4])/(24*c^2) + (B*x^4*Sqrt[b*x^2 + c*x^4])/(6*c) - (b^2*(5*b*
B - 6*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)$$

$$= \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} + \frac{\left(2(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx,x,x^2\right.}{6c}$$

$$= -\frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} + \frac{(b(5bB-6Ac))\text{Subst}\left(\int \frac{x}{\sqrt{bx+}}\right.}{16c^2}$$

$$= \frac{b(5bB-6Ac)\sqrt{bx^2+cx^4}}{16c^3} - \frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} - \frac{(b^2(}{}$$

$$= \frac{b(5bB-6Ac)\sqrt{bx^2+cx^4}}{16c^3} - \frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} - \frac{(b^2(}{}$$

$$= \frac{b(5bB-6Ac)\sqrt{bx^2+cx^4}}{16c^3} - \frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} - \frac{b^2(5}{}$$

Mathematica [A]
time = 0.15, size = 125, normalized size = 0.90

$$\frac{x\left(\sqrt{c}\,x(b+cx^2)\,(15b^2B+4c^2x^2(3A+2Bx^2)-2bc(9A+5Bx^2))+3b^2(5bB-6Ac)\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{48c^{7/2}\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(15*b^2*B + 4*c^2*x^2*(3*A + 2*B*x^2) - 2*b*c*(9*
A + 5*B*x^2)) + 3*b^2*(5*b*B - 6*A*c)*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sq
rt[b + c*x^2]]))/(48*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.44, size = 169, normalized size = 1.22

method	result
risch	$-\frac{x^2\left(-8B\,c^2x^4-12A\,c^2x^2+10bB\,x^2c+18Abc-15b^2B\right)\left(c\,x^2+b\right)}{48c^3\sqrt{x^2\left(c\,x^2+b\right)}}+\frac{\left(\frac{3b^2\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+b}\right)A}{8c^{\frac{5}{2}}}-\frac{5b^3\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+\right.}{16c^{\frac{7}{2}}}\right.}{\sqrt{x^2\left(c\,x^2+b\right)}}$

default	$\frac{x\sqrt{c\,x^{2}+b}\,\left(8B\sqrt{c\,x^{2}+b}\,c^{\frac{7}{2}}x^{5}+12A\sqrt{c\,x^{2}+b}\,c^{\frac{7}{2}}x^{3}-10B\sqrt{c\,x^{2}+b}\,c^{\frac{5}{2}}b\,x^{3}-18A\sqrt{c\,x^{2}+b}\,c^{\frac{5}{2}}b x+15B\sqrt{c\,x}\right.}{48\sqrt{x^{4}c+b\,x^{2}}\,c^{\frac{9}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*x*(c*x^2+b)^(1/2)*(8*B*(c*x^2+b)^(1/2)*c^(7/2)*x^5+12*A*(c*x^2+b)^(1/2
)*c^(7/2)*x^3-10*B*(c*x^2+b)^(1/2)*c^(5/2)*b*x^3-18*A*(c*x^2+b)^(1/2)*c^(5/
2)*b*x+15*B*(c*x^2+b)^(1/2)*c^(3/2)*b^2*x+18*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2)
)*b^2*c^2-15*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^3*c)/(c*x^4+b*x^2)^(1/2)/c^(
9/2)
```

Maxima [A]
time = 0.28, size = 183, normalized size = 1.32

$$\frac{1}{16}\left(\frac{4\sqrt{cx^{4}+bx^{2}}\,x^{2}}{c}+\frac{3\,b^{2}\log\left(2\,cx^{2}+b+2\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{c^{\frac{5}{2}}}-\frac{6\sqrt{cx^{4}+bx^{2}}\,b}{c^{2}}\right)A+\frac{1}{96}\left(\frac{16\sqrt{cx^{4}+bx^{2}}\,x^{4}}{c}-\frac{20\sqrt{cx^{4}+bx^{2}}\,bx^{2}}{c^{2}}-\frac{15\,b^{3}\log\left(2\,cx^{2}+b+2\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{c^{\frac{7}{2}}}+\frac{30\sqrt{cx^{4}+bx^{2}}\,b^{2}}{c^{3}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*sqrt(c*x^4 + b*x^2)*x^2/c + 3*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 +
b*x^2)*sqrt(c))/c^(5/2) - 6*sqrt(c*x^4 + b*x^2)*b/c^2)*A + 1/96*(16*sqrt(c*
x^4 + b*x^2)*x^4/c - 20*sqrt(c*x^4 + b*x^2)*b*x^2/c^2 - 15*b^3*log(2*c*x^2
+ b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*sqrt(c*x^4 + b*x^2)*b^2/c
^3)*B
```

Fricas [A]
time = 1.41, size = 226, normalized size = 1.63

$$\left[-\frac{3\,(5\,Bb^{3}-6\,Ab^{2}c)\sqrt{c}\,\log\left(-2\,cx^{2}-b-2\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)-2\,(8\,Bc^{3}x^{4}+15\,Bb^{2}c-18\,Abc^{2}-2\,(5\,Bbc^{2}-6\,Ac^{3})x^{2})\sqrt{cx^{4}+bx^{2}}}{96\,c^{4}},\ \frac{3\,(5\,Bb^{3}-6\,Ab^{2}c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)+(8\,Bc^{3}x^{4}+15\,Bb^{2}c-18\,Abc^{2}-2\,(5\,Bbc^{2}-6\,Ac^{3})x^{2})\sqrt{cx^{4}+bx^{2}}}{48\,c^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*B*b^3 - 6*A*b^2*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b
*x^2)*sqrt(c)) - 2*(8*B*c^3*x^4 + 15*B*b^2*c - 18*A*b*c^2 - 2*(5*B*b*c^2 -
6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4, 1/48*(3*(5*B*b^3 - 6*A*b^2*c)*sqrt(
-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*B*c^3*x^4 + 15*B*
b^2*c - 18*A*b*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}(A+Bx^{2})}{\sqrt{x^{2}\,(b+cx^{2})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**5*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 0.71, size = 150, normalized size = 1.08

$$\frac{1}{48}\sqrt{cx^2+b}\left(2x^2\left(\frac{4Bx^2}{c\operatorname{sgn}(x)}-\frac{5Bbc^3\operatorname{sgn}(x)-6Ac^4\operatorname{sgn}(x)}{c^5}\right)+\frac{3\left(5Bb^2c^2\operatorname{sgn}(x)-6Abc^3\operatorname{sgn}(x)\right)}{c^5}\right)x-\frac{\left(5Bb^3\log\left(|b|\right)-6Ab^2c\log\left(|b|\right)\right)\operatorname{sgn}(x)}{32c^{\frac{7}{2}}}+\frac{\left(5Bb^3-6Ab^2c\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{16c^{\frac{7}{2}}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^2 + b)*(2*x^2*(4*B*x^2/(c*sgn(x)) - (5*B*b*c^3*sgn(x) - 6*A*c
^4*sgn(x))/c^5) + 3*(5*B*b^2*c^2*sgn(x) - 6*A*b*c^3*sgn(x))/c^5)*x - 1/32*(
5*B*b^3*log(abs(b)) - 6*A*b^2*c*log(abs(b)))*sgn(x)/c^(7/2) + 1/16*(5*B*b^3
 - 6*A*b^2*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(7/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.132 $\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=83

$$-\frac{(3bB-4Ac-2Bcx^2)\sqrt{bx^2+cx^4}}{8c^2}+\frac{b(3bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}$$

```
[Out] 1/8*b*(-4*A*c+3*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(5/2)-1/8*(
-2*B*c*x^2-4*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/c^2
```

Rubi [A]
time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2059, 793, 634, 212}

$$\frac{b(3bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{\sqrt{bx^2+cx^4}\,(-4Ac+3bB-2Bcx^2)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -1/8*((3*b*B - 4*A*c - 2*B*c*x^2)*Sqrt[b*x^2 + c*x^4])/c^2 + (b*(3*b*B - 4*
A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{(3bB-4Ac-2Bcx^2)\sqrt{bx^2+cx^4}}{8c^2} + \frac{(b(3bB-4Ac))\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x\right.}{16c^2}$$

$$= -\frac{(3bB-4Ac-2Bcx^2)\sqrt{bx^2+cx^4}}{8c^2} + \frac{(b(3bB-4Ac))\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+}}\right.}{8c^2}$$

$$= -\frac{(3bB-4Ac-2Bcx^2)\sqrt{bx^2+cx^4}}{8c^2} + \frac{b(3bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}$$

Mathematica [A]
time = 0.10, size = 99, normalized size = 1.19

$$\frac{x\left(\sqrt{c}\,x(b+cx^2)\left(-3bB+4Ac+2Bcx^2\right)+b(-3bB+4Ac)\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{8c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(-3*b*B + 4*A*c + 2*B*c*x^2) + b*(-3*b*B + 4*A*c)
*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(8*c^(5/2)*Sqrt[x^2*
(b + c*x^2)])
```

Maple [A]
time = 0.39, size = 127, normalized size = 1.53

method	result

risch	$\frac{x^{2} (2 B c x^{2}+4 A c-3 B b) (c x^{2}+b)}{8 c^{2} \sqrt{x^{2} (c x^{2}+b)}}+\frac{\left(-\frac{b \ln \left(\sqrt{c}\, x+\sqrt{c x^{2}+b}\right) A}{2 c^{\frac{3}{2}}}+\frac{3 b^{2} \ln \left(\sqrt{c}\, x+\sqrt{c x^{2}+b}\right) B}{8 c^{\frac{5}{2}}}\right) x \sqrt{c x^{2}+b}}{\sqrt{x^{2} (c x^{2}+b)}}$
default	$\frac{x \sqrt{c x^{2}+b}\, \left(2 B \sqrt{c x^{2}+b}\, c^{\frac{5}{2}} x^{3}+4 A \sqrt{c x^{2}+b}\, c^{\frac{5}{2}} x-3 B \sqrt{c x^{2}+b}\, c^{\frac{3}{2}} b x-4 A \ln \left(\sqrt{c}\, x+\sqrt{c x^{2}+b}\right) b c^{2}+3 B \ln\right.}{8 \sqrt{x^{4} c+b x^{2}}\, c^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x*(c*x^2+b)^(1/2)*(2*B*(c*x^2+b)^(1/2)*c^(5/2)*x^3+4*A*(c*x^2+b)^(1/2)*
c^(5/2)*x-3*B*(c*x^2+b)^(1/2)*c^(3/2)*b*x-4*A*ln(c^(1/2)*x+(c*x^2+b)^(1/2))
*b*c^2+3*B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2*c)/(c*x^4+b*x^2)^(1/2)/c^(7/2)
```

Maxima [A]
time = 0.29, size = 134, normalized size = 1.61

$$\frac{1}{16} \left(\frac{4 \sqrt{cx^4+bx^2}\, x^2}{c}+\frac{3 b^2 \log \left(2 cx^2+b+2 \sqrt{cx^4+bx^2}\, \sqrt{c}\right)}{c^{\frac{5}{2}}}-\frac{6 \sqrt{cx^4+bx^2}\, b}{c^2}\right) B-\frac{1}{4} A \left(\frac{b \log \left(2 cx^2+b+2 \sqrt{cx^4+bx^2}\, \sqrt{c}\right)}{c^{\frac{3}{2}}}-\frac{2 \sqrt{cx^4+bx^2}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*sqrt(c*x^4 + b*x^2)*x^2/c + 3*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 +
b*x^2)*sqrt(c))/c^(5/2) - 6*sqrt(c*x^4 + b*x^2)*b/c^2)*B - 1/4*A*(b*log(2*c
*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) - 2*sqrt(c*x^4 + b*x^2)/c
)
```

Fricas [A]
time = 1.67, size = 177, normalized size = 2.13

$$\left[-\frac{(3 Bb^2-4 Abc)\sqrt{c}\, \log \left(-2 cx^2-b+2 \sqrt{cx^4+bx^2}\, \sqrt{c}\right)-2 (2 Bc^2x^2-3 Bbc+4 Ac^2)\sqrt{cx^4+bx^2}}{16 c^3}, -\frac{(3 Bb^2-4 Abc)\sqrt{-c}\, \arctan \left(\frac{\sqrt{cx^4+bx^2}\, \sqrt{-c}}{cx^2+b}\right)-(2 Bc^2x^2-3 Bbc+4 Ac^2)\sqrt{cx^4+bx^2}}{8 c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*B*b^2 - 4*A*b*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2
)*sqrt(c)) - 2*(2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^3,
-1/8*((3*B*b^2 - 4*A*b*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x
^2 + b)) - (2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(A+Bx^2)}{\sqrt{x^2\,(b+cx^2)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 0.62, size = 112, normalized size = 1.35

$$\frac{1}{8}\sqrt{cx^2+b}\,x\left(\frac{2\,Bx^2}{c\mathrm{sgn}\,(x)}-\frac{3\,Bbc\mathrm{sgn}(x)-4\,Ac^2\mathrm{sgn}(x)}{c^3}\right)+\frac{(3\,Bb^2\log\left(|b|\right)-4\,Abc\log\left(|b|\right))\mathrm{sgn}(x)}{16\,c^{\frac{5}{2}}}-\frac{(3\,Bb^2-4\,Abc)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{8\,c^{\frac{5}{2}}\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^2 + b)*x*(2*B*x^2/(c*sgn(x)) - (3*B*b*c*sgn(x) - 4*A*c^2*sgn(x
))/c^3) + 1/16*(3*B*b^2*log(abs(b)) - 4*A*b*c*log(abs(b)))*sgn(x)/c^(5/2) -
 1/8*(3*B*b^2 - 4*A*b*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(5/2)*sg
n(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.133 $\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=66

$$\frac{B\sqrt{bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[Out] -1/2*(-2*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)+1/2*B*(c
*x^4+b*x^2)^(1/2)/c
```

Rubi [A]

time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2059, 654, 634, 212}

$$\frac{B\sqrt{bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (B*Sqrt[b*x^2 + c*x^4])/(2*c) - ((b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b
*x^2 + c*x^4]])/(2*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\ &= \frac{B\sqrt{bx^2+cx^4}}{2c}+\frac{(-bB+2Ac)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{4c}\\ &= \frac{B\sqrt{bx^2+cx^4}}{2c}+\frac{(-bB+2Ac)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c}\\ &= \frac{B\sqrt{bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 82, normalized size = 1.24

$$\frac{x\left(B\sqrt{c}\,x(b+cx^2)+(bB-2Ac)\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{2c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(B*Sqrt[c]*x*(b + c*x^2) + (b*B - 2*A*c)*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x
) + Sqrt[b + c*x^2]]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.40, size = 88, normalized size = 1.33

method	result	size
default	$\frac{x\sqrt{c\,x^2+b}\,\left(B\,c^{\frac{3}{2}}\sqrt{c\,x^2+b}\,x+2A\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)c^2-B\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)bc\right)}{2\sqrt{x^4c+b\,x^2}\,c^{\frac{5}{2}}}$	88

risch	$\frac{B\,x^{2}\left(c\,x^{2}+b\right)}{2c\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(\frac{A\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)}{\sqrt{c}}-\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)Bb}{2c^{\frac{3}{2}}}\right)x\sqrt{c\,x^{2}+b}}{\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(c*x^2+b)^(1/2)*(B*c^(3/2)*(c*x^2+b)^(1/2)*x+2*A*ln(c^(1/2)*x+(c*x^2+
b)^(1/2))*c^2-B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b*c)/(c*x^4+b*x^2)^(1/2)/c^(5
/2)
```

Maxima [A]
time = 0.31, size = 88, normalized size = 1.33

$$-\frac{1}{4}\,B\left(\frac{b\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{c^{\frac{3}{2}}}-\frac{2\,\sqrt{cx^{4}+bx^{2}}}{c}\right)+\frac{A\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*B*(b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) - 2*sqrt
(c*x^4 + b*x^2)/c) + 1/2*A*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))
/sqrt(c)
```

Fricas [A]
time = 0.93, size = 131, normalized size = 1.98

$$\left[\frac{2\,\sqrt{cx^{4}+bx^{2}}\,Bc-(Bb-2\,Ac)\sqrt{c}\,\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{4\,c^{2}},\ \frac{\sqrt{cx^{4}+bx^{2}}\,Bc+(Bb-2\,Ac)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)}{2\,c^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*x^4 + b*x^2)*B*c - (B*b - 2*A*c)*sqrt(c)*log(-2*c*x^2 - b -
2*sqrt(c*x^4 + b*x^2)*sqrt(c)))/c^2, 1/2*(sqrt(c*x^4 + b*x^2)*B*c + (B*b -
2*A*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)))/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x(A+Bx^{2})}{\sqrt{x^{2}\left(b+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 0.93, size = 77, normalized size = 1.17

$$\frac{\sqrt{cx^2+b}\,Bx}{2\,c\mathrm{sgn}\,(x)} - \frac{(Bb\log(|b|) - 2\,Ac\log(|b|))\mathrm{sgn}(x)}{4\,c^{\frac{3}{2}}} + \frac{(Bb-2\,Ac)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{2\,c^{\frac{3}{2}}\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + b)*B*x/(c*sgn(x)) - 1/4*(B*b*log(abs(b)) - 2*A*c*log(abs(b
)))*sgn(x)/c^(3/2) + 1/2*(B*b - 2*A*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)
))/(c^(3/2)*sgn(x))
```

Mupad [B]
time = 0.81, size = 89, normalized size = 1.35

$$\frac{B\,\sqrt{c\,x^4+b\,x^2}}{2\,c} + \frac{A\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,\sqrt{c}} - \frac{B\,b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (B*(b*x^2 + c*x^4)^(1/2))/(2*c) + (A*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c
*x^4)^(1/2)))/(2*c^(1/2)) - (B*b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4
)^(1/2)))/(4*c^(3/2))
```

3.134 $\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=57

$$-\frac{A\sqrt{bx^2+cx^4}}{bx^2}+\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

```
[Out] B*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(1/2)-A*(c*x^4+b*x^2)^(1/2)/b/
x^2
```

Rubi [A]
time = 0.09, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2059, 806, 634, 212}

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}-\frac{A\sqrt{bx^2+cx^4}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -((A*Sqrt[b*x^2 + c*x^4])/(b*x^2)) + (B*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
c*x^4]])/Sqrt[c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
```

```
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= -\frac{A\sqrt{bx^2+cx^4}}{bx^2}+\frac{1}{2}B\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= -\frac{A\sqrt{bx^2+cx^4}}{bx^2}+B\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\
&= -\frac{A\sqrt{bx^2+cx^4}}{bx^2}+\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 77, normalized size = 1.35

$$\frac{-A\sqrt{c}\,(b+cx^2)-bBx\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)}{b\sqrt{c}\,\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-(A*Sqrt[c]*(b + c*x^2)) - b*B*x*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b
 + c*x^2]])/(b*Sqrt[c]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.38, size = 67, normalized size = 1.18

method	result	size

default	$-\frac{\sqrt{c\,x^{2}+b}\,\left(-B\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)bx+A\sqrt{c\,x^{2}+b}\,\sqrt{c}\right)}{\sqrt{x^{4}c+b\,x^{2}}\,\sqrt{c}\,b}$	67
risch	$-\frac{A(c\,x^{2}+b)}{b\sqrt{x^{2}\,(c\,x^{2}+b)}}+\frac{B\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)x\sqrt{c\,x^{2}+b}}{\sqrt{c}\,\sqrt{x^{2}\,(c\,x^{2}+b)}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2+b)^(1/2)*(-B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b*x+A*(c*x^2+b)^(1/2)*c^
(1/2))/(c*x^4+b*x^2)^(1/2)/c^(1/2)/b
```

Maxima [A]
time = 0.31, size = 56, normalized size = 0.98

$$\frac{B\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{2\,\sqrt{c}}-\frac{\sqrt{cx^{4}+bx^{2}}\,A}{bx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*B*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) - sqrt(c*x^4
 + b*x^2)*A/(b*x^2)
```

Fricas [A]
time = 1.67, size = 136, normalized size = 2.39

$$\left[\frac{Bb\sqrt{c}\,x^{2}\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)-2\,\sqrt{cx^{4}+bx^{2}}\,Ac}{2\,bcx^{2}},-\frac{Bb\sqrt{-c}\,x^{2}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)+\sqrt{cx^{4}+bx^{2}}\,Ac}{bcx^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*b*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2
*sqrt(c*x^4 + b*x^2)*A*c)/(b*c*x^2), -(B*b*sqrt(-c)*x^2*arctan(sqrt(c*x^4 +
 b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*A*c)/(b*c*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+Bx^{2}}{x\sqrt{x^{2}\,(b+cx^{2})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A]
time = 0.56, size = 66, normalized size = 1.16

$$-\frac{B\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2\right)}{2\sqrt{c}\,\mathrm{sgn}\left(x\right)}+\frac{2\,A\sqrt{c}}{\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*B*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)/(sqrt(c)*sgn(x)) + 2*A*sqrt(c)/
(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)*sgn(x))
```

Mupad [B]
time = 0.51, size = 57, normalized size = 1.00

$$\frac{B\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,\sqrt{c}}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] (B*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(2*c^(1/2)) - (A*(b*
x^2 + c*x^4)^(1/2))/(b*x^2)
```

3.135 $\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=61

$$-\frac{A\sqrt{bx^2+cx^4}}{3bx^4}-\frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{3b^2x^2}$$

```
[Out] -1/3*A*(c*x^4+b*x^2)^(1/2)/b/x^4-1/3*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/b^2
/x^2
```

Rubi [A]
time = 0.11, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2059, 806, 664}

$$-\frac{\sqrt{bx^2+cx^4}\,(3bB-2Ac)}{3b^2x^2}-\frac{A\sqrt{bx^2+cx^4}}{3bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/3*(A*Sqrt[b*x^2 + c*x^4])/(b*x^4) - ((3*b*B - 2*A*c)*Sqrt[b*x^2 + c*x^4]
)/(3*b^2*x^2)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
```

```
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{3bx^4} + \frac{\left(-2(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+cx^2}}\,dx, x\right.}{3b}$$
$$= -\frac{A\sqrt{bx^2+cx^4}}{3bx^4} - \frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{3b^2x^2}$$

Mathematica [**A**]
time = 0.07, size = 43, normalized size = 0.70

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(3bBx^2+A(b-2cx^2)\right)}{3b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*Sqrt[b*x^2 + c*x^4]),x]
[Out] -1/3*(Sqrt[x^2*(b + c*x^2)]*(3*b*B*x^2 + A*(b - 2*c*x^2)))/(b^2*x^4)
```

Maple [**A**]
time = 0.39, size = 47, normalized size = 0.77

method	result	size
trager	$-\frac{\left(-2Ac\,x^2+3bB\,x^2+Ab\right)\sqrt{x^4c+b\,x^2}}{3b^2x^4}$	40
gosper	$-\frac{\left(c\,x^2+b\right)\left(-2Ac\,x^2+3bB\,x^2+Ab\right)}{3x^2b^2\sqrt{x^4c+b\,x^2}}$	47
default	$-\frac{\left(c\,x^2+b\right)\left(-2Ac\,x^2+3bB\,x^2+Ab\right)}{3x^2b^2\sqrt{x^4c+b\,x^2}}$	47
risch	$-\frac{\left(c\,x^2+b\right)\left(-2Ac\,x^2+3bB\,x^2+Ab\right)}{3x^2\sqrt{x^2\left(c\,x^2+b\right)}\,b^2}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*x^2+b)*(-2*A*c*x^2+3*B*b*x^2+A*b)/x^2/b^2/(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 70, normalized size = 1.15

$$\frac{1}{3}A\left(\frac{2\sqrt{cx^4+bx^2}\,c}{b^2x^2}-\frac{\sqrt{cx^4+bx^2}}{bx^4}\right)-\frac{\sqrt{cx^4+bx^2}\,B}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*A*(2*sqrt(c*x^4 + b*x^2)*c/(b^2*x^2) - sqrt(c*x^4 + b*x^2)/(b*x^4)) - s
qrt(c*x^4 + b*x^2)*B/(b*x^2)
```

Fricas [A]
time = 0.91, size = 38, normalized size = 0.62

$$-\frac{\sqrt{cx^4+bx^2}\,((3\,Bb-2\,Ac)x^2+Ab)}{3\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(c*x^4 + b*x^2)*((3*B*b - 2*A*c)*x^2 + A*b)/(b^2*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+Bx^2}{x^3\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral((A + B*x**2)/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(53) = 106.
time = 0.66, size = 124, normalized size = 2.03

$$\frac{2\left(3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4B\sqrt{c}\;-6\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2Bb\sqrt{c}\;+6\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2Ac^{\frac{3}{2}}+3\,Bb^2\sqrt{c}\;-2\,Abc^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^3\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*sqrt(c) - 6*(sqrt(c)*x - sqrt(c*x^
2 + b))^2*B*b*sqrt(c) + 6*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*c^(3/2) + 3*B*b
^2*sqrt(c) - 2*A*b*c^(3/2))/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3*sgn(x)
)
```

Mupad [B]
time = 0.20, size = 39, normalized size = 0.64

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(A\,b-2\,A\,c\,x^2+3\,B\,b\,x^2\right)}{3\,b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(A*b - 2*A*c*x^2 + 3*B*b*x^2))/(3*b^2*x^4)
```

3.136 $\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=96

$$-\frac{A\sqrt{bx^2+cx^4}}{5bx^6}-\frac{(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^2x^4}+\frac{2c(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^3x^2}$$

```
[Out] -1/5*A*(c*x^4+b*x^2)^(1/2)/b/x^6-1/15*(-4*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/b^
2/x^4+2/15*c*(-4*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^2
```

Rubi [A]
time = 0.14, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2059, 806, 672, 664}

$$\frac{2c\sqrt{bx^2+cx^4}(5bB-4Ac)}{15b^3x^2}-\frac{\sqrt{bx^2+cx^4}(5bB-4Ac)}{15b^2x^4}-\frac{A\sqrt{bx^2+cx^4}}{5bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/5*(A*Sqrt[b*x^2 + c*x^4])/(b*x^6) - ((5*b*B - 4*A*c)*Sqrt[b*x^2 + c*x^4]
)/(15*b^2*x^4) + (2*c*(5*b*B - 4*A*c)*Sqrt[b*x^2 + c*x^4])/(15*b^3*x^2)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
```

```
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\ &= -\frac{A\sqrt{bx^2+cx^4}}{5bx^6}+\frac{\left(-3(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+cx^2}}\,dx,\right.}{5b}\\ &= -\frac{A\sqrt{bx^2+cx^4}}{5bx^6}-\frac{(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^2x^4}-\frac{(c(5bB-4Ac))\text{Subst}\left(\int \frac{1}{x\sqrt{bx+c}}\right.}{15b^2}\\ &= -\frac{A\sqrt{bx^2+cx^4}}{5bx^6}-\frac{(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^2x^4}+\frac{2c(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^3x^2}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 64, normalized size = 0.67

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(-5bBx^2(b-2cx^2)+A(-3b^2+4bcx^2-8c^2x^4)\right)}{15b^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*Sqrt[b*x^2 + c*x^4]),x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(-5*b*B*x^2*(b - 2*c*x^2) + A*(-3*b^2 + 4*b*c*x^2 -
8*c^2*x^4)))/(15*b^3*x^6)
```

Maple [A]
time = 0.38, size = 70, normalized size = 0.73

method	result	size
trager	$-\frac{\left(8A\,c^{2}x^{4}-10x^{4}bBc-4Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A\right)\sqrt{x^{4}c+b\,x^{2}}}{15b^{3}x^{6}}$	63
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(8A\,c^{2}x^{4}-10x^{4}bBc-4Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A\right)}{15x^{4}b^{3}\sqrt{x^{4}c+b\,x^{2}}}$	70
default	$-\frac{\left(c\,x^{2}+b\right)\left(8A\,c^{2}x^{4}-10x^{4}bBc-4Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A\right)}{15x^{4}b^{3}\sqrt{x^{4}c+b\,x^{2}}}$	70
risch	$-\frac{\left(c\,x^{2}+b\right)\left(8A\,c^{2}x^{4}-10x^{4}bBc-4Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A\right)}{15x^{4}\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,b^{3}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(c*x^2+b)*(8*A*c^2*x^4-10*B*b*c*x^4-4*A*b*c*x^2+5*B*b^2*x^2+3*A*b^2)/
x^4/b^3/(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.34, size = 119, normalized size = 1.24

$$\frac{1}{3}\,B\left(\frac{2\,\sqrt{cx^4+bx^2}\,c}{b^2x^2}-\frac{\sqrt{cx^4+bx^2}}{bx^4}\right)-\frac{1}{15}\,A\left(\frac{8\,\sqrt{cx^4+bx^2}\,c^2}{b^3x^2}-\frac{4\,\sqrt{cx^4+bx^2}\,c}{b^2x^4}+\frac{3\,\sqrt{cx^4+bx^2}}{bx^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*B*(2*sqrt(c*x^4 + b*x^2)*c/(b^2*x^2) - sqrt(c*x^4 + b*x^2)/(b*x^4)) - 1
/15*A*(8*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^2) - 4*sqrt(c*x^4 + b*x^2)*c/(b^2*x
^4) + 3*sqrt(c*x^4 + b*x^2)/(b*x^6))
```

Fricas [A]
time = 1.67, size = 62, normalized size = 0.65

$$\frac{\left(2\,(5\,Bbc-4\,Ac^2)x^4-3\,Ab^2-(5\,Bb^2-4\,Abc)x^2\right)\sqrt{cx^4+bx^2}}{15\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*B*b*c - 4*A*c^2)*x^4 - 3*A*b^2 - (5*B*b^2 - 4*A*b*c)*x^2)*sqrt(c
*x^4 + b*x^2)/(b^3*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+Bx^2}{x^5\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**5*sqrt(x**2*(b + c*x**2))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(84) = 168.
time = 0.76, size = 180, normalized size = 1.88

$$\frac{4\left(15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6Bc^{\frac{3}{2}}-35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Bbc^{\frac{3}{2}}+40\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Ac^{\frac{5}{2}}+25\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2Bb^2c^{\frac{3}{2}}-20\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2Abc^{\frac{5}{2}}-5Bb^3c^{\frac{3}{2}}+4Ab^2c^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^5\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*c^(3/2) - 35*(sqrt(c)*x - sqrt(c
*x^2 + b))^4*B*b*c^(3/2) + 40*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*c^(5/2) + 2
5*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^2*c^(3/2) - 20*(sqrt(c)*x - sqrt(c*x^
2 + b))^2*A*b*c^(5/2) - 5*B*b^3*c^(3/2) + 4*A*b^2*c^(5/2))/(((sqrt(c)*x - s
qrt(c*x^2 + b))^2 - b)^5*sgn(x))
```

Mupad [B]
time = 0.25, size = 62, normalized size = 0.65

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(5\,B\,b^2\,x^2+3\,A\,b^2-10\,B\,b\,c\,x^4-4\,A\,b\,c\,x^2+8\,A\,c^2\,x^4\right)}{15\,b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(3*A*b^2 + 5*B*b^2*x^2 + 8*A*c^2*x^4 - 4*A*b*c*x^2
- 10*B*b*c*x^4))/(15*b^3*x^6)
```

3.137 $\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=133

$$-\frac{A\sqrt{bx^2+cx^4}}{7bx^8}-\frac{(7bB-6Ac)\sqrt{bx^2+cx^4}}{35b^2x^6}+\frac{4c(7bB-6Ac)\sqrt{bx^2+cx^4}}{105b^3x^4}-\frac{8c^2(7bB-6Ac)\sqrt{bx^2+cx^4}}{105b^4x^2}$$

```
[Out] -1/7*A*(c*x^4+b*x^2)^(1/2)/b/x^8-1/35*(-6*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b^
2/x^6+4/105*c*(-6*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^4-8/105*c^2*(-6*A*c+
7*B*b)*(c*x^4+b*x^2)^(1/2)/b^4/x^2
```

Rubi [A]
time = 0.17, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2059, 806, 672, 664}

$$-\frac{8c^2\sqrt{bx^2+cx^4}\,(7bB-6Ac)}{105b^4x^2}+\frac{4c\sqrt{bx^2+cx^4}\,(7bB-6Ac)}{105b^3x^4}-\frac{\sqrt{bx^2+cx^4}\,(7bB-6Ac)}{35b^2x^6}-\frac{A\sqrt{bx^2+cx^4}}{7bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/7*(A*Sqrt[b*x^2 + c*x^4])/(b*x^8) - ((7*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4]
)/(35*b^2*x^6) + (4*c*(7*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/(105*b^3*x^4) -
(8*c^2*(7*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/(105*b^4*x^2)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
```

```
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4\sqrt{bx+cx^2}}\,dx,x,x^2\right)$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} + \frac{\left(-4(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+cx^2}}\,dx,\right.}{7b}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(7bB-6Ac)\sqrt{bx^2+cx^4}}{35b^2x^6} - \frac{(2c(7bB-6Ac))\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+}}\right.}{35b^2}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(7bB-6Ac)\sqrt{bx^2+cx^4}}{35b^2x^6} + \frac{4c(7bB-6Ac)\sqrt{bx^2+cx^4}}{105b^3x^4} + \frac{(4c^2}{}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(7bB-6Ac)\sqrt{bx^2+cx^4}}{35b^2x^6} + \frac{4c(7bB-6Ac)\sqrt{bx^2+cx^4}}{105b^3x^4} - \frac{8c^2(}{}$$

Mathematica [A]
time = 0.12, size = 89, normalized size = 0.67

$$-\frac{\sqrt{x^2(b+cx^2)}\,(7bBx^2(3b^2-4bcx^2+8c^2x^4)+3A(5b^3-6b^2cx^2+8bc^2x^4-16c^3x^6))}{105b^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/105*(Sqrt[x^2*(b + c*x^2)]*(7*b*B*x^2*(3*b^2 - 4*b*c*x^2 + 8*c^2*x^4) +
3*A*(5*b^3 - 6*b^2*c*x^2 + 8*b*c^2*x^4 - 16*c^3*x^6)))/(b^4*x^8)
```

Maple [A]
time = 0.39, size = 94, normalized size = 0.71

method	result	size
trager	$-\frac{\left(-48A\,c^{3}x^{6}+56x^{6}Bb\,c^{2}+24Ab\,c^{2}x^{4}-28x^{4}B\,b^{2}c-18A\,b^{2}c\,x^{2}+21x^{2}B\,b^{3}+15A\,b^{3}\right)\sqrt{x^{4}c+b\,x^{2}}}{105b^{4}x^{8}}$	87
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-48A\,c^{3}x^{6}+56x^{6}Bb\,c^{2}+24Ab\,c^{2}x^{4}-28x^{4}B\,b^{2}c-18A\,b^{2}c\,x^{2}+21x^{2}B\,b^{3}+15A\,b^{3}\right)}{105x^{6}b^{4}\sqrt{x^{4}c+b\,x^{2}}}$	94
default	$-\frac{\left(c\,x^{2}+b\right)\left(-48A\,c^{3}x^{6}+56x^{6}Bb\,c^{2}+24Ab\,c^{2}x^{4}-28x^{4}B\,b^{2}c-18A\,b^{2}c\,x^{2}+21x^{2}B\,b^{3}+15A\,b^{3}\right)}{105x^{6}b^{4}\sqrt{x^{4}c+b\,x^{2}}}$	94
risch	$-\frac{\left(c\,x^{2}+b\right)\left(-48A\,c^{3}x^{6}+56x^{6}Bb\,c^{2}+24Ab\,c^{2}x^{4}-28x^{4}B\,b^{2}c-18A\,b^{2}c\,x^{2}+21x^{2}B\,b^{3}+15A\,b^{3}\right)}{105x^{6}\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,b^{4}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(c*x^2+b)*(-48*A*c^3*x^6+56*B*b*c^2*x^6+24*A*b*c^2*x^4-28*B*b^2*c*x^
4-18*A*b^2*c*x^2+21*B*b^3*x^2+15*A*b^3)/x^6/b^4/(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.41, size = 167, normalized size = 1.26

$$-\frac{1}{15}\,B\left(\frac{8\,\sqrt{cx^4+bx^2}\,c^2}{b^3x^2}-\frac{4\,\sqrt{cx^4+bx^2}\,c}{b^2x^4}+\frac{3\,\sqrt{cx^4+bx^2}}{bx^6}\right)+\frac{1}{35}\,A\left(\frac{16\,\sqrt{cx^4+bx^2}\,c^3}{b^4x^2}-\frac{8\,\sqrt{cx^4+bx^2}\,c^2}{b^3x^4}+\frac{6\,\sqrt{cx^4+bx^2}\,c}{b^2x^6}-\frac{5\,\sqrt{cx^4+bx^2}}{bx^8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*B*(8*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^2) - 4*sqrt(c*x^4 + b*x^2)*c/(b^2
*x^4) + 3*sqrt(c*x^4 + b*x^2)/(b*x^6)) + 1/35*A*(16*sqrt(c*x^4 + b*x^2)*c^3
/(b^4*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^4) + 6*sqrt(c*x^4 + b*x^2)*c/
(b^2*x^6) - 5*sqrt(c*x^4 + b*x^2)/(b*x^8))
```

Fricas [A]
time = 1.28, size = 86, normalized size = 0.65

$$-\frac{\left(8\left(7\,Bbc^2-6\,Ac^3\right)x^6-4\left(7\,Bb^2c-6\,Abc^2\right)x^4+15\,Ab^3+3\left(7\,Bb^3-6\,Ab^2c\right)x^2\right)\sqrt{cx^4+bx^2}}{105\,b^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/105*(8*(7*B*b*c^2 - 6*A*c^3)*x^6 - 4*(7*B*b^2*c - 6*A*b*c^2)*x^4 + 15*A*
b^3 + 3*(7*B*b^3 - 6*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^4*x^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^7 \sqrt{x^2 (b + cx^2)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**7*sqrt(x**2*(b + c*x**2))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(117) = 234.
time = 1.08, size = 236, normalized size = 1.77

$$\frac{16\left(70\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^8 Bc^{\frac{5}{2}}-175\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^6 Bbc^{\frac{5}{2}}+210\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^6 Ac^{\frac{7}{2}}+147\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4 Bb^2c^{\frac{5}{2}}-126\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4 Abc^{\frac{7}{2}}-49\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2 Bb^3c^{\frac{5}{2}}+42\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2 Ab^2c^{\frac{7}{2}}+7\,Bb^4c^{\frac{5}{2}}-6\,Ab^3c^{\frac{7}{2}}\right)}{105\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^7 \operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*c^(5/2) - 175*(sqrt(c)*x - sqr
t(c*x^2 + b))^6*B*b*c^(5/2) + 210*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*c^(7/2)
 + 147*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^2*c^(5/2) - 126*(sqrt(c)*x - sqr
t(c*x^2 + b))^4*A*b*c^(7/2) - 49*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^3*c^(5
/2) + 42*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^2*c^(7/2) + 7*B*b^4*c^(5/2) -
6*A*b^3*c^(7/2))/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7*sgn(x))
```

Mupad [B]
time = 0.30, size = 121, normalized size = 0.91

$$\frac{(6\,A\,c-7\,B\,b)\,\sqrt{c\,x^4+b\,x^2}}{35\,b^2\,x^6}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{7\,b\,x^8}-\frac{(24\,A\,c^2-28\,B\,b\,c)\,\sqrt{c\,x^4+b\,x^2}}{105\,b^3\,x^4}+\frac{(48\,A\,c^3-56\,B\,b\,c^2)\,\sqrt{c\,x^4+b\,x^2}}{105\,b^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^7*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] ((6*A*c - 7*B*b)*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^6) - (A*(b*x^2 + c*x^4)^(
1/2))/(7*b*x^8) - ((24*A*c^2 - 28*B*b*c)*(b*x^2 + c*x^4)^(1/2))/(105*b^3*x^
4) + ((48*A*c^3 - 56*B*b*c^2)*(b*x^2 + c*x^4)^(1/2))/(105*b^4*x^2)
```

3.138 $\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=170

$$-\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}}-\frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8}+\frac{2c(9bB-8Ac)\sqrt{bx^2+cx^4}}{105b^3x^6}-\frac{8c^2(9bB-8Ac)\sqrt{bx^2+cx^4}}{315b^4x^4}+\frac{16}{}$$

```
[Out] -1/9*A*(c*x^4+b*x^2)^(1/2)/b/x^10-1/63*(-8*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b
^2/x^8+2/105*c*(-8*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^6-8/315*c^2*(-8*A*c
+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^4/x^4+16/315*c^3*(-8*A*c+9*B*b)*(c*x^4+b*x^2)
^(1/2)/b^5/x^2
```

Rubi [A]

time = 0.19, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2059, 806, 672, 664}

$$\frac{16c^3\sqrt{bx^2+cx^4}(9bB-8Ac)}{315b^5x^2}-\frac{8c^2\sqrt{bx^2+cx^4}(9bB-8Ac)}{315b^4x^4}+\frac{2c\sqrt{bx^2+cx^4}(9bB-8Ac)}{105b^3x^6}-\frac{\sqrt{bx^2+cx^4}(9bB-8Ac)}{63b^2x^8}-\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^9*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/9*(A*Sqrt[b*x^2 + c*x^4])/(b*x^10) - ((9*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4
])/(63*b^2*x^8) + (2*c*(9*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(105*b^3*x^6) -
 (8*c^2*(9*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(315*b^4*x^4) + (16*c^3*(9*b*B
 - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(315*b^5*x^2)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^5\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} + \frac{\left(-5(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+cx^2}}\,dx, \right.}{9b}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} - \frac{(c(9bB-8Ac))\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+}}\right.}{21b^2}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} + \frac{2c(9bB-8Ac)\sqrt{bx^2+cx^4}}{105b^3x^6} + \frac{(4c}{}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} + \frac{2c(9bB-8Ac)\sqrt{bx^2+cx^4}}{105b^3x^6} - \frac{8c^2(}{}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} + \frac{2c(9bB-8Ac)\sqrt{bx^2+cx^4}}{105b^3x^6} - \frac{8c^2(}{}$$

Mathematica [A]
time = 0.14, size = 110, normalized size = 0.65

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(9bBx^2(-5b^3+6b^2cx^2-8bc^2x^4+16c^3x^6)+A(-35b^4+40b^3cx^2-48b^2c^2x^4+64bc^3x^6-128c^4x^8)\right)}{315b^5x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^9*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(9*b*B*x^2*(-5*b^3 + 6*b^2*c*x^2 - 8*b*c^2*x^4 + 16*
c^3*x^6) + A*(-35*b^4 + 40*b^3*c*x^2 - 48*b^2*c^2*x^4 + 64*b*c^3*x^6 - 128*
c^4*x^8)))/(315*b^5*x^10)
```

Maple [A]
time = 0.39, size = 118, normalized size = 0.69

method	result	s
trager	$-\frac{\left(128 A \,c^{4} x^{8}-144 B b \,c^{3} x^{8}-64 A b \,c^{3} x^{6}+72 B \,b^{2} c^{2} x^{6}+48 A \,b^{2} c^{2} x^{4}-54 B \,b^{3} c \,x^{4}-40 A \,b^{3} c \,x^{2}+45 B \,b^{4} x^{2}+35 A \,b^{4}\right) \sqrt{x^{4} c +b \,x^{2}}}{315 b^{5} x^{10}}$	1
gosper	$-\frac{\left(c \,x^{2}+b \right) \left(128 A \,c^{4} x^{8}-144 B b \,c^{3} x^{8}-64 A b \,c^{3} x^{6}+72 B \,b^{2} c^{2} x^{6}+48 A \,b^{2} c^{2} x^{4}-54 B \,b^{3} c \,x^{4}-40 A \,b^{3} c \,x^{2}+45 B \,b^{4} x^{2}+35 A \,b^{4}\right)}{315 x^{8} b^{5} \sqrt{x^{4} c +b \,x^{2}}}$	1
default	$-\frac{\left(c \,x^{2}+b \right) \left(128 A \,c^{4} x^{8}-144 B b \,c^{3} x^{8}-64 A b \,c^{3} x^{6}+72 B \,b^{2} c^{2} x^{6}+48 A \,b^{2} c^{2} x^{4}-54 B \,b^{3} c \,x^{4}-40 A \,b^{3} c \,x^{2}+45 B \,b^{4} x^{2}+35 A \,b^{4}\right)}{315 x^{8} b^{5} \sqrt{x^{4} c +b \,x^{2}}}$	1
risch	$-\frac{\left(c \,x^{2}+b \right) \left(128 A \,c^{4} x^{8}-144 B b \,c^{3} x^{8}-64 A b \,c^{3} x^{6}+72 B \,b^{2} c^{2} x^{6}+48 A \,b^{2} c^{2} x^{4}-54 B \,b^{3} c \,x^{4}-40 A \,b^{3} c \,x^{2}+45 B \,b^{4} x^{2}+35 A \,b^{4}\right)}{315 x^{8} \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, b^{5}}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(c*x^2+b)*(128*A*c^4*x^8-144*B*b*c^3*x^8-64*A*b*c^3*x^6+72*B*b^2*c^2
*x^6+48*A*b^2*c^2*x^4-54*B*b^3*c*x^4-40*A*b^3*c*x^2+45*B*b^4*x^2+35*A*b^4)/
x^8/b^5/(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 215, normalized size = 1.26

$$\frac{1}{35}\,B\left(\frac{16\,\sqrt{cx^4+bx^2}\,c^3}{b^4x^2}-\frac{8\,\sqrt{cx^4+bx^2}\,c^2}{b^3x^4}+\frac{6\,\sqrt{cx^4+bx^2}\,c}{b^2x^6}-\frac{5\,\sqrt{cx^4+bx^2}}{bx^8}\right)-\frac{1}{315}\,A\left(\frac{128\,\sqrt{cx^4+bx^2}\,c^4}{b^5x^2}-\frac{64\,\sqrt{cx^4+bx^2}\,c^3}{b^4x^4}+\frac{48\,\sqrt{cx^4+bx^2}\,c^2}{b^3x^6}-\frac{40\,\sqrt{cx^4+bx^2}\,c}{b^2x^8}+\frac{35\,\sqrt{cx^4+bx^2}}{bx^{10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/35*B*(16*sqrt(c*x^4 + b*x^2)*c^3/(b^4*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^2/(b
^3*x^4) + 6*sqrt(c*x^4 + b*x^2)*c/(b^2*x^6) - 5*sqrt(c*x^4 + b*x^2)/(b*x^8)
) - 1/315*A*(128*sqrt(c*x^4 + b*x^2)*c^4/(b^5*x^2) - 64*sqrt(c*x^4 + b*x^2)
*c^3/(b^4*x^4) + 48*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^6) - 40*sqrt(c*x^4 + b*x
^2)*c/(b^2*x^8) + 35*sqrt(c*x^4 + b*x^2)/(b*x^10))
```

Fricas [A]
time = 1.74, size = 110, normalized size = 0.65

$$\frac{\left(16\,(9\,Bbc^3-8\,Ac^4)x^8-8\,(9\,Bb^2c^2-8\,Abc^3)x^6-35\,Ab^4+6\,(9\,Bb^3c-8\,Ab^2c^2)x^4-5\,(9\,Bb^4-8\,Ab^3c)x^2\right)\sqrt{cx^4+bx^2}}{315\,b^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(16*(9*B*b*c^3 - 8*A*c^4)*x^8 - 8*(9*B*b^2*c^2 - 8*A*b*c^3)*x^6 - 35*
A*b^4 + 6*(9*B*b^3*c - 8*A*b^2*c^2)*x^4 - 5*(9*B*b^4 - 8*A*b^3*c)*x^2)*sqrt
(c*x^4 + b*x^2)/(b^5*x^10)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^9\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**9/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**9*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A]
time = 1.44, size = 292, normalized size = 1.72

$$\frac{32\left(315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bc^{\frac{7}{2}}-819\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Bbc^{\frac{7}{2}}+1008\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Ac^{\frac{9}{2}}+756\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}Bb^2c^{\frac{7}{2}}-672\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}Abc^{\frac{9}{2}}-324\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}Bb^3c^{\frac{7}{2}}+288\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{4}Ab^2c^{\frac{9}{2}}+81\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}Bb^4c^{\frac{7}{2}}-72\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{2}Ab^3c^{\frac{9}{2}}-9Bb^5c^{\frac{7}{2}}+8Ab^4c^{\frac{9}{2}}\right)}{315\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^9\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 32/315*(315*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*c^(7/2) - 819*(sqrt(c)*x - s
qrt(c*x^2 + b))^8*B*b*c^(7/2) + 1008*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*c^(9
/2) + 756*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^2*c^(7/2) - 672*(sqrt(c)*x -
sqrt(c*x^2 + b))^6*A*b*c^(9/2) - 324*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^3*
c^(7/2) + 288*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^2*c^(9/2) + 81*(sqrt(c)*x
 - sqrt(c*x^2 + b))^2*B*b^4*c^(7/2) - 72*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*
b^3*c^(9/2) - 9*B*b^5*c^(7/2) + 8*A*b^4*c^(9/2))/(((sqrt(c)*x - sqrt(c*x^2
+ b))^2 - b)^9*sgn(x))
```

Mupad [B]
time = 0.32, size = 156, normalized size = 0.92

$$\frac{(8\,A\,c-9\,B\,b)\,\sqrt{c\,x^4+b\,x^2}}{63\,b^2\,x^8}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{9\,b\,x^{10}}-\frac{(16\,A\,c^2-18\,B\,b\,c)\,\sqrt{c\,x^4+b\,x^2}}{105\,b^3\,x^6}+\frac{(64\,A\,c^3-72\,B\,b\,c^2)\,\sqrt{c\,x^4+b\,x^2}}{315\,b^4\,x^4}-\frac{(128\,A\,c^4-144\,B\,b\,c^3)\,\sqrt{c\,x^4+b\,x^2}}{315\,b^5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^9*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] ((8*A*c - 9*B*b)*(b*x^2 + c*x^4)^(1/2))/(63*b^2*x^8) - (A*(b*x^2 + c*x^4)^(
1/2))/(9*b*x^10) - ((16*A*c^2 - 18*B*b*c)*(b*x^2 + c*x^4)^(1/2))/(105*b^3*x
^6) + ((64*A*c^3 - 72*B*b*c^2)*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^4) - ((128
*A*c^4 - 144*B*b*c^3)*(b*x^2 + c*x^4)^(1/2))/(315*b^5*x^2)
```

3.139 $\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=131

$$-\frac{8b^2(6bB-7Ac)\sqrt{bx^2+cx^4}}{105c^4x}+\frac{4b(6bB-7Ac)x\sqrt{bx^2+cx^4}}{105c^3}-\frac{(6bB-7Ac)x^3\sqrt{bx^2+cx^4}}{35c^2}+\frac{Bx^5\sqrt{bx^2+cx^4}}{7c}$$

```
[Out] -8/105*b^2*(-7*A*c+6*B*b)*(c*x^4+b*x^2)^(1/2)/c^4/x+4/105*b*(-7*A*c+6*B*b)*
x*(c*x^4+b*x^2)^(1/2)/c^3-1/35*(-7*A*c+6*B*b)*x^3*(c*x^4+b*x^2)^(1/2)/c^2+1
/7*B*x^5*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A]
time = 0.15, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2064, 2041, 1602}

$$-\frac{8b^2\sqrt{bx^2+cx^4}\,(6bB-7Ac)}{105c^4x}+\frac{4bx\sqrt{bx^2+cx^4}\,(6bB-7Ac)}{105c^3}-\frac{x^3\sqrt{bx^2+cx^4}\,(6bB-7Ac)}{35c^2}+\frac{Bx^5\sqrt{bx^2+cx^4}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-8*b^2*(6*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(105*c^4*x) + (4*b*(6*b*B - 7*
A*c)*x*Sqrt[b*x^2 + c*x^4])/(105*c^3) - ((6*b*B - 7*A*c)*x^3*Sqrt[b*x^2 + c
*x^4])/(35*c^2) + (B*x^5*Sqrt[b*x^2 + c*x^4])/(7*c)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
```

```
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx &= \frac{Bx^5\sqrt{bx^2+cx^4}}{7c}-\frac{(6bB-7Ac)\int \frac{x^6}{\sqrt{bx^2+cx^4}}\,dx}{7c}\\ &= -\frac{(6bB-7Ac)x^3\sqrt{bx^2+cx^4}}{35c^2}+\frac{Bx^5\sqrt{bx^2+cx^4}}{7c}+\frac{(4b(6bB-7Ac))\int \frac{x^4}{\sqrt{bx^2+cx^4}}}{35c^2}\\ &= \frac{4b(6bB-7Ac)x\sqrt{bx^2+cx^4}}{105c^3}-\frac{(6bB-7Ac)x^3\sqrt{bx^2+cx^4}}{35c^2}+\frac{Bx^5\sqrt{bx^2+cx^4}}{7c}-\\ &= -\frac{8b^2(6bB-7Ac)\sqrt{bx^2+cx^4}}{105c^4x}+\frac{4b(6bB-7Ac)x\sqrt{bx^2+cx^4}}{105c^3}-\frac{(6bB-7Ac)x^3\sqrt{b}}{35c^2}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 85, normalized size = 0.65

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(-48b^3B+8b^2c(7A+3Bx^2)+3c^3x^4(7A+5Bx^2)-2bc^2x^2(14A+9Bx^2)\right)}{105c^4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(-48*b^3*B + 8*b^2*c*(7*A + 3*B*x^2) + 3*c^3*x^4*(7*
A + 5*B*x^2) - 2*b*c^2*x^2*(14*A + 9*B*x^2)))/(105*c^4*x)
```

Maple [A]
time = 0.37, size = 89, normalized size = 0.68

method	result	size
trager	$\frac{(15Bc^3x^6+21Ac^3x^4-18Bbc^2x^4-28Abc^2x^2+24Bb^2cx^2+56Ab^2c-48Bb^3)\sqrt{x^4c+bx^2}}{105c^4x}$	84
gosper	$\frac{(cx^2+b)(15Bc^3x^6+21Ac^3x^4-18Bbc^2x^4-28Abc^2x^2+24Bb^2cx^2+56Ab^2c-48Bb^3)x}{105c^4\sqrt{x^4c+bx^2}}$	89
default	$\frac{(cx^2+b)(15Bc^3x^6+21Ac^3x^4-18Bbc^2x^4-28Abc^2x^2+24Bb^2cx^2+56Ab^2c-48Bb^3)x}{105c^4\sqrt{x^4c+bx^2}}$	89

risch	$\frac{x(c\,x^2+b)\left(15B\,c^3x^6+21A\,c^3x^4-18Bb\,c^2x^4-28Ab\,c^2x^2+24B\,b^2c\,x^2+56A\,b^2c-48B\,b^3\right)}{105\sqrt{x^2\left(c\,x^2+b\right)}\;c^4}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(c*x^2+b)*(15*B*c^3*x^6+21*A*c^3*x^4-18*B*b*c^2*x^4-28*A*b*c^2*x^2+24
*B*b^2*c*x^2+56*A*b^2*c-48*B*b^3)*x/c^4/(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 106, normalized size = 0.81

$$\frac{(3\,c^3x^6-bc^2x^4+4\,b^2cx^2+8\,b^3)A}{15\,\sqrt{cx^2+b}\,c^3}+\frac{(5\,c^4x^8-bc^3x^6+2\,b^2c^2x^4-8\,b^3cx^2-16\,b^4)B}{35\,\sqrt{cx^2+b}\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^3*x^6 - b*c^2*x^4 + 4*b^2*c*x^2 + 8*b^3)*A/(sqrt(c*x^2 + b)*c^3)
+ 1/35*(5*c^4*x^8 - b*c^3*x^6 + 2*b^2*c^2*x^4 - 8*b^3*c*x^2 - 16*b^4)*B/(sq
rt(c*x^2 + b)*c^4)
```

Fricas [A]
time = 1.02, size = 83, normalized size = 0.63

$$\frac{(15\,Bc^3x^6-3\,(6\,Bbc^2-7\,Ac^3)x^4-48\,Bb^3+56\,Ab^2c+4\,(6\,Bb^2c-7\,Abc^2)x^2)\sqrt{cx^4+bx^2}}{105\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*c^3*x^6 - 3*(6*B*b*c^2 - 7*A*c^3)*x^4 - 48*B*b^3 + 56*A*b^2*c +
 4*(6*B*b^2*c - 7*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^6(A+Bx^2)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**6*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 0.47, size = 130, normalized size = 0.99

$$\frac{8\left(6\,Bb^{\frac{7}{2}}-7\,Ab^{\frac{5}{2}}c\right)\mathrm{sgn}(x)}{105\,c^4}-\frac{\left(Bb^3-Ab^2c\right)\sqrt{cx^2+b}}{c^4\mathrm{sgn}\left(x\right)}+\frac{15\left(cx^2+b\right)^{\frac{7}{2}}B-63\left(cx^2+b\right)^{\frac{5}{2}}Bb+105\left(cx^2+b\right)^{\frac{3}{2}}Bb^2+21\left(cx^2+b\right)^{\frac{5}{2}}Ac-70\left(cx^2+b\right)^{\frac{3}{2}}Abc}{105\,c^4\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 8/105*(6*B*b^(7/2) - 7*A*b^(5/2)*c)*sgn(x)/c^4 - (B*b^3 - A*b^2*c)*sqrt(c*x
^2 + b)/(c^4*sgn(x)) + 1/105*(15*(c*x^2 + b)^(7/2)*B - 63*(c*x^2 + b)^(5/2)
*B*b + 105*(c*x^2 + b)^(3/2)*B*b^2 + 21*(c*x^2 + b)^(5/2)*A*c - 70*(c*x^2 +
 b)^(3/2)*A*b*c)/(c^4*sgn(x))
```

Mupad [B]
time = 0.26, size = 87, normalized size = 0.66

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{48\,B\,b^3-56\,A\,b^2\,c}{105\,c^4}-\frac{B\,x^6}{7\,c}-\frac{x^4\,\left(21\,A\,c^3-18\,B\,b\,c^2\right)}{105\,c^4}+\frac{4\,b\,x^2\,\left(7\,A\,c-6\,B\,b\right)}{105\,c^3}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*((48*B*b^3 - 56*A*b^2*c)/(105*c^4) - (B*x^6)/(7*c)
- (x^4*(21*A*c^3 - 18*B*b*c^2))/(105*c^4) + (4*b*x^2*(7*A*c - 6*B*b))/(105*
c^3)))/x
```

3.140 $\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$

Optimal. Leaf size=94

$$\frac{2b(4bB-5Ac)\sqrt{bx^2+cx^4}}{15c^3x}-\frac{(4bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^2}+\frac{Bx^3\sqrt{bx^2+cx^4}}{5c}$$

```
[Out] 2/15*b*(-5*A*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x-1/15*(-5*A*c+4*B*b)*x*(c*x^
4+b*x^2)^(1/2)/c^2+1/5*B*x^3*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A]
time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2064, 2041, 1602}

$$\frac{2b\sqrt{bx^2+cx^4}(4bB-5Ac)}{15c^3x}-\frac{x\sqrt{bx^2+cx^4}(4bB-5Ac)}{15c^2}+\frac{Bx^3\sqrt{bx^2+cx^4}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*b*(4*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(15*c^3*x) - ((4*b*B - 5*A*c)*x*S
qrt[b*x^2 + c*x^4])/(15*c^2) + (B*x^3*Sqrt[b*x^2 + c*x^4])/(5*c)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
```

```
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{Bx^3\sqrt{bx^2+cx^4}}{5c} - \frac{(4bB-5Ac)\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx}{5c}$$
$$= -\frac{(4bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{bx^2+cx^4}}{5c} + \frac{(2b(4bB-5Ac))\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{15c^2}$$
$$= \frac{2b(4bB-5Ac)\sqrt{bx^2+cx^4}}{15c^3x} - \frac{(4bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{bx^2+cx^4}}{5c}$$

Mathematica [A]
time = 0.04, size = 63, normalized size = 0.67

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\,(8b^2B-2bc(5A+2Bx^2)+c^2x^2(5A+3Bx^2))}{15c^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(8*b^2*B - 2*b*c*(5*A + 2*B*x^2) + c^2*x^2*(5*A + 3*
B*x^2)))/(15*c^3*x)
```

Maple [A]
time = 0.38, size = 65, normalized size = 0.69

method	result	size
trager	$-\frac{\left(-3B\,c^2x^4-5A\,c^2x^2+4bB\,x^2c+10Abc-8b^2B\right)\sqrt{x^4c+b\,x^2}}{15c^3x}$	60
gosper	$-\frac{\left(c\,x^2+b\right)\left(-3B\,c^2x^4-5A\,c^2x^2+4bB\,x^2c+10Abc-8b^2B\right)x}{15c^3\sqrt{x^4c+b\,x^2}}$	65
default	$-\frac{\left(c\,x^2+b\right)\left(-3B\,c^2x^4-5A\,c^2x^2+4bB\,x^2c+10Abc-8b^2B\right)x}{15c^3\sqrt{x^4c+b\,x^2}}$	65
risch	$-\frac{x\left(c\,x^2+b\right)\left(-3B\,c^2x^4-5A\,c^2x^2+4bB\,x^2c+10Abc-8b^2B\right)}{15\sqrt{x^2\left(c\,x^2+b\right)}\,c^3}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(c*x^2+b)*(-3*B*c^2*x^4-5*A*c^2*x^2+4*B*b*c*x^2+10*A*b*c-8*B*b^2)*x/c
^3/(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 83, normalized size = 0.88

$$\frac{(c^2x^4 - bcx^2 - 2\,b^2)A}{3\,\sqrt{cx^2 + b}\,c^2} + \frac{(3\,c^3x^6 - bc^2x^4 + 4\,b^2cx^2 + 8\,b^3)B}{15\,\sqrt{cx^2 + b}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)*A/(sqrt(c*x^2 + b)*c^2) + 1/15*(3*c^3*x^6 -
 b*c^2*x^4 + 4*b^2*c*x^2 + 8*b^3)*B/(sqrt(c*x^2 + b)*c^3)
```

Fricas [A]
time = 1.38, size = 59, normalized size = 0.63

$$\frac{(3\,Bc^2x^4 + 8\,Bb^2 - 10\,Abc - (4\,Bbc - 5\,Ac^2)x^2)\sqrt{cx^4 + bx^2}}{15\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*c^2*x^4 + 8*B*b^2 - 10*A*b*c - (4*B*b*c - 5*A*c^2)*x^2)*sqrt(c*x^
4 + b*x^2)/(c^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(A + Bx^2)}{\sqrt{x^2\,(b + cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 0.52, size = 98, normalized size = 1.04

$$-\frac{2\left(4\,Bb^{\frac{5}{2}} - 5\,Ab^{\frac{3}{2}}c\right)\mathrm{sgn}(x)}{15\,c^3} + \frac{(Bb^2 - Abc)\sqrt{cx^2 + b}}{c^3\mathrm{sgn}\,(x)} + \frac{3\,(cx^2 + b)^{\frac{5}{2}}B - 10\,(cx^2 + b)^{\frac{3}{2}}Bb + 5\,(cx^2 + b)^{\frac{3}{2}}Ac}{15\,c^3\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(4*B*b^(5/2) - 5*A*b^(3/2)*c)*sgn(x)/c^3 + (B*b^2 - A*b*c)*sqrt(c*x^2
 + b)/(c^3*sgn(x)) + 1/15*(3*(c*x^2 + b)^(5/2)*B - 10*(c*x^2 + b)^(3/2)*B*b
 + 5*(c*x^2 + b)^(3/2)*A*c)/(c^3*sgn(x))
```

Mupad [B]
time = 0.23, size = 64, normalized size = 0.68

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{8\,B\,b^2-10\,A\,b\,c}{15\,c^3}+\frac{x^2\,\left(5\,A\,c^2-4\,B\,b\,c\right)}{15\,c^3}+\frac{B\,x^4}{5\,c}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((8*B*b^2 - 10*A*b*c)/(15*c^3) + (x^2*(5*A*c^2 - 4*B
*b*c))/(15*c^3) + (B*x^4)/(5*c)))/x
```

3.141 $\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=59

$$-\frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{3c^2x}+\frac{Bx\sqrt{bx^2+cx^4}}{3c}$$

```
[Out] -1/3*(-3*A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x+1/3*B*x*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A]
time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2064, 1602}

$$\frac{Bx\sqrt{bx^2+cx^4}}{3c}-\frac{\sqrt{bx^2+cx^4}\,(2bB-3Ac)}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -1/3*((2*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(c^2*x) + (B*x*Sqrt[b*x^2 + c*x^
4])/(3*c)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{Bx\sqrt{bx^2+cx^4}}{3c} - \frac{(2bB-3Ac)\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{3c}$$
$$= -\frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{3c^2x} + \frac{Bx\sqrt{bx^2+cx^4}}{3c}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 0.68

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\,\left(-2bB+3Ac+Bcx^2\right)}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-2*b*B + 3*A*c + B*c*x^2))/(3*c^2*x)
```

Maple [A]
time = 0.37, size = 42, normalized size = 0.71

method	result	size
trager	$\frac{\left(Bc\,x^2+3Ac-2Bb\right)\sqrt{x^4c+b\,x^2}}{3c^2x}$	37
gosper	$\frac{\left(c\,x^2+b\right)\left(Bc\,x^2+3Ac-2Bb\right)x}{3c^2\sqrt{x^4c+b\,x^2}}$	42
default	$\frac{\left(c\,x^2+b\right)\left(Bc\,x^2+3Ac-2Bb\right)x}{3c^2\sqrt{x^4c+b\,x^2}}$	42
risch	$\frac{x\left(c\,x^2+b\right)\left(Bc\,x^2+3Ac-2Bb\right)}{3\sqrt{x^2\left(c\,x^2+b\right)}\,c^2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c*x^2+b)*(B*c*x^2+3*A*c-2*B*b)*x/c^2/(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.29, size = 50, normalized size = 0.85

$$\frac{\sqrt{cx^2+b}\,A}{c} + \frac{\left(c^2x^4-bcx^2-2\,b^2\right)B}{3\,\sqrt{cx^2+b}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2 + b)*A/c + 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)*B/(sqrt(c*x^2 + b)*c^
2)
```

Fricas [A]
time = 0.88, size = 36, normalized size = 0.61

$$\frac{\sqrt{cx^4+bx^2}\,(Bcx^2-2\,Bb+3\,Ac)}{3\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(B*c*x^2 - 2*B*b + 3*A*c)/(c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(A+Bx^2)}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 0.48, size = 67, normalized size = 1.14

$$\frac{\left(2\,Bb^{\frac{3}{2}}-3\,A\sqrt{b}\,c\right)\mathrm{sgn}(x)}{3\,c^2}+\frac{(cx^2+b)^{\frac{3}{2}}B}{3\,c^2\mathrm{sgn}\,(x)}-\frac{\sqrt{cx^2+b}\,(Bb-Ac)}{c^2\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*B*b^(3/2) - 3*A*sqrt(b)*c)*sgn(x)/c^2 + 1/3*(c*x^2 + b)^(3/2)*B/(c^2
*sgn(x)) - sqrt(c*x^2 + b)*(B*b - A*c)/(c^2*sgn(x))
```

Mupad [B]
time = 0.19, size = 41, normalized size = 0.69

$$\frac{\left(\frac{3\,A\,c-2\,B\,b}{3\,c^2}+\frac{B\,x^2}{3\,c}\right)\,\sqrt{c\,x^4+b\,x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (((3*A*c - 2*B*b)/(3*c^2) + (B*x^2)/(3*c))*(b*x^2 + c*x^4)^(1/2))/x
```

3.142 $\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=55

$$\frac{B\sqrt{bx^2+cx^4}}{cx}-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

```
[Out] -A*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(1/2)+B*(c*x^4+b*x^2)^(1/2)/c/x
```

Rubi [A]
time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1159, 2033, 212}

$$\frac{B\sqrt{bx^2+cx^4}}{cx}-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (B*Sqrt[b*x^2 + c*x^4])/(c*x) - (A*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
)/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[e*((b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 3)*x)), x] - Dist[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3)), Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
c*d*(4*p + 3), 0]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx &= \frac{B\sqrt{bx^2+cx^4}}{cx}+A\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\ &= \frac{B\sqrt{bx^2+cx^4}}{cx}-A\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)\\ &= \frac{B\sqrt{bx^2+cx^4}}{cx}-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 73, normalized size = 1.33

$$\frac{x\left(\sqrt{b}\,B(b+cx^2)-Ac\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{\sqrt{b}\,c\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[b]*B*(b + c*x^2) - A*c*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqr
t[b]]))/(Sqrt[b]*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.37, size = 72, normalized size = 1.31

method	result	size
default	$-\frac{x\sqrt{c\,x^2+b}\,\left(A\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c-B\sqrt{c\,x^2+b}\,\sqrt{b}\right)}{\sqrt{x^4c+b\,x^2}\,c\sqrt{b}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x*(c*x^2+b)^(1/2)*(A*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c-B*(c*x^2+b)^(1/
2)*b^(1/2))/(c*x^4+b*x^2)^(1/2)/c/b^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [A]
time = 1.29, size = 138, normalized size = 2.51

$$\left[\frac{A\sqrt{b}\,cx\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\,Bb}{2\,bcx}, \frac{A\sqrt{-b}\,cx\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\,Bb}{bcx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*c*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^
3) + 2*sqrt(c*x^4 + b*x^2)*B*b)/(b*c*x), (A*sqrt(-b)*c*x*arctan(sqrt(c*x^4
+ b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*B*b)/(b*c*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 0.48, size = 80, normalized size = 1.45

$$-\frac{\left(Ac\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+B\sqrt{-b}\,\sqrt{b}\right)\operatorname{sgn}(x)}{\sqrt{-b}\,c}+\frac{\frac{A\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\frac{\sqrt{cx^2+b}\,B}{c}}{\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(A*c*arctan(sqrt(b)/sqrt(-b)) + B*sqrt(-b)*sqrt(b))*sgn(x)/(sqrt(-b)*c) +
(A*arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) + sqrt(c*x^2 + b)*B/c)/sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{B\,x^2+A}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(b*x^2 + c*x^4)^(1/2),x)
[Out] int((A + B*x^2)/(b*x^2 + c*x^4)^(1/2), x)
```

3.143 $\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=68

$$-\frac{A\sqrt{bx^2+cx^4}}{2bx^3}-\frac{(2bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*(-A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-1/2*A*(c*x
^4+b*x^2)^(1/2)/b/x^3
```

Rubi [A]
time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2063, 2033, 212}

$$-\frac{(2bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{A\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*(A*Sqrt[b*x^2 + c*x^4])/(b*x^3) - ((2*b*B - A*c)*ArcTanh[(Sqrt[b]*x)/S
qrt[b*x^2 + c*x^4]])/(2*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
```

```
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx &= -\frac{A\sqrt{bx^2+cx^4}}{2bx^3}-\frac{(-2bB+Ac)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b}\\ &= -\frac{A\sqrt{bx^2+cx^4}}{2bx^3}+\frac{(-2bB+Ac)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b}\\ &= -\frac{A\sqrt{bx^2+cx^4}}{2bx^3}-\frac{(2bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 86, normalized size = 1.26

$$\frac{-A\sqrt{b}\,(b+cx^2)-(2bB-Ac)x^2\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{2b^{3/2}x\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-(A*Sqrt[b]*(b + c*x^2)) - (2*b*B - A*c)*x^2*Sqrt[b + c*x^2]*ArcTanh[Sqrt[
b + c*x^2]/Sqrt[b]])/(2*b^(3/2)*x*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.52, size = 105, normalized size = 1.54

method	result	size
default	$-\frac{\sqrt{c\,x^2+b}\,\left(2B\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)b^2x^2-A\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)bc\,x^2+A\sqrt{c\,x^2+b}\,b^{\frac{3}{2}}\right)}{2x\sqrt{x^4c+b\,x^2}\,b^{\frac{5}{2}}}$	105
risch	$-\frac{A(c\,x^2+b)}{2bx\sqrt{x^2\,(c\,x^2+b)}}+\frac{\left(\frac{\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)Ac}{2b^{\frac{3}{2}}}-\frac{B\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)}{\sqrt{b}}\right)x\sqrt{c\,x^2+b}}{\sqrt{x^2\,(c\,x^2+b)}}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x*(c*x^2+b)^(1/2)*(2*B*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b^2*x^2-A*l
n(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*c*x^2+A*(c*x^2+b)^(1/2)*b^(3/2))/(c*x^
4+b*x^2)^(1/2)/b^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^2), x)
```

Fricas [A]
time = 1.72, size = 152, normalized size = 2.24

$$\left[-\frac{(2\,Bb-Ac)\sqrt{b}\,x^3\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\,Ab}{4\,b^2x^3},\ \frac{(2\,Bb-Ac)\sqrt{-b}\,x^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)-\sqrt{cx^4+bx^2}\,Ab}{2\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*B*b - A*c)*sqrt(b)*x^3*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2
)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*A*b)/(b^2*x^3), 1/2*((2*B*b - A*c)*
sqrt(-b)*x^3*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^
4 + b*x^2)*A*b)/(b^2*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^2\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A]
time = 0.50, size = 66, normalized size = 0.97

$$\frac{\frac{(2\,Bbc-Ac^2)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}-\frac{\sqrt{cx^2+b}\,Ac}{bx^2}}{2\,c\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/2*((2*B*b*c - A*c^2)*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b) - sqrt
(c*x^2 + b)*A*c/(b*x^2))/(c*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{x^2\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(1/2)), x)
```

3.144 $\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=103

$$-\frac{A\sqrt{bx^2+cx^4}}{4bx^5}-\frac{(4bB-3Ac)\sqrt{bx^2+cx^4}}{8b^2x^3}+\frac{c(4bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}$$

```
[Out] 1/8*c*(-3*A*c+4*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2)-1/4*A*(
c*x^4+b*x^2)^(1/2)/b/x^5-1/8*(-3*A*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^3
```

Rubi [A]
time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2063, 2050, 2033, 212}

$$\frac{c(4bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}-\frac{\sqrt{bx^2+cx^4}\,(4bB-3Ac)}{8b^2x^3}-\frac{A\sqrt{bx^2+cx^4}}{4bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/4*(A*Sqrt[b*x^2 + c*x^4])/(b*x^5) - ((4*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4]
)/(8*b^2*x^3) + (c*(4*b*B - 3*A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
)/(8*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
```

```
 + j*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx = -\frac{A\sqrt{bx^2+cx^4}}{4bx^5} - \frac{(-4bB+3Ac)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{4bx^5} - \frac{(4bB-3Ac)\sqrt{bx^2+cx^4}}{8b^2x^3} - \frac{(c(4bB-3Ac))\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^2}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{4bx^5} - \frac{(4bB-3Ac)\sqrt{bx^2+cx^4}}{8b^2x^3} + \frac{(c(4bB-3Ac))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x\right.}{8b^2}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{4bx^5} - \frac{(4bB-3Ac)\sqrt{bx^2+cx^4}}{8b^2x^3} + \frac{c(4bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right.}{8b^{5/2}}$$

Mathematica [A]
time = 0.12, size = 104, normalized size = 1.01

$$\frac{-\sqrt{b}\,(b+cx^2)\,(2Ab+4bBx^2-3Acx^2)+c(4bB-3Ac)x^4\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{8b^{5/2}x^3\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-(Sqrt[b]*(b + c*x^2)*(2*A*b + 4*b*B*x^2 - 3*A*c*x^2)) + c*(4*b*B - 3*A*c)
*x^4*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(8*b^(5/2)*x^3*Sqrt[
x^2*(b + c*x^2)])
```

Maple [A]
time = 0.44, size = 146, normalized size = 1.42

method	result
risch	$-\frac{(c\,x^2+b)\left(-3Ac\,x^2+4bB\,x^2+2Ab\right)}{8b^2x^3\sqrt{x^2\left(c\,x^2+b\right)}}+\frac{\left(-\frac{3c^2\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)A}{8b^{\frac{5}{2}}}+\frac{c\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)B}{2b^{\frac{3}{2}}}\right)x\sqrt{c\,x^2+\cdots}}{\sqrt{x^2\left(c\,x^2+b\right)}}$
default	$-\frac{\sqrt{c\,x^2+b}\,\left(3A\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)b\,c^2x^4-4B\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)b^2cx^4-3A\sqrt{c\,x^2+b}\,b^{\frac{3}{2}}c\,x^2+\cdots\right.}{8x^3\sqrt{x^4c+b\,x^2}\,b^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/x^3*(c*x^2+b)^(1/2)*(3*A*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*c^2*x^4
-4*B*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b^2*c*x^4-3*A*(c*x^2+b)^(1/2)*b^(3
/2)*c*x^2+4*B*(c*x^2+b)^(1/2)*b^(5/2)*x^2+2*A*(c*x^2+b)^(1/2)*b^(5/2))/(c*x
^4+b*x^2)^(1/2)/b^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^4), x)
```

Fricas [A]
time = 1.04, size = 199, normalized size = 1.93

$$\left[-\frac{\left(4\,Bbc-3\,Ac^2\right)\sqrt{b}\,x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\,\left(2\,Ab^2+\left(4\,Bb^2-3\,Abc\right)x^2\right)}{16\,b^3x^5},\ -\frac{\left(4\,Bbc-3\,Ac^2\right)\sqrt{-b}\,x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\,\left(2\,Ab^2+\left(4\,Bb^2-3\,Abc\right)x^2\right)}{8\,b^3x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*B*b*c - 3*A*c^2)*sqrt(b)*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4
+ b*x^2)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^2 - 3*A*b*
c)*x^2))/(b^3*x^5), -1/8*((4*B*b*c - 3*A*c^2)*sqrt(-b)*x^5*arctan(sqrt(c*x^
4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^
2 - 3*A*b*c)*x^2))/(b^3*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^4 \sqrt{x^2 (b + cx^2)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A]
time = 0.47, size = 125, normalized size = 1.21

$$-\frac{\frac{(4\,Bbc^2-3\,Ac^3)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^2}+\frac{4\,(cx^2+b)^{\frac{3}{2}}\,Bbc^2-4\,\sqrt{cx^2+b}\,Bb^2c^2-3\,(cx^2+b)^{\frac{3}{2}}\,Ac^3+5\,\sqrt{cx^2+b}\,Abc^3}{b^2c^2x^4}}{8\,c\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*((4*B*b*c^2 - 3*A*c^3)*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2)
 + (4*(c*x^2 + b)^(3/2)*B*b*c^2 - 4*sqrt(c*x^2 + b)*B*b^2*c^2 - 3*(c*x^2 +
b)^(3/2)*A*c^3 + 5*sqrt(c*x^2 + b)*A*b*c^3)/(b^2*c^2*x^4))/(c*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2 + A}{x^4\,\sqrt{c\,x^4 + b\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^4*(b*x^2 + c*x^4)^(1/2)), x)
```

3.145 $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}}+\frac{5b(7bB-6Ac)\sqrt{bx^2+cx^4}}{16c^4}-\frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3}+\frac{(7bB-6Ac)x^4\sqrt{bx^2+cx^4}}{6bc^2}$$

```
[Out] -5/16*b^2*(-6*A*c+7*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(9/2)-(
-A*c+B*b)*x^8/b/c/(c*x^4+b*x^2)^(1/2)+5/16*b*(-6*A*c+7*B*b)*(c*x^4+b*x^2)^(
1/2)/c^4-5/24*(-6*A*c+7*B*b)*x^2*(c*x^4+b*x^2)^(1/2)/c^3+1/6*(-6*A*c+7*B*b)
*x^4*(c*x^4+b*x^2)^(1/2)/b/c^2
```

Rubi [A]
time = 0.22, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2059, 802, 684, 654, 634, 212}

$$-\frac{5b^2(7bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{9/2}}+\frac{5b\sqrt{bx^2+cx^4}\,(7bB-6Ac)}{16c^4}-\frac{5x^2\sqrt{bx^2+cx^4}\,(7bB-6Ac)}{24c^3}+\frac{x^4\sqrt{bx^2+cx^4}\,(7bB-6Ac)}{6bc^2}-\frac{x^8(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^8)/(b*c*Sqrt[b*x^2 + c*x^4])) + (5*b*(7*b*B - 6*A*c)*Sqrt[
b*x^2 + c*x^4])/(16*c^4) - (5*(7*b*B - 6*A*c)*x^2*Sqrt[b*x^2 + c*x^4])/(24*
c^3) + ((7*b*B - 6*A*c)*x^4*Sqrt[b*x^2 + c*x^4])/(6*b*c^2) - (5*b^2*(7*b*B
- 6*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 802

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Dist[e*((m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e))), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4(A+Bx)}{(bx+cx^2)^{3/2}}\,dx,x,x^2\right)$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{1}{2}\left(-\frac{6A}{b}+\frac{7B}{c}\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{(7bB-6Ac)x^4\sqrt{bx^2+cx^4}}{6bc^2} - \frac{(5(7bB-6Ac))\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+}}\right.}{12c^2}$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3} + \frac{(7bB-6Ac)x^4\sqrt{bx^2+cx^4}}{6bc^2} +$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{5b(7bB-6Ac)\sqrt{bx^2+cx^4}}{16c^4} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3} +$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{5b(7bB-6Ac)\sqrt{bx^2+cx^4}}{16c^4} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3} +$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{5b(7bB-6Ac)\sqrt{bx^2+cx^4}}{16c^4} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3} +$$

Mathematica [A]
time = 0.19, size = 139, normalized size = 0.76

$$\frac{x\left(\sqrt{c}\,x(105b^3B+4c^3x^4(3A+2Bx^2)-2bc^2x^2(15A+7Bx^2)+b^2(-90Ac+35Bcx^2))+15b^2(7bB-6Ac)\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{48c^{9/2}\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(105*b^3*B + 4*c^3*x^4*(3*A + 2*B*x^2) - 2*b*c^2*x^2*(15*A +
7*B*x^2) + b^2*(-90*A*c + 35*B*c*x^2)) + 15*b^2*(7*b*B - 6*A*c)*Sqrt[b + c*
x^2]*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(48*c^(9/2)*Sqrt[x^2*(b + c*x^2)
])
```

Maple [A]
time = 0.39, size = 166, normalized size = 0.90

method	result

default	$\frac{x^{3}(c\,x^{2}+b)\left(8B\,c^{\frac{9}{2}}x^{7}+12A\,c^{\frac{9}{2}}x^{5}-14B\,c^{\frac{7}{2}}b\,x^{5}-30A\,c^{\frac{7}{2}}b\,x^{3}+35B\,c^{\frac{5}{2}}b^{2}x^{3}-90A\,c^{\frac{5}{2}}b^{2}x+105B\,c^{\frac{3}{2}}b^{3}x+90A\sqrt{c\,x^{2}+b}\,\ln\left(\sqrt{c}\,x+\ldots\right.\right.}{48(x^{4}c+b\,x^{2})^{\frac{3}{2}}c^{\frac{11}{2}}}$
risch	$-\frac{x^{2}\left(-8B\,c^{2}x^{4}-12A\,c^{2}x^{2}+22bB\,x^{2}c+42Abc-57b^{2}B\right)(c\,x^{2}+b)}{48c^{4}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(-\frac{b^{2}xA}{c^{3}\sqrt{c\,x^{2}+b}}+\frac{b^{3}xB}{c^{4}\sqrt{c\,x^{2}+b}}+\frac{15b^{2}\ln\left(\sqrt{c}\,x+\sqrt{c\,x\ldots}\right.}{8c^{\frac{7}{2}}}\right.}{\sqrt{x^{2}\left(c\ldots\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*x^3*(c*x^2+b)*(8*B*c^(9/2)*x^7+12*A*c^(9/2)*x^5-14*B*c^(7/2)*b*x^5-30*
A*c^(7/2)*b*x^3+35*B*c^(5/2)*b^2*x^3-90*A*c^(5/2)*b^2*x+105*B*c^(3/2)*b^3*x
+90*A*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2*c^2-105*B*(c*x^2+b)
^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^3*c)/(c*x^4+b*x^2)^(3/2)/c^(11/2)
```

Maxima [A]
time = 0.28, size = 237, normalized size = 1.29

$$\frac{1}{16}\left(\frac{4\,x^{6}}{\sqrt{cx^{4}+bx^{2}}\,c}-\frac{10\,bx^{4}}{\sqrt{cx^{4}+bx^{2}}\,c^{2}}-\frac{30\,b^{2}x^{2}}{\sqrt{cx^{4}+bx^{2}}\,c^{3}}+\frac{15\,b^{2}\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{c^{\frac{7}{2}}}\right)A+\frac{1}{96}\left(\frac{16\,x^{8}}{\sqrt{cx^{4}+bx^{2}}\,c}-\frac{28\,bx^{6}}{\sqrt{cx^{4}+bx^{2}}\,c^{2}}+\frac{70\,b^{2}x^{4}}{\sqrt{cx^{4}+bx^{2}}\,c^{3}}+\frac{210\,b^{3}x^{2}}{\sqrt{cx^{4}+bx^{2}}\,c^{4}}-\frac{105\,b^{3}\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{c^{\frac{9}{2}}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*x^6/(sqrt(c*x^4 + b*x^2)*c) - 10*b*x^4/(sqrt(c*x^4 + b*x^2)*c^2) -
30*b^2*x^2/(sqrt(c*x^4 + b*x^2)*c^3) + 15*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^
4 + b*x^2)*sqrt(c))/c^(7/2))*A + 1/96*(16*x^8/(sqrt(c*x^4 + b*x^2)*c) - 28*
b*x^6/(sqrt(c*x^4 + b*x^2)*c^2) + 70*b^2*x^4/(sqrt(c*x^4 + b*x^2)*c^3) + 21
0*b^3*x^2/(sqrt(c*x^4 + b*x^2)*c^4) - 105*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^
4 + b*x^2)*sqrt(c))/c^(9/2))*B
```

Fricas [A]
time = 1.06, size = 340, normalized size = 1.85

$$\left[-\frac{15\,(7\,Bb^{4}-6\,Ab^{3}c+(7\,Bb^{3}c-6\,Ab^{2}c^{2})x^{2})\sqrt{c}\,\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)-2\,(8\,Bc^{4}x^{6}+105\,Bb^{3}c-90\,Ab^{2}c^{2}-2\,(7\,Bbc^{3}-6\,Ac^{4})x^{4}+5\,(7\,Bb^{2}c^{2}-6\,Abc^{3})x^{2})\sqrt{cx^{4}+bx^{2}}}{96\,(c^{6}x^{2}+bc^{5})},\ \frac{15\,(7\,Bb^{4}-6\,Ab^{3}c+(7\,Bb^{3}c-6\,Ab^{2}c^{2})x^{2})\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)+(8\,Bc^{4}x^{6}+105\,Bb^{3}c-90\,Ab^{2}c^{2}-2\,(7\,Bbc^{3}-6\,Ac^{4})x^{4}+5\,(7\,Bb^{2}c^{2}-6\,Abc^{3})x^{2})\sqrt{cx^{4}+bx^{2}}}{48\,(c^{6}x^{2}+bc^{5})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(7*B*b^4 - 6*A*b^3*c + (7*B*b^3*c - 6*A*b^2*c^2)*x^2)*sqrt(c)*lo
g(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*(8*B*c^4*x^6 + 105*B*b^
3*c - 90*A*b^2*c^2 - 2*(7*B*b*c^3 - 6*A*c^4)*x^4 + 5*(7*B*b^2*c^2 - 6*A*b*c
^3)*x^2)*sqrt(c*x^4 + b*x^2))/(c^6*x^2 + b*c^5), 1/48*(15*(7*B*b^4 - 6*A*b^
3*c + (7*B*b^3*c - 6*A*b^2*c^2)*x^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sq
```

```
rt(-c)/(c*x^2 + b)) + (8*B*c^4*x^6 + 105*B*b^3*c - 90*A*b^2*c^2 - 2*(7*B*b*
c^3 - 6*A*c^4)*x^4 + 5*(7*B*b^2*c^2 - 6*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/
(c^6*x^2 + b*c^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**9*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 0.47, size = 183, normalized size = 0.99

$$\frac{\left(\left(2\,x^2\left(\frac{4\,Bx^2}{c\mathrm{sgn}(x)}-\frac{7\,Bbc^5\mathrm{sgn}(x)-6\,Ac^6\mathrm{sgn}(x)}{c^7}\right)+\frac{5\,(7\,Bb^2c^4\mathrm{sgn}(x)-6\,Abc^5\mathrm{sgn}(x))}{c^7}\right)x^2+\frac{15\,(7\,Bb^3c^3\mathrm{sgn}(x)-6\,Ab^2c^4\mathrm{sgn}(x))}{c^7}\right)x}{48\,\sqrt{cx^2+b}}-\frac{5\,(7\,Bb^3\log\left(|b|\right)-6\,Ab^2c\log\left(|b|\right))\mathrm{sgn}(x)}{32\,c^{\frac{9}{2}}}+\frac{5\,(7\,Bb^3-6\,Ab^2c)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{16\,c^{\frac{9}{2}}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*((2*x^2*(4*B*x^2/(c*sgn(x)) - (7*B*b*c^5*sgn(x) - 6*A*c^6*sgn(x))/c^7)
 + 5*(7*B*b^2*c^4*sgn(x) - 6*A*b*c^5*sgn(x))/c^7)*x^2 + 15*(7*B*b^3*c^3*sgn
(x) - 6*A*b^2*c^4*sgn(x))/c^7)*x/sqrt(c*x^2 + b) - 5/32*(7*B*b^3*log(abs(b)
) - 6*A*b^2*c*log(abs(b)))*sgn(x)/c^(9/2) + 5/16*(7*B*b^3 - 6*A*b^2*c)*log(
abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(9/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.146 $\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}}-\frac{3(5bB-4Ac)\sqrt{bx^2+cx^4}}{8c^3}+\frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2}+\frac{3b(5bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}$$

```
[Out] 3/8*b*(-4*A*c+5*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)-(-A*c
+B*b)*x^6/b/c/(c*x^4+b*x^2)^(1/2)-3/8*(-4*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/c^
3+1/4*(-4*A*c+5*B*b)*x^2*(c*x^4+b*x^2)^(1/2)/b/c^2
```

Rubi [A]
time = 0.18, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2059, 802, 684, 654, 634, 212}

$$\frac{3b(5bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}-\frac{3\sqrt{bx^2+cx^4}\,(5bB-4Ac)}{8c^3}+\frac{x^2\sqrt{bx^2+cx^4}\,(5bB-4Ac)}{4bc^2}-\frac{x^6(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^6)/(b*c*Sqrt[b*x^2 + c*x^4])) - (3*(5*b*B - 4*A*c)*Sqrt[b*
x^2 + c*x^4])/(8*c^3) + ((5*b*B - 4*A*c)*x^2*Sqrt[b*x^2 + c*x^4])/(4*b*c^2)
 + (3*b*(5*b*B - 4*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(7
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 802

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Dist[e*((m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e))), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(bx+cx^2)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} + \frac{1}{2}\left(-\frac{4A}{b}+\frac{5B}{c}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2} - \frac{(3(5bB-4Ac))\text{Subst}\left(\int \frac{x}{\sqrt{bx+c\ldots}}\right.}{8c^2}$$

$$= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} - \frac{3(5bB-4Ac)\sqrt{bx^2+cx^4}}{8c^3} + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2} + \frac{(3b(\ldots}{\ldots}$$

$$= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} - \frac{3(5bB-4Ac)\sqrt{bx^2+cx^4}}{8c^3} + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2} + \frac{(3b(\ldots}{\ldots}$$

$$= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} - \frac{3(5bB-4Ac)\sqrt{bx^2+cx^4}}{8c^3} + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2} + \frac{3b(5\ldots}{\ldots}$$

Mathematica [A]
time = 0.16, size = 114, normalized size = 0.78

$$\frac{x\left(\sqrt{c}\,x(-15b^2B+bc(12A-5Bx^2)+2c^2x^2(2A+Bx^2))-3b(5bB-4Ac)\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{8c^{7/2}\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(-15*b^2*B + b*c*(12*A - 5*B*x^2) + 2*c^2*x^2*(2*A + B*x^2))
- 3*b*(5*b*B - 4*A*c)*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))
/(8*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.39, size = 140, normalized size = 0.95

method	result
default	$-\frac{x^3(c\,x^2+b)\left(-2B\,c^{\frac{7}{2}}x^5-4A\,c^{\frac{7}{2}}x^3+5B\,c^{\frac{5}{2}}b\,x^3-12A\,c^{\frac{5}{2}}bx+15B\,c^{\frac{3}{2}}b^2x+12A\sqrt{c\,x^2+b}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)b\,c^2-15B\ldots\right.}{8(x^4c+b\,x^2)^{\frac{3}{2}}c^{\frac{9}{2}}}$

risch	$\frac{x^2(2Bcx^2+4Ac-7Bb)(cx^2+b)}{8c^3\sqrt{x^2(cx^2+b)}}+\frac{\left(\frac{bxA}{c^2\sqrt{cx^2+b}}-\frac{b^2xB}{c^3\sqrt{cx^2+b}}-\frac{3b\ln\left(\sqrt{c}\,x+\sqrt{cx^2+b}\right)A}{2c^{\frac{5}{2}}}+\frac{15b^2\ln\left(\sqrt{c}\,x+\sqrt{c}\right.}{8c^{\frac{7}{2}}}\right.}{\sqrt{x^2(cx^2+b)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x^3*(c*x^2+b)*(-2*B*c^(7/2)*x^5-4*A*c^(7/2)*x^3+5*B*c^(5/2)*b*x^3-12*A
*c^(5/2)*b*x+15*B*c^(3/2)*b^2*x+12*A*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c*x^2+b)
^(1/2))*b*c^2-15*B*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2*c)/(c*
x^4+b*x^2)^(3/2)/c^(9/2)
```

Maxima [A]
time = 0.28, size = 187, normalized size = 1.27

$$\frac{1}{4}\left(\frac{2x^4}{\sqrt{cx^4+bx^2}\,c}+\frac{6bx^2}{\sqrt{cx^4+bx^2}\,c^2}-\frac{3b\log\left(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}}\right)A+\frac{1}{16}\left(\frac{4x^6}{\sqrt{cx^4+bx^2}\,c}-\frac{10bx^4}{\sqrt{cx^4+bx^2}\,c^2}-\frac{30b^2x^2}{\sqrt{cx^4+bx^2}\,c^3}+\frac{15b^2\log\left(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{7}{2}}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*x^4/(sqrt(c*x^4 + b*x^2)*c) + 6*b*x^2/(sqrt(c*x^4 + b*x^2)*c^2) - 3*
b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2))*A + 1/16*(4*x^6
/(sqrt(c*x^4 + b*x^2)*c) - 10*b*x^4/(sqrt(c*x^4 + b*x^2)*c^2) - 30*b^2*x^2/
(sqrt(c*x^4 + b*x^2)*c^3) + 15*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*
sqrt(c))/c^(7/2))*B
```

Fricas [A]
time = 1.20, size = 289, normalized size = 1.97

$$\left[-\frac{3\left(5Bb^3-4Ab^2c+\left(5Bb^2c-4Abc^2\right)x^2\right)\sqrt{c}\log\left(-2cx^2-b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(2Bc^3x^4-15Bb^2c+12Abc^2-\left(5Bbc^2-4Ac^3\right)x^2\right)\sqrt{cx^4+bx^2}}{16\left(c^5x^2+bc^4\right)},-\frac{3\left(5Bb^3-4Ab^2c+\left(5Bb^2c-4Abc^2\right)x^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-\left(2Bc^3x^4-15Bb^2c+12Abc^2-\left(5Bbc^2-4Ac^3\right)x^2\right)\sqrt{cx^4+bx^2}}{8\left(c^5x^2+bc^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(5*B*b^3 - 4*A*b^2*c + (5*B*b^2*c - 4*A*b*c^2)*x^2)*sqrt(c)*log(-
2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*(2*B*c^3*x^4 - 15*B*b^2*c
+ 12*A*b*c^2 - (5*B*b*c^2 - 4*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/(c^5*x^2 + b
*c^4), -1/8*(3*(5*B*b^3 - 4*A*b^2*c + (5*B*b^2*c - 4*A*b*c^2)*x^2)*sqrt(-c)
*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (2*B*c^3*x^4 - 15*B*b^2
*c + 12*A*b*c^2 - (5*B*b*c^2 - 4*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/(c^5*x^2
+ b*c^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**7*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 0.47, size = 145, normalized size = 0.99

$$\frac{\left(x^2\left(\frac{2\,Bx^2}{c\mathrm{sgn}(x)}-\frac{5\,Bbc^3\mathrm{sgn}(x)-4\,Ac^4\mathrm{sgn}(x)}{c^5}\right)-\frac{3\,(5\,Bb^2c^2\mathrm{sgn}(x)-4\,Abc^3\mathrm{sgn}(x))}{c^5}\right)x}{8\,\sqrt{cx^2+b}}+\frac{3\,(5\,Bb^2\log\left(|b|\right)-4\,Abc\log\left(|b|\right))\mathrm{sgn}(x)}{16\,c^{\frac{7}{2}}}-\frac{3\,(5\,Bb^2-4\,Abc)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{8\,c^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(x^2*(2*B*x^2/(c*sgn(x)) - (5*B*b*c^3*sgn(x) - 4*A*c^4*sgn(x))/c^5) - 3
*(5*B*b^2*c^2*sgn(x) - 4*A*b*c^3*sgn(x))/c^5)*x/sqrt(c*x^2 + b) + 3/16*(5*B
*b^2*log(abs(b)) - 4*A*b*c*log(abs(b)))*sgn(x)/c^(7/2) - 3/8*(5*B*b^2 - 4*A
*b*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(7/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.147 $\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}}+\frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{2bc^2}-\frac{(3bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}$$

```
[Out] -1/2*(-2*A*c+3*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(5/2)-(-A*c+
B*b)*x^4/b/c/(c*x^4+b*x^2)^(1/2)+1/2*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/b/c
^2
```

Rubi [A]
time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2059, 802, 654, 634, 212}

$$-\frac{(3bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}+\frac{\sqrt{bx^2+cx^4}\,(3bB-2Ac)}{2bc^2}-\frac{x^4(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^4)/(b*c*Sqrt[b*x^2 + c*x^4])) + ((3*b*B - 2*A*c)*Sqrt[b*x^
2 + c*x^4])/(2*b*c^2) - ((3*b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
 c*x^4]])/(2*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 802

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Dist[e*((m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e))), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(bx+cx^2)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}} + \frac{1}{2}\left(-\frac{2A}{b}+\frac{3B}{c}\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= -\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}} + \frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{2bc^2} - \frac{(3bB-2Ac)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx \ldots\right)}{4c^2}\\
&= -\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}} + \frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{2bc^2} - \frac{(3bB-2Ac)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \ldots\right)}{2c^2}\\
&= -\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}} + \frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{2bc^2} - \frac{(3bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 90, normalized size = 0.80

$$\frac{x\left(\sqrt{c}\,x(3bB-2Ac+Bcx^2)+(3bB-2Ac)\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{2c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(3*b*B - 2*A*c + B*c*x^2) + (3*b*B - 2*A*c)*Sqrt[b + c*x^2]*L
og[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(2*c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.40, size = 114, normalized size = 1.02

method	result
default	$\frac{x^{3}\left(c\,x^{2}+b\right)\left(B\,c^{\frac{5}{2}}x^{3}-2A\,c^{\frac{5}{2}}x+3B\,c^{\frac{3}{2}}bx+2A\sqrt{c\,x^{2}+b}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)c^{2}-3B\sqrt{c\,x^{2}+b}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}\right.}\right.}{2\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}c^{\frac{7}{2}}}$
risch	$\frac{B\,x^{2}\left(c\,x^{2}+b\right)}{2c^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(-\frac{xA}{c\sqrt{c\,x^{2}+b}}+\frac{xbB}{c^{2}\sqrt{c\,x^{2}+b}}+\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)A}{c^{\frac{3}{2}}}-\frac{3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)B}{2c^{\frac{5}{2}}}\right.}{\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^3*(c*x^2+b)*(B*c^(5/2)*x^3-2*A*c^(5/2)*x+3*B*c^(3/2)*b*x+2*A*(c*x^2+b
)^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*c^2-3*B*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+
(c*x^2+b)^(1/2))*b*c)/(c*x^4+b*x^2)^(3/2)/c^(7/2)
```

Maxima [A]
time = 0.28, size = 138, normalized size = 1.23

$$\frac{1}{4}\left(\frac{2\,x^{4}}{\sqrt{cx^{4}+bx^{2}}\,c}+\frac{6\,bx^{2}}{\sqrt{cx^{4}+bx^{2}}\,c^{2}}-\frac{3\,b\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{c^{\frac{5}{2}}}\right)B-\frac{1}{2}\,A\left(\frac{2\,x^{2}}{\sqrt{cx^{4}+bx^{2}}\,c}-\frac{\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{c^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*x^4/(sqrt(c*x^4 + b*x^2)*c) + 6*b*x^2/(sqrt(c*x^4 + b*x^2)*c^2) - 3*
b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2))*B - 1/2*A*(2*x^
2/(sqrt(c*x^4 + b*x^2)*c) - log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)
)/c^(3/2))
```

Fricas [A]
time = 2.56, size = 230, normalized size = 2.05

$$\left[-\frac{\left(3\,Bb^{2}-2\,Abc+\left(3\,Bbc-2\,Ac^{2}\right)x^{2}\right)\sqrt{c}\,\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)-2\left(Bc^{2}x^{2}+3\,Bbc-2\,Ac^{2}\right)\sqrt{cx^{4}+bx^{2}}}{4\left(c^{4}x^{2}+bc^{3}\right)},\ \frac{\left(3\,Bb^{2}-2\,Abc+\left(3\,Bbc-2\,Ac^{2}\right)x^{2}\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)+\left(Bc^{2}x^{2}+3\,Bbc-2\,Ac^{2}\right)\sqrt{cx^{4}+bx^{2}}}{2\left(c^{4}x^{2}+bc^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*B*b^2 - 2*A*b*c + (3*B*b*c - 2*A*c^2)*x^2)*sqrt(c)*log(-2*c*x^2 -
 b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*(B*c^2*x^2 + 3*B*b*c - 2*A*c^2)*sqr
t(c*x^4 + b*x^2))/(c^4*x^2 + b*c^3), 1/2*((3*B*b^2 - 2*A*b*c + (3*B*b*c - 2
*A*c^2)*x^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (B
*c^2*x^2 + 3*B*b*c - 2*A*c^2)*sqrt(c*x^4 + b*x^2))/(c^4*x^2 + b*c^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**5*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 0.43, size = 104, normalized size = 0.93

$$\frac{x\left(\frac{Bx^2}{c\operatorname{sgn}(x)}+\frac{3\,Bbc\operatorname{sgn}(x)-2\,Ac^2\operatorname{sgn}(x)}{c^3}\right)}{2\sqrt{cx^2+b}}-\frac{(3\,Bb\log(|b|)-2\,Ac\log(|b|))\operatorname{sgn}(x)}{4\,c^{\frac{5}{2}}}+\frac{(3\,Bb-2\,Ac)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\,\right|\right)}{2\,c^{\frac{5}{2}}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x*(B*x^2/(c*sgn(x)) + (3*B*b*c*sgn(x) - 2*A*c^2*sgn(x))/c^3)/sqrt(c*x^2
 + b) - 1/4*(3*B*b*log(abs(b)) - 2*A*c*log(abs(b)))*sgn(x)/c^(5/2) + 1/2*(3
*B*b - 2*A*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(5/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.148 $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}$$

```
[Out] B*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)-(-A*c+B*b)*x^2/b/c/(c*x^
4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2059, 791, 634, 212}

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}-\frac{x^2(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^2)/(b*c*Sqrt[b*x^2 + c*x^4])) + (B*ArcTanh[(Sqrt[c]*x^2)/S
qrt[b*x^2 + c*x^4]])/c^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && LtQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(bx+cx^2)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}} + \frac{B\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{2c}\\
&= -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}} + \frac{B\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{c}\\
&= -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}} + \frac{B\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 77, normalized size = 1.15

$$-\frac{x\left(\sqrt{c}\,(bB-Ac)x+bB\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{bc^{3/2}\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((x*(Sqrt[c]*(b*B - A*c)*x + b*B*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b
 + c*x^2]]))/(b*c^(3/2)*Sqrt[x^2*(b + c*x^2)]))
```

Maple [A]
time = 0.37, size = 75, normalized size = 1.12

method	result	size
default	$\frac{x^{3}\left(c\,x^{2}+b\right)\left(A\,c^{\frac{5}{2}}x-B\,c^{\frac{3}{2}}bx+B\sqrt{c\,x^{2}+b}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)bc\right)}{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}c^{\frac{5}{2}}b}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^3*(c*x^2+b)*(A*c^(5/2)*x-B*c^(3/2)*b*x+B*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c*
x^2+b)^(1/2))*b*c)/(c*x^4+b*x^2)^(3/2)/c^(5/2)/b
```

Maxima [A]
time = 0.29, size = 79, normalized size = 1.18

$$-\frac{1}{2}\,B\left(\frac{2\,x^{2}}{\sqrt{cx^{4}+bx^{2}}\,c}-\frac{\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{c^{\frac{3}{2}}}\right)+\frac{Ax^{2}}{\sqrt{cx^{4}+bx^{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*B*(2*x^2/(sqrt(c*x^4 + b*x^2)*c) - log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*
x^2)*sqrt(c))/c^(3/2)) + A*x^2/(sqrt(c*x^4 + b*x^2)*b)
```

Fricas [A]
time = 2.74, size = 188, normalized size = 2.81

$$\left[\frac{\left(Bbcx^{2}+Bb^{2}\right)\sqrt{c}\,\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)-2\,\sqrt{cx^{4}+bx^{2}}\,\left(Bbc-Ac^{2}\right)}{2\,\left(bc^{3}x^{2}+b^{2}c^{2}\right)},\,-\frac{\left(Bbcx^{2}+Bb^{2}\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)+\sqrt{cx^{4}+bx^{2}}\,\left(Bbc-Ac^{2}\right)}{bc^{3}x^{2}+b^{2}c^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b*c*x^2 + B*b^2)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*
sqrt(c)) - 2*sqrt(c*x^4 + b*x^2)*(B*b*c - A*c^2))/(b*c^3*x^2 + b^2*c^2), -(
(B*b*c*x^2 + B*b^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b
)) + sqrt(c*x^4 + b*x^2)*(B*b*c - A*c^2))/(b*c^3*x^2 + b^2*c^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3}(A+Bx^{2})}{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**3*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 0.44, size = 70, normalized size = 1.04

$$\frac{B\log\left(|b|\right)\operatorname{sgn}(x)}{2\,c^{\frac{3}{2}}} - \frac{(Bb\operatorname{sgn}(x) - Ac\operatorname{sgn}(x))x}{\sqrt{cx^2+b}\,bc} - \frac{B\log\left(\left|-\sqrt{c}\,x + \sqrt{cx^2+b}\right|\right)}{c^{\frac{3}{2}}\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*B*log(abs(b))*sgn(x)/c^(3/2) - (B*b*sgn(x) - A*c*sgn(x))*x/(sqrt(c*x^2
+ b)*b*c) - B*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(3/2)*sgn(x))
```

Mupad [B]
time = 0.63, size = 78, normalized size = 1.16

$$\frac{B\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,c^{3/2}}+\frac{A\,x^2}{b\,\sqrt{c\,x^4+b\,x^2}}-\frac{B\,x^2}{c\,\sqrt{c\,x^4+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (B*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(2*c^(3/2)) + (A*x^2
)/(b*(b*x^2 + c*x^4)^(1/2)) - (B*x^2)/(c*(b*x^2 + c*x^4)^(1/2))
```

3.149 $\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{Ab-(bB-2Ac)x^2}{b^2\sqrt{bx^2+cx^4}}$$

```
[Out] (-A*b+(-2*A*c+B*b)*x^2)/b^2/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2059, 650}

$$-\frac{Ab-x^2(bB-2Ac)}{b^2\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((A*b - (b*B - 2*A*c)*x^2)/(b^2*Sqrt[b*x^2 + c*x^4]))
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(bx+cx^2)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{Ab-(bB-2Ac)x^2}{b^2\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 37, normalized size = 1.00

$$\frac{bBx^2 - A(b + 2cx^2)}{b^2\sqrt{x^2(b + cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
[Out] (b*B*x^2 - A*(b + 2*c*x^2))/(b^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.39, size = 47, normalized size = 1.27

method	result	size
gosper	$-\frac{x^{2} \left(c\,x^{2}+b\right) \left(2 A c\,x^{2}-b B\,x^{2}+A b\right)}{b^{2} \left(x^{4} c+b\,x^{2}\right)^{\frac{3}{2}}}$	47
default	$-\frac{x^{2} \left(c\,x^{2}+b\right) \left(2 A c\,x^{2}-b B\,x^{2}+A b\right)}{b^{2} \left(x^{4} c+b\,x^{2}\right)^{\frac{3}{2}}}$	47
trager	$-\frac{\left(2 A c\,x^{2}-b B\,x^{2}+A b\right) \sqrt{x^{4} c+b\,x^{2}}}{\left(c\,x^{2}+b\right) b^{2} x^{2}}$	49
risch	$-\frac{A \left(c\,x^{2}+b\right)}{b^{2} \sqrt{x^{2} \left(c\,x^{2}+b\right)}}-\frac{x^{2} \left(A c -B b\right)}{b^{2} \sqrt{x^{2} \left(c\,x^{2}+b\right)}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -x^2*(c*x^2+b)*(2*A*c*x^2-B*b*x^2+A*b)/b^2/(c*x^4+b*x^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 65, normalized size = 1.76

$$-A\left(\frac{2\,cx^2}{\sqrt{cx^4 + bx^2}\,b^2} + \frac{1}{\sqrt{cx^4 + bx^2}\,b}\right) + \frac{Bx^2}{\sqrt{cx^4 + bx^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] -A*(2*c*x^2/(sqrt(c*x^4 + b*x^2)*b^2) + 1/(sqrt(c*x^4 + b*x^2)*b)) + B*x^2/
(sqrt(c*x^4 + b*x^2)*b)
```

Fricas [A]
time = 1.86, size = 49, normalized size = 1.32

$$\frac{\sqrt{cx^4 + bx^2}\,((Bb - 2\,Ac)x^2 - Ab)}{b^2cx^4 + b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2)*((B*b - 2*A*c)*x^2 - A*b)/(b^2*c*x^4 + b^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(A + Bx^2)}{(x^2 (b + cx^2))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 0.46, size = 65, normalized size = 1.76

$$\frac{2 A\sqrt{c}}{\left(\left(\sqrt{c}\, x - \sqrt{cx^2 + b}\right)^2 - b\right) b\mathrm{sgn}\,(x)} + \frac{(Bb - Ac)x}{\sqrt{cx^2 + b}\, b^2 \mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2*A*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)*b*sgn(x)) + (B*b - A*c)*
x/(sqrt(c*x^2 + b)*b^2*sgn(x))
```

Mupad [B]
time = 0.18, size = 53, normalized size = 1.43

$$-\frac{\left(\frac{A}{b} - x^2 \left(\frac{B}{b} - \frac{2\, A\, c}{b^2}\right)\right)\, \sqrt{c\, x^4 + b\, x^2}}{x\, \left(c\, x^3 + b\, x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] -((A/b - x^2*(B/b - (2*A*c)/b^2))*(b*x^2 + c*x^4)^(1/2))/(x*(b*x + c*x^3))
```

3.150 $\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{A}{3bx^2\sqrt{bx^2+cx^4}}-\frac{(3bB-4Ac)\,(b+2cx^2)}{3b^3\sqrt{bx^2+cx^4}}$$

```
[Out] -1/3*A/b/x^2/(c*x^4+b*x^2)^(1/2)-1/3*(-4*A*c+3*B*b)*(2*c*x^2+b)/b^3/(c*x^4+
b*x^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2059, 806, 627}

$$-\frac{(b+2cx^2)\,(3bB-4Ac)}{3b^3\sqrt{bx^2+cx^4}}-\frac{A}{3bx^2\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/3*A/(b*x^2*Sqrt[b*x^2 + c*x^4]) - ((3*b*B - 4*A*c)*(b + 2*c*x^2))/(3*b^3
*Sqrt[b*x^2 + c*x^4])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
```

```
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\ &= -\frac{A}{3bx^2\sqrt{bx^2+cx^4}}+\frac{\left(bB-Ac+\frac{1}{2}(bB-2Ac)\right)\text{Subst}\left(\int \frac{1}{\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)}{3b}\\ &= -\frac{A}{3bx^2\sqrt{bx^2+cx^4}}-\frac{(3bB-4Ac)\left(b+2cx^2\right)}{3b^3\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [**A**]
time = 0.10, size = 64, normalized size = 0.97

$$\frac{-3bBx^2(b+2cx^2)+A(-b^2+4bcx^2+8c^2x^4)}{3b^3x^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-3*b*B*x^2*(b + 2*c*x^2) + A*(-b^2 + 4*b*c*x^2 + 8*c^2*x^4))/(3*b^3*x^2*Sq
rt[x^2*(b + c*x^2)])
```

Maple [**A**]
time = 0.39, size = 66, normalized size = 1.00

method	result	size
gosper	$-\frac{(c\,x^2+b)\left(-8A\,c^2x^4+6x^4bBc-4Abc\,x^2+3b^2B\,x^2+b^2A\right)}{3b^3\left(x^4c+b\,x^2\right)^{\frac{3}{2}}}$	66
default	$-\frac{(c\,x^2+b)\left(-8A\,c^2x^4+6x^4bBc-4Abc\,x^2+3b^2B\,x^2+b^2A\right)}{3b^3\left(x^4c+b\,x^2\right)^{\frac{3}{2}}}$	66
trager	$-\frac{\left(-8A\,c^2x^4+6x^4bBc-4Abc\,x^2+3b^2B\,x^2+b^2A\right)\sqrt{x^4c+b\,x^2}}{3(c\,x^2+b)b^3x^4}$	71
risch	$-\frac{(c\,x^2+b)\left(-5Ac\,x^2+3bB\,x^2+Ab\right)}{3b^3x^2\sqrt{x^2\left(c\,x^2+b\right)}}+\frac{x^2(Ac-Bb)c}{b^3\sqrt{x^2\left(c\,x^2+b\right)}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*x^2+b)*(-8*A*c^2*x^4+6*B*b*c*x^4-4*A*b*c*x^2+3*B*b^2*x^2+A*b^2)/b^3
/(c*x^4+b*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 112, normalized size = 1.70

$$-B\left(\frac{2\,cx^2}{\sqrt{cx^4+bx^2}\,b^2}+\frac{1}{\sqrt{cx^4+bx^2}\,b}\right)+\frac{1}{3}\,A\left(\frac{8\,c^2x^2}{\sqrt{cx^4+bx^2}\,b^3}+\frac{4\,c}{\sqrt{cx^4+bx^2}\,b^2}-\frac{1}{\sqrt{cx^4+bx^2}\,bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -B*(2*c*x^2/(sqrt(c*x^4 + b*x^2)*b^2) + 1/(sqrt(c*x^4 + b*x^2)*b)) + 1/3*A*
(8*c^2*x^2/(sqrt(c*x^4 + b*x^2)*b^3) + 4*c/(sqrt(c*x^4 + b*x^2)*b^2) - 1/(s
qrt(c*x^4 + b*x^2)*b*x^2))
```

Fricas [A]
time = 1.39, size = 72, normalized size = 1.09

$$-\frac{(2\,(3\,Bbc-4\,Ac^2)x^4+Ab^2+(3\,Bb^2-4\,Abc)x^2)\sqrt{cx^4+bx^2}}{3\,(b^3cx^6+b^4x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*B*b*c - 4*A*c^2)*x^4 + A*b^2 + (3*B*b^2 - 4*A*b*c)*x^2)*sqrt(c*x
^4 + b*x^2)/(b^3*c*x^6 + b^4*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+Bx^2}{x\,(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(58) = 116.
time = 0.67, size = 189, normalized size = 2.86

$$-\frac{(Bbc-Ac^2)x}{\sqrt{cx^2+b}\,b^3\mathrm{sgn}\,(x)}+\frac{2\left(3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4Bb\sqrt{c}-3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4Ac^{\frac{3}{2}}-6\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2Bb^2\sqrt{c}+12\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2Abc^{\frac{3}{2}}+3\,Bb^3\sqrt{c}-5\,Ab^2c^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^3b^2\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*b*c - A*c^2)*x/(sqrt(c*x^2 + b)*b^3*sgn(x)) + 2/3*(3*(sqrt(c)*x - sqrt(
c*x^2 + b))^4*B*b*sqrt(c) - 3*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*c^(3/2) - 6
*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^2*sqrt(c) + 12*(sqrt(c)*x - sqrt(c*x^2
 + b))^2*A*b*c^(3/2) + 3*B*b^3*sqrt(c) - 5*A*b^2*c^(3/2))/(((sqrt(c)*x - sq
rt(c*x^2 + b))^2 - b)^3*b^2*sgn(x))
```

Mupad [B]
time = 0.27, size = 70, normalized size = 1.06

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(3\,B\,b^2\,x^2+A\,b^2+6\,B\,b\,c\,x^4-4\,A\,b\,c\,x^2-8\,A\,c^2\,x^4\right)}{3\,b^3\,x^4\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(A*b^2 + 3*B*b^2*x^2 - 8*A*c^2*x^4 - 4*A*b*c*x^2 +
6*B*b*c*x^4))/(3*b^3*x^4*(b + c*x^2))
```

3.151 $\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{A}{5bx^4\sqrt{bx^2+cx^4}}-\frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}}+\frac{4c(5bB-6Ac)\left(b+2cx^2\right)}{15b^4\sqrt{bx^2+cx^4}}$$

```
[Out] -1/5*A/b/x^4/(c*x^4+b*x^2)^(1/2)+1/15*(6*A*c-5*B*b)/b^2/x^2/(c*x^4+b*x^2)^(
1/2)+4/15*c*(-6*A*c+5*B*b)*(2*c*x^2+b)/b^4/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2059, 806, 672, 627}

$$\frac{4c\left(b+2cx^2\right)(5bB-6Ac)}{15b^4\sqrt{bx^2+cx^4}}-\frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}}-\frac{A}{5bx^4\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/5*A/(b*x^4*Sqrt[b*x^2 + c*x^4]) - (5*b*B - 6*A*c)/(15*b^2*x^2*Sqrt[b*x^2
 + c*x^4]) + (4*c*(5*b*B - 6*A*c)*(b + 2*c*x^2))/(15*b^4*Sqrt[b*x^2 + c*x^4
])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
```

```
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{A}{5bx^4\sqrt{bx^2+cx^4}} + \frac{\left(\frac{1}{2}(bB-2Ac)-2(-bB+Ac)\right)\text{Subst}\left(\int \frac{1}{x(bx+cx^2)^{3/2}}\,dx, x, \ldots\right.}{5b}\\
&= -\frac{A}{5bx^4\sqrt{bx^2+cx^4}} - \frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}} - \frac{(2c(5bB-6Ac))\text{Subst}\left(\int \frac{1}{(bx+cx^2)^{3/2}} \ldots\right.}{15b^2}\\
&= -\frac{A}{5bx^4\sqrt{bx^2+cx^4}} - \frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}} + \frac{4c(5bB-6Ac)\left(b+2cx^2\right)}{15b^4\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [**A**]
time = 0.13, size = 85, normalized size = 0.84

$$\frac{-5bBx^2(b^2-4bcx^2-8c^2x^4)-3A(b^3-2b^2cx^2+8bc^2x^4+16c^3x^6)}{15b^4x^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-5*b*B*x^2*(b^2 - 4*b*c*x^2 - 8*c^2*x^4) - 3*A*(b^3 - 2*b^2*c*x^2 + 8*b*c^
2*x^4 + 16*c^3*x^6))/(15*b^4*x^4*Sqrt[x^2*(b + c*x^2)])
```

Maple [**A**]
time = 0.39, size = 94, normalized size = 0.93

method	result	size
gosper	$-\frac{(c\,x^{2}+b)\,(48A\,c^{3}x^{6}-40x^{6}Bb\,c^{2}+24Ab\,c^{2}x^{4}-20x^{4}B\,b^{2}c-6A\,b^{2}c\,x^{2}+5x^{2}B\,b^{3}+3A\,b^{3})}{15x^{2}b^{4}(x^{4}c+b\,x^{2})^{\frac{3}{2}}}$	94
default	$-\frac{(c\,x^{2}+b)\,(48A\,c^{3}x^{6}-40x^{6}Bb\,c^{2}+24Ab\,c^{2}x^{4}-20x^{4}B\,b^{2}c-6A\,b^{2}c\,x^{2}+5x^{2}B\,b^{3}+3A\,b^{3})}{15x^{2}b^{4}(x^{4}c+b\,x^{2})^{\frac{3}{2}}}$	94
trager	$-\frac{(48A\,c^{3}x^{6}-40x^{6}Bb\,c^{2}+24Ab\,c^{2}x^{4}-20x^{4}B\,b^{2}c-6A\,b^{2}c\,x^{2}+5x^{2}B\,b^{3}+3A\,b^{3})\sqrt{x^{4}c+b\,x^{2}}}{15(c\,x^{2}+b)b^{4}x^{6}}$	96
risch	$-\frac{(c\,x^{2}+b)\,(33A\,c^{2}x^{4}-25x^{4}bBc-9Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A)}{15b^{4}x^{4}\sqrt{x^{2}\,(c\,x^{2}+b)}}-\frac{x^{2}c^{2}(Ac-Bb)}{b^{4}\sqrt{x^{2}\,(c\,x^{2}+b)}}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(c*x^2+b)*(48*A*c^3*x^6-40*B*b*c^2*x^6+24*A*b*c^2*x^4-20*B*b^2*c*x^4-
6*A*b^2*c*x^2+5*B*b^3*x^2+3*A*b^3)/x^2/b^4/(c*x^4+b*x^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 160, normalized size = 1.58

$$\frac{1}{3}\,B\left(\frac{8\,c^{2}x^{2}}{\sqrt{cx^{4}+bx^{2}}\,b^{3}}+\frac{4\,c}{\sqrt{cx^{4}+bx^{2}}\,b^{2}}-\frac{1}{\sqrt{cx^{4}+bx^{2}}\,bx^{2}}\right)-\frac{1}{5}\,A\left(\frac{16\,c^{3}x^{2}}{\sqrt{cx^{4}+bx^{2}}\,b^{4}}+\frac{8\,c^{2}}{\sqrt{cx^{4}+bx^{2}}\,b^{3}}-\frac{2\,c}{\sqrt{cx^{4}+bx^{2}}\,b^{2}x^{2}}+\frac{1}{\sqrt{cx^{4}+bx^{2}}\,bx^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*B*(8*c^2*x^2/(sqrt(c*x^4 + b*x^2)*b^3) + 4*c/(sqrt(c*x^4 + b*x^2)*b^2)
- 1/(sqrt(c*x^4 + b*x^2)*b*x^2)) - 1/5*A*(16*c^3*x^2/(sqrt(c*x^4 + b*x^2)*b
^4) + 8*c^2/(sqrt(c*x^4 + b*x^2)*b^3) - 2*c/(sqrt(c*x^4 + b*x^2)*b^2*x^2) +
 1/(sqrt(c*x^4 + b*x^2)*b*x^4))
```

Fricas [A]
time = 1.80, size = 98, normalized size = 0.97

$$\frac{(8\,(5\,Bbc^{2}-6\,Ac^{3})x^{6}+4\,(5\,Bb^{2}c-6\,Abc^{2})x^{4}-3\,Ab^{3}-(5\,Bb^{3}-6\,Ab^{2}c)x^{2})\sqrt{cx^{4}+bx^{2}}}{15\,(b^{4}cx^{8}+b^{5}x^{6})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*(5*B*b*c^2 - 6*A*c^3)*x^6 + 4*(5*B*b^2*c - 6*A*b*c^2)*x^4 - 3*A*b^3
 - (5*B*b^3 - 6*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^4*c*x^8 + b^5*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^{2}}{x^{3}\,(x^{2}\,(b+cx^{2}))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**3*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(89) = 178.
time = 1.26, size = 302, normalized size = 2.99

$$\frac{(Bbc^2 - Ac^3)x}{\sqrt{cx^2+b}\,b^4\mathrm{sgn}(x)} - \frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8Bbc^{\frac{3}{2}} - 15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8Ac^{\frac{5}{2}} - 90\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6Bb^2c^{\frac{3}{2}} + 90\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6Abc^{\frac{5}{2}} + 160\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Bb^3c^{\frac{3}{2}} - 240\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Ab^2c^{\frac{5}{2}} - 110\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2Bb^4c^{\frac{3}{2}} + 150\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2Ab^3c^{\frac{5}{2}} + 25Bb^5c^{\frac{3}{2}} - 33Ab^4c^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^5b^3\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] (B*b*c^2 - A*c^3)*x/(sqrt(c*x^2 + b)*b^4*sgn(x)) - 2/15*(15*(sqrt(c)*x - sq
rt(c*x^2 + b))^8*B*b*c^(3/2) - 15*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*c^(5/2)
 - 90*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^2*c^(3/2) + 90*(sqrt(c)*x - sqrt(
c*x^2 + b))^6*A*b*c^(5/2) + 160*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^3*c^(3/
2) - 240*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^2*c^(5/2) - 110*(sqrt(c)*x - s
qrt(c*x^2 + b))^2*B*b^4*c^(3/2) + 150*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^3
*c^(5/2) + 25*B*b^5*c^(3/2) - 33*A*b^4*c^(5/2))/(((sqrt(c)*x - sqrt(c*x^2 +
 b))^2 - b)^5*b^3*sgn(x))
```

Mupad [B]
time = 0.42, size = 95, normalized size = 0.94

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(5\,B\,b^3\,x^2+3\,A\,b^3-20\,B\,b^2\,c\,x^4-6\,A\,b^2\,c\,x^2-40\,B\,b\,c^2\,x^6+24\,A\,b\,c^2\,x^4+48\,A\,c^3\,x^6\right)}{15\,b^4\,x^6\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(3*A*b^3 + 5*B*b^3*x^2 + 48*A*c^3*x^6 - 6*A*b^2*c*x
^2 + 24*A*b*c^2*x^4 - 20*B*b^2*c*x^4 - 40*B*b*c^2*x^6))/(15*b^4*x^6*(b + c*
x^2))
```

3.152 $\int \frac{A+Bx^2}{x^5(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{A}{7bx^6\sqrt{bx^2+cx^4}}-\frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}}+\frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}}-\frac{8c^2(7bB-8Ac)\left(b+2cx^2\right)}{35b^5\sqrt{bx^2+cx^4}}$$

```
[Out] -1/7*A/b/x^6/(c*x^4+b*x^2)^(1/2)+1/35*(8*A*c-7*B*b)/b^2/x^4/(c*x^4+b*x^2)^(
1/2)+2/35*c*(-8*A*c+7*B*b)/b^3/x^2/(c*x^4+b*x^2)^(1/2)-8/35*c^2*(-8*A*c+7*B
*b)*(2*c*x^2+b)/b^5/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2059, 806, 672, 627}

$$-\frac{8c^2\left(b+2cx^2\right)(7bB-8Ac)}{35b^5\sqrt{bx^2+cx^4}}+\frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}}-\frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}}-\frac{A}{7bx^6\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/7*A/(b*x^6*Sqrt[b*x^2 + c*x^4]) - (7*b*B - 8*A*c)/(35*b^2*x^4*Sqrt[b*x^2
 + c*x^4]) + (2*c*(7*b*B - 8*A*c))/(35*b^3*x^2*Sqrt[b*x^2 + c*x^4]) - (8*c^
2*(7*b*B - 8*A*c)*(b + 2*c*x^2))/(35*b^5*Sqrt[b*x^2 + c*x^4])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
```

```
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{A}{7bx^6\sqrt{bx^2+cx^4}} + \frac{\left(\frac{1}{2}(bB-2Ac)-3(-bB+Ac)\right)\text{Subst}\left(\int \frac{1}{x^2(bx+cx^2)^{3/2}}\,dx, x,\right.}{7b}$$

$$= -\frac{A}{7bx^6\sqrt{bx^2+cx^4}} - \frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}} - \frac{(3c(7bB-8Ac))\text{Subst}\left(\int \frac{1}{x(bx+cx^2)^{3/2}}\right.}{35b^2}$$

$$= -\frac{A}{7bx^6\sqrt{bx^2+cx^4}} - \frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}} + \frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}} + \frac{(4c^2(7bB-8\ldots}{}$$

$$= -\frac{A}{7bx^6\sqrt{bx^2+cx^4}} - \frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}} + \frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}} - \frac{8c^2(7bB-8A\ldots}{35b^5\sqrt{bx\ldots}}$$

Mathematica [A]
time = 0.16, size = 108, normalized size = 0.78

$$\frac{-7bBx^2(b^3-2b^2cx^2+8bc^2x^4+16c^3x^6)+A(-5b^4+8b^3cx^2-16b^2c^2x^4+64bc^3x^6+128c^4x^8)}{35b^5x^6\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-7*b*B*x^2*(b^3 - 2*b^2*c*x^2 + 8*b*c^2*x^4 + 16*c^3*x^6) + A*(-5*b^4 + 8*
b^3*c*x^2 - 16*b^2*c^2*x^4 + 64*b*c^3*x^6 + 128*c^4*x^8))/(35*b^5*x^6*Sqrt[
x^2*(b + c*x^2)])
```

Maple [A]
time = 0.39, size = 118, normalized size = 0.86

method	result	siz
gosper	$-\frac{(c\,x^2+b)\,(-128A\,c^4x^8+112Bb\,c^3x^8-64Ab\,c^3x^6+56B\,b^2c^2x^6+16A\,b^2c^2x^4-14B\,b^3c\,x^4-8A\,b^3c\,x^2+7B\,b^4x^2+5A\,b^4)}{35x^4b^5(x^4c+b\,x^2)^{\frac{3}{2}}}$	11
default	$-\frac{(c\,x^2+b)\,(-128A\,c^4x^8+112Bb\,c^3x^8-64Ab\,c^3x^6+56B\,b^2c^2x^6+16A\,b^2c^2x^4-14B\,b^3c\,x^4-8A\,b^3c\,x^2+7B\,b^4x^2+5A\,b^4)}{35x^4b^5(x^4c+b\,x^2)^{\frac{3}{2}}}$	11
trager	$-\frac{(-128A\,c^4x^8+112Bb\,c^3x^8-64Ab\,c^3x^6+56B\,b^2c^2x^6+16A\,b^2c^2x^4-14B\,b^3c\,x^4-8A\,b^3c\,x^2+7B\,b^4x^2+5A\,b^4)\sqrt{x^4c+b\,x^2}}{35(c\,x^2+b)b^5x^8}$	12
risch	$-\frac{(c\,x^2+b)\,(-93A\,c^3x^6+77x^6Bb\,c^2+29Ab\,c^2x^4-21x^4B\,b^2c-13A\,b^2c\,x^2+7x^2B\,b^3+5A\,b^3)}{35b^5x^6\sqrt{x^2\,(c\,x^2+b)}}+\frac{x^2c^3(Ac-Bb)}{b^5\sqrt{x^2\,(c\,x^2+b)}}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*(c*x^2+b)*(-128*A*c^4*x^8+112*B*b*c^3*x^8-64*A*b*c^3*x^6+56*B*b^2*c^2
*x^6+16*A*b^2*c^2*x^4-14*B*b^3*c*x^4-8*A*b^3*c*x^2+7*B*b^4*x^2+5*A*b^4)/x^4
/b^5/(c*x^4+b*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 208, normalized size = 1.51

$$-\frac{1}{5}B\left(\frac{16\,c^3x^2}{\sqrt{cx^4+bx^2}\,b^4}+\frac{8\,c^2}{\sqrt{cx^4+bx^2}\,b^3}-\frac{2\,c}{\sqrt{cx^4+bx^2}\,b^2x^2}+\frac{1}{\sqrt{cx^4+bx^2}\,bx^4}\right)+\frac{1}{35}A\left(\frac{128\,c^4x^2}{\sqrt{cx^4+bx^2}\,b^5}+\frac{64\,c^3}{\sqrt{cx^4+bx^2}\,b^4}-\frac{16\,c^2}{\sqrt{cx^4+bx^2}\,b^3x^2}+\frac{8\,c}{\sqrt{cx^4+bx^2}\,b^2x^4}-\frac{5}{\sqrt{cx^4+bx^2}\,bx^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5*B*(16*c^3*x^2/(sqrt(c*x^4 + b*x^2)*b^4) + 8*c^2/(sqrt(c*x^4 + b*x^2)*b
^3) - 2*c/(sqrt(c*x^4 + b*x^2)*b^2*x^2) + 1/(sqrt(c*x^4 + b*x^2)*b*x^4)) +
1/35*A*(128*c^4*x^2/(sqrt(c*x^4 + b*x^2)*b^5) + 64*c^3/(sqrt(c*x^4 + b*x^2)
*b^4) - 16*c^2/(sqrt(c*x^4 + b*x^2)*b^3*x^2) + 8*c/(sqrt(c*x^4 + b*x^2)*b^2
*x^4) - 5/(sqrt(c*x^4 + b*x^2)*b*x^6))
```

Fricas [A]
time = 1.00, size = 121, normalized size = 0.88

$$-\frac{(16\,(7\,Bbc^3-8\,Ac^4)x^8+8\,(7\,Bb^2c^2-8\,Abc^3)x^6+5\,Ab^4-2\,(7\,Bb^3c-8\,Ab^2c^2)x^4+(7\,Bb^4-8\,Ab^3c)x^2)\sqrt{cx^4+bx^2}}{35\,(b^5cx^{10}+b^6x^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/35*(16*(7*B*b*c^3 - 8*A*c^4)*x^8 + 8*(7*B*b^2*c^2 - 8*A*b*c^3)*x^6 + 5*A
*b^4 - 2*(7*B*b^3*c - 8*A*b^2*c^2)*x^4 + (7*B*b^4 - 8*A*b^3*c)*x^2)*sqrt(c*
x^4 + b*x^2)/(b^5*c*x^10 + b^6*x^8)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^5 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**5*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(122) = 244.
time = 2.03, size = 415, normalized size = 3.01

$$-\frac{(Bbc^3 - Ac^4)x}{\sqrt{cx^2+b}\, b^5 \mathrm{sgn}\left(x\right)} + \frac{2 \left(35 \left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{12} Bbc^{\frac{5}{2}} - 35 \left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{12} Ac^{\frac{7}{2}} - 280 \left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{10} Bb^2c^{\frac{5}{2}} + 280 \left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{10} Abc^{\frac{7}{2}} + 1015 \left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{8} Bb^3c^{\frac{5}{2}} - 1015 \left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{8} Ab^2c^{\frac{7}{2}} - 1680 \left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{6} Bb^4c^{\frac{5}{2}} + 2240 \left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{6} Ab^3c^{\frac{7}{2}} + 1337 \left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{4} Bb^5c^{\frac{5}{2}} - 1673 \left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{4} Ab^4c^{\frac{7}{2}} - 504 \left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{2} Bb^6c^{\frac{5}{2}} + 616 \left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{2} Ab^5c^{\frac{7}{2}} + 77\, Bb^7c^{\frac{5}{2}} - 93\, Ab^6c^{\frac{7}{2}}\right)}{35 \left(\left(\sqrt{c}x - \sqrt{cx^2+b}\right)^2 - b\right)^7 b^4 \mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*b*c^3 - A*c^4)*x/(sqrt(c*x^2 + b)*b^5*sgn(x)) + 2/35*(35*(sqrt(c)*x - s
qrt(c*x^2 + b))^12*B*b*c^(5/2) - 35*(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*c^(7
/2) - 280*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^2*c^(5/2) + 280*(sqrt(c)*x -
 sqrt(c*x^2 + b))^10*A*b*c^(7/2) + 1015*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b
^3*c^(5/2) - 1015*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b^2*c^(7/2) - 1680*(sqr
t(c)*x - sqrt(c*x^2 + b))^6*B*b^4*c^(5/2) + 2240*(sqrt(c)*x - sqrt(c*x^2 +
b))^6*A*b^3*c^(7/2) + 1337*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^5*c^(5/2) -
1673*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^4*c^(7/2) - 504*(sqrt(c)*x - sqrt(
c*x^2 + b))^2*B*b^6*c^(5/2) + 616*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^5*c^(
7/2) + 77*B*b^7*c^(5/2) - 93*A*b^6*c^(7/2))/(((sqrt(c)*x - sqrt(c*x^2 + b))
^2 - b)^7*b^4*sgn(x))
```

Mupad [**B**]
time = 0.56, size = 173, normalized size = 1.25

$$-\frac{\left(7\,B\,b^2-13\,A\,b\,c\right)\,\sqrt{c\,x^4+b\,x^2}}{35\,b^4\,x^6}-\frac{\left(x^2\,\left(\frac{58\,A\,c^4-42\,B\,b\,c^3}{35\,b^5}-\frac{2\,c^3\,\left(93\,A\,c-77\,B\,b\right)}{35\,b^5}\right)-\frac{c^2\,\left(93\,A\,c-77\,B\,b\right)}{35\,b^4}\right)\,\sqrt{c\,x^4+b\,x^2}}{x^2\,\left(c\,x^2+b\right)}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{7\,b^2\,x^8}-\frac{c\,\left(29\,A\,c-21\,B\,b\right)\,\sqrt{c\,x^4+b\,x^2}}{35\,b^4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] - ((7*B*b^2 - 13*A*b*c)*(b*x^2 + c*x^4)^(1/2))/(35*b^4*x^6) - ((x^2*((58*A*
c^4 - 42*B*b*c^3)/(35*b^5) - (2*c^3*(93*A*c - 77*B*b))/(35*b^5)) - (c^2*(93
*A*c - 77*B*b))/(35*b^4))*(b*x^2 + c*x^4)^(1/2))/(x^2*(b + c*x^2)) - (A*(b*
x^2 + c*x^4)^(1/2))/(7*b^2*x^8) - (c*(29*A*c - 21*B*b)*(b*x^2 + c*x^4)^(1/2
))/(35*b^4*x^4)
```

3.153 $\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}}+\frac{8b(6bB-5Ac)\sqrt{bx^2+cx^4}}{15c^4x}-\frac{4(6bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^3}+\frac{(6bB-5Ac)x^3\sqrt{bx^2+cx^4}}{5bc^2}$$

```
[Out] -(-A*c+B*b)*x^7/b/c/(c*x^4+b*x^2)^(1/2)+8/15*b*(-5*A*c+6*B*b)*(c*x^4+b*x^2)
^(1/2)/c^4/x-4/15*(-5*A*c+6*B*b)*x*(c*x^4+b*x^2)^(1/2)/c^3+1/5*(-5*A*c+6*B*
b)*x^3*(c*x^4+b*x^2)^(1/2)/b/c^2
```

Rubi [A]
time = 0.16, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2062, 2041, 1602}

$$\frac{8b\sqrt{bx^2+cx^4}(6bB-5Ac)}{15c^4x}-\frac{4x\sqrt{bx^2+cx^4}(6bB-5Ac)}{15c^3}+\frac{x^3\sqrt{bx^2+cx^4}(6bB-5Ac)}{5bc^2}-\frac{x^7(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^7)/(b*c*Sqrt[b*x^2 + c*x^4])) + (8*b*(6*b*B - 5*A*c)*Sqrt[
b*x^2 + c*x^4])/(15*c^4*x) - (4*(6*b*B - 5*A*c)*x*Sqrt[b*x^2 + c*x^4])/(15*
c^3) + ((6*b*B - 5*A*c)*x^3*Sqrt[b*x^2 + c*x^4])/(5*b*c^2)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}}+\frac{(6bB-5Ac)\int\frac{x^6}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}}+\frac{(6bB-5Ac)x^3\sqrt{bx^2+cx^4}}{5bc^2}-\frac{(4(6bB-5Ac))\int\frac{x^4}{\sqrt{bx^2+cx^4}}\,d}{5c^2}$$

$$= -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}}-\frac{4(6bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^3}+\frac{(6bB-5Ac)x^3\sqrt{bx^2+cx^4}}{5bc^2}+\frac{(8}{}$$

$$= -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}}+\frac{8b(6bB-5Ac)\sqrt{bx^2+cx^4}}{15c^4x}-\frac{4(6bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^3}+\frac{(6}{}$$

Mathematica [A]
time = 0.06, size = 82, normalized size = 0.59

$$\frac{x(48b^3B-8b^2c(5A-3Bx^2)+c^3x^4(5A+3Bx^2)-2bc^2x^2(10A+3Bx^2))}{15c^4\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(48*b^3*B - 8*b^2*c*(5*A - 3*B*x^2) + c^3*x^4*(5*A + 3*B*x^2) - 2*b*c^2*
x^2*(10*A + 3*B*x^2)))/(15*c^4*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.38, size = 91, normalized size = 0.65

method	result	size
gosper	$-\frac{(c\,x^2+b)\left(-3B\,c^3x^6-5A\,c^3x^4+6Bb\,c^2x^4+20Ab\,c^2x^2-24B\,b^2c\,x^2+40A\,b^2c-48B\,b^3\right)x^3}{15c^4\left(x^4c+b\,x^2\right)^{\frac{3}{2}}}$	91
default	$-\frac{(c\,x^2+b)\left(-3B\,c^3x^6-5A\,c^3x^4+6Bb\,c^2x^4+20Ab\,c^2x^2-24B\,b^2c\,x^2+40A\,b^2c-48B\,b^3\right)x^3}{15c^4\left(x^4c+b\,x^2\right)^{\frac{3}{2}}}$	91

trager	$-\frac{(-3B\,c^3x^6-5A\,c^3x^4+6Bb\,c^2x^4+20Ab\,c^2x^2-24B\,b^2c\,x^2+40A\,b^2c-48B\,b^3)\sqrt{x^4c+b\,x^2}}{15(c\,x^2+b)c^4x}$	93
risch	$-\frac{(-3B\,c^2x^4-5A\,c^2x^2+9bB\,x^2c+25Abc-33b^2B)(c\,x^2+b)x}{15c^4\sqrt{x^2\,(c\,x^2+b)}}-\frac{b^2(Ac-Bb)x}{c^4\sqrt{x^2\,(c\,x^2+b)}}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(c*x^2+b)*(-3*B*c^3*x^6-5*A*c^3*x^4+6*B*b*c^2*x^4+20*A*b*c^2*x^2-24*B
*b^2*c*x^2+40*A*b^2*c-48*B*b^3)*x^3/c^4/(c*x^4+b*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 82, normalized size = 0.59

$$\frac{(c^2x^4-4\,bcx^2-8\,b^2)A}{3\,\sqrt{cx^2+b}\,c^3}+\frac{(c^3x^6-2\,bc^2x^4+8\,b^2cx^2+16\,b^3)B}{5\,\sqrt{cx^2+b}\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*x^4 - 4*b*c*x^2 - 8*b^2)*A/(sqrt(c*x^2 + b)*c^3) + 1/5*(c^3*x^6 -
2*b*c^2*x^4 + 8*b^2*c*x^2 + 16*b^3)*B/(sqrt(c*x^2 + b)*c^4)
```

Fricas [A]
time = 1.84, size = 93, normalized size = 0.67

$$\frac{(3\,Bc^3x^6-(6\,Bbc^2-5\,Ac^3)x^4+48\,Bb^3-40\,Ab^2c+4\,(6\,Bb^2c-5\,Abc^2)x^2)\sqrt{cx^4+bx^2}}{15\,(c^5x^3+bc^4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*c^3*x^6 - (6*B*b*c^2 - 5*A*c^3)*x^4 + 48*B*b^3 - 40*A*b^2*c + 4*(
6*B*b^2*c - 5*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^5*x^3 + b*c^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^8(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**8*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 0.84, size = 145, normalized size = 1.04

$$-\frac{8\,(6\,Bb^3-5\,Ab^2c)\mathrm{sgn}(x)}{15\,\sqrt{b}\,c^4}+\frac{Bb^3-Ab^2c}{\sqrt{cx^2+b}\,c^4\mathrm{sgn}\,(x)}+\frac{3\,(cx^2+b)^{\frac{5}{2}}Bc^{16}-15\,(cx^2+b)^{\frac{3}{2}}Bbc^{16}+45\,\sqrt{cx^2+b}\,Bb^2c^{16}+5\,(cx^2+b)^{\frac{3}{2}}Ac^{17}-30\,\sqrt{cx^2+b}\,Abc^{17}}{15\,c^{20}\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -8/15*(6*B*b^3 - 5*A*b^2*c)*sgn(x)/(sqrt(b)*c^4) + (B*b^3 - A*b^2*c)/(sqrt(
c*x^2 + b)*c^4*sgn(x)) + 1/15*(3*(c*x^2 + b)^(5/2)*B*c^16 - 15*(c*x^2 + b)^
(3/2)*B*b*c^16 + 45*sqrt(c*x^2 + b)*B*b^2*c^16 + 5*(c*x^2 + b)^(3/2)*A*c^17
 - 30*sqrt(c*x^2 + b)*A*b*c^17)/(c^20*sgn(x))
```

Mupad [B]
time = 0.40, size = 92, normalized size = 0.66

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(48\,B\,b^3+24\,B\,b^2\,c\,x^2-40\,A\,b^2\,c-6\,B\,b\,c^2\,x^4-20\,A\,b\,c^2\,x^2+3\,B\,c^3\,x^6+5\,A\,c^3\,x^4\right)}{15\,c^4\,x\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*(48*B*b^3 + 5*A*c^3*x^4 + 3*B*c^3*x^6 - 40*A*b^2*c -
 20*A*b*c^2*x^2 + 24*B*b^2*c*x^2 - 6*B*b*c^2*x^4))/(15*c^4*x*(b + c*x^2))
```

3.154 $\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{(bB-Ac)x^5}{bc\sqrt{bx^2+cx^4}}-\frac{2(4bB-3Ac)\sqrt{bx^2+cx^4}}{3c^3x}+\frac{(4bB-3Ac)x\sqrt{bx^2+cx^4}}{3bc^2}$$

```
[Out] -(-A*c+B*b)*x^5/b/c/(c*x^4+b*x^2)^(1/2)-2/3*(-3*A*c+4*B*b)*(c*x^4+b*x^2)^(1
/2)/c^3/x+1/3*(-3*A*c+4*B*b)*x*(c*x^4+b*x^2)^(1/2)/b/c^2
```

Rubi [A]
time = 0.13, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2062, 2041, 1602}

$$-\frac{2\sqrt{bx^2+cx^4}\,(4bB-3Ac)}{3c^3x}+\frac{x\sqrt{bx^2+cx^4}\,(4bB-3Ac)}{3bc^2}-\frac{x^5(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^5)/(b*c*Sqrt[b*x^2 + c*x^4])) - (2*(4*b*B - 3*A*c)*Sqrt[b*
x^2 + c*x^4])/(3*c^3*x) + ((4*b*B - 3*A*c)*x*Sqrt[b*x^2 + c*x^4])/(3*b*c^2)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
```

```
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^5}{bc\sqrt{bx^2+cx^4}} + \frac{(4bB-3Ac)\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^5}{bc\sqrt{bx^2+cx^4}} + \frac{(4bB-3Ac)x\sqrt{bx^2+cx^4}}{3bc^2} - \frac{(2(4bB-3Ac))\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{3c^2}$$

$$= -\frac{(bB-Ac)x^5}{bc\sqrt{bx^2+cx^4}} - \frac{2(4bB-3Ac)\sqrt{bx^2+cx^4}}{3c^3x} + \frac{(4bB-3Ac)x\sqrt{bx^2+cx^4}}{3bc^2}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.58

$$\frac{x(-8b^2B+c^2x^2(3A+Bx^2)+b(6Ac-4Bcx^2))}{3c^3\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(-8*b^2*B + c^2*x^2*(3*A + B*x^2) + b*(6*A*c - 4*B*c*x^2)))/(3*c^3*Sqrt[
x^2*(b + c*x^2)])
```

Maple [A]
time = 0.38, size = 66, normalized size = 0.63

method	result	size
gosper	$\frac{(c\,x^2+b)\left(B\,c^2x^4+3A\,c^2x^2-4bB\,x^2c+6Abc-8b^2B\right)x^3}{3c^3\left(x^4c+b\,x^2\right)^{\frac{3}{2}}}$	66
default	$\frac{(c\,x^2+b)\left(B\,c^2x^4+3A\,c^2x^2-4bB\,x^2c+6Abc-8b^2B\right)x^3}{3c^3\left(x^4c+b\,x^2\right)^{\frac{3}{2}}}$	66
trager	$\frac{\left(B\,c^2x^4+3A\,c^2x^2-4bB\,x^2c+6Abc-8b^2B\right)\sqrt{x^4c+b\,x^2}}{3(c\,x^2+b)c^3x}$	68
risch	$\frac{(Bc\,x^2+3Ac-5Bb)\left(c\,x^2+b\right)x}{3c^3\sqrt{x^2\left(c\,x^2+b\right)}}+\frac{b(Ac-Bb)x}{c^3\sqrt{x^2\left(c\,x^2+b\right)}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c*x^2+b)*(B*c^2*x^4+3*A*c^2*x^2-4*B*b*c*x^2+6*A*b*c-8*B*b^2)*x^3/c^3/(
c*x^4+b*x^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 59, normalized size = 0.57

$$\frac{(cx^2 + 2\,b)A}{\sqrt{cx^2 + b}\,c^2} + \frac{(c^2x^4 - 4\,bcx^2 - 8\,b^2)B}{3\,\sqrt{cx^2 + b}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (c*x^2 + 2*b)*A/(sqrt(c*x^2 + b)*c^2) + 1/3*(c^2*x^4 - 4*b*c*x^2 - 8*b^2)*B
/(sqrt(c*x^2 + b)*c^3)
```

Fricas [A]
time = 1.48, size = 68, normalized size = 0.65

$$\frac{(Bc^2x^4 - 8\,Bb^2 + 6\,Abc - (4\,Bbc - 3\,Ac^2)x^2)\sqrt{cx^4 + bx^2}}{3\,(c^4x^3 + bc^3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*c^2*x^4 - 8*B*b^2 + 6*A*b*c - (4*B*b*c - 3*A*c^2)*x^2)*sqrt(c*x^4 +
b*x^2)/(c^4*x^3 + b*c^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6(A + Bx^2)}{(x^2\,(b + cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**6*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 1.39, size = 107, normalized size = 1.03

$$\frac{2\,(4\,Bb^2 - 3\,Abc)\mathrm{sgn}(x)}{3\,\sqrt{b}\,c^3} - \frac{Bb^2 - Abc}{\sqrt{cx^2 + b}\,c^3\mathrm{sgn}\,(x)} + \frac{(cx^2 + b)^{\frac{3}{2}}Bc^6 - 6\,\sqrt{cx^2 + b}\,Bbc^6 + 3\,\sqrt{cx^2 + b}\,Ac^7}{3\,c^9\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(4*B*b^2 - 3*A*b*c)*sgn(x)/(sqrt(b)*c^3) - (B*b^2 - A*b*c)/(sqrt(c*x^2
+ b)*c^3*sgn(x)) + 1/3*((c*x^2 + b)^(3/2)*B*c^6 - 6*sqrt(c*x^2 + b)*B*b*c^6
 + 3*sqrt(c*x^2 + b)*A*c^7)/(c^9*sgn(x))
```

Mupad [B]
time = 0.28, size = 67, normalized size = 0.64

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-8\,B\,b^2-4\,B\,b\,c\,x^2+6\,A\,b\,c+B\,c^2\,x^4+3\,A\,c^2\,x^2\right)}{3\,c^3\,x\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*(3*A*c^2*x^2 - 8*B*b^2 + B*c^2*x^4 + 6*A*b*c - 4*B*b
*c*x^2))/(3*c^3*x*(b + c*x^2))
```

3.155 $\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=69

$$-\frac{(bB-Ac)x^3}{bc\sqrt{bx^2+cx^4}}+\frac{(2bB-Ac)\sqrt{bx^2+cx^4}}{bc^2x}$$

```
[Out] -(-A*c+B*b)*x^3/b/c/(c*x^4+b*x^2)^(1/2)+(-A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b/
c^2/x
```

Rubi [A]
time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2062, 1602}

$$\frac{\sqrt{bx^2+cx^4}\,(2bB-Ac)}{bc^2x}-\frac{x^3(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^3)/(b*c*Sqrt[b*x^2 + c*x^4])) + ((2*b*B - A*c)*Sqrt[b*x^2
+ c*x^4])/(b*c^2*x)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^3}{bc\sqrt{bx^2+cx^4}} + \frac{(2bB-Ac)\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$
$$= -\frac{(bB-Ac)x^3}{bc\sqrt{bx^2+cx^4}} + \frac{(2bB-Ac)\sqrt{bx^2+cx^4}}{bc^2x}$$

Mathematica [**A**]
time = 0.03, size = 35, normalized size = 0.51

$$\frac{x(2bB-Ac+Bcx^2)}{c^2\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*b*B - A*c + B*c*x^2))/(c^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [**A**]
time = 0.38, size = 44, normalized size = 0.64

method	result	size
gosper	$-\frac{(c\,x^2+b)\,(-Bc\,x^2+Ac-2Bb)\,x^3}{c^2(x^4c+b\,x^2)^{\frac{3}{2}}}$	44
default	$-\frac{(c\,x^2+b)\,(-Bc\,x^2+Ac-2Bb)\,x^3}{c^2(x^4c+b\,x^2)^{\frac{3}{2}}}$	44
trager	$-\frac{(-Bc\,x^2+Ac-2Bb)\sqrt{x^4c+b\,x^2}}{(c\,x^2+b)c^2x}$	46
risch	$\frac{B(c\,x^2+b)x}{c^2\sqrt{x^2\,(c\,x^2+b)}} - \frac{(Ac-Bb)x}{c^2\sqrt{x^2\,(c\,x^2+b)}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2+b)*(-B*c*x^2+A*c-2*B*b)*x^3/c^2/(c*x^4+b*x^2)^(3/2)
```

Maxima [**A**]
time = 0.28, size = 39, normalized size = 0.57

$$\frac{(cx^2+2\,b)B}{\sqrt{cx^2+b}\,c^2} - \frac{A}{\sqrt{cx^2+b}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] (c*x^2 + 2*b)*B/(sqrt(c*x^2 + b)*c^2) - A/(sqrt(c*x^2 + b)*c)
```

Fricas [A]
time = 1.56, size = 45, normalized size = 0.65

$$\frac{\sqrt{cx^4 + bx^2}\,(Bcx^2 + 2\,Bb - Ac)}{c^3x^3 + bc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^4 + b*x^2)*(B*c*x^2 + 2*B*b - A*c)/(c^3*x^3 + b*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(A + Bx^2)}{(x^2\,(b + cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**4*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 1.24, size = 63, normalized size = 0.91

$$-\frac{(2\,Bb - Ac)\text{sgn}(x)}{\sqrt{b}\,c^2} + \frac{\sqrt{cx^2 + b}\,B}{c^2\text{sgn}\,(x)} + \frac{Bb - Ac}{\sqrt{cx^2 + b}\,c^2\text{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] -(2*B*b - A*c)*sgn(x)/(sqrt(b)*c^2) + sqrt(c*x^2 + b)*B/(c^2*sgn(x)) + (B*b
 - A*c)/(sqrt(c*x^2 + b)*c^2*sgn(x))
```

Mupad [B]
time = 0.21, size = 44, normalized size = 0.64

$$\frac{\sqrt{c\,x^4 + b\,x^2}\,\left(B\,c\,x^2 - A\,c + 2\,B\,b\right)}{c^2\,x\,\left(c\,x^2 + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] ((b*x^2 + c*x^4)^(1/2)*(2*B*b - A*c + B*c*x^2))/(c^2*x*(b + c*x^2))
```

3.156 $\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=64

$$-\frac{(bB-Ac)x}{bc\sqrt{bx^2+cx^4}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

```
[Out] -A*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-(-A*c+B*b)*x/b/c/(c*x^4+b
*x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {2062, 2033, 212}

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}-\frac{x(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x)/(b*c*Sqrt[b*x^2 + c*x^4])) - (A*ArcTanh[(Sqrt[b]*x)/Sqrt[
b*x^2 + c*x^4]])/b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
```

```
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx &= -\frac{(bB-Ac)x}{bc\sqrt{bx^2+cx^4}}+\frac{A\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{b}\\ &= -\frac{(bB-Ac)x}{bc\sqrt{bx^2+cx^4}}-\frac{A\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{b}\\ &= -\frac{(bB-Ac)x}{bc\sqrt{bx^2+cx^4}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 73, normalized size = 1.14

$$-\frac{x\left(\sqrt{b}\,(bB-Ac)+Ac\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{b^{3/2}c\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((x*(Sqrt[b]*(b*B - A*c) + A*c*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqr
t[b]]))/(b^(3/2)*c*Sqrt[x^2*(b + c*x^2)]))
```

Maple [A]
time = 0.53, size = 79, normalized size = 1.23

method	result	size
default	$-\frac{x^{3}\left(c\,x^{2}+b\right)\left(A\sqrt{c\,x^{2}+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)bc-A\,b^{\frac{3}{2}}c+B\,b^{\frac{5}{2}}\right)}{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}c\,b^{\frac{5}{2}}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x^3*(c*x^2+b)*(A*(c*x^2+b)^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*c-A
*b^(3/2)*c+B*b^(5/2))/(c*x^4+b*x^2)^(3/2)/c/b^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^2/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [A]
time = 1.65, size = 199, normalized size = 3.11

$$\left[\frac{\left(Ac^{2}x^{3}+Abcx\right)\sqrt{b}\log\left(-\frac{cx^{3}+2\,bx-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{b}}{x^{3}}\right)-2\,\sqrt{cx^{4}+bx^{2}}\,\left(Bb^{2}-Abc\right)}{2\,\left(b^{2}c^{2}x^{3}+b^{3}cx\right)},\ \frac{\left(Ac^{2}x^{3}+Abcx\right)\sqrt{-b}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-b}}{cx^{3}+bx}\right)-\sqrt{cx^{4}+bx^{2}}\,\left(Bb^{2}-Abc\right)}{b^{2}c^{2}x^{3}+b^{3}cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((A*c^2*x^3 + A*b*c*x)*sqrt(b)*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*
x^2)*sqrt(b))/x^3) - 2*sqrt(c*x^4 + b*x^2)*(B*b^2 - A*b*c))/(b^2*c^2*x^3 +
b^3*c*x), ((A*c^2*x^3 + A*b*c*x)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-
b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*(B*b^2 - A*b*c))/(b^2*c^2*x^3 + b^3
*c*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\left(A+Bx^{2}\right)}{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 0.93, size = 109, normalized size = 1.70

$$\frac{A\arctan\left(\frac{\sqrt{cx^{2}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b\operatorname{sgn}\left(x\right)}-\frac{\left(A\sqrt{b}\,c\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)-B\sqrt{-b}\,b+A\sqrt{-b}\,c\right)\operatorname{sgn}(x)}{\sqrt{-b}\,b^{\frac{3}{2}}c}-\frac{Bb-Ac}{\sqrt{cx^{2}+b}\,bc\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] A*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b*sgn(x)) - (A*sqrt(b)*c*arcta
n(sqrt(b)/sqrt(-b)) - B*sqrt(-b)*b + A*sqrt(-b)*c)*sgn(x)/(sqrt(-b)*b^(3/2)
*c) - (B*b - A*c)/(sqrt(c*x^2 + b)*b*c*sgn(x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.157 $\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{B}{3cx\sqrt{bx^2+cx^4}}-\frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}}+\frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{2b^2cx^3}-\frac{(2bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}$$

```
[Out] -1/2*(-3*A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2)-1/3*B/c/
x/(c*x^4+b*x^2)^(1/2)+1/3*(3*A*c-2*B*b)/b/c/x/(c*x^4+b*x^2)^(1/2)+1/2*(-3*A
*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/c/x^3
```

Rubi [A]
time = 0.06, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1159, 2031, 2050, 2033, 212}

$$-\frac{(2bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}+\frac{\sqrt{bx^2+cx^4}\,(2bB-3Ac)}{2b^2cx^3}-\frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}}-\frac{B}{3cx\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -1/3*B/(c*x*Sqrt[b*x^2 + c*x^4]) - (2*b*B - 3*A*c)/(3*b*c*x*Sqrt[b*x^2 + c*
x^4]) + ((2*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(2*b^2*c*x^3) - ((2*b*B - 3*A
*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[e*((b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 3)*x)), x] - Dist[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3)), Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
c*d*(4*p + 3), 0]
```

Rule 2031

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
```

```
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= -\frac{B}{3cx\sqrt{bx^2+cx^4}}+\frac{(-2bB+3Ac)\int \frac{1}{\left(bx^2+cx^4\right)^{3/2}}\,dx}{3c}\\ &= -\frac{B}{3cx\sqrt{bx^2+cx^4}}-\frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}}+\frac{(-2bB+3Ac)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{bc}\\ &= -\frac{B}{3cx\sqrt{bx^2+cx^4}}-\frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}}+\frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{2b^2cx^3}+\frac{(2bB-3Ac)\int}{2}\\ &= -\frac{B}{3cx\sqrt{bx^2+cx^4}}-\frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}}+\frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{2b^2cx^3}-\frac{(2bB-3Ac)\mathrm{Su}}{}\\ &= -\frac{B}{3cx\sqrt{bx^2+cx^4}}-\frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}}+\frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{2b^2cx^3}-\frac{(2bB-3Ac)\,\mathrm{ta}}{}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 96, normalized size = 0.68

$$\frac{\sqrt{b}\left(2bBx^2-A(b+3cx^2)\right)-(2bB-3Ac)x^2\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{2b^{5/2}x\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[b]*(2*b*B*x^2 - A*(b + 3*c*x^2)) - (2*b*B - 3*A*c)*x^2*Sqrt[b + c*x^2
]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(2*b^(5/2)*x*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.45, size = 130, normalized size = 0.92

method	result
default	$\frac{x\left(c\,x^{2}+b\right)\left(3A\sqrt{c\,x^{2}+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)b c\,x^{2}-3A\,b^{\frac{3}{2}}c\,x^{2}-2B\sqrt{c\,x^{2}+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)b^{2}}{2\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}b^{\frac{7}{2}}}$
risch	$-\frac{A\left(c\,x^{2}+b\right)}{2b^{2}x\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(-\frac{Ac}{b^{2}\sqrt{c\,x^{2}+b}}+\frac{B}{b\sqrt{c\,x^{2}+b}}+\frac{3\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)Ac}{2b^{\frac{5}{2}}}-\frac{\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}}}{x}\right)}{b^{\frac{3}{2}}}\right.}{\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(c*x^2+b)*(3*A*(c*x^2+b)^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*
c*x^2-3*A*b^(3/2)*c*x^2-2*B*(c*x^2+b)^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b
)/x)*b^2*x^2+2*B*b^(5/2)*x^2-A*b^(5/2))/(c*x^4+b*x^2)^(3/2)/b^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [A]
time = 1.63, size = 260, normalized size = 1.83

$$\left[-\frac{\left(\left(2\,Bbc-3\,Ac^{2}\right)x^{5}+\left(2\,Bb^{2}-3\,Abc\right)x^{3}\right)\sqrt{b}\,\log\left(-\frac{cx^{3}+2\,bx+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{b}}{x^{3}}\right)+2\,\sqrt{cx^{4}+bx^{2}}\left(Ab^{2}-\left(2\,Bb^{2}-3\,Abc\right)x^{2}\right)}{4\left(b^{3}cx^{5}+b^{4}x^{3}\right)},\ \frac{\left(\left(2\,Bbc-3\,Ac^{2}\right)x^{5}+\left(2\,Bb^{2}-3\,Abc\right)x^{3}\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-b}}{cx^{3}+bx}\right)-\sqrt{cx^{4}+bx^{2}}\left(Ab^{2}-\left(2\,Bb^{2}-3\,Abc\right)x^{2}\right)}{2\left(b^{3}cx^{5}+b^{4}x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*B*b*c - 3*A*c^2)*x^5 + (2*B*b^2 - 3*A*b*c)*x^3)*sqrt(b)*log(-(c*
x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(
A*b^2 - (2*B*b^2 - 3*A*b*c)*x^2))/(b^3*c*x^5 + b^4*x^3), 1/2*(((2*B*b*c - 3
*A*c^2)*x^5 + (2*B*b^2 - 3*A*b*c)*x^3)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*
sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*(A*b^2 - (2*B*b^2 - 3*A*b*c)*
x^2))/(b^3*c*x^5 + b^4*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 0.64, size = 107, normalized size = 0.75

$$\frac{(2\,Bb-3\,Ac)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{2\,\sqrt{-b}\,b^2\mathrm{sgn}\left(x\right)}+\frac{2\left(cx^2+b\right)Bb-2\,Bb^2-3\left(cx^2+b\right)Ac+2\,Abc}{2\left(\left(cx^2+b\right)^{\frac{3}{2}}-\sqrt{cx^2+b}\,b\right)b^2\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*B*b - 3*A*c)*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2*sgn(x))
+ 1/2*(2*(c*x^2 + b)*B*b - 2*B*b^2 - 3*(c*x^2 + b)*A*c + 2*A*b*c)/(((c*x^2
+ b)^(3/2) - sqrt(c*x^2 + b)*b)*b^2*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((A + B*x^2)/(b*x^2 + c*x^4)^(3/2), x)
```

3.158 $\int \frac{A+Bx^2}{x^2(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{A}{4bx^3\sqrt{bx^2+cx^4}}+\frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}}-\frac{3(4bB-5Ac)\sqrt{bx^2+cx^4}}{8b^3x^3}+\frac{3c(4bB-5Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}$$

```
[Out] 3/8*c*(-5*A*c+4*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(7/2)-1/4*A/b
/x^3/(c*x^4+b*x^2)^(1/2)+1/4*(-5*A*c+4*B*b)/b^2/x/(c*x^4+b*x^2)^(1/2)-3/8*(
-5*A*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^3
```

Rubi [A]
time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2063, 2031, 2050, 2033, 212}

$$\frac{3c(4bB-5Ac)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}-\frac{3\sqrt{bx^2+cx^4}\,(4bB-5Ac)}{8b^3x^3}+\frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}}-\frac{A}{4bx^3\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/4*A/(b*x^3*Sqrt[b*x^2 + c*x^4]) + (4*b*B - 5*A*c)/(4*b^2*x*Sqrt[b*x^2 +
c*x^4]) - (3*(4*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(8*b^3*x^3) + (3*c*(4*b*B
 - 5*A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2031

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx &= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}}-\frac{(-4bB+5Ac)\int \frac{1}{\left(bx^2+cx^4\right)^{3/2}}\,dx}{4b}\\
&= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}}+\frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}}+\frac{(3(4bB-5Ac))\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b^2}\\
&= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}}+\frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}}-\frac{3(4bB-5Ac)\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{(3c(4bB-}{}\\
&= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}}+\frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}}-\frac{3(4bB-5Ac)\sqrt{bx^2+cx^4}}{8b^3x^3}+\frac{(3c(4bB-}{}\\
&= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}}+\frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}}-\frac{3(4bB-5Ac)\sqrt{bx^2+cx^4}}{8b^3x^3}+\frac{3c(4bB-}{}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 117, normalized size = 0.85

$$\frac{\sqrt{b}\left(-4bBx^2(b+3cx^2)+A(-2b^2+5bcx^2+15c^2x^4)\right)+3c(4bB-5Ac)x^4\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{8b^{7/2}x^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[b]*(-4*b*B*x^2*(b + 3*c*x^2) + A*(-2*b^2 + 5*b*c*x^2 + 15*c^2*x^4)) +
 3*c*(4*b*B - 5*A*c)*x^4*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/
(8*b^(7/2)*x^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.40, size = 157, normalized size = 1.15

method	result
default	$-\frac{(c x^2+b)\left(15A\sqrt{c x^2+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c x^2+b}}{x}\right) b c^2 x^4-15A b^{\frac{3}{2}} c^2 x^4-12B\sqrt{c x^2+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c x^2+}}{x}\right.\right.}{8x(x^4c+b x^2)^{\frac{3}{2}} b^{\frac{9}{2}}}$
risch	$-\frac{(c x^2+b)(-7Ac x^2+4bB x^2+2Ab)}{8b^3x^3\sqrt{x^2(c x^2+b)}}+\frac{\left(\frac{c^2A}{b^3\sqrt{c x^2+b}}-\frac{cB}{b^2\sqrt{c x^2+b}}-\frac{15c^2\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c x^2+b}}{x}\right)A}{8b^{\frac{7}{2}}}+\frac{3c\ln\left(\frac{2b+2}{}\right.}{}\right.}{\sqrt{x^2(c x^2+b)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/x*(c*x^2+b)*(15*A*(c*x^2+b)^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*
b*c^2*x^4-15*A*b^(3/2)*c^2*x^4-12*B*(c*x^2+b)^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)
^(1/2)+b)/x)*b^2*c*x^4+12*B*b^(5/2)*c*x^4-5*A*b^(5/2)*c*x^2+4*B*b^(7/2)*x^2
+2*A*b^(7/2))/(c*x^4+b*x^2)^(3/2)/b^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^2), x)
```

Fricas [A]
time = 1.18, size = 315, normalized size = 2.30

$$\left[-\frac{3\left((4\,Bbc^2-5\,Ac^3)x^7+(4\,Bb^2c-5\,Abc^2)x^5\right)\sqrt{b}\,\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\left(3\,(4\,Bb^2c-5\,Abc^2)x^4+2\,Ab^3+(4\,Bb^3-5\,Ab^2c)x^2\right)\sqrt{cx^4+bx^2}}{16\,(b^4cx^7+b^5x^5)},-\frac{3\left((4\,Bbc^2-5\,Ac^3)x^7+(4\,Bb^2c-5\,Abc^2)x^5\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\left(3\,(4\,Bb^2c-5\,Abc^2)x^4+2\,Ab^3+(4\,Bb^3-5\,Ab^2c)x^2\right)\sqrt{cx^4+bx^2}}{8\,(b^4cx^7+b^5x^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((4*B*b*c^2 - 5*A*c^3)*x^7 + (4*B*b^2*c - 5*A*b*c^2)*x^5)*sqrt(b)
*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(3*(4*B*b^2*
c - 5*A*b*c^2)*x^4 + 2*A*b^3 + (4*B*b^3 - 5*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^
2))/(b^4*c*x^7 + b^5*x^5), -1/8*(3*((4*B*b*c^2 - 5*A*c^3)*x^7 + (4*B*b^2*c
- 5*A*b*c^2)*x^5)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x
)) + (3*(4*B*b^2*c - 5*A*b*c^2)*x^4 + 2*A*b^3 + (4*B*b^3 - 5*A*b^2*c)*x^2)*
sqrt(c*x^4 + b*x^2))/(b^4*c*x^7 + b^5*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^2\,(x^2\,(b + cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**2*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [A]
time = 0.52, size = 149, normalized size = 1.09

$$-\frac{3\,(4\,Bbc - 5\,Ac^2)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{8\,\sqrt{-b}\,b^3\mathrm{sgn}\,(x)} - \frac{Bbc - Ac^2}{\sqrt{cx^2+b}\,b^3\mathrm{sgn}\,(x)} - \frac{4\,(cx^2+b)^{\frac{3}{2}}Bbc - 4\,\sqrt{cx^2+b}\,Bb^2c - 7\,(cx^2+b)^{\frac{3}{2}}Ac^2 + 9\,\sqrt{cx^2+b}\,Abc^2}{8\,b^3c^2x^4\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -3/8*(4*B*b*c - 5*A*c^2)*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^3*sgn
(x)) - (B*b*c - A*c^2)/(sqrt(c*x^2 + b)*b^3*sgn(x)) - 1/8*(4*(c*x^2 + b)^(3
/2)*B*b*c - 4*sqrt(c*x^2 + b)*B*b^2*c - 7*(c*x^2 + b)^(3/2)*A*c^2 + 9*sqrt(
c*x^2 + b)*A*b*c^2)/(b^3*c^2*x^4*sgn(x))
```

Mupad [B]
time = 1.26, size = 89, normalized size = 0.65

$$-\frac{A\left(\frac{b}{c\,x^2}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{b}{c\,x^2}\right)}{7\,x\,\left(c\,x^4+b\,x^2\right)^{3/2}} - \frac{B\,x\,\left(\frac{b}{c\,x^2}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{b}{c\,x^2}\right)}{5\,\left(c\,x^4+b\,x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] - (A*(b/(c*x^2) + 1)^(3/2)*hypergeom([3/2, 7/2], 9/2, -b/(c*x^2)))/(7*x*(b*
x^2 + c*x^4)^(3/2)) - (B*x*(b/(c*x^2) + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2,
 -b/(c*x^2)))/(5*(b*x^2 + c*x^4)^(3/2))
```

3.159 $\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{13}Abx^{13/2}+\frac{2}{17}(bB+Ac)x^{17/2}+\frac{2}{21}Bcx^{21/2}$$

```
[Out] 2/13*A*b*x^(13/2)+2/17*(A*c+B*b)*x^(17/2)+2/21*B*c*x^(21/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1598, 459}

$$\frac{2}{17}x^{17/2}(Ac+bB)+\frac{2}{13}Abx^{13/2}+\frac{2}{21}Bcx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*A*b*x^(13/2))/13 + (2*(b*B + A*c)*x^(17/2))/17 + (2*B*c*x^(21/2))/21
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx &= \int x^{11/2}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Abx^{11/2}+(bB+Ac)x^{15/2}+Bcx^{19/2}\right)\,dx\\ &= \frac{2}{13}Abx^{13/2}+\frac{2}{17}(bB+Ac)x^{17/2}+\frac{2}{21}Bcx^{21/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.90

$$\frac{2x^{13/2}(357Ab+273bBx^2+273Acx^2+221Bcx^4)}{4641}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(13/2)*(357*A*b + 273*b*B*x^2 + 273*A*c*x^2 + 221*B*c*x^4))/4641
```

Maple [A]
time = 0.14, size = 28, normalized size = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{13}{2}}}{13}+\frac{2(Ac+Bb)x^{\frac{17}{2}}}{17}+\frac{2Bc\,x^{\frac{21}{2}}}{21}$	28
default	$\frac{2Ab\,x^{\frac{13}{2}}}{13}+\frac{2(Ac+Bb)x^{\frac{17}{2}}}{17}+\frac{2Bc\,x^{\frac{21}{2}}}{21}$	28
gosper	$\frac{2x^{\frac{13}{2}}\left(221Bc\,x^4+273Ac\,x^2+273bB\,x^2+357Ab\right)}{4641}$	32
trager	$\frac{2x^{\frac{13}{2}}\left(221Bc\,x^4+273Ac\,x^2+273bB\,x^2+357Ab\right)}{4641}$	32
risch	$\frac{2x^{\frac{13}{2}}\left(221Bc\,x^4+273Ac\,x^2+273bB\,x^2+357Ab\right)}{4641}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/13*A*b*x^(13/2)+2/17*(A*c+B*b)*x^(17/2)+2/21*B*c*x^(21/2)
```

Maxima [A]
time = 0.27, size = 27, normalized size = 0.69

$$\frac{2}{21}\,Bcx^{\frac{21}{2}}+\frac{2}{17}\,(Bb+Ac)x^{\frac{17}{2}}+\frac{2}{13}\,Abx^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/21*B*c*x^(21/2) + 2/17*(B*b + A*c)*x^(17/2) + 2/13*A*b*x^(13/2)
```

Fricas [A]
time = 1.36, size = 32, normalized size = 0.82

$$\frac{2}{4641}\left(221\,Bcx^{10}+273\,(Bb+Ac)x^8+357\,Abx^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/4641*(221*B*c*x^10 + 273*(B*b + A*c)*x^8 + 357*A*b*x^6)*sqrt(x)
```

Sympy [A]
time = 1.09, size = 46, normalized size = 1.18

$$\frac{2Abx^{\frac{13}{2}}}{13} + \frac{2Acx^{\frac{17}{2}}}{17} + \frac{2Bbx^{\frac{17}{2}}}{17} + \frac{2Bcx^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] 2*A*b*x**(13/2)/13 + 2*A*c*x**(17/2)/17 + 2*B*b*x**(17/2)/17 + 2*B*c*x**(21
/2)/21
```

Giac [A]
time = 0.46, size = 29, normalized size = 0.74

$$\frac{2}{21}\,Bcx^{\frac{21}{2}} + \frac{2}{17}\,Bbx^{\frac{17}{2}} + \frac{2}{17}\,Acx^{\frac{17}{2}} + \frac{2}{13}\,Abx^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/21*B*c*x^(21/2) + 2/17*B*b*x^(17/2) + 2/17*A*c*x^(17/2) + 2/13*A*b*x^(13/
2)
```

Mupad [B]
time = 0.06, size = 31, normalized size = 0.79

$$\frac{2\,x^{13/2}\,\left(357\,A\,b+273\,A\,c\,x^2+273\,B\,b\,x^2+221\,B\,c\,x^4\right)}{4641}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(13/2)*(357*A*b + 273*A*c*x^2 + 273*B*b*x^2 + 221*B*c*x^4))/4641
```

3.160 $\int x^{5/2}(A+Bx^2)(bx^2+cx^4)\,dx$

Optimal. Leaf size=39

$$\frac{2}{11}Abx^{11/2}+\frac{2}{15}(bB+Ac)x^{15/2}+\frac{2}{19}Bcx^{19/2}$$

```
[Out] 2/11*A*b*x^(11/2)+2/15*(A*c+B*b)*x^(15/2)+2/19*B*c*x^(19/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1598, 459}

$$\frac{2}{15}x^{15/2}(Ac+bB)+\frac{2}{11}Abx^{11/2}+\frac{2}{19}Bcx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*A*b*x^(11/2))/11 + (2*(b*B + A*c)*x^(15/2))/15 + (2*B*c*x^(19/2))/19
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx &= \int x^{9/2}(A+Bx^2)\left(b+cx^2\right)\,dx\\ &= \int \left(Abx^{9/2}+(bB+Ac)x^{13/2}+Bcx^{17/2}\right)\,dx\\ &= \frac{2}{11}Abx^{11/2}+\frac{2}{15}(bB+Ac)x^{15/2}+\frac{2}{19}Bcx^{19/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 0.90

$$\frac{2x^{11/2}(285Ab + 209bBx^2 + 209Acx^2 + 165Bcx^4)}{3135}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
[Out] (2*x^(11/2)*(285*A*b + 209*b*B*x^2 + 209*A*c*x^2 + 165*B*c*x^4))/3135
```

Maple [A]
time = 0.14, size = 28, normalized size = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{11}{2}}}{11}+\frac{2(Ac+Bb)x^{\frac{15}{2}}}{15}+\frac{2Bc\,x^{\frac{19}{2}}}{19}$	28
default	$\frac{2Ab\,x^{\frac{11}{2}}}{11}+\frac{2(Ac+Bb)x^{\frac{15}{2}}}{15}+\frac{2Bc\,x^{\frac{19}{2}}}{19}$	28
gosper	$\frac{2x^{\frac{11}{2}}\left(165Bc\,x^{4}+209Ac\,x^{2}+209bB\,x^{2}+285Ab\right)}{3135}$	32
trager	$\frac{2x^{\frac{11}{2}}\left(165Bc\,x^{4}+209Ac\,x^{2}+209bB\,x^{2}+285Ab\right)}{3135}$	32
risch	$\frac{2x^{\frac{11}{2}}\left(165Bc\,x^{4}+209Ac\,x^{2}+209bB\,x^{2}+285Ab\right)}{3135}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 2/11*A*b*x^(11/2)+2/15*(A*c+B*b)*x^(15/2)+2/19*B*c*x^(19/2)
```

Maxima [A]
time = 0.26, size = 27, normalized size = 0.69

$$\frac{2}{19}\,Bcx^{\frac{19}{2}} + \frac{2}{15}\,(Bb + Ac)x^{\frac{15}{2}} + \frac{2}{11}\,Abx^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 2/19*B*c*x^(19/2) + 2/15*(B*b + A*c)*x^(15/2) + 2/11*A*b*x^(11/2)
```

Fricas [A]
time = 2.26, size = 32, normalized size = 0.82

$$\frac{2}{3135}\left(165\,Bcx^9 + 209\,(Bb + Ac)x^7 + 285\,Abx^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/3135*(165*B*c*x^9 + 209*(B*b + A*c)*x^7 + 285*A*b*x^5)*sqrt(x)
```

Sympy [A]
time = 0.73, size = 46, normalized size = 1.18

$$\frac{2Abx^{\frac{11}{2}}}{11}+\frac{2Acx^{\frac{15}{2}}}{15}+\frac{2Bbx^{\frac{15}{2}}}{15}+\frac{2Bcx^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] 2*A*b*x**(11/2)/11 + 2*A*c*x**(15/2)/15 + 2*B*b*x**(15/2)/15 + 2*B*c*x**(19
/2)/19
```

Giac [A]
time = 0.47, size = 29, normalized size = 0.74

$$\frac{2}{19}\,Bcx^{\frac{19}{2}}+\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{15}\,Acx^{\frac{15}{2}}+\frac{2}{11}\,Abx^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/19*B*c*x^(19/2) + 2/15*B*b*x^(15/2) + 2/15*A*c*x^(15/2) + 2/11*A*b*x^(11/
2)
```

Mupad [B]
time = 0.11, size = 31, normalized size = 0.79

$$\frac{2\,x^{11/2}\,\left(285\,A\,b+209\,A\,c\,x^2+209\,B\,b\,x^2+165\,B\,c\,x^4\right)}{3135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(11/2)*(285*A*b + 209*A*c*x^2 + 209*B*b*x^2 + 165*B*c*x^4))/3135
```

3.161 $\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{9}Abx^{9/2}+\frac{2}{13}(bB+Ac)x^{13/2}+\frac{2}{17}Bcx^{17/2}$$

```
[Out] 2/9*A*b*x^(9/2)+2/13*(A*c+B*b)*x^(13/2)+2/17*B*c*x^(17/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1598, 459}

$$\frac{2}{13}x^{13/2}(Ac+bB)+\frac{2}{9}Abx^{9/2}+\frac{2}{17}Bcx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*A*b*x^(9/2))/9 + (2*(b*B + A*c)*x^(13/2))/13 + (2*B*c*x^(17/2))/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx &= \int x^{7/2}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\
&= \int \left(Abx^{7/2}+(bB+Ac)x^{11/2}+Bcx^{15/2}\right)\,dx\\
&= \frac{2}{9}Abx^{9/2}+\frac{2}{13}(bB+Ac)x^{13/2}+\frac{2}{17}Bcx^{17/2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.05

$$\frac{2\left(221Abx^{9/2}+153bBx^{13/2}+153Acx^{13/2}+117Bcx^{17/2}\right)}{1989}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*(221*A*b*x^(9/2) + 153*b*B*x^(13/2) + 153*A*c*x^(13/2) + 117*B*c*x^(17/2
)))/1989
```

Maple [A]
time = 0.13, size = 28, normalized size = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{9}{2}}}{9}+\frac{2(Ac+Bb)x^{\frac{13}{2}}}{13}+\frac{2Bc\,x^{\frac{17}{2}}}{17}$	28
default	$\frac{2Ab\,x^{\frac{9}{2}}}{9}+\frac{2(Ac+Bb)x^{\frac{13}{2}}}{13}+\frac{2Bc\,x^{\frac{17}{2}}}{17}$	28
gosper	$\frac{2x^{\frac{9}{2}}\left(117Bc\,x^{4}+153Ac\,x^{2}+153bB\,x^{2}+221Ab\right)}{1989}$	32
trager	$\frac{2x^{\frac{9}{2}}\left(117Bc\,x^{4}+153Ac\,x^{2}+153bB\,x^{2}+221Ab\right)}{1989}$	32
risch	$\frac{2x^{\frac{9}{2}}\left(117Bc\,x^{4}+153Ac\,x^{2}+153bB\,x^{2}+221Ab\right)}{1989}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*A*b*x^(9/2)+2/13*(A*c+B*b)*x^(13/2)+2/17*B*c*x^(17/2)
```

Maxima [A]
time = 0.26, size = 27, normalized size = 0.69

$$\frac{2}{17}\,Bcx^{\frac{17}{2}}+\frac{2}{13}\,(Bb+Ac)x^{\frac{13}{2}}+\frac{2}{9}\,Abx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/17*B*c*x^(17/2) + 2/13*(B*b + A*c)*x^(13/2) + 2/9*A*b*x^(9/2)
```

Fricas [A]
time = 2.50, size = 32, normalized size = 0.82

$$\frac{2}{1989}\left(117\,Bcx^{8}+153\,(Bb+Ac)x^{6}+221\,Abx^{4}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/1989*(117*B*c*x^8 + 153*(B*b + A*c)*x^6 + 221*A*b*x^4)*sqrt(x)
```

Sympy [A]
time = 0.47, size = 46, normalized size = 1.18

$$\frac{2Abx^{\frac{9}{2}}}{9}+\frac{2Acx^{\frac{13}{2}}}{13}+\frac{2Bbx^{\frac{13}{2}}}{13}+\frac{2Bcx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] 2*A*b*x**(9/2)/9 + 2*A*c*x**(13/2)/13 + 2*B*b*x**(13/2)/13 + 2*B*c*x**(17/2
)/17
```

Giac [A]
time = 0.47, size = 29, normalized size = 0.74

$$\frac{2}{17}Bcx^{\frac{17}{2}}+\frac{2}{13}Bbx^{\frac{13}{2}}+\frac{2}{13}Acx^{\frac{13}{2}}+\frac{2}{9}Abx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/17*B*c*x^(17/2) + 2/13*B*b*x^(13/2) + 2/13*A*c*x^(13/2) + 2/9*A*b*x^(9/2)
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.79

$$\frac{2\,x^{9/2}\,\left(221\,A\,b+153\,A\,c\,x^2+153\,B\,b\,x^2+117\,B\,c\,x^4\right)}{1989}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(9/2)*(221*A*b + 153*A*c*x^2 + 153*B*b*x^2 + 117*B*c*x^4))/1989
```

3.162 $\int \sqrt{x}\,(A+Bx^2)\,(bx^2+cx^4)\,dx$

Optimal. Leaf size=39

$$\frac{2}{7}Abx^{7/2}+\frac{2}{11}(bB+Ac)x^{11/2}+\frac{2}{15}Bcx^{15/2}$$

```
[Out] 2/7*A*b*x^(7/2)+2/11*(A*c+B*b)*x^(11/2)+2/15*B*c*x^(15/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\frac{2}{11}x^{11/2}(Ac+bB)+\frac{2}{7}Abx^{7/2}+\frac{2}{15}Bcx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*A*b*x^(7/2))/7 + (2*(b*B + A*c)*x^(11/2))/11 + (2*B*c*x^(15/2))/15
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\,\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx &= \int x^{5/2}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Abx^{5/2}+(bB+Ac)x^{9/2}+Bcx^{13/2}\right)\,dx\\ &= \frac{2}{7}Abx^{7/2}+\frac{2}{11}(bB+Ac)x^{11/2}+\frac{2}{15}Bcx^{15/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.90

$$\frac{2x^{7/2}(165Ab + 105bBx^2 + 105Acx^2 + 77Bcx^4)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4),x]
[Out] (2*x^(7/2)*(165*A*b + 105*b*B*x^2 + 105*A*c*x^2 + 77*B*c*x^4))/1155
```

Maple [A]
time = 0.14, size = 28, normalized size = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{7}{2}}}{7}+\frac{2(Ac+Bb)x^{\frac{11}{2}}}{11}+\frac{2Bc\,x^{\frac{15}{2}}}{15}$	28
default	$\frac{2Ab\,x^{\frac{7}{2}}}{7}+\frac{2(Ac+Bb)x^{\frac{11}{2}}}{11}+\frac{2Bc\,x^{\frac{15}{2}}}{15}$	28
gosper	$\frac{2x^{\frac{7}{2}}\left(77Bc\,x^{4}+105Ac\,x^{2}+105bB\,x^{2}+165Ab\right)}{1155}$	32
trager	$\frac{2x^{\frac{7}{2}}\left(77Bc\,x^{4}+105Ac\,x^{2}+105bB\,x^{2}+165Ab\right)}{1155}$	32
risch	$\frac{2x^{\frac{7}{2}}\left(77Bc\,x^{4}+105Ac\,x^{2}+105bB\,x^{2}+165Ab\right)}{1155}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/7*A*b*x^(7/2)+2/11*(A*c+B*b)*x^(11/2)+2/15*B*c*x^(15/2)
```

Maxima [A]
time = 0.27, size = 27, normalized size = 0.69

$$\frac{2}{15}\,Bcx^{\frac{15}{2}}+\frac{2}{11}\,(Bb+Ac)x^{\frac{11}{2}}+\frac{2}{7}\,Abx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x, algorithm="maxima")
[Out] 2/15*B*c*x^(15/2) + 2/11*(B*b + A*c)*x^(11/2) + 2/7*A*b*x^(7/2)
```

Fricas [A]
time = 1.88, size = 32, normalized size = 0.82

$$\frac{2}{1155}\left(77\,Bcx^7+105\,(Bb+Ac)x^5+165\,Abx^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*c*x^7 + 105*(B*b + A*c)*x^5 + 165*A*b*x^3)*sqrt(x)
```

Sympy [A]
time = 1.12, size = 37, normalized size = 0.95

$$\frac{2Abx^{\frac{7}{2}}}{7} + \frac{2Bcx^{\frac{15}{2}}}{15} + \frac{2x^{\frac{11}{2}}(Ac + Bb)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)*x**(1/2),x)
```

```
[Out] 2*A*b*x**(7/2)/7 + 2*B*c*x**(15/2)/15 + 2*x**(11/2)*(A*c + B*b)/11
```

Giac [A]
time = 0.45, size = 29, normalized size = 0.74

$$\frac{2}{15}\,Bcx^{\frac{15}{2}} + \frac{2}{11}\,Bbx^{\frac{11}{2}} + \frac{2}{11}\,Acx^{\frac{11}{2}} + \frac{2}{7}\,Abx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*B*c*x^(15/2) + 2/11*B*b*x^(11/2) + 2/11*A*c*x^(11/2) + 2/7*A*b*x^(7/2)
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.79

$$\frac{2\,x^{7/2}\,\left(165\,A\,b+105\,A\,c\,x^2+105\,B\,b\,x^2+77\,B\,c\,x^4\right)}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(7/2)*(165*A*b + 105*A*c*x^2 + 105*B*b*x^2 + 77*B*c*x^4))/1155
```

3.163 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}} \, dx$

Optimal. Leaf size=39

$$\frac{2}{5}Abx^{5/2}+\frac{2}{9}(bB+Ac)x^{9/2}+\frac{2}{13}Bcx^{13/2}$$

```
[Out] 2/5*A*b*x^(5/2)+2/9*(A*c+B*b)*x^(9/2)+2/13*B*c*x^(13/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1598, 459}

$$\frac{2}{9}x^{9/2}(Ac+bB)+\frac{2}{5}Abx^{5/2}+\frac{2}{13}Bcx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/Sqrt[x],x]
```

```
[Out] (2*A*b*x^(5/2))/5 + (2*(b*B + A*c)*x^(9/2))/9 + (2*B*c*x^(13/2))/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}} \, dx &= \int x^{3/2}\left(A+Bx^2\right)\left(b+cx^2\right) \, dx \\
&= \int \left(Abx^{3/2}+(bB+Ac)x^{7/2}+Bcx^{11/2}\right) \, dx \\
&= \frac{2}{5}Abx^{5/2}+\frac{2}{9}(bB+Ac)x^{9/2}+\frac{2}{13}Bcx^{13/2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.90

$$\frac{2}{585}x^{5/2}\left(117Ab+65bBx^2+65Acx^2+45Bcx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/Sqrt[x],x]
```

```
[Out] (2*x^(5/2)*(117*A*b + 65*b*B*x^2 + 65*A*c*x^2 + 45*B*c*x^4))/585
```

Maple [A]
time = 0.13, size = 28, normalized size = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{5}{2}}}{5}+\frac{2(Ac+Bb)x^{\frac{9}{2}}}{9}+\frac{2Bc\,x^{\frac{13}{2}}}{13}$	28
default	$\frac{2Ab\,x^{\frac{5}{2}}}{5}+\frac{2(Ac+Bb)x^{\frac{9}{2}}}{9}+\frac{2Bc\,x^{\frac{13}{2}}}{13}$	28
gosper	$\frac{2x^{\frac{5}{2}}\left(45Bc\,x^4+65Ac\,x^2+65bB\,x^2+117Ab\right)}{585}$	32
trager	$\frac{2x^{\frac{5}{2}}\left(45Bc\,x^4+65Ac\,x^2+65bB\,x^2+117Ab\right)}{585}$	32
risch	$\frac{2x^{\frac{5}{2}}\left(45Bc\,x^4+65Ac\,x^2+65bB\,x^2+117Ab\right)}{585}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*A*b*x^(5/2)+2/9*(A*c+B*b)*x^(9/2)+2/13*B*c*x^(13/2)
```

Maxima [A]
time = 0.27, size = 27, normalized size = 0.69

$$\frac{2}{13}\,Bcx^{\frac{13}{2}}+\frac{2}{9}\,(Bb+Ac)x^{\frac{9}{2}}+\frac{2}{5}\,Abx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*c*x^(13/2) + 2/9*(B*b + A*c)*x^(9/2) + 2/5*A*b*x^(5/2)
```

Fricas [A]
time = 1.75, size = 32, normalized size = 0.82

$$\frac{2}{585}\left(45\,Bcx^6+65\,(Bb+Ac)x^4+117\,Abx^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
[Out] 2/585*(45*B*c*x^6 + 65*(B*b + A*c)*x^4 + 117*A*b*x^2)*sqrt(x)
```

Sympy [A]
time = 0.27, size = 46, normalized size = 1.18

$$\frac{2Abx^{\frac{5}{2}}}{5} + \frac{2Acx^{\frac{9}{2}}}{9} + \frac{2Bbx^{\frac{9}{2}}}{9} + \frac{2Bcx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(1/2),x)
[Out] 2*A*b*x**(5/2)/5 + 2*A*c*x**(9/2)/9 + 2*B*b*x**(9/2)/9 + 2*B*c*x**(13/2)/13
```

Giac [A]
time = 0.46, size = 29, normalized size = 0.74

$$\frac{2}{13}\,Bcx^{\frac{13}{2}} + \frac{2}{9}\,Bbx^{\frac{9}{2}} + \frac{2}{9}\,Acx^{\frac{9}{2}} + \frac{2}{5}\,Abx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
[Out] 2/13*B*c*x^(13/2) + 2/9*B*b*x^(9/2) + 2/9*A*c*x^(9/2) + 2/5*A*b*x^(5/2)
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.79

$$\frac{2\,x^{5/2}\,\left(117\,A\,b+65\,A\,c\,x^2+65\,B\,b\,x^2+45\,B\,c\,x^4\right)}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^(1/2),x)
[Out] (2*x^(5/2)*(117*A*b + 65*A*c*x^2 + 65*B*b*x^2 + 45*B*c*x^4))/585
```

3.164 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2}{3}Abx^{3/2}+\frac{2}{7}(bB+Ac)x^{7/2}+\frac{2}{11}Bcx^{11/2}$$

```
[Out] 2/3*A*b*x^(3/2)+2/7*(A*c+B*b)*x^(7/2)+2/11*B*c*x^(11/2)
```

Rubi [A]

time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\frac{2}{7}x^{7/2}(Ac+bB)+\frac{2}{3}Abx^{3/2}+\frac{2}{11}Bcx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^(3/2),x]
```

```
[Out] (2*A*b*x^(3/2))/3 + (2*(b*B + A*c)*x^(7/2))/7 + (2*B*c*x^(11/2))/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}}\,dx &= \int \sqrt{x}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Ab\sqrt{x}+(bB+Ac)x^{5/2}+Bcx^{9/2}\right)\,dx\\ &= \frac{2}{3}Abx^{3/2}+\frac{2}{7}(bB+Ac)x^{7/2}+\frac{2}{11}Bcx^{11/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.90

$$\frac{2}{231}x^{3/2}\left(77Ab+33bBx^2+33Acx^2+21Bcx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^(3/2),x]
```

```
[Out] (2*x^(3/2)*(77*A*b + 33*b*B*x^2 + 33*A*c*x^2 + 21*B*c*x^4))/231
```

Maple [A]
time = 0.13, size = 28, normalized size = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2(Ac+Bb)x^{\frac{7}{2}}}{7}+\frac{2Bc\,x^{\frac{11}{2}}}{11}$	28
default	$\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2(Ac+Bb)x^{\frac{7}{2}}}{7}+\frac{2Bc\,x^{\frac{11}{2}}}{11}$	28
gosper	$\frac{2x^{\frac{3}{2}}\left(21Bc\,x^{4}+33Ac\,x^{2}+33bB\,x^{2}+77Ab\right)}{231}$	32
trager	$\frac{2x^{\frac{3}{2}}\left(21Bc\,x^{4}+33Ac\,x^{2}+33bB\,x^{2}+77Ab\right)}{231}$	32
risch	$\frac{2x^{\frac{3}{2}}\left(21Bc\,x^{4}+33Ac\,x^{2}+33bB\,x^{2}+77Ab\right)}{231}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*A*b*x^(3/2)+2/7*(A*c+B*b)*x^(7/2)+2/11*B*c*x^(11/2)
```

Maxima [A]
time = 0.27, size = 27, normalized size = 0.69

$$\frac{2}{11}\,Bcx^{\frac{11}{2}}+\frac{2}{7}\,(Bb+Ac)x^{\frac{7}{2}}+\frac{2}{3}\,Abx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*B*c*x^(11/2) + 2/7*(B*b + A*c)*x^(7/2) + 2/3*A*b*x^(3/2)
```

Fricas [A]
time = 1.87, size = 30, normalized size = 0.77

$$\frac{2}{231}\,\left(21\,Bcx^5+33\,(Bb+Ac)x^3+77\,Abx\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*B*c*x^5 + 33*(B*b + A*c)*x^3 + 77*A*b*x)*sqrt(x)
```

Sympy [A]
time = 0.32, size = 46, normalized size = 1.18

$$\frac{2Abx^{\frac{3}{2}}}{3}+\frac{2Acx^{\frac{7}{2}}}{7}+\frac{2Bbx^{\frac{7}{2}}}{7}+\frac{2Bcx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(3/2),x)
```

```
[Out] 2*A*b*x**(3/2)/3 + 2*A*c*x**(7/2)/7 + 2*B*b*x**(7/2)/7 + 2*B*c*x**(11/2)/11
```

Giac [A]
time = 0.46, size = 29, normalized size = 0.74

$$\frac{2}{11}\,Bcx^{\frac{11}{2}}+\frac{2}{7}\,Bbx^{\frac{7}{2}}+\frac{2}{7}\,Acx^{\frac{7}{2}}+\frac{2}{3}\,Abx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/11*B*c*x^(11/2) + 2/7*B*b*x^(7/2) + 2/7*A*c*x^(7/2) + 2/3*A*b*x^(3/2)
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.79

$$\frac{2\,x^{3/2}\,\left(77\,A\,b+33\,A\,c\,x^2+33\,B\,b\,x^2+21\,B\,c\,x^4\right)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^(3/2),x)
```

```
[Out] (2*x^(3/2)*(77*A*b + 33*A*c*x^2 + 33*B*b*x^2 + 21*B*c*x^4))/231
```

3.165 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}} \, dx$

Optimal. Leaf size=37

$$2Ab\sqrt{x}+\frac{2}{5}(bB+Ac)x^{5/2}+\frac{2}{9}Bcx^{9/2}$$

```
[Out] 2/5*(A*c+B*b)*x^(5/2)+2/9*B*c*x^(9/2)+2*A*b*x^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\frac{2}{5}x^{5/2}(Ac+bB)+2Ab\sqrt{x}+\frac{2}{9}Bcx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^(5/2),x]
```

```
[Out] 2*A*b*Sqrt[x] + (2*(b*B + A*c)*x^(5/2))/5 + (2*B*c*x^(9/2))/9
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx &= \int \frac{(A+Bx^2)(b+cx^2)}{\sqrt{x}}\,dx\\ &= \int \left(\frac{Ab}{\sqrt{x}}+(bB+Ac)x^{3/2}+Bcx^{7/2}\right)dx\\ &= 2Ab\sqrt{x}+\frac{2}{5}(bB+Ac)x^{5/2}+\frac{2}{9}Bcx^{9/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.95

$$\frac{2}{45}\sqrt{x}\left(45Ab+9bBx^2+9Acx^2+5Bcx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^(5/2),x]
```

```
[Out] (2*Sqrt[x]*(45*A*b + 9*b*B*x^2 + 9*A*c*x^2 + 5*B*c*x^4))/45
```

Maple [A]
time = 0.13, size = 28, normalized size = 0.76

method	result	size
derivativedivides	$\frac{2(Ac+Bb)x^{\frac{5}{2}}}{5}+\frac{2Bc\,x^{\frac{9}{2}}}{9}+2Ab\sqrt{x}$	28
default	$\frac{2(Ac+Bb)x^{\frac{5}{2}}}{5}+\frac{2Bc\,x^{\frac{9}{2}}}{9}+2Ab\sqrt{x}$	28
trager	$\left(\frac{2}{9}Bc\,x^4+\frac{2}{5}Ac\,x^2+\frac{2}{5}bB\,x^2+2Ab\right)\sqrt{x}$	31
gosper	$\frac{2\sqrt{x}\,\left(5Bc\,x^4+9Ac\,x^2+9bB\,x^2+45Ab\right)}{45}$	32
risch	$\frac{2\sqrt{x}\,\left(5Bc\,x^4+9Ac\,x^2+9bB\,x^2+45Ab\right)}{45}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(A*c+B*b)*x^(5/2)+2/9*B*c*x^(9/2)+2*A*b*x^(1/2)
```

Maxima [A]
time = 0.27, size = 27, normalized size = 0.73

$$\frac{2}{9}Bcx^{\frac{9}{2}}+\frac{2}{5}(Bb+Ac)x^{\frac{5}{2}}+2\,Ab\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*c*x^(9/2) + 2/5*(B*b + A*c)*x^(5/2) + 2*A*b*sqrt(x)
```

Fricas [A]
time = 1.63, size = 29, normalized size = 0.78

$$\frac{2}{45}\left(5\,Bcx^4+9\,(Bb+Ac)x^2+45\,Ab\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*B*c*x^4 + 9*(B*b + A*c)*x^2 + 45*A*b)*sqrt(x)
```

Sympy [A]
time = 0.37, size = 44, normalized size = 1.19

$$2Ab\sqrt{x} + \frac{2Acx^{\frac{5}{2}}}{5} + \frac{2Bbx^{\frac{5}{2}}}{5} + \frac{2Bcx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(5/2),x)
```

```
[Out] 2*A*b*sqrt(x) + 2*A*c*x**(5/2)/5 + 2*B*b*x**(5/2)/5 + 2*B*c*x**(9/2)/9
```

Giac [A]
time = 0.57, size = 29, normalized size = 0.78

$$\frac{2}{9}\,Bcx^{\frac{9}{2}} + \frac{2}{5}\,Bbx^{\frac{5}{2}} + \frac{2}{5}\,Acx^{\frac{5}{2}} + 2\,Ab\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*B*c*x^(9/2) + 2/5*B*b*x^(5/2) + 2/5*A*c*x^(5/2) + 2*A*b*sqrt(x)
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.84

$$\frac{2\,\sqrt{x}\,\left(45\,A\,b+9\,A\,c\,x^2+9\,B\,b\,x^2+5\,B\,c\,x^4\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^(5/2),x)
```

```
[Out] (2*x^(1/2)*(45*A*b + 9*A*c*x^2 + 9*B*b*x^2 + 5*B*c*x^4))/45
```

3.166 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}} \, dx$

Optimal. Leaf size=37

$$-\frac{2Ab}{\sqrt{x}}+\frac{2}{3}(bB+Ac)x^{3/2}+\frac{2}{7}Bcx^{7/2}$$

```
[Out] 2/3*(A*c+B*b)*x^(3/2)+2/7*B*c*x^(7/2)-2*A*b/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1598, 459}

$$\frac{2}{3}x^{3/2}(Ac+bB)-\frac{2Ab}{\sqrt{x}}+\frac{2}{7}Bcx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^(7/2),x]
```

```
[Out] (-2*A*b)/Sqrt[x] + (2*(b*B + A*c)*x^(3/2))/3 + (2*B*c*x^(7/2))/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}} \, dx &= \int \frac{(A+Bx^2)(b+cx^2)}{x^{3/2}} \, dx \\
&= \int \left(\frac{Ab}{x^{3/2}}+(bB+Ac)\sqrt{x}+Bcx^{5/2}\right) dx \\
&= -\frac{2Ab}{\sqrt{x}}+\frac{2}{3}(bB+Ac)x^{3/2}+\frac{2}{7}Bcx^{7/2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.95

$$\frac{2(-21Ab+7bBx^2+7Acx^2+3Bcx^4)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^(7/2),x]
[Out] (2*(-21*A*b + 7*b*B*x^2 + 7*A*c*x^2 + 3*B*c*x^4))/(21*Sqrt[x])
```

Maple [A]
time = 0.06, size = 30, normalized size = 0.81

method	result	size
derivativedivides	$\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2Ac\,x^{\frac{3}{2}}}{3}+\frac{2bB\,x^{\frac{3}{2}}}{3}-\frac{2Ab}{\sqrt{x}}$	30
default	$\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2Ac\,x^{\frac{3}{2}}}{3}+\frac{2bB\,x^{\frac{3}{2}}}{3}-\frac{2Ab}{\sqrt{x}}$	30
gosper	$-\frac{2(-3Bc\,x^4-7Ac\,x^2-7bB\,x^2+21Ab)}{21\sqrt{x}}$	32
trager	$-\frac{2(-3Bc\,x^4-7Ac\,x^2-7bB\,x^2+21Ab)}{21\sqrt{x}}$	32
risch	$-\frac{2(-3Bc\,x^4-7Ac\,x^2-7bB\,x^2+21Ab)}{21\sqrt{x}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/7*B*c*x^(7/2)+2/3*A*c*x^(3/2)+2/3*b*B*x^(3/2)-2*A*b/x^(1/2)
```

Maxima [A]
time = 0.27, size = 27, normalized size = 0.73

$$\frac{2}{7}\,Bcx^{\frac{7}{2}}+\frac{2}{3}\,(Bb+Ac)x^{\frac{3}{2}}-\frac{2\,Ab}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x, algorithm="maxima")
[Out] 2/7*B*c*x^(7/2) + 2/3*(B*b + A*c)*x^(3/2) - 2*A*b/sqrt(x)
```

Fricas [A]
time = 1.93, size = 29, normalized size = 0.78

$$\frac{2\,(3\,Bcx^4+7\,(Bb+Ac)x^2-21\,Ab)}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*B*c*x^4 + 7*(B*b + A*c)*x^2 - 21*A*b)/sqrt(x)
```

Sympy [A]
time = 0.54, size = 44, normalized size = 1.19

$$-\frac{2Ab}{\sqrt{x}}+\frac{2Acx^{\frac{3}{2}}}{3}+\frac{2Bbx^{\frac{3}{2}}}{3}+\frac{2Bcx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(7/2),x)
```

```
[Out] -2*A*b/sqrt(x) + 2*A*c*x**(3/2)/3 + 2*B*b*x**(3/2)/3 + 2*B*c*x**(7/2)/7
```

Giac [A]
time = 0.57, size = 29, normalized size = 0.78

$$\frac{2}{7}\,Bcx^{\frac{7}{2}}+\frac{2}{3}\,Bbx^{\frac{3}{2}}+\frac{2}{3}\,Acx^{\frac{3}{2}}-\frac{2\,Ab}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/7*B*c*x^(7/2) + 2/3*B*b*x^(3/2) + 2/3*A*c*x^(3/2) - 2*A*b/sqrt(x)
```

Mupad [B]
time = 0.10, size = 31, normalized size = 0.84

$$\frac{14\,A\,c\,x^2-42\,A\,b+14\,B\,b\,x^2+6\,B\,c\,x^4}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^(7/2),x)
```

```
[Out] (14*A*c*x^2 - 42*A*b + 14*B*b*x^2 + 6*B*c*x^4)/(21*x^(1/2))
```

3.167 $\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=63

$$\frac{2}{17}Ab^2x^{17/2}+\frac{2}{21}b(bB+2Ac)x^{21/2}+\frac{2}{25}c(2bB+Ac)x^{25/2}+\frac{2}{29}Bc^2x^{29/2}$$

```
[Out] 2/17*A*b^2*x^(17/2)+2/21*b*(2*A*c+B*b)*x^(21/2)+2/25*c*(A*c+2*B*b)*x^(25/2)
+2/29*B*c^2*x^(29/2)
```

Rubi [A]

time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1598, 459}

$$\frac{2}{17}Ab^2x^{17/2}+\frac{2}{25}cx^{25/2}(Ac+2bB)+\frac{2}{21}bx^{21/2}(2Ac+bB)+\frac{2}{29}Bc^2x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(17/2))/17 + (2*b*(b*B + 2*A*c)*x^(21/2))/21 + (2*c*(2*b*B + A*c
)*x^(25/2))/25 + (2*B*c^2*x^(29/2))/29
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx &= \int x^{15/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\
&= \int \left(Ab^2x^{15/2}+b(bB+2Ac)x^{19/2}+c(2bB+Ac)x^{23/2}+Bc^2x^{27/2}\right)dx\\
&= \frac{2}{17}Ab^2x^{17/2}+\frac{2}{21}b(bB+2Ac)x^{21/2}+\frac{2}{25}c(2bB+Ac)x^{25/2}+\frac{2}{29}Bc^2x^{2}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.97

$$\frac{2x^{17/2}(29A(525b^2+850bcx^2+357c^2x^4)+17Bx^2(725b^2+1218bcx^2+525c^2x^4))}{258825}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(17/2)*(29*A*(525*b^2 + 850*b*c*x^2 + 357*c^2*x^4) + 17*B*x^2*(725*b^2
 + 1218*b*c*x^2 + 525*c^2*x^4)))/258825
```

Maple [A]
time = 0.35, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2Bc^2x^{\frac{29}{2}}}{29}+\frac{2(Ac^2+2bBc)x^{\frac{25}{2}}}{25}+\frac{2(2Abc+b^2B)x^{\frac{21}{2}}}{21}+\frac{2Ab^2x^{\frac{17}{2}}}{17}$	52
default	$\frac{2Bc^2x^{\frac{29}{2}}}{29}+\frac{2(Ac^2+2bBc)x^{\frac{25}{2}}}{25}+\frac{2(2Abc+b^2B)x^{\frac{21}{2}}}{21}+\frac{2Ab^2x^{\frac{17}{2}}}{17}$	52
gosper	$\frac{2x^{\frac{17}{2}}(8925Bc^2x^6+10353Ac^2x^4+20706x^4bBc+24650Abcx^2+12325b^2Bx^2+15225b^2A)}{258825}$	56
trager	$\frac{2x^{\frac{17}{2}}(8925Bc^2x^6+10353Ac^2x^4+20706x^4bBc+24650Abcx^2+12325b^2Bx^2+15225b^2A)}{258825}$	56
risch	$\frac{2x^{\frac{17}{2}}(8925Bc^2x^6+10353Ac^2x^4+20706x^4bBc+24650Abcx^2+12325b^2Bx^2+15225b^2A)}{258825}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/29*B*c^2*x^(29/2)+2/25*(A*c^2+2*B*b*c)*x^(25/2)+2/21*(2*A*b*c+B*b^2)*x^(2
1/2)+2/17*A*b^2*x^(17/2)
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.81

$$\frac{2}{29}Bc^2x^{\frac{29}{2}}+\frac{2}{25}\left(2Bbc+Ac^2\right)x^{\frac{25}{2}}+\frac{2}{17}Ab^2x^{\frac{17}{2}}+\frac{2}{21}\left(Bb^2+2Abc\right)x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/29*B*c^2*x^(29/2) + 2/25*(2*B*b*c + A*c^2)*x^(25/2) + 2/17*A*b^2*x^(17/2)
 + 2/21*(B*b^2 + 2*A*b*c)*x^(21/2)
```

Fricas [A]
time = 1.87, size = 56, normalized size = 0.89

$$\frac{2}{258825}\left(8925\,Bc^2x^{14}+10353\left(2\,Bbc+Ac^2\right)x^{12}+15225\,Ab^2x^8+12325\left(Bb^2+2\,Abc\right)x^{10}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/258825*(8925*B*c^2*x^14 + 10353*(2*B*b*c + A*c^2)*x^12 + 15225*A*b^2*x^8
+ 12325*(B*b^2 + 2*A*b*c)*x^10)*sqrt(x)
```

Sympy [A]
time = 2.24, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{17}{2}}}{17} + \frac{4Abcx^{\frac{21}{2}}}{21} + \frac{2Ac^2x^{\frac{25}{2}}}{25} + \frac{2Bb^2x^{\frac{21}{2}}}{21} + \frac{4Bbcx^{\frac{25}{2}}}{25} + \frac{2Bc^2x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*A*b**2*x**(17/2)/17 + 4*A*b*c*x**(21/2)/21 + 2*A*c**2*x**(25/2)/25 + 2*B*
b**2*x**(21/2)/21 + 4*B*b*c*x**(25/2)/25 + 2*B*c**2*x**(29/2)/29
```

Giac [A]
time = 0.49, size = 53, normalized size = 0.84

$$\frac{2}{29}\,Bc^2x^{\frac{29}{2}} + \frac{4}{25}\,Bbcx^{\frac{25}{2}} + \frac{2}{25}\,Ac^2x^{\frac{25}{2}} + \frac{2}{21}\,Bb^2x^{\frac{21}{2}} + \frac{4}{21}\,Abcx^{\frac{21}{2}} + \frac{2}{17}\,Ab^2x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/29*B*c^2*x^(29/2) + 4/25*B*b*c*x^(25/2) + 2/25*A*c^2*x^(25/2) + 2/21*B*b^
2*x^(21/2) + 4/21*A*b*c*x^(21/2) + 2/17*A*b^2*x^(17/2)
```

Mupad [B]
time = 0.13, size = 51, normalized size = 0.81

$$x^{21/2}\left(\frac{2\,B\,b^2}{21}+\frac{4\,A\,c\,b}{21}\right)+x^{25/2}\left(\frac{2\,A\,c^2}{25}+\frac{4\,B\,b\,c}{25}\right)+\frac{2\,A\,b^2\,x^{17/2}}{17}+\frac{2\,B\,c^2\,x^{29/2}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(21/2)*((2*B*b^2)/21 + (4*A*b*c)/21) + x^(25/2)*((2*A*c^2)/25 + (4*B*b*c)
/25) + (2*A*b^2*x^(17/2))/17 + (2*B*c^2*x^(29/2))/29
```

3.168 $\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=63

$$\frac{2}{15}Ab^2x^{15/2}+\frac{2}{19}b(bB+2Ac)x^{19/2}+\frac{2}{23}c(2bB+Ac)x^{23/2}+\frac{2}{27}Bc^2x^{27/2}$$

```
[Out] 2/15*A*b^2*x^(15/2)+2/19*b*(2*A*c+B*b)*x^(19/2)+2/23*c*(A*c+2*B*b)*x^(23/2)
+2/27*B*c^2*x^(27/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1598, 459}

$$\frac{2}{15}Ab^2x^{15/2}+\frac{2}{23}cx^{23/2}(Ac+2bB)+\frac{2}{19}bx^{19/2}(2Ac+bB)+\frac{2}{27}Bc^2x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(15/2))/15 + (2*b*(b*B + 2*A*c)*x^(19/2))/19 + (2*c*(2*b*B + A*c
)*x^(23/2))/23 + (2*B*c^2*x^(27/2))/27
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx &= \int x^{13/2}(A+Bx^2)\left(b+cx^2\right)^2\,dx\\ &= \int \left(Ab^2x^{13/2}+b(bB+2Ac)x^{17/2}+c(2bB+Ac)x^{21/2}+Bc^2x^{25/2}\right)dx\\ &= \frac{2}{15}Ab^2x^{15/2}+\frac{2}{19}b(bB+2Ac)x^{19/2}+\frac{2}{23}c(2bB+Ac)x^{23/2}+\frac{2}{27}Bc^2x^{27/}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 0.97

$$\frac{2x^{15/2}(9A(437b^2+690bcx^2+285c^2x^4)+5Bx^2(621b^2+1026bcx^2+437c^2x^4))}{58995}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(15/2)*(9*A*(437*b^2 + 690*b*c*x^2 + 285*c^2*x^4) + 5*B*x^2*(621*b^2 +
 1026*b*c*x^2 + 437*c^2*x^4)))/58995
```

Maple [A]
time = 0.36, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2B\,c^2x^{\frac{27}{2}}}{27}+\frac{2(A\,c^2+2bBc)x^{\frac{23}{2}}}{23}+\frac{2(2Abc+b^2B)x^{\frac{19}{2}}}{19}+\frac{2A\,b^2x^{\frac{15}{2}}}{15}$	52
default	$\frac{2B\,c^2x^{\frac{27}{2}}}{27}+\frac{2(A\,c^2+2bBc)x^{\frac{23}{2}}}{23}+\frac{2(2Abc+b^2B)x^{\frac{19}{2}}}{19}+\frac{2A\,b^2x^{\frac{15}{2}}}{15}$	52
gosper	$\frac{2x^{\frac{15}{2}}(2185B\,c^2x^6+2565A\,c^2x^4+5130x^4bBc+6210Abc\,x^2+3105b^2B\,x^2+3933b^2A)}{58995}$	56
trager	$\frac{2x^{\frac{15}{2}}(2185B\,c^2x^6+2565A\,c^2x^4+5130x^4bBc+6210Abc\,x^2+3105b^2B\,x^2+3933b^2A)}{58995}$	56
risch	$\frac{2x^{\frac{15}{2}}(2185B\,c^2x^6+2565A\,c^2x^4+5130x^4bBc+6210Abc\,x^2+3105b^2B\,x^2+3933b^2A)}{58995}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/27*B*c^2*x^(27/2)+2/23*(A*c^2+2*B*b*c)*x^(23/2)+2/19*(2*A*b*c+B*b^2)*x^(1
9/2)+2/15*A*b^2*x^(15/2)
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.81

$$\frac{2}{27}\,Bc^2x^{\frac{27}{2}}+\frac{2}{23}\left(2\,Bbc+Ac^2\right)x^{\frac{23}{2}}+\frac{2}{15}\,Ab^2x^{\frac{15}{2}}+\frac{2}{19}\left(Bb^2+2\,Abc\right)x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/27*B*c^2*x^(27/2) + 2/23*(2*B*b*c + A*c^2)*x^(23/2) + 2/15*A*b^2*x^(15/2)
 + 2/19*(B*b^2 + 2*A*b*c)*x^(19/2)
```

Fricas [A]
time = 1.60, size = 56, normalized size = 0.89

$$\frac{2}{58995}\left(2185\,Bc^2x^{13}+2565\left(2\,Bbc+Ac^2\right)x^{11}+3933\,Ab^2x^7+3105\left(Bb^2+2\,Abc\right)x^9\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/58995*(2185*B*c^2*x^13 + 2565*(2*B*b*c + A*c^2)*x^11 + 3933*A*b^2*x^7 + 3
105*(B*b^2 + 2*A*b*c)*x^9)*sqrt(x)
```

Sympy [A]
time = 1.57, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{15}{2}}}{15} + \frac{4Abcx^{\frac{19}{2}}}{19} + \frac{2Ac^2x^{\frac{23}{2}}}{23} + \frac{2Bb^2x^{\frac{19}{2}}}{19} + \frac{4Bbcx^{\frac{23}{2}}}{23} + \frac{2Bc^2x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*A*b**2*x**(15/2)/15 + 4*A*b*c*x**(19/2)/19 + 2*A*c**2*x**(23/2)/23 + 2*B*
b**2*x**(19/2)/19 + 4*B*b*c*x**(23/2)/23 + 2*B*c**2*x**(27/2)/27
```

Giac [A]
time = 0.44, size = 53, normalized size = 0.84

$$\frac{2}{27}\,Bc^2x^{\frac{27}{2}} + \frac{4}{23}\,Bbcx^{\frac{23}{2}} + \frac{2}{23}\,Ac^2x^{\frac{23}{2}} + \frac{2}{19}\,Bb^2x^{\frac{19}{2}} + \frac{4}{19}\,Abcx^{\frac{19}{2}} + \frac{2}{15}\,Ab^2x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/27*B*c^2*x^(27/2) + 4/23*B*b*c*x^(23/2) + 2/23*A*c^2*x^(23/2) + 2/19*B*b^
2*x^(19/2) + 4/19*A*b*c*x^(19/2) + 2/15*A*b^2*x^(15/2)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.81

$$x^{19/2}\left(\frac{2\,B\,b^2}{19}+\frac{4\,A\,c\,b}{19}\right)+x^{23/2}\left(\frac{2\,A\,c^2}{23}+\frac{4\,B\,b\,c}{23}\right)+\frac{2\,A\,b^2\,x^{15/2}}{15}+\frac{2\,B\,c^2\,x^{27/2}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(19/2)*((2*B*b^2)/19 + (4*A*b*c)/19) + x^(23/2)*((2*A*c^2)/23 + (4*B*b*c)
/23) + (2*A*b^2*x^(15/2))/15 + (2*B*c^2*x^(27/2))/27
```

3.169 $\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=63

$$\frac{2}{13}Ab^2x^{13/2}+\frac{2}{17}b(bB+2Ac)x^{17/2}+\frac{2}{21}c(2bB+Ac)x^{21/2}+\frac{2}{25}Bc^2x^{25/2}$$

```
[Out] 2/13*A*b^2*x^(13/2)+2/17*b*(2*A*c+B*b)*x^(17/2)+2/21*c*(A*c+2*B*b)*x^(21/2)
+2/25*B*c^2*x^(25/2)
```

Rubi [A]

time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {1598, 459}

$$\frac{2}{13}Ab^2x^{13/2}+\frac{2}{21}cx^{21/2}(Ac+2bB)+\frac{2}{17}bx^{17/2}(2Ac+bB)+\frac{2}{25}Bc^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(13/2))/13 + (2*b*(b*B + 2*A*c)*x^(17/2))/17 + (2*c*(2*b*B + A*c
)*x^(21/2))/21 + (2*B*c^2*x^(25/2))/25
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx&=\int x^{11/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\&=\int\left(Ab^2x^{11/2}+b(bB+2Ac)x^{15/2}+c(2bB+Ac)x^{19/2}+Bc^2x^{23/2}\right)dx\\&=\frac{2}{13}Ab^2x^{13/2}+\frac{2}{17}b(bB+2Ac)x^{17/2}+\frac{2}{21}c(2bB+Ac)x^{21/2}+\frac{2}{25}Bc^2x^{2\cdot}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 0.97

$$\frac{2x^{13/2}(25A(357b^2+546bcx^2+221c^2x^4)+13Bx^2(525b^2+850bcx^2+357c^2x^4))}{116025}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(13/2)*(25*A*(357*b^2 + 546*b*c*x^2 + 221*c^2*x^4) + 13*B*x^2*(525*b^2
 + 850*b*c*x^2 + 357*c^2*x^4)))/116025
```

Maple [A]
time = 0.35, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2B\,c^2x^{\frac{25}{2}}}{25}+\frac{2(A\,c^2+2bBc)x^{\frac{21}{2}}}{21}+\frac{2(2Abc+b^2B)x^{\frac{17}{2}}}{17}+\frac{2A\,b^2x^{\frac{13}{2}}}{13}$	52
default	$\frac{2B\,c^2x^{\frac{25}{2}}}{25}+\frac{2(A\,c^2+2bBc)x^{\frac{21}{2}}}{21}+\frac{2(2Abc+b^2B)x^{\frac{17}{2}}}{17}+\frac{2A\,b^2x^{\frac{13}{2}}}{13}$	52
gosper	$\frac{2x^{\frac{13}{2}}(4641B\,c^2x^6+5525A\,c^2x^4+11050x^4bBc+13650Abc\,x^2+6825b^2B\,x^2+8925b^2A)}{116025}$	56
trager	$\frac{2x^{\frac{13}{2}}(4641B\,c^2x^6+5525A\,c^2x^4+11050x^4bBc+13650Abc\,x^2+6825b^2B\,x^2+8925b^2A)}{116025}$	56
risch	$\frac{2x^{\frac{13}{2}}(4641B\,c^2x^6+5525A\,c^2x^4+11050x^4bBc+13650Abc\,x^2+6825b^2B\,x^2+8925b^2A)}{116025}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/25*B*c^2*x^(25/2)+2/21*(A*c^2+2*B*b*c)*x^(21/2)+2/17*(2*A*b*c+B*b^2)*x^(1
7/2)+2/13*A*b^2*x^(13/2)
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.81

$$\frac{2}{25}\,Bc^2x^{\frac{25}{2}}+\frac{2}{21}\left(2\,Bbc+Ac^2\right)x^{\frac{21}{2}}+\frac{2}{13}\,Ab^2x^{\frac{13}{2}}+\frac{2}{17}\left(Bb^2+2\,Abc\right)x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/25*B*c^2*x^(25/2) + 2/21*(2*B*b*c + A*c^2)*x^(21/2) + 2/13*A*b^2*x^(13/2)
 + 2/17*(B*b^2 + 2*A*b*c)*x^(17/2)
```

Fricas [A]
time = 1.55, size = 56, normalized size = 0.89

$$\frac{2}{116025}\left(4641\,Bc^2x^{12}+5525\left(2\,Bbc+Ac^2\right)x^{10}+8925\,Ab^2x^6+6825\left(Bb^2+2\,Abc\right)x^8\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/116025*(4641*B*c^2*x^12 + 5525*(2*B*b*c + A*c^2)*x^10 + 8925*A*b^2*x^6 +
6825*(B*b^2 + 2*A*b*c)*x^8)*sqrt(x)
```

Sympy [A]
time = 1.10, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{13}{2}}}{13}+\frac{4Abcx^{\frac{17}{2}}}{17}+\frac{2Ac^2x^{\frac{21}{2}}}{21}+\frac{2Bb^2x^{\frac{17}{2}}}{17}+\frac{4Bbcx^{\frac{21}{2}}}{21}+\frac{2Bc^2x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*A*b**2*x**(13/2)/13 + 4*A*b*c*x**(17/2)/17 + 2*A*c**2*x**(21/2)/21 + 2*B*
b**2*x**(17/2)/17 + 4*B*b*c*x**(21/2)/21 + 2*B*c**2*x**(25/2)/25
```

Giac [A]
time = 0.47, size = 53, normalized size = 0.84

$$\frac{2}{25}\,Bc^2x^{\frac{25}{2}}+\frac{4}{21}\,Bbcx^{\frac{21}{2}}+\frac{2}{21}\,Ac^2x^{\frac{21}{2}}+\frac{2}{17}\,Bb^2x^{\frac{17}{2}}+\frac{4}{17}\,Abcx^{\frac{17}{2}}+\frac{2}{13}\,Ab^2x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/25*B*c^2*x^(25/2) + 4/21*B*b*c*x^(21/2) + 2/21*A*c^2*x^(21/2) + 2/17*B*b^
2*x^(17/2) + 4/17*A*b*c*x^(17/2) + 2/13*A*b^2*x^(13/2)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.81

$$x^{17/2}\left(\frac{2\,B\,b^2}{17}+\frac{4\,A\,c\,b}{17}\right)+x^{21/2}\left(\frac{2\,A\,c^2}{21}+\frac{4\,B\,b\,c}{21}\right)+\frac{2\,A\,b^2\,x^{13/2}}{13}+\frac{2\,B\,c^2\,x^{25/2}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(17/2)*((2*B*b^2)/17 + (4*A*b*c)/17) + x^(21/2)*((2*A*c^2)/21 + (4*B*b*c)
/21) + (2*A*b^2*x^(13/2))/13 + (2*B*c^2*x^(25/2))/25
```

3.170 $\int \sqrt{x} \left(A+Bx^2\right) \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=63

$$\frac{2}{11}Ab^2x^{11/2}+\frac{2}{15}b(bB+2Ac)x^{15/2}+\frac{2}{19}c(2bB+Ac)x^{19/2}+\frac{2}{23}Bc^2x^{23/2}$$

```
[Out] 2/11*A*b^2*x^(11/2)+2/15*b*(2*A*c+B*b)*x^(15/2)+2/19*c*(A*c+2*B*b)*x^(19/2)
+2/23*B*c^2*x^(23/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1598, 459}

$$\frac{2}{11}Ab^2x^{11/2}+\frac{2}{19}cx^{19/2}(Ac+2bB)+\frac{2}{15}bx^{15/2}(2Ac+bB)+\frac{2}{23}Bc^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(11/2))/11 + (2*b*(b*B + 2*A*c)*x^(15/2))/15 + (2*c*(2*b*B + A*c
)*x^(19/2))/19 + (2*B*c^2*x^(23/2))/23
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2 \, dx &= \int x^{9/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2 \, dx\\
&= \int \left(Ab^2x^{9/2}+b(bB+2Ac)x^{13/2}+c(2bB+Ac)x^{17/2}+Bc^2x^{21/2}\right) dx\\
&= \frac{2}{11}Ab^2x^{11/2}+\frac{2}{15}b(bB+2Ac)x^{15/2}+\frac{2}{19}c(2bB+Ac)x^{19/2}+\frac{2}{23}Bc^2x^{23/
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 0.97

$$\frac{2x^{11/2}(23A(285b^2+418bcx^2+165c^2x^4)+11Bx^2(437b^2+690bcx^2+285c^2x^4))}{72105}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(11/2)*(23*A*(285*b^2 + 418*b*c*x^2 + 165*c^2*x^4) + 11*B*x^2*(437*b^2
 + 690*b*c*x^2 + 285*c^2*x^4)))/72105
```

Maple [A]
time = 0.39, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2B\,c^2x^{\frac{23}{2}}}{23}+\frac{2(A\,c^2+2bBc)x^{\frac{19}{2}}}{19}+\frac{2(2Abc+b^2B)x^{\frac{15}{2}}}{15}+\frac{2A\,b^2x^{\frac{11}{2}}}{11}$	52
default	$\frac{2B\,c^2x^{\frac{23}{2}}}{23}+\frac{2(A\,c^2+2bBc)x^{\frac{19}{2}}}{19}+\frac{2(2Abc+b^2B)x^{\frac{15}{2}}}{15}+\frac{2A\,b^2x^{\frac{11}{2}}}{11}$	52
gosper	$\frac{2x^{\frac{11}{2}}(3135B\,c^2x^6+3795A\,c^2x^4+7590x^4bBc+9614Abc\,x^2+4807b^2B\,x^2+6555b^2A)}{72105}$	56
trager	$\frac{2x^{\frac{11}{2}}(3135B\,c^2x^6+3795A\,c^2x^4+7590x^4bBc+9614Abc\,x^2+4807b^2B\,x^2+6555b^2A)}{72105}$	56
risch	$\frac{2x^{\frac{11}{2}}(3135B\,c^2x^6+3795A\,c^2x^4+7590x^4bBc+9614Abc\,x^2+4807b^2B\,x^2+6555b^2A)}{72105}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/23*B*c^2*x^(23/2)+2/19*(A*c^2+2*B*b*c)*x^(19/2)+2/15*(2*A*b*c+B*b^2)*x^(1
5/2)+2/11*A*b^2*x^(11/2)
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.81

$$\frac{2}{23}\,Bc^2x^{\frac{23}{2}}+\frac{2}{19}\left(2\,Bbc+Ac^2\right)x^{\frac{19}{2}}+\frac{2}{11}\,Ab^2x^{\frac{11}{2}}+\frac{2}{15}\left(Bb^2+2\,Abc\right)x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/23*B*c^2*x^(23/2) + 2/19*(2*B*b*c + A*c^2)*x^(19/2) + 2/11*A*b^2*x^(11/2)
 + 2/15*(B*b^2 + 2*A*b*c)*x^(15/2)
```

Fricas [A]
time = 1.68, size = 56, normalized size = 0.89

$$\frac{2}{72105}\left(3135\,Bc^2x^{11}+3795\left(2\,Bbc+Ac^2\right)x^9+6555\,Ab^2x^5+4807\left(Bb^2+2\,Abc\right)x^7\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/72105*(3135*B*c^2*x^11 + 3795*(2*B*b*c + A*c^2)*x^9 + 6555*A*b^2*x^5 + 48
07*(B*b^2 + 2*A*b*c)*x^7)*sqrt(x)
```

Sympy [A]
time = 1.78, size = 66, normalized size = 1.05

$$\frac{2Ab^2x^{\frac{11}{2}}}{11} + \frac{2Bc^2x^{\frac{23}{2}}}{23} + \frac{2x^{\frac{19}{2}}(Ac^2 + 2Bbc)}{19} + \frac{2x^{\frac{15}{2}} \cdot (2Abc + Bb^2)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2*x**(1/2),x)
```

```
[Out] 2*A*b**2*x**(11/2)/11 + 2*B*c**2*x**(23/2)/23 + 2*x**(19/2)*(A*c**2 + 2*B*b
*c)/19 + 2*x**(15/2)*(2*A*b*c + B*b**2)/15
```

Giac [A]
time = 0.46, size = 53, normalized size = 0.84

$$\frac{2}{23}\,Bc^2x^{\frac{23}{2}} + \frac{4}{19}\,Bbcx^{\frac{19}{2}} + \frac{2}{19}\,Ac^2x^{\frac{19}{2}} + \frac{2}{15}\,Bb^2x^{\frac{15}{2}} + \frac{4}{15}\,Abcx^{\frac{15}{2}} + \frac{2}{11}\,Ab^2x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/23*B*c^2*x^(23/2) + 4/19*B*b*c*x^(19/2) + 2/19*A*c^2*x^(19/2) + 2/15*B*b^
2*x^(15/2) + 4/15*A*b*c*x^(15/2) + 2/11*A*b^2*x^(11/2)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.81

$$x^{15/2}\left(\frac{2\,B\,b^2}{15} + \frac{4\,A\,c\,b}{15}\right) + x^{19/2}\left(\frac{2\,A\,c^2}{19} + \frac{4\,B\,b\,c}{19}\right) + \frac{2\,A\,b^2\,x^{11/2}}{11} + \frac{2\,B\,c^2\,x^{23/2}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(15/2)*((2*B*b^2)/15 + (4*A*b*c)/15) + x^(19/2)*((2*A*c^2)/19 + (4*B*b*c)
/19) + (2*A*b^2*x^(11/2))/11 + (2*B*c^2*x^(23/2))/23
```

3.171 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}} \, dx$

Optimal. Leaf size=63

$$\frac{2}{9}Ab^2x^{9/2}+\frac{2}{13}b(bB+2Ac)x^{13/2}+\frac{2}{17}c(2bB+Ac)x^{17/2}+\frac{2}{21}Bc^2x^{21/2}$$

```
[Out] 2/9*A*b^2*x^(9/2)+2/13*b*(2*A*c+B*b)*x^(13/2)+2/17*c*(A*c+2*B*b)*x^(17/2)+2
/21*B*c^2*x^(21/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1598, 459}

$$\frac{2}{9}Ab^2x^{9/2}+\frac{2}{17}cx^{17/2}(Ac+2bB)+\frac{2}{13}bx^{13/2}(2Ac+bB)+\frac{2}{21}Bc^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/Sqrt[x],x]
```

```
[Out] (2*A*b^2*x^(9/2))/9 + (2*b*(b*B + 2*A*c)*x^(13/2))/13 + (2*c*(2*b*B + A*c)*
x^(17/2))/17 + (2*B*c^2*x^(21/2))/21
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}} \, dx &= \int x^{7/2}(A+Bx^2)\left(b+cx^2\right)^2 \, dx \\
&= \int \left(Ab^2x^{7/2}+b(bB+2Ac)x^{11/2}+c(2bB+Ac)x^{15/2}+Bc^2x^{19/2}\right) dx \\
&= \frac{2}{9}Ab^2x^{9/2}+\frac{2}{13}b(bB+2Ac)x^{13/2}+\frac{2}{17}c(2bB+Ac)x^{17/2}+\frac{2}{21}Bc^2x^{21/2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 0.97

$$\frac{2x^{9/2}(7A(221b^2+306bcx^2+117c^2x^4)+3Bx^2(357b^2+546bcx^2+221c^2x^4))}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/Sqrt[x],x]
[Out] (2*x^(9/2)*(7*A*(221*b^2 + 306*b*c*x^2 + 117*c^2*x^4) + 3*B*x^2*(357*b^2 +
546*b*c*x^2 + 221*c^2*x^4)))/13923
```

Maple [A]
time = 0.36, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2Bc^2x^{\frac{21}{2}}}{21}+\frac{2(Ac^2+2bBc)x^{\frac{17}{2}}}{17}+\frac{2(2Abc+b^2B)x^{\frac{13}{2}}}{13}+\frac{2Ab^2x^{\frac{9}{2}}}{9}$	52
default	$\frac{2Bc^2x^{\frac{21}{2}}}{21}+\frac{2(Ac^2+2bBc)x^{\frac{17}{2}}}{17}+\frac{2(2Abc+b^2B)x^{\frac{13}{2}}}{13}+\frac{2Ab^2x^{\frac{9}{2}}}{9}$	52
gosper	$\frac{2x^{\frac{9}{2}}(663Bc^2x^6+819Ac^2x^4+1638x^4bBc+2142Abcx^2+1071b^2Bx^2+1547b^2A)}{13923}$	56
trager	$\frac{2x^{\frac{9}{2}}(663Bc^2x^6+819Ac^2x^4+1638x^4bBc+2142Abcx^2+1071b^2Bx^2+1547b^2A)}{13923}$	56
risch	$\frac{2x^{\frac{9}{2}}(663Bc^2x^6+819Ac^2x^4+1638x^4bBc+2142Abcx^2+1071b^2Bx^2+1547b^2A)}{13923}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/21*B*c^2*x^(21/2)+2/17*(A*c^2+2*B*b*c)*x^(17/2)+2/13*(2*A*b*c+B*b^2)*x^(1
3/2)+2/9*A*b^2*x^(9/2)
```

Maxima [A]
time = 0.26, size = 51, normalized size = 0.81

$$\frac{2}{21}Bc^2x^{\frac{21}{2}}+\frac{2}{17}\left(2Bbc+Ac^2\right)x^{\frac{17}{2}}+\frac{2}{9}Ab^2x^{\frac{9}{2}}+\frac{2}{13}\left(Bb^2+2Abc\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
[Out] 2/21*B*c^2*x^(21/2) + 2/17*(2*B*b*c + A*c^2)*x^(17/2) + 2/9*A*b^2*x^(9/2) +
 2/13*(B*b^2 + 2*A*b*c)*x^(13/2)
```

Fricas [A]
time = 1.74, size = 56, normalized size = 0.89

$$\frac{2}{13923}\left(663Bc^2x^{10}+819\left(2Bbc+Ac^2\right)x^8+1547Ab^2x^4+1071\left(Bb^2+2Abc\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*B*c^2*x^10 + 819*(2*B*b*c + A*c^2)*x^8 + 1547*A*b^2*x^4 + 1071
*(B*b^2 + 2*A*b*c)*x^6)*sqrt(x)
```

Sympy [A]
time = 0.74, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{9}{2}}}{9}+\frac{4Abcx^{\frac{13}{2}}}{13}+\frac{2Ac^2x^{\frac{17}{2}}}{17}+\frac{2Bb^2x^{\frac{13}{2}}}{13}+\frac{4Bbcx^{\frac{17}{2}}}{17}+\frac{2Bc^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] 2*A*b**2*x**(9/2)/9 + 4*A*b*c*x**(13/2)/13 + 2*A*c**2*x**(17/2)/17 + 2*B*b*
*2*x**(13/2)/13 + 4*B*b*c*x**(17/2)/17 + 2*B*c**2*x**(21/2)/21
```

Giac [A]
time = 0.44, size = 53, normalized size = 0.84

$$\frac{2}{21}\,Bc^2x^{\frac{21}{2}}+\frac{4}{17}\,Bbcx^{\frac{17}{2}}+\frac{2}{17}\,Ac^2x^{\frac{17}{2}}+\frac{2}{13}\,Bb^2x^{\frac{13}{2}}+\frac{4}{13}\,Abcx^{\frac{13}{2}}+\frac{2}{9}\,Ab^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*B*c^2*x^(21/2) + 4/17*B*b*c*x^(17/2) + 2/17*A*c^2*x^(17/2) + 2/13*B*b^
2*x^(13/2) + 4/13*A*b*c*x^(13/2) + 2/9*A*b^2*x^(9/2)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.81

$$x^{13/2}\left(\frac{2\,B\,b^2}{13}+\frac{4\,A\,c\,b}{13}\right)+x^{17/2}\left(\frac{2\,A\,c^2}{17}+\frac{4\,B\,b\,c}{17}\right)+\frac{2\,A\,b^2\,x^{9/2}}{9}+\frac{2\,B\,c^2\,x^{21/2}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(1/2),x)
```

```
[Out] x^(13/2)*((2*B*b^2)/13 + (4*A*b*c)/13) + x^(17/2)*((2*A*c^2)/17 + (4*B*b*c)
/17) + (2*A*b^2*x^(9/2))/9 + (2*B*c^2*x^(21/2))/21
```

3.172 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}} \, dx$

Optimal. Leaf size=63

$$\frac{2}{7}Ab^2x^{7/2}+\frac{2}{11}b(bB+2Ac)x^{11/2}+\frac{2}{15}c(2bB+Ac)x^{15/2}+\frac{2}{19}Bc^2x^{19/2}$$

```
[Out] 2/7*A*b^2*x^(7/2)+2/11*b*(2*A*c+B*b)*x^(11/2)+2/15*c*(A*c+2*B*b)*x^(15/2)+2
/19*B*c^2*x^(19/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1598, 459}

$$\frac{2}{7}Ab^2x^{7/2}+\frac{2}{15}cx^{15/2}(Ac+2bB)+\frac{2}{11}bx^{11/2}(2Ac+bB)+\frac{2}{19}Bc^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(3/2),x]
```

```
[Out] (2*A*b^2*x^(7/2))/7 + (2*b*(b*B + 2*A*c)*x^(11/2))/11 + (2*c*(2*b*B + A*c)*
x^(15/2))/15 + (2*B*c^2*x^(19/2))/19
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}} \, dx &= \int x^{5/2}(A+Bx^2)\left(b+cx^2\right)^2 \, dx\\ &= \int \left(Ab^2x^{5/2}+b(bB+2Ac)x^{9/2}+c(2bB+Ac)x^{13/2}+Bc^2x^{17/2}\right) dx\\ &= \frac{2}{7}Ab^2x^{7/2}+\frac{2}{11}b(bB+2Ac)x^{11/2}+\frac{2}{15}c(2bB+Ac)x^{15/2}+\frac{2}{19}Bc^2x^{19/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 0.97

$$\frac{2x^{7/2}(19A(165b^2+210bcx^2+77c^2x^4)+7Bx^2(285b^2+418bcx^2+165c^2x^4))}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(3/2),x]
```

```
[Out] (2*x^(7/2)*(19*A*(165*b^2 + 210*b*c*x^2 + 77*c^2*x^4) + 7*B*x^2*(285*b^2 +
418*b*c*x^2 + 165*c^2*x^4)))/21945
```

Maple [A]
time = 0.35, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2Bc^2x^{\frac{19}{2}}}{19}+\frac{2(Ac^2+2bBc)x^{\frac{15}{2}}}{15}+\frac{2(2Abc+b^2B)x^{\frac{11}{2}}}{11}+\frac{2Ab^2x^{\frac{7}{2}}}{7}$	52
default	$\frac{2Bc^2x^{\frac{19}{2}}}{19}+\frac{2(Ac^2+2bBc)x^{\frac{15}{2}}}{15}+\frac{2(2Abc+b^2B)x^{\frac{11}{2}}}{11}+\frac{2Ab^2x^{\frac{7}{2}}}{7}$	52
gosper	$\frac{2x^{\frac{7}{2}}(1155Bc^2x^6+1463Ac^2x^4+2926x^4bBc+3990Abcx^2+1995b^2Bx^2+3135b^2A)}{21945}$	56
trager	$\frac{2x^{\frac{7}{2}}(1155Bc^2x^6+1463Ac^2x^4+2926x^4bBc+3990Abcx^2+1995b^2Bx^2+3135b^2A)}{21945}$	56
risch	$\frac{2x^{\frac{7}{2}}(1155Bc^2x^6+1463Ac^2x^4+2926x^4bBc+3990Abcx^2+1995b^2Bx^2+3135b^2A)}{21945}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/19*B*c^2*x^(19/2)+2/15*(A*c^2+2*B*b*c)*x^(15/2)+2/11*(2*A*b*c+B*b^2)*x^(1
1/2)+2/7*A*b^2*x^(7/2)
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.81

$$\frac{2}{19}\,Bc^2x^{\frac{19}{2}}+\frac{2}{15}\,(2\,Bbc+Ac^2)x^{\frac{15}{2}}+\frac{2}{7}\,Ab^2x^{\frac{7}{2}}+\frac{2}{11}\,(Bb^2+2\,Abc)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*c^2*x^(19/2) + 2/15*(2*B*b*c + A*c^2)*x^(15/2) + 2/7*A*b^2*x^(7/2) +
 2/11*(B*b^2 + 2*A*b*c)*x^(11/2)
```

Fricas [A]
time = 1.63, size = 56, normalized size = 0.89

$$\frac{2}{21945}\,(1155\,Bc^2x^9+1463\,(2\,Bbc+Ac^2)x^7+3135\,Ab^2x^3+1995\,(Bb^2+2\,Abc)x^5)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*B*c^2*x^9 + 1463*(2*B*b*c + A*c^2)*x^7 + 3135*A*b^2*x^3 + 199
5*(B*b^2 + 2*A*b*c)*x^5)*sqrt(x)
```

Sympy [A]
time = 0.86, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{7}{2}}}{7}+\frac{4Abcx^{\frac{11}{2}}}{11}+\frac{2Ac^2x^{\frac{15}{2}}}{15}+\frac{2Bb^2x^{\frac{11}{2}}}{11}+\frac{4Bbcx^{\frac{15}{2}}}{15}+\frac{2Bc^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(3/2),x)
```

```
[Out] 2*A*b**2*x**(7/2)/7 + 4*A*b*c*x**(11/2)/11 + 2*A*c**2*x**(15/2)/15 + 2*B*b*
*2*x**(11/2)/11 + 4*B*b*c*x**(15/2)/15 + 2*B*c**2*x**(19/2)/19
```

Giac [A]
time = 0.47, size = 53, normalized size = 0.84

$$\frac{2}{19}\,Bc^2x^{\frac{19}{2}}+\frac{4}{15}\,Bbcx^{\frac{15}{2}}+\frac{2}{15}\,Ac^2x^{\frac{15}{2}}+\frac{2}{11}\,Bb^2x^{\frac{11}{2}}+\frac{4}{11}\,Abcx^{\frac{11}{2}}+\frac{2}{7}\,Ab^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/19*B*c^2*x^(19/2) + 4/15*B*b*c*x^(15/2) + 2/15*A*c^2*x^(15/2) + 2/11*B*b^
2*x^(11/2) + 4/11*A*b*c*x^(11/2) + 2/7*A*b^2*x^(7/2)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.81

$$x^{11/2}\left(\frac{2\,B\,b^2}{11}+\frac{4\,A\,c\,b}{11}\right)+x^{15/2}\left(\frac{2\,A\,c^2}{15}+\frac{4\,B\,b\,c}{15}\right)+\frac{2\,A\,b^2\,x^{7/2}}{7}+\frac{2\,B\,c^2\,x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(3/2),x)
```

```
[Out] x^(11/2)*((2*B*b^2)/11 + (4*A*b*c)/11) + x^(15/2)*((2*A*c^2)/15 + (4*B*b*c)
/15) + (2*A*b^2*x^(7/2))/7 + (2*B*c^2*x^(19/2))/19
```

3.173 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}} dx$

Optimal. Leaf size=63

$$\frac{2}{5}Ab^2x^{5/2}+\frac{2}{9}b(bB+2Ac)x^{9/2}+\frac{2}{13}c(2bB+Ac)x^{13/2}+\frac{2}{17}Bc^2x^{17/2}$$

```
[Out] 2/5*A*b^2*x^(5/2)+2/9*b*(2*A*c+B*b)*x^(9/2)+2/13*c*(A*c+2*B*b)*x^(13/2)+2/1
7*B*c^2*x^(17/2)
```

Rubi [A]

time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1598, 459}

$$\frac{2}{5}Ab^2x^{5/2}+\frac{2}{13}cx^{13/2}(Ac+2bB)+\frac{2}{9}bx^{9/2}(2Ac+bB)+\frac{2}{17}Bc^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(5/2),x]
```

```
[Out] (2*A*b^2*x^(5/2))/5 + (2*b*(b*B + 2*A*c)*x^(9/2))/9 + (2*c*(2*b*B + A*c)*x^
(13/2))/13 + (2*B*c^2*x^(17/2))/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}}\,dx &= \int x^{3/2}(A+Bx^2)\left(b+cx^2\right)^2\,dx\\ &= \int \left(Ab^2x^{3/2}+b(bB+2Ac)x^{7/2}+c(2bB+Ac)x^{11/2}+Bc^2x^{15/2}\right)dx\\ &= \frac{2}{5}Ab^2x^{5/2}+\frac{2}{9}b(bB+2Ac)x^{9/2}+\frac{2}{13}c(2bB+Ac)x^{13/2}+\frac{2}{17}Bc^2x^{17/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 0.97

$$\frac{2x^{5/2}(17A(117b^2+130bcx^2+45c^2x^4)+5Bx^2(221b^2+306bcx^2+117c^2x^4))}{9945}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(5/2),x]
```

```
[Out] (2*x^(5/2)*(17*A*(117*b^2 + 130*b*c*x^2 + 45*c^2*x^4) + 5*B*x^2*(221*b^2 +
306*b*c*x^2 + 117*c^2*x^4)))/9945
```

Maple [A]
time = 0.36, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2Bc^2x^{\frac{17}{2}}}{17}+\frac{2(Ac^2+2bBc)x^{\frac{13}{2}}}{13}+\frac{2(2Abc+b^2B)x^{\frac{9}{2}}}{9}+\frac{2Ab^2x^{\frac{5}{2}}}{5}$	52
default	$\frac{2Bc^2x^{\frac{17}{2}}}{17}+\frac{2(Ac^2+2bBc)x^{\frac{13}{2}}}{13}+\frac{2(2Abc+b^2B)x^{\frac{9}{2}}}{9}+\frac{2Ab^2x^{\frac{5}{2}}}{5}$	52
gosper	$\frac{2x^{\frac{5}{2}}(585Bc^2x^6+765Ac^2x^4+1530x^4bBc+2210Abcx^2+1105b^2Bx^2+1989b^2A)}{9945}$	56
trager	$\frac{2x^{\frac{5}{2}}(585Bc^2x^6+765Ac^2x^4+1530x^4bBc+2210Abcx^2+1105b^2Bx^2+1989b^2A)}{9945}$	56
risch	$\frac{2x^{\frac{5}{2}}(585Bc^2x^6+765Ac^2x^4+1530x^4bBc+2210Abcx^2+1105b^2Bx^2+1989b^2A)}{9945}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/17*B*c^2*x^(17/2)+2/13*(A*c^2+2*B*b*c)*x^(13/2)+2/9*(2*A*b*c+B*b^2)*x^(9/
2)+2/5*A*b^2*x^(5/2)
```

Maxima [A]
time = 0.26, size = 51, normalized size = 0.81

$$\frac{2}{17}Bc^2x^{\frac{17}{2}}+\frac{2}{13}\left(2Bbc+Ac^2\right)x^{\frac{13}{2}}+\frac{2}{5}Ab^2x^{\frac{5}{2}}+\frac{2}{9}\left(Bb^2+2Abc\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/17*B*c^2*x^(17/2) + 2/13*(2*B*b*c + A*c^2)*x^(13/2) + 2/5*A*b^2*x^(5/2) +
 2/9*(B*b^2 + 2*A*b*c)*x^(9/2)
```

Fricas [A]
time = 1.55, size = 56, normalized size = 0.89

$$\frac{2}{9945}\left(585Bc^2x^8+765\left(2Bbc+Ac^2\right)x^6+1989Ab^2x^2+1105\left(Bb^2+2Abc\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*B*c^2*x^8 + 765*(2*B*b*c + A*c^2)*x^6 + 1989*A*b^2*x^2 + 1105*(
B*b^2 + 2*A*b*c)*x^4)*sqrt(x)
```

Sympy [A]
time = 1.02, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{5}{2}}}{5}+\frac{4Abcx^{\frac{9}{2}}}{9}+\frac{2Ac^2x^{\frac{13}{2}}}{13}+\frac{2Bb^2x^{\frac{9}{2}}}{9}+\frac{4Bbcx^{\frac{13}{2}}}{13}+\frac{2Bc^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(5/2),x)
```

```
[Out] 2*A*b**2*x**(5/2)/5 + 4*A*b*c*x**(9/2)/9 + 2*A*c**2*x**(13/2)/13 + 2*B*b**2
*x**(9/2)/9 + 4*B*b*c*x**(13/2)/13 + 2*B*c**2*x**(17/2)/17
```

Giac [A]
time = 0.43, size = 53, normalized size = 0.84

$$\frac{2}{17}\,Bc^2x^{\frac{17}{2}}+\frac{4}{13}\,Bbcx^{\frac{13}{2}}+\frac{2}{13}\,Ac^2x^{\frac{13}{2}}+\frac{2}{9}\,Bb^2x^{\frac{9}{2}}+\frac{4}{9}\,Abcx^{\frac{9}{2}}+\frac{2}{5}\,Ab^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/17*B*c^2*x^(17/2) + 4/13*B*b*c*x^(13/2) + 2/13*A*c^2*x^(13/2) + 2/9*B*b^2
*x^(9/2) + 4/9*A*b*c*x^(9/2) + 2/5*A*b^2*x^(5/2)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.81

$$x^{9/2}\,\left(\frac{2\,B\,b^2}{9}+\frac{4\,A\,c\,b}{9}\right)+x^{13/2}\,\left(\frac{2\,A\,c^2}{13}+\frac{4\,B\,b\,c}{13}\right)+\frac{2\,A\,b^2\,x^{5/2}}{5}+\frac{2\,B\,c^2\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(5/2),x)
```

```
[Out] x^(9/2)*((2*B*b^2)/9 + (4*A*b*c)/9) + x^(13/2)*((2*A*c^2)/13 + (4*B*b*c)/13
) + (2*A*b^2*x^(5/2))/5 + (2*B*c^2*x^(17/2))/17
```

3.174 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2}{3}Ab^2x^{3/2}+\frac{2}{7}b(bB+2Ac)x^{7/2}+\frac{2}{11}c(2bB+Ac)x^{11/2}+\frac{2}{15}Bc^2x^{15/2}$$

```
[Out] 2/3*A*b^2*x^(3/2)+2/7*b*(2*A*c+B*b)*x^(7/2)+2/11*c*(A*c+2*B*b)*x^(11/2)+2/1
5*B*c^2*x^(15/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1598, 459}

$$\frac{2}{3}Ab^2x^{3/2}+\frac{2}{11}cx^{11/2}(Ac+2bB)+\frac{2}{7}bx^{7/2}(2Ac+bB)+\frac{2}{15}Bc^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(7/2),x]
```

```
[Out] (2*A*b^2*x^(3/2))/3 + (2*b*(b*B + 2*A*c)*x^(7/2))/7 + (2*c*(2*b*B + A*c)*x^
(11/2))/11 + (2*B*c^2*x^(15/2))/15
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}}\,dx &= \int \sqrt{x}\,(A+Bx^2)\,(b+cx^2)^2\,dx\\ &= \int \left(Ab^2\sqrt{x}+b(bB+2Ac)x^{5/2}+c(2bB+Ac)x^{9/2}+Bc^2x^{13/2}\right)dx\\ &= \frac{2}{3}Ab^2x^{3/2}+\frac{2}{7}b(bB+2Ac)x^{7/2}+\frac{2}{11}c(2bB+Ac)x^{11/2}+\frac{2}{15}Bc^2x^{15/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 0.95

$$\frac{2x^{3/2}(5A(77b^2+66bcx^2+21c^2x^4)+Bx^2(165b^2+210bcx^2+77c^2x^4))}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(7/2),x]
```

```
[Out] (2*x^(3/2)*(5*A*(77*b^2 + 66*b*c*x^2 + 21*c^2*x^4) + B*x^2*(165*b^2 + 210*b
*c*x^2 + 77*c^2*x^4)))/1155
```

Maple [A]
time = 0.35, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2B\,c^2x^{\frac{15}{2}}}{15}+\frac{2(A\,c^2+2bBc)x^{\frac{11}{2}}}{11}+\frac{2(2Abc+b^2B)x^{\frac{7}{2}}}{7}+\frac{2A\,b^2x^{\frac{3}{2}}}{3}$	52
default	$\frac{2B\,c^2x^{\frac{15}{2}}}{15}+\frac{2(A\,c^2+2bBc)x^{\frac{11}{2}}}{11}+\frac{2(2Abc+b^2B)x^{\frac{7}{2}}}{7}+\frac{2A\,b^2x^{\frac{3}{2}}}{3}$	52
gosper	$\frac{2x^{\frac{3}{2}}(77B\,c^2x^6+105A\,c^2x^4+210x^4bBc+330Abc\,x^2+165b^2B\,x^2+385b^2A)}{1155}$	56
trager	$\frac{2x^{\frac{3}{2}}(77B\,c^2x^6+105A\,c^2x^4+210x^4bBc+330Abc\,x^2+165b^2B\,x^2+385b^2A)}{1155}$	56
risch	$\frac{2x^{\frac{3}{2}}(77B\,c^2x^6+105A\,c^2x^4+210x^4bBc+330Abc\,x^2+165b^2B\,x^2+385b^2A)}{1155}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*B*c^2*x^(15/2)+2/11*(A*c^2+2*B*b*c)*x^(11/2)+2/7*(2*A*b*c+B*b^2)*x^(7/
2)+2/3*A*b^2*x^(3/2)
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.81

$$\frac{2}{15}\,Bc^2x^{\frac{15}{2}}+\frac{2}{11}\,(2\,Bbc+Ac^2)x^{\frac{11}{2}}+\frac{2}{3}\,Ab^2x^{\frac{3}{2}}+\frac{2}{7}\,(Bb^2+2\,Abc)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*c^2*x^(15/2) + 2/11*(2*B*b*c + A*c^2)*x^(11/2) + 2/3*A*b^2*x^(3/2) +
 2/7*(B*b^2 + 2*A*b*c)*x^(7/2)
```

Fricas [A]
time = 1.94, size = 54, normalized size = 0.86

$$\frac{2}{1155}\,(77\,Bc^2x^7+105\,(2\,Bbc+Ac^2)x^5+385\,Ab^2x+165\,(Bb^2+2\,Abc)x^3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*c^2*x^7 + 105*(2*B*b*c + A*c^2)*x^5 + 385*A*b^2*x + 165*(B*b^2
 + 2*A*b*c)*x^3)*sqrt(x)
```

Sympy [A]
time = 1.32, size = 80, normalized size = 1.27

$$\frac{2Ab^2x^{\frac{3}{2}}}{3}+\frac{4Abcx^{\frac{7}{2}}}{7}+\frac{2Ac^2x^{\frac{11}{2}}}{11}+\frac{2Bb^2x^{\frac{7}{2}}}{7}+\frac{4Bbcx^{\frac{11}{2}}}{11}+\frac{2Bc^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(7/2),x)
```

```
[Out] 2*A*b**2*x**(3/2)/3 + 4*A*b*c*x**(7/2)/7 + 2*A*c**2*x**(11/2)/11 + 2*B*b**2
*x**(7/2)/7 + 4*B*b*c*x**(11/2)/11 + 2*B*c**2*x**(15/2)/15
```

Giac [A]
time = 0.44, size = 53, normalized size = 0.84

$$\frac{2}{15}\,Bc^2x^{\frac{15}{2}}+\frac{4}{11}\,Bbcx^{\frac{11}{2}}+\frac{2}{11}\,Ac^2x^{\frac{11}{2}}+\frac{2}{7}\,Bb^2x^{\frac{7}{2}}+\frac{4}{7}\,Abcx^{\frac{7}{2}}+\frac{2}{3}\,Ab^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*B*c^2*x^(15/2) + 4/11*B*b*c*x^(11/2) + 2/11*A*c^2*x^(11/2) + 2/7*B*b^2
*x^(7/2) + 4/7*A*b*c*x^(7/2) + 2/3*A*b^2*x^(3/2)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.81

$$x^{7/2}\left(\frac{2\,B\,b^2}{7}+\frac{4\,A\,c\,b}{7}\right)+x^{11/2}\left(\frac{2\,A\,c^2}{11}+\frac{4\,B\,b\,c}{11}\right)+\frac{2\,A\,b^2\,x^{3/2}}{3}+\frac{2\,B\,c^2\,x^{15/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(7/2),x)
```

```
[Out] x^(7/2)*((2*B*b^2)/7 + (4*A*b*c)/7) + x^(11/2)*((2*A*c^2)/11 + (4*B*b*c)/11
) + (2*A*b^2*x^(3/2))/3 + (2*B*c^2*x^(15/2))/15
```

3.175 $\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=85

$$\frac{2}{21}Ab^3x^{21/2}+\frac{2}{25}b^2(bB+3Ac)x^{25/2}+\frac{6}{29}bc(bB+Ac)x^{29/2}+\frac{2}{33}c^2(3bB+Ac)x^{33/2}+\frac{2}{37}Bc^3x^{37/2}$$

```
[Out] 2/21*A*b^3*x^(21/2)+2/25*b^2*(3*A*c+B*b)*x^(25/2)+6/29*b*c*(A*c+B*b)*x^(29/
2)+2/33*c^2*(A*c+3*B*b)*x^(33/2)+2/37*B*c^3*x^(37/2)
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 459}

$$\frac{2}{21}Ab^3x^{21/2}+\frac{2}{25}b^2x^{25/2}(3Ac+bB)+\frac{2}{33}c^2x^{33/2}(Ac+3bB)+\frac{6}{29}bcx^{29/2}(Ac+bB)+\frac{2}{37}Bc^3x^{37/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(21/2))/21 + (2*b^2*(b*B + 3*A*c)*x^(25/2))/25 + (6*b*c*(b*B + A
*c)*x^(29/2))/29 + (2*c^2*(3*b*B + A*c)*x^(33/2))/33 + (2*B*c^3*x^(37/2))/3
7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx&=\int x^{19/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx\\&=\int\left(Ab^3x^{19/2}+b^2(bB+3Ac)x^{23/2}+3bc(bB+Ac)x^{27/2}+c^2(3bB+A\right.\\&=\frac{2}{21}Ab^3x^{21/2}+\frac{2}{25}b^2(bB+3Ac)x^{25/2}+\frac{6}{29}bc(bB+Ac)x^{29/2}+\frac{2}{33}c^2(3b\end{aligned}$$

Mathematica [A]
time = 0.07, size = 85, normalized size = 1.00

$$\frac{2}{21}Ab^3x^{21/2}+\frac{2}{25}b^2(bB+3Ac)x^{25/2}+\frac{6}{29}bc(bB+Ac)x^{29/2}+\frac{2}{33}c^2(3bB+Ac)x^{33/2}+\frac{2}{37}Bc^3x^{37/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(21/2))/21 + (2*b^2*(b*B + 3*A*c)*x^(25/2))/25 + (6*b*c*(b*B + A
*c)*x^(29/2))/29 + (2*c^2*(3*b*B + A*c)*x^(33/2))/33 + (2*B*c^3*x^(37/2))/3
7
```

Maple [A]
time = 0.36, size = 76, normalized size = 0.89

method	result
derivativedivides	$\frac{2Bc^3x^{\frac{37}{2}}}{37}+\frac{2(Ac^3+3Bbc^2)x^{\frac{33}{2}}}{33}+\frac{2(3Abc^2+3Bb^2c)x^{\frac{29}{2}}}{29}+\frac{2(3Ab^2c+Bb^3)x^{\frac{25}{2}}}{25}+\frac{2Ab^3x^{\frac{21}{2}}}{21}$
default	$\frac{2Bc^3x^{\frac{37}{2}}}{37}+\frac{2(Ac^3+3Bbc^2)x^{\frac{33}{2}}}{33}+\frac{2(3Abc^2+3Bb^2c)x^{\frac{29}{2}}}{29}+\frac{2(3Ab^2c+Bb^3)x^{\frac{25}{2}}}{25}+\frac{2Ab^3x^{\frac{21}{2}}}{21}$
gosper	$\frac{2x^{\frac{21}{2}}(167475Bc^3x^8+187775Ac^3x^6+563325x^6Bbc^2+641025Abc^2x^4+641025x^4Bb^2c+743589Ab^2cx^2+247863x^2Bb^3+}{6196575}$
trager	$\frac{2x^{\frac{21}{2}}(167475Bc^3x^8+187775Ac^3x^6+563325x^6Bbc^2+641025Abc^2x^4+641025x^4Bb^2c+743589Ab^2cx^2+247863x^2Bb^3+}{6196575}$
risch	$\frac{2x^{\frac{21}{2}}(167475Bc^3x^8+187775Ac^3x^6+563325x^6Bbc^2+641025Abc^2x^4+641025x^4Bb^2c+743589Ab^2cx^2+247863x^2Bb^3+}{6196575}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/37*B*c^3*x^(37/2)+2/33*(A*c^3+3*B*b*c^2)*x^(33/2)+2/29*(3*A*b*c^2+3*B*b^2
*c)*x^(29/2)+2/25*(3*A*b^2*c+B*b^3)*x^(25/2)+2/21*A*b^3*x^(21/2)
```

Maxima [A]
time = 0.27, size = 73, normalized size = 0.86

$$\frac{2}{37}Bc^3x^{\frac{37}{2}}+\frac{2}{33}\left(3Bbc^2+Ac^3\right)x^{\frac{33}{2}}+\frac{6}{29}\left(Bb^2c+Abc^2\right)x^{\frac{29}{2}}+\frac{2}{21}Ab^3x^{\frac{21}{2}}+\frac{2}{25}\left(Bb^3+3Ab^2c\right)x^{\frac{25}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/37*B*c^3*x^(37/2) + 2/33*(3*B*b*c^2 + A*c^3)*x^(33/2) + 6/29*(B*b^2*c + A
*b*c^2)*x^(29/2) + 2/21*A*b^3*x^(21/2) + 2/25*(B*b^3 + 3*A*b^2*c)*x^(25/2)
```

Fricas [A]
time = 1.83, size = 78, normalized size = 0.92

$$\frac{2}{6196575}\left(167475Bc^3x^{18}+187775\left(3Bbc^2+Ac^3\right)x^{16}+641025\left(Bb^2c+Abc^2\right)x^{14}+295075Ab^3x^{10}+247863\left(Bb^3+3Ab^2c\right)x^{12}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/6196575*(167475*B*c^3*x^18 + 187775*(3*B*b*c^2 + A*c^3)*x^16 + 641025*(B*
b^2*c + A*b*c^2)*x^14 + 295075*A*b^3*x^10 + 247863*(B*b^3 + 3*A*b^2*c)*x^12
)*sqrt(x)
```

Sympy [A]
time = 3.96, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{21}{2}}}{21}+\frac{6Ab^2cx^{\frac{25}{2}}}{25}+\frac{6Abc^2x^{\frac{29}{2}}}{29}+\frac{2Ac^3x^{\frac{33}{2}}}{33}+\frac{2Bb^3x^{\frac{25}{2}}}{25}+\frac{6Bb^2cx^{\frac{29}{2}}}{29}+\frac{2Bbc^2x^{\frac{33}{2}}}{11}+\frac{2Bc^3x^{\frac{37}{2}}}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*A*b**3*x**(21/2)/21 + 6*A*b**2*c*x**(25/2)/25 + 6*A*b*c**2*x**(29/2)/29 +
 2*A*c**3*x**(33/2)/33 + 2*B*b**3*x**(25/2)/25 + 6*B*b**2*c*x**(29/2)/29 +
2*B*b*c**2*x**(33/2)/11 + 2*B*c**3*x**(37/2)/37
```

Giac [A]
time = 0.46, size = 77, normalized size = 0.91

$$\frac{2}{37}Bc^3x^{\frac{37}{2}}+\frac{2}{11}Bbc^2x^{\frac{33}{2}}+\frac{2}{33}Ac^3x^{\frac{33}{2}}+\frac{6}{29}Bb^2cx^{\frac{29}{2}}+\frac{6}{29}Abc^2x^{\frac{29}{2}}+\frac{2}{25}Bb^3x^{\frac{25}{2}}+\frac{6}{25}Ab^2cx^{\frac{25}{2}}+\frac{2}{21}Ab^3x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/37*B*c^3*x^(37/2) + 2/11*B*b*c^2*x^(33/2) + 2/33*A*c^3*x^(33/2) + 6/29*B*
b^2*c*x^(29/2) + 6/29*A*b*c^2*x^(29/2) + 2/25*B*b^3*x^(25/2) + 6/25*A*b^2*c
*x^(25/2) + 2/21*A*b^3*x^(21/2)
```

Mupad [B]
time = 0.10, size = 69, normalized size = 0.81

$$x^{25/2}\left(\frac{2\,B\,b^3}{25}+\frac{6\,A\,c\,b^2}{25}\right)+x^{33/2}\left(\frac{2\,A\,c^3}{33}+\frac{2\,B\,b\,c^2}{11}\right)+\frac{2\,A\,b^3\,x^{21/2}}{21}+\frac{2\,B\,c^3\,x^{37/2}}{37}+\frac{6\,b\,c\,x^{29/2}\,(A\,c+B\,b)}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(25/2)*((2*B*b^3)/25 + (6*A*b^2*c)/25) + x^(33/2)*((2*A*c^3)/33 + (2*B*b*
c^2)/11) + (2*A*b^3*x^(21/2))/21 + (2*B*c^3*x^(37/2))/37 + (6*b*c*x^(29/2)*
(A*c + B*b))/29
```

3.176 $\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=85

$$\frac{2}{19}Ab^3x^{19/2}+\frac{2}{23}b^2(bB+3Ac)x^{23/2}+\frac{2}{9}bc(bB+Ac)x^{27/2}+\frac{2}{31}c^2(3bB+Ac)x^{31/2}+\frac{2}{35}Bc^3x^{35/2}$$

```
[Out] 2/19*A*b^3*x^(19/2)+2/23*b^2*(3*A*c+B*b)*x^(23/2)+2/9*b*c*(A*c+B*b)*x^(27/2
)+2/31*c^2*(A*c+3*B*b)*x^(31/2)+2/35*B*c^3*x^(35/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1598, 459}

$$\frac{2}{19}Ab^3x^{19/2}+\frac{2}{23}b^2x^{23/2}(3Ac+bB)+\frac{2}{31}c^2x^{31/2}(Ac+3bB)+\frac{2}{9}bcx^{27/2}(Ac+bB)+\frac{2}{35}Bc^3x^{35/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(19/2))/19 + (2*b^2*(b*B + 3*A*c)*x^(23/2))/23 + (2*b*c*(b*B + A
*c)*x^(27/2))/9 + (2*c^2*(3*b*B + A*c)*x^(31/2))/31 + (2*B*c^3*x^(35/2))/35
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx &= \int x^{17/2}(A+Bx^2)\left(b+cx^2\right)^3\,dx\\ &= \int \left(Ab^3x^{17/2}+b^2(bB+3Ac)x^{21/2}+3bc(bB+Ac)x^{25/2}+c^2(3bB+Ac\right.\\ &= \frac{2}{19}Ab^3x^{19/2}+\frac{2}{23}b^2(bB+3Ac)x^{23/2}+\frac{2}{9}bc(bB+Ac)x^{27/2}+\frac{2}{31}c^2(3bB\end{aligned}$$

Mathematica [A]
time = 0.07, size = 85, normalized size = 1.00

$$\frac{2}{19}Ab^3x^{19/2}+\frac{2}{23}b^2(bB+3Ac)x^{23/2}+\frac{2}{9}bc(bB+Ac)x^{27/2}+\frac{2}{31}c^2(3bB+Ac)x^{31/2}+\frac{2}{35}Bc^3x^{35/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(19/2))/19 + (2*b^2*(b*B + 3*A*c)*x^(23/2))/23 + (2*b*c*(b*B + A
*c)*x^(27/2))/9 + (2*c^2*(3*b*B + A*c)*x^(31/2))/31 + (2*B*c^3*x^(35/2))/35
```

Maple [A]
time = 0.36, size = 76, normalized size = 0.89

method	result
derivativedivides	$\frac{2B\,c^3x^{\frac{35}{2}}}{35}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{31}{2}}}{31}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{27}{2}}}{27}+\frac{2(3A\,b^2c+B\,b^3)x^{\frac{23}{2}}}{23}+\frac{2A\,b^3x^{\frac{19}{2}}}{19}$
default	$\frac{2B\,c^3x^{\frac{35}{2}}}{35}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{31}{2}}}{31}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{27}{2}}}{27}+\frac{2(3A\,b^2c+B\,b^3)x^{\frac{23}{2}}}{23}+\frac{2A\,b^3x^{\frac{19}{2}}}{19}$
gosper	$\frac{2x^{\frac{19}{2}}(121923B\,c^3x^8+137655A\,c^3x^6+412965x^6Bb\,c^2+474145Ab\,c^2x^4+474145x^4B\,b^2c+556605A\,b^2c\,x^2+185535x^2B\,b^3}{4267305}$
trager	$\frac{2x^{\frac{19}{2}}(121923B\,c^3x^8+137655A\,c^3x^6+412965x^6Bb\,c^2+474145Ab\,c^2x^4+474145x^4B\,b^2c+556605A\,b^2c\,x^2+185535x^2B\,b^3}{4267305}$
risch	$\frac{2x^{\frac{19}{2}}(121923B\,c^3x^8+137655A\,c^3x^6+412965x^6Bb\,c^2+474145Ab\,c^2x^4+474145x^4B\,b^2c+556605A\,b^2c\,x^2+185535x^2B\,b^3}{4267305}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/35*B*c^3*x^(35/2)+2/31*(A*c^3+3*B*b*c^2)*x^(31/2)+2/27*(3*A*b*c^2+3*B*b^2
*c)*x^(27/2)+2/23*(3*A*b^2*c+B*b^3)*x^(23/2)+2/19*A*b^3*x^(19/2)
```

Maxima [A]
time = 0.27, size = 73, normalized size = 0.86

$$\frac{2}{35}Bc^3x^{\frac{35}{2}}+\frac{2}{31}\left(3\,Bbc^2+Ac^3\right)x^{\frac{31}{2}}+\frac{2}{9}\left(Bb^2c+Abc^2\right)x^{\frac{27}{2}}+\frac{2}{19}Ab^3x^{\frac{19}{2}}+\frac{2}{23}\left(Bb^3+3\,Ab^2c\right)x^{\frac{23}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/35*B*c^3*x^(35/2) + 2/31*(3*B*b*c^2 + A*c^3)*x^(31/2) + 2/9*(B*b^2*c + A*
b*c^2)*x^(27/2) + 2/19*A*b^3*x^(19/2) + 2/23*(B*b^3 + 3*A*b^2*c)*x^(23/2)
```

Fricas [A]
time = 2.19, size = 78, normalized size = 0.92

$$\frac{2}{4267305}\left(121923\,Bc^3x^{17}+137655\left(3\,Bbc^2+Ac^3\right)x^{15}+474145\left(Bb^2c+Abc^2\right)x^{13}+224595\,Ab^3x^9+185535\left(Bb^3+3\,Ab^2c\right)x^{11}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/4267305*(121923*B*c^3*x^17 + 137655*(3*B*b*c^2 + A*c^3)*x^15 + 474145*(B*
b^2*c + A*b*c^2)*x^13 + 224595*A*b^3*x^9 + 185535*(B*b^3 + 3*A*b^2*c)*x^11)
*sqrt(x)
```

Sympy [A]
time = 2.92, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{19}{2}}}{19} + \frac{6Ab^2cx^{\frac{23}{2}}}{23} + \frac{2Abc^2x^{\frac{27}{2}}}{9} + \frac{2Ac^3x^{\frac{31}{2}}}{31} + \frac{2Bb^3x^{\frac{23}{2}}}{23} + \frac{2Bb^2cx^{\frac{27}{2}}}{9} + \frac{6Bbc^2x^{\frac{31}{2}}}{31} + \frac{2Bc^3x^{\frac{35}{2}}}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*A*b**3*x**(19/2)/19 + 6*A*b**2*c*x**(23/2)/23 + 2*A*b*c**2*x**(27/2)/9 +
2*A*c**3*x**(31/2)/31 + 2*B*b**3*x**(23/2)/23 + 2*B*b**2*c*x**(27/2)/9 + 6*
B*b*c**2*x**(31/2)/31 + 2*B*c**3*x**(35/2)/35
```

Giac [A]
time = 0.44, size = 77, normalized size = 0.91

$$\frac{2}{35}\,Bc^3x^{\frac{35}{2}} + \frac{6}{31}\,Bbc^2x^{\frac{31}{2}} + \frac{2}{31}\,Ac^3x^{\frac{31}{2}} + \frac{2}{9}\,Bb^2cx^{\frac{27}{2}} + \frac{2}{9}\,Abc^2x^{\frac{27}{2}} + \frac{2}{23}\,Bb^3x^{\frac{23}{2}} + \frac{6}{23}\,Ab^2cx^{\frac{23}{2}} + \frac{2}{19}\,Ab^3x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/35*B*c^3*x^(35/2) + 6/31*B*b*c^2*x^(31/2) + 2/31*A*c^3*x^(31/2) + 2/9*B*b
^2*c*x^(27/2) + 2/9*A*b*c^2*x^(27/2) + 2/23*B*b^3*x^(23/2) + 6/23*A*b^2*c*x
^(23/2) + 2/19*A*b^3*x^(19/2)
```

Mupad [B]
time = 0.03, size = 69, normalized size = 0.81

$$x^{23/2}\left(\frac{2\,B\,b^3}{23}+\frac{6\,A\,c\,b^2}{23}\right)+x^{31/2}\left(\frac{2\,A\,c^3}{31}+\frac{6\,B\,b\,c^2}{31}\right)+\frac{2\,A\,b^3\,x^{19/2}}{19}+\frac{2\,B\,c^3\,x^{35/2}}{35}+\frac{2\,b\,c\,x^{27/2}\,(A\,c+B\,b)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(23/2)*((2*B*b^3)/23 + (6*A*b^2*c)/23) + x^(31/2)*((2*A*c^3)/31 + (6*B*b*
c^2)/31) + (2*A*b^3*x^(19/2))/19 + (2*B*c^3*x^(35/2))/35 + (2*b*c*x^(27/2)*
(A*c + B*b))/9
```

3.177 $\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=85

$$\frac{2}{17}Ab^3x^{17/2}+\frac{2}{21}b^2(bB+3Ac)x^{21/2}+\frac{6}{25}bc(bB+Ac)x^{25/2}+\frac{2}{29}c^2(3bB+Ac)x^{29/2}+\frac{2}{33}Bc^3x^{33/2}$$

```
[Out] 2/17*A*b^3*x^(17/2)+2/21*b^2*(3*A*c+B*b)*x^(21/2)+6/25*b*c*(A*c+B*b)*x^(25/
2)+2/29*c^2*(A*c+3*B*b)*x^(29/2)+2/33*B*c^3*x^(33/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {1598, 459}

$$\frac{2}{17}Ab^3x^{17/2}+\frac{2}{21}b^2x^{21/2}(3Ac+bB)+\frac{2}{29}c^2x^{29/2}(Ac+3bB)+\frac{6}{25}bcx^{25/2}(Ac+bB)+\frac{2}{33}Bc^3x^{33/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(17/2))/17 + (2*b^2*(b*B + 3*A*c)*x^(21/2))/21 + (6*b*c*(b*B + A
*c)*x^(25/2))/25 + (2*c^2*(3*b*B + A*c)*x^(29/2))/29 + (2*B*c^3*x^(33/2))/3
3
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx &= \int x^{15/2}(A+Bx^2)\left(b+cx^2\right)^3\,dx\\ &= \int \left(Ab^3x^{15/2}+b^2(bB+3Ac)x^{19/2}+3bc(bB+Ac)x^{23/2}+c^2(3bB+A\right.\\ &= \frac{2}{17}Ab^3x^{17/2}+\frac{2}{21}b^2(bB+3Ac)x^{21/2}+\frac{6}{25}bc(bB+Ac)x^{25/2}+\frac{2}{29}c^2(3b\end{aligned}$$

Mathematica [A]
time = 0.05, size = 97, normalized size = 1.14

$$\frac{2\left(167475Ab^3x^{17/2}+135575b^3Bx^{21/2}+406725Ab^2cx^{21/2}+341649b^2Bcx^{25/2}+341649Abc^2x^{25/2}+294525bBc^2x^{29/2}+98175Ac^3x^{29/2}+86275Bc^3x^{33/2}\right)}{2847075}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*(167475*A*b^3*x^(17/2) + 135575*b^3*B*x^(21/2) + 406725*A*b^2*c*x^(21/2)
 + 341649*b^2*B*c*x^(25/2) + 341649*A*b*c^2*x^(25/2) + 294525*b*B*c^2*x^(29
/2) + 98175*A*c^3*x^(29/2) + 86275*B*c^3*x^(33/2)))/2847075
```

Maple [A]
time = 0.36, size = 76, normalized size = 0.89

method	result
derivativedivides	$\frac{2Bc^3x^{\frac{33}{2}}}{33}+\frac{2(Ac^3+3Bbc^2)x^{\frac{29}{2}}}{29}+\frac{2(3Abc^2+3Bb^2c)x^{\frac{25}{2}}}{25}+\frac{2(3Ab^2c+Bb^3)x^{\frac{21}{2}}}{21}+\frac{2Ab^3x^{\frac{17}{2}}}{17}$
default	$\frac{2Bc^3x^{\frac{33}{2}}}{33}+\frac{2(Ac^3+3Bbc^2)x^{\frac{29}{2}}}{29}+\frac{2(3Abc^2+3Bb^2c)x^{\frac{25}{2}}}{25}+\frac{2(3Ab^2c+Bb^3)x^{\frac{21}{2}}}{21}+\frac{2Ab^3x^{\frac{17}{2}}}{17}$
gosper	$\frac{2x^{\frac{17}{2}}(86275Bc^3x^8+98175Ac^3x^6+294525x^6Bbc^2+341649Abc^2x^4+341649x^4Bb^2c+406725Ab^2cx^2+135575x^2Bb^3+16}{2847075}$
trager	$\frac{2x^{\frac{17}{2}}(86275Bc^3x^8+98175Ac^3x^6+294525x^6Bbc^2+341649Abc^2x^4+341649x^4Bb^2c+406725Ab^2cx^2+135575x^2Bb^3+16}{2847075}$
risch	$\frac{2x^{\frac{17}{2}}(86275Bc^3x^8+98175Ac^3x^6+294525x^6Bbc^2+341649Abc^2x^4+341649x^4Bb^2c+406725Ab^2cx^2+135575x^2Bb^3+16}{2847075}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/33*B*c^3*x^(33/2)+2/29*(A*c^3+3*B*b*c^2)*x^(29/2)+2/25*(3*A*b*c^2+3*B*b^2
*c)*x^(25/2)+2/21*(3*A*b^2*c+B*b^3)*x^(21/2)+2/17*A*b^3*x^(17/2)
```

Maxima [A]
time = 0.27, size = 73, normalized size = 0.86

$$\frac{2}{33}\,Bc^3x^{\frac{33}{2}}+\frac{2}{29}\left(3\,Bbc^2+Ac^3\right)x^{\frac{29}{2}}+\frac{6}{25}\left(Bb^2c+Abc^2\right)x^{\frac{25}{2}}+\frac{2}{17}\,Ab^3x^{\frac{17}{2}}+\frac{2}{21}\left(Bb^3+3\,Ab^2c\right)x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/33*B*c^3*x^(33/2) + 2/29*(3*B*b*c^2 + A*c^3)*x^(29/2) + 6/25*(B*b^2*c + A
*b*c^2)*x^(25/2) + 2/17*A*b^3*x^(17/2) + 2/21*(B*b^3 + 3*A*b^2*c)*x^(21/2)
```

Fricas [A]
time = 1.43, size = 78, normalized size = 0.92

$$\frac{2}{2847075}\left(86275\,Bc^3x^{16}+98175\left(3\,Bbc^2+Ac^3\right)x^{14}+341649\left(Bb^2c+Abc^2\right)x^{12}+167475\,Ab^3x^8+135575\left(Bb^3+3\,Ab^2c\right)x^{10}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/2847075*(86275*B*c^3*x^16 + 98175*(3*B*b*c^2 + A*c^3)*x^14 + 341649*(B*b^
2*c + A*b*c^2)*x^12 + 167475*A*b^3*x^8 + 135575*(B*b^3 + 3*A*b^2*c)*x^10)*s
qrt(x)
```

Sympy [A]
time = 2.19, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{17}{2}}}{17} + \frac{2Ab^2cx^{\frac{21}{2}}}{7} + \frac{6Abc^2x^{\frac{25}{2}}}{25} + \frac{2Ac^3x^{\frac{29}{2}}}{29} + \frac{2Bb^3x^{\frac{21}{2}}}{21} + \frac{6Bb^2cx^{\frac{25}{2}}}{25} + \frac{6Bbc^2x^{\frac{29}{2}}}{29} + \frac{2Bc^3x^{\frac{33}{2}}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*A*b**3*x**(17/2)/17 + 2*A*b**2*c*x**(21/2)/7 + 6*A*b*c**2*x**(25/2)/25 +
2*A*c**3*x**(29/2)/29 + 2*B*b**3*x**(21/2)/21 + 6*B*b**2*c*x**(25/2)/25 + 6
*B*b*c**2*x**(29/2)/29 + 2*B*c**3*x**(33/2)/33
```

Giac [A]
time = 0.47, size = 77, normalized size = 0.91

$$\frac{2}{33}\,Bc^3x^{\frac{33}{2}} + \frac{6}{29}\,Bbc^2x^{\frac{29}{2}} + \frac{2}{29}\,Ac^3x^{\frac{29}{2}} + \frac{6}{25}\,Bb^2cx^{\frac{25}{2}} + \frac{6}{25}\,Abc^2x^{\frac{25}{2}} + \frac{2}{21}\,Bb^3x^{\frac{21}{2}} + \frac{2}{7}\,Ab^2cx^{\frac{21}{2}} + \frac{2}{17}\,Ab^3x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/33*B*c^3*x^(33/2) + 6/29*B*b*c^2*x^(29/2) + 2/29*A*c^3*x^(29/2) + 6/25*B*
b^2*c*x^(25/2) + 6/25*A*b*c^2*x^(25/2) + 2/21*B*b^3*x^(21/2) + 2/7*A*b^2*c*
x^(21/2) + 2/17*A*b^3*x^(17/2)
```

Mupad [B]
time = 0.04, size = 69, normalized size = 0.81

$$x^{21/2}\left(\frac{2\,B\,b^3}{21}+\frac{2\,A\,c\,b^2}{7}\right)+x^{29/2}\left(\frac{2\,A\,c^3}{29}+\frac{6\,B\,b\,c^2}{29}\right)+\frac{2\,A\,b^3\,x^{17/2}}{17}+\frac{2\,B\,c^3\,x^{33/2}}{33}+\frac{6\,b\,c\,x^{25/2}\,\left(A\,c+B\,b\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(21/2)*((2*B*b^3)/21 + (2*A*b^2*c)/7) + x^(29/2)*((2*A*c^3)/29 + (6*B*b*c
^2)/29) + (2*A*b^3*x^(17/2))/17 + (2*B*c^3*x^(33/2))/33 + (6*b*c*x^(25/2)*(
A*c + B*b))/25
```

3.178 $\int \sqrt{x} \left(A+Bx^2\right) \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=85

$$\frac{2}{15}Ab^3x^{15/2}+\frac{2}{19}b^2(bB+3Ac)x^{19/2}+\frac{6}{23}bc(bB+Ac)x^{23/2}+\frac{2}{27}c^2(3bB+Ac)x^{27/2}+\frac{2}{31}Bc^3x^{31/2}$$

```
[Out] 2/15*A*b^3*x^(15/2)+2/19*b^2*(3*A*c+B*b)*x^(19/2)+6/23*b*c*(A*c+B*b)*x^(23/
2)+2/27*c^2*(A*c+3*B*b)*x^(27/2)+2/31*B*c^3*x^(31/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1598, 459}

$$\frac{2}{15}Ab^3x^{15/2}+\frac{2}{19}b^2x^{19/2}(3Ac+bB)+\frac{2}{27}c^2x^{27/2}(Ac+3bB)+\frac{6}{23}bcx^{23/2}(Ac+bB)+\frac{2}{31}Bc^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(15/2))/15 + (2*b^2*(b*B + 3*A*c)*x^(19/2))/19 + (6*b*c*(b*B + A
*c)*x^(23/2))/23 + (2*c^2*(3*b*B + A*c)*x^(27/2))/27 + (2*B*c^3*x^(31/2))/3
1
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx &= \int x^{13/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx\\ &= \int \left(Ab^3x^{13/2}+b^2(bB+3Ac)x^{17/2}+3bc(bB+Ac)x^{21/2}+c^2(3bB+Ac\right.\\ &= \frac{2}{15}Ab^3x^{15/2}+\frac{2}{19}b^2(bB+3Ac)x^{19/2}+\frac{6}{23}bc(bB+Ac)x^{23/2}+\frac{2}{27}c^2(3bB\end{aligned}$$

Mathematica [A]
time = 0.05, size = 83, normalized size = 0.98

$$\frac{2x^{15/2}(31A(3933b^3+9315b^2cx^2+7695bc^2x^4+2185c^3x^6)+15Bx^2(6417b^3+15903b^2cx^2+13547bc^2x^4+3933c^3x^6))}{1828845}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*x^(15/2)*(31*A*(3933*b^3 + 9315*b^2*c*x^2 + 7695*b*c^2*x^4 + 2185*c^3*x^
6) + 15*B*x^2*(6417*b^3 + 15903*b^2*c*x^2 + 13547*b*c^2*x^4 + 3933*c^3*x^6)
))/1828845
```

Maple [A]
time = 0.35, size = 76, normalized size = 0.89

method	result
derivativedivides	$\frac{2Bc^3x^{\frac{31}{2}}}{31}+\frac{2(Ac^3+3Bbc^2)x^{\frac{27}{2}}}{27}+\frac{2(3Abc^2+3Bb^2c)x^{\frac{23}{2}}}{23}+\frac{2(3Ab^2c+Bb^3)x^{\frac{19}{2}}}{19}+\frac{2Ab^3x^{\frac{15}{2}}}{15}$
default	$\frac{2Bc^3x^{\frac{31}{2}}}{31}+\frac{2(Ac^3+3Bbc^2)x^{\frac{27}{2}}}{27}+\frac{2(3Abc^2+3Bb^2c)x^{\frac{23}{2}}}{23}+\frac{2(3Ab^2c+Bb^3)x^{\frac{19}{2}}}{19}+\frac{2Ab^3x^{\frac{15}{2}}}{15}$
gosper	$\frac{2x^{\frac{15}{2}}(58995Bc^3x^8+67735Ac^3x^6+203205x^6Bbc^2+238545Abc^2x^4+238545x^4Bb^2c+288765Ab^2cx^2+96255x^2Bb^3+1}{1828845}$
trager	$\frac{2x^{\frac{15}{2}}(58995Bc^3x^8+67735Ac^3x^6+203205x^6Bbc^2+238545Abc^2x^4+238545x^4Bb^2c+288765Ab^2cx^2+96255x^2Bb^3+1}{1828845}$
risch	$\frac{2x^{\frac{15}{2}}(58995Bc^3x^8+67735Ac^3x^6+203205x^6Bbc^2+238545Abc^2x^4+238545x^4Bb^2c+288765Ab^2cx^2+96255x^2Bb^3+1}{1828845}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/31*B*c^3*x^(31/2)+2/27*(A*c^3+3*B*b*c^2)*x^(27/2)+2/23*(3*A*b*c^2+3*B*b^2
*c)*x^(23/2)+2/19*(3*A*b^2*c+B*b^3)*x^(19/2)+2/15*A*b^3*x^(15/2)
```

Maxima [A]
time = 0.32, size = 73, normalized size = 0.86

$$\frac{2}{31}\,Bc^3x^{\frac{31}{2}}+\frac{2}{27}\,\left(3\,Bbc^2+Ac^3\right)x^{\frac{27}{2}}+\frac{6}{23}\,\left(Bb^2c+Abc^2\right)x^{\frac{23}{2}}+\frac{2}{15}\,Ab^3x^{\frac{15}{2}}+\frac{2}{19}\,\left(Bb^3+3\,Ab^2c\right)x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/31*B*c^3*x^(31/2) + 2/27*(3*B*b*c^2 + A*c^3)*x^(27/2) + 6/23*(B*b^2*c + A
*b*c^2)*x^(23/2) + 2/15*A*b^3*x^(15/2) + 2/19*(B*b^3 + 3*A*b^2*c)*x^(19/2)
```

Fricas [A]
time = 1.83, size = 78, normalized size = 0.92

$$\frac{2}{1828845}\left(58995\,Bc^3x^{15}+67735\,\left(3\,Bbc^2+Ac^3\right)x^{13}+238545\,\left(Bb^2c+Abc^2\right)x^{11}+121923\,Ab^3x^7+96255\,\left(Bb^3+3\,Ab^2c\right)x^9\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1828845*(58995*B*c^3*x^15 + 67735*(3*B*b*c^2 + A*c^3)*x^13 + 238545*(B*b^
2*c + A*b*c^2)*x^11 + 121923*A*b^3*x^7 + 96255*(B*b^3 + 3*A*b^2*c)*x^9)*sqr
t(x)
```

Sympy [A]
time = 2.65, size = 95, normalized size = 1.12

$$\frac{2Ab^3x^{\frac{15}{2}}}{15}+\frac{2Bc^3x^{\frac{31}{2}}}{31}+\frac{2x^{\frac{27}{2}}(Ac^3+3Bbc^2)}{27}+\frac{2x^{\frac{23}{2}}\cdot(3Abc^2+3Bb^2c)}{23}+\frac{2x^{\frac{19}{2}}\cdot(3Ab^2c+Bb^3)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3*x**(1/2),x)
```

```
[Out] 2*A*b**3*x**(15/2)/15 + 2*B*c**3*x**(31/2)/31 + 2*x**(27/2)*(A*c**3 + 3*B*b
*c**2)/27 + 2*x**(23/2)*(3*A*b*c**2 + 3*B*b**2*c)/23 + 2*x**(19/2)*(3*A*b**
2*c + B*b**3)/19
```

Giac [A]
time = 0.43, size = 77, normalized size = 0.91

$$\frac{2}{31}Bc^3x^{\frac{31}{2}}+\frac{2}{9}Bbc^2x^{\frac{27}{2}}+\frac{2}{27}Ac^3x^{\frac{27}{2}}+\frac{6}{23}Bb^2cx^{\frac{23}{2}}+\frac{6}{23}Abc^2x^{\frac{23}{2}}+\frac{2}{19}Bb^3x^{\frac{19}{2}}+\frac{6}{19}Ab^2cx^{\frac{19}{2}}+\frac{2}{15}Ab^3x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/31*B*c^3*x^(31/2) + 2/9*B*b*c^2*x^(27/2) + 2/27*A*c^3*x^(27/2) + 6/23*B*b
^2*c*x^(23/2) + 6/23*A*b*c^2*x^(23/2) + 2/19*B*b^3*x^(19/2) + 6/19*A*b^2*c*
x^(19/2) + 2/15*A*b^3*x^(15/2)
```

Mupad [B]
time = 0.03, size = 69, normalized size = 0.81

$$x^{19/2}\left(\frac{2\,B\,b^3}{19}+\frac{6\,A\,c\,b^2}{19}\right)+x^{27/2}\left(\frac{2\,A\,c^3}{27}+\frac{2\,B\,b\,c^2}{9}\right)+\frac{2\,A\,b^3\,x^{15/2}}{15}+\frac{2\,B\,c^3\,x^{31/2}}{31}+\frac{6\,b\,c\,x^{23/2}\,(A\,c+B\,b)}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(19/2)*((2*B*b^3)/19 + (6*A*b^2*c)/19) + x^(27/2)*((2*A*c^3)/27 + (2*B*b*
c^2)/9) + (2*A*b^3*x^(15/2))/15 + (2*B*c^3*x^(31/2))/31 + (6*b*c*x^(23/2)*(
A*c + B*b))/23
```

3.179 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=85

$$\frac{2}{13}Ab^3x^{13/2}+\frac{2}{17}b^2(bB+3Ac)x^{17/2}+\frac{2}{7}bc(bB+Ac)x^{21/2}+\frac{2}{25}c^2(3bB+Ac)x^{25/2}+\frac{2}{29}Bc^3x^{29/2}$$

```
[Out] 2/13*A*b^3*x^(13/2)+2/17*b^2*(3*A*c+B*b)*x^(17/2)+2/7*b*c*(A*c+B*b)*x^(21/2
)+2/25*c^2*(A*c+3*B*b)*x^(25/2)+2/29*B*c^3*x^(29/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1598, 459}

$$\frac{2}{13}Ab^3x^{13/2}+\frac{2}{17}b^2x^{17/2}(3Ac+bB)+\frac{2}{25}c^2x^{25/2}(Ac+3bB)+\frac{2}{7}bcx^{21/2}(Ac+bB)+\frac{2}{29}Bc^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/Sqrt[x],x]
```

```
[Out] (2*A*b^3*x^(13/2))/13 + (2*b^2*(b*B + 3*A*c)*x^(17/2))/17 + (2*b*c*(b*B + A
*c)*x^(21/2))/7 + (2*c^2*(3*b*B + A*c)*x^(25/2))/25 + (2*B*c^3*x^(29/2))/29
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx &= \int x^{11/2}(A+Bx^2)\left(b+cx^2\right)^3\,dx\\ &= \int \left(Ab^3x^{11/2}+b^2(bB+3Ac)x^{15/2}+3bc(bB+Ac)x^{19/2}+c^2(3bB+Ac)x^{23}\right.\\ &= \frac{2}{13}Ab^3x^{13/2}+\frac{2}{17}b^2(bB+3Ac)x^{17/2}+\frac{2}{7}bc(bB+Ac)x^{21/2}+\frac{2}{25}c^2(3bB+A\end{aligned}$$

Mathematica [A]
time = 0.05, size = 83, normalized size = 0.98

$$\frac{2x^{13/2}(29A(2975b^3+6825b^2cx^2+5525bc^2x^4+1547c^3x^6)+13Bx^2(5075b^3+12325b^2cx^2+10353bc^2x^4+2975c^3x^6))}{1121575}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/Sqrt[x],x]
```

```
[Out] (2*x^(13/2)*(29*A*(2975*b^3 + 6825*b^2*c*x^2 + 5525*b*c^2*x^4 + 1547*c^3*x^
6) + 13*B*x^2*(5075*b^3 + 12325*b^2*c*x^2 + 10353*b*c^2*x^4 + 2975*c^3*x^6)
))/1121575
```

Maple [A]
time = 0.36, size = 76, normalized size = 0.89

method	result
derivativedivides	$\frac{2B\,c^3x^{\frac{29}{2}}}{29}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{25}{2}}}{25}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{21}{2}}}{21}+\frac{2(3A\,b^2c+B\,b^3)x^{\frac{17}{2}}}{17}+\frac{2A\,b^3x^{\frac{13}{2}}}{13}$
default	$\frac{2B\,c^3x^{\frac{29}{2}}}{29}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{25}{2}}}{25}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{21}{2}}}{21}+\frac{2(3A\,b^2c+B\,b^3)x^{\frac{17}{2}}}{17}+\frac{2A\,b^3x^{\frac{13}{2}}}{13}$
gosper	$\frac{2x^{\frac{13}{2}}(38675B\,c^3x^8+44863A\,c^3x^6+134589x^6Bb\,c^2+160225Ab\,c^2x^4+160225x^4B\,b^2c+197925A\,b^2c\,x^2+65975x^2B\,b^3+862}{1121575}$
trager	$\frac{2x^{\frac{13}{2}}(38675B\,c^3x^8+44863A\,c^3x^6+134589x^6Bb\,c^2+160225Ab\,c^2x^4+160225x^4B\,b^2c+197925A\,b^2c\,x^2+65975x^2B\,b^3+862}{1121575}$
risch	$\frac{2x^{\frac{13}{2}}(38675B\,c^3x^8+44863A\,c^3x^6+134589x^6Bb\,c^2+160225Ab\,c^2x^4+160225x^4B\,b^2c+197925A\,b^2c\,x^2+65975x^2B\,b^3+862}{1121575}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/29*B*c^3*x^(29/2)+2/25*(A*c^3+3*B*b*c^2)*x^(25/2)+2/21*(3*A*b*c^2+3*B*b^2
*c)*x^(21/2)+2/17*(3*A*b^2*c+B*b^3)*x^(17/2)+2/13*A*b^3*x^(13/2)
```

Maxima [A]
time = 0.32, size = 73, normalized size = 0.86

$$\frac{2}{29}\,Bc^3x^{\frac{29}{2}}+\frac{2}{25}\left(3\,Bbc^2+Ac^3\right)x^{\frac{25}{2}}+\frac{2}{7}\left(Bb^2c+Abc^2\right)x^{\frac{21}{2}}+\frac{2}{13}\,Ab^3x^{\frac{13}{2}}+\frac{2}{17}\left(Bb^3+3\,Ab^2c\right)x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/29*B*c^3*x^(29/2) + 2/25*(3*B*b*c^2 + A*c^3)*x^(25/2) + 2/7*(B*b^2*c + A*
b*c^2)*x^(21/2) + 2/13*A*b^3*x^(13/2) + 2/17*(B*b^3 + 3*A*b^2*c)*x^(17/2)
```

Fricas [A]
time = 1.35, size = 78, normalized size = 0.92

$$\frac{2}{1121575}\left(38675\,Bc^3x^{14}+44863\left(3\,Bbc^2+Ac^3\right)x^{12}+160225\left(Bb^2c+Abc^2\right)x^{10}+86275\,Ab^3x^6+65975\left(Bb^3+3\,Ab^2c\right)x^8\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1121575*(38675*B*c^3*x^14 + 44863*(3*B*b*c^2 + A*c^3)*x^12 + 160225*(B*b^
2*c + A*b*c^2)*x^10 + 86275*A*b^3*x^6 + 65975*(B*b^3 + 3*A*b^2*c)*x^8)*sqrt
(x)
```

Sympy [A]
time = 1.63, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{13}{2}}}{13}+\frac{6Ab^2cx^{\frac{17}{2}}}{17}+\frac{2Abc^2x^{\frac{21}{2}}}{7}+\frac{2Ac^3x^{\frac{25}{2}}}{25}+\frac{2Bb^3x^{\frac{17}{2}}}{17}+\frac{2Bb^2cx^{\frac{21}{2}}}{7}+\frac{6Bbc^2x^{\frac{25}{2}}}{25}+\frac{2Bc^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] 2*A*b**3*x**(13/2)/13 + 6*A*b**2*c*x**(17/2)/17 + 2*A*b*c**2*x**(21/2)/7 +
2*A*c**3*x**(25/2)/25 + 2*B*b**3*x**(17/2)/17 + 2*B*b**2*c*x**(21/2)/7 + 6*
B*b*c**2*x**(25/2)/25 + 2*B*c**3*x**(29/2)/29
```

Giac [A]
time = 0.48, size = 77, normalized size = 0.91

$$\frac{2}{29}\,Bc^3x^{\frac{29}{2}}+\frac{6}{25}\,Bbc^2x^{\frac{25}{2}}+\frac{2}{25}\,Ac^3x^{\frac{25}{2}}+\frac{2}{7}\,Bb^2cx^{\frac{21}{2}}+\frac{2}{7}\,Abc^2x^{\frac{21}{2}}+\frac{2}{17}\,Bb^3x^{\frac{17}{2}}+\frac{6}{17}\,Ab^2cx^{\frac{17}{2}}+\frac{2}{13}\,Ab^3x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/29*B*c^3*x^(29/2) + 6/25*B*b*c^2*x^(25/2) + 2/25*A*c^3*x^(25/2) + 2/7*B*b
^2*c*x^(21/2) + 2/7*A*b*c^2*x^(21/2) + 2/17*B*b^3*x^(17/2) + 6/17*A*b^2*c*x
^(17/2) + 2/13*A*b^3*x^(13/2)
```

Mupad [B]
time = 0.03, size = 69, normalized size = 0.81

$$x^{17/2}\left(\frac{2\,B\,b^3}{17}+\frac{6\,A\,c\,b^2}{17}\right)+x^{25/2}\left(\frac{2\,A\,c^3}{25}+\frac{6\,B\,b\,c^2}{25}\right)+\frac{2\,A\,b^3\,x^{13/2}}{13}+\frac{2\,B\,c^3\,x^{29/2}}{29}+\frac{2\,b\,c\,x^{21/2}\,\left(A\,c+B\,b\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(1/2),x)
```

```
[Out] x^(17/2)*((2*B*b^3)/17 + (6*A*b^2*c)/17) + x^(25/2)*((2*A*c^3)/25 + (6*B*b*
c^2)/25) + (2*A*b^3*x^(13/2))/13 + (2*B*c^3*x^(29/2))/29 + (2*b*c*x^(21/2)*
(A*c + B*b))/7
```

3.180 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2}{11}Ab^3x^{11/2}+\frac{2}{15}b^2(bB+3Ac)x^{15/2}+\frac{6}{19}bc(bB+Ac)x^{19/2}+\frac{2}{23}c^2(3bB+Ac)x^{23/2}+\frac{2}{27}Bc^3x^{27/2}$$

```
[Out] 2/11*A*b^3*x^(11/2)+2/15*b^2*(3*A*c+B*b)*x^(15/2)+6/19*b*c*(A*c+B*b)*x^(19/
2)+2/23*c^2*(A*c+3*B*b)*x^(23/2)+2/27*B*c^3*x^(27/2)
```

Rubi [A]

time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1598, 459}

$$\frac{2}{11}Ab^3x^{11/2}+\frac{2}{15}b^2x^{15/2}(3Ac+bB)+\frac{2}{23}c^2x^{23/2}(Ac+3bB)+\frac{6}{19}bcx^{19/2}(Ac+bB)+\frac{2}{27}Bc^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(3/2),x]
```

```
[Out] (2*A*b^3*x^(11/2))/11 + (2*b^2*(b*B + 3*A*c)*x^(15/2))/15 + (6*b*c*(b*B + A
*c)*x^(19/2))/19 + (2*c^2*(3*b*B + A*c)*x^(23/2))/23 + (2*B*c^3*x^(27/2))/2
7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx &= \int x^{9/2}(A+Bx^2)\left(b+cx^2\right)^3\,dx\\ &= \int \left(Ab^3x^{9/2}+b^2(bB+3Ac)x^{13/2}+3bc(bB+Ac)x^{17/2}+c^2(3bB+Ac)x^{21/2}\right.\\ &= \frac{2}{11}Ab^3x^{11/2}+\frac{2}{15}b^2(bB+3Ac)x^{15/2}+\frac{6}{19}bc(bB+Ac)x^{19/2}+\frac{2}{23}c^2(3bB+A\end{aligned}$$

Mathematica [A]
time = 0.04, size = 83, normalized size = 0.98

$$\frac{2x^{11/2}(27A(2185b^3+4807b^2cx^2+3795bc^2x^4+1045c^3x^6)+11Bx^2(3933b^3+9315b^2cx^2+7695bc^2x^4+2185c^3x^6))}{648945}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(3/2),x]
```

```
[Out] (2*x^(11/2)*(27*A*(2185*b^3 + 4807*b^2*c*x^2 + 3795*b*c^2*x^4 + 1045*c^3*x^
6) + 11*B*x^2*(3933*b^3 + 9315*b^2*c*x^2 + 7695*b*c^2*x^4 + 2185*c^3*x^6)))
/648945
```

Maple [A]
time = 0.35, size = 76, normalized size = 0.89

method	result
derivativedivides	$\frac{2B\,c^3x^{\frac{27}{2}}}{27}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{23}{2}}}{23}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{19}{2}}}{19}+\frac{2(3A\,b^2c+B\,b^3)x^{\frac{15}{2}}}{15}+\frac{2A\,b^3x^{\frac{11}{2}}}{11}$
default	$\frac{2B\,c^3x^{\frac{27}{2}}}{27}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{23}{2}}}{23}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{19}{2}}}{19}+\frac{2(3A\,b^2c+B\,b^3)x^{\frac{15}{2}}}{15}+\frac{2A\,b^3x^{\frac{11}{2}}}{11}$
gosper	$\frac{2x^{\frac{11}{2}}(24035B\,c^3x^8+28215A\,c^3x^6+84645x^6Bb\,c^2+102465Ab\,c^2x^4+102465x^4B\,b^2c+129789A\,b^2c\,x^2+43263x^2B\,b^3+58}{648945}$
trager	$\frac{2x^{\frac{11}{2}}(24035B\,c^3x^8+28215A\,c^3x^6+84645x^6Bb\,c^2+102465Ab\,c^2x^4+102465x^4B\,b^2c+129789A\,b^2c\,x^2+43263x^2B\,b^3+58}{648945}$
risch	$\frac{2x^{\frac{11}{2}}(24035B\,c^3x^8+28215A\,c^3x^6+84645x^6Bb\,c^2+102465Ab\,c^2x^4+102465x^4B\,b^2c+129789A\,b^2c\,x^2+43263x^2B\,b^3+58}{648945}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/27*B*c^3*x^(27/2)+2/23*(A*c^3+3*B*b*c^2)*x^(23/2)+2/19*(3*A*b*c^2+3*B*b^2
*c)*x^(19/2)+2/15*(3*A*b^2*c+B*b^3)*x^(15/2)+2/11*A*b^3*x^(11/2)
```

Maxima [A]
time = 0.27, size = 73, normalized size = 0.86

$$\frac{2}{27}\,Bc^3x^{\frac{27}{2}}+\frac{2}{23}\left(3\,Bbc^2+Ac^3\right)x^{\frac{23}{2}}+\frac{6}{19}\left(Bb^2c+Abc^2\right)x^{\frac{19}{2}}+\frac{2}{11}\,Ab^3x^{\frac{11}{2}}+\frac{2}{15}\left(Bb^3+3\,Ab^2c\right)x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/27*B*c^3*x^(27/2) + 2/23*(3*B*b*c^2 + A*c^3)*x^(23/2) + 6/19*(B*b^2*c + A
*b*c^2)*x^(19/2) + 2/11*A*b^3*x^(11/2) + 2/15*(B*b^3 + 3*A*b^2*c)*x^(15/2)
```

Fricas [A]
time = 1.69, size = 78, normalized size = 0.92

$$\frac{2}{648945}\left(24035\,Bc^3x^{13}+28215\left(3\,Bbc^2+Ac^3\right)x^{11}+102465\left(Bb^2c+Abc^2\right)x^9+58995\,Ab^3x^5+43263\left(Bb^3+3\,Ab^2c\right)x^7\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/648945*(24035*B*c^3*x^13 + 28215*(3*B*b*c^2 + A*c^3)*x^11 + 102465*(B*b^2
*c + A*b*c^2)*x^9 + 58995*A*b^3*x^5 + 43263*(B*b^3 + 3*A*b^2*c)*x^7)*sqrt(x
)
```

Sympy [A]
time = 1.88, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{11}{2}}}{11}+\frac{2Ab^2cx^{\frac{15}{2}}}{5}+\frac{6Abc^2x^{\frac{19}{2}}}{19}+\frac{2Ac^3x^{\frac{23}{2}}}{23}+\frac{2Bb^3x^{\frac{15}{2}}}{15}+\frac{6Bb^2cx^{\frac{19}{2}}}{19}+\frac{6Bbc^2x^{\frac{23}{2}}}{23}+\frac{2Bc^3x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(3/2),x)
```

```
[Out] 2*A*b**3*x**(11/2)/11 + 2*A*b**2*c*x**(15/2)/5 + 6*A*b*c**2*x**(19/2)/19 +
2*A*c**3*x**(23/2)/23 + 2*B*b**3*x**(15/2)/15 + 6*B*b**2*c*x**(19/2)/19 + 6
*B*b*c**2*x**(23/2)/23 + 2*B*c**3*x**(27/2)/27
```

Giac [A]
time = 0.43, size = 77, normalized size = 0.91

$$\frac{2}{27}\,Bc^3x^{\frac{27}{2}}+\frac{6}{23}\,Bbc^2x^{\frac{23}{2}}+\frac{2}{23}\,Ac^3x^{\frac{23}{2}}+\frac{6}{19}\,Bb^2cx^{\frac{19}{2}}+\frac{6}{19}\,Abc^2x^{\frac{19}{2}}+\frac{2}{15}\,Bb^3x^{\frac{15}{2}}+\frac{2}{5}\,Ab^2cx^{\frac{15}{2}}+\frac{2}{11}\,Ab^3x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/27*B*c^3*x^(27/2) + 6/23*B*b*c^2*x^(23/2) + 2/23*A*c^3*x^(23/2) + 6/19*B*
b^2*c*x^(19/2) + 6/19*A*b*c^2*x^(19/2) + 2/15*B*b^3*x^(15/2) + 2/5*A*b^2*c*
x^(15/2) + 2/11*A*b^3*x^(11/2)
```

Mupad [B]
time = 0.03, size = 69, normalized size = 0.81

$$x^{15/2}\,\left(\frac{2\,B\,b^3}{15}+\frac{2\,A\,c\,b^2}{5}\right)+x^{23/2}\,\left(\frac{2\,A\,c^3}{23}+\frac{6\,B\,b\,c^2}{23}\right)+\frac{2\,A\,b^3\,x^{11/2}}{11}+\frac{2\,B\,c^3\,x^{27/2}}{27}+\frac{6\,b\,c\,x^{19/2}\,\left(A\,c+B\,b\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(3/2),x)
```

```
[Out] x^(15/2)*((2*B*b^3)/15 + (2*A*b^2*c)/5) + x^(23/2)*((2*A*c^3)/23 + (6*B*b*c
^2)/23) + (2*A*b^3*x^(11/2))/11 + (2*B*c^3*x^(27/2))/27 + (6*b*c*x^(19/2)*(
A*c + B*b))/19
```

3.181 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2}{9}Ab^3x^{9/2}+\frac{2}{13}b^2(bB+3Ac)x^{13/2}+\frac{6}{17}bc(bB+Ac)x^{17/2}+\frac{2}{21}c^2(3bB+Ac)x^{21/2}+\frac{2}{25}Bc^3x^{25/2}$$

```
[Out] 2/9*A*b^3*x^(9/2)+2/13*b^2*(3*A*c+B*b)*x^(13/2)+6/17*b*c*(A*c+B*b)*x^(17/2)
+2/21*c^2*(A*c+3*B*b)*x^(21/2)+2/25*B*c^3*x^(25/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1598, 459}

$$\frac{2}{9}Ab^3x^{9/2}+\frac{2}{13}b^2x^{13/2}(3Ac+bB)+\frac{2}{21}c^2x^{21/2}(Ac+3bB)+\frac{6}{17}bcx^{17/2}(Ac+bB)+\frac{2}{25}Bc^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(5/2),x]
```

```
[Out] (2*A*b^3*x^(9/2))/9 + (2*b^2*(b*B + 3*A*c)*x^(13/2))/13 + (6*b*c*(b*B + A*c
)*x^(17/2))/17 + (2*c^2*(3*b*B + A*c)*x^(21/2))/21 + (2*B*c^3*x^(25/2))/25
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}}\,dx &= \int x^{7/2}(A+Bx^2)\left(b+cx^2\right)^3\,dx\\ &= \int \left(Ab^3x^{7/2}+b^2(bB+3Ac)x^{11/2}+3bc(bB+Ac)x^{15/2}+c^2(3bB+Ac)x^{19/}\right.\\ &= \frac{2}{9}Ab^3x^{9/2}+\frac{2}{13}b^2(bB+3Ac)x^{13/2}+\frac{6}{17}bc(bB+Ac)x^{17/2}+\frac{2}{21}c^2(3bB+Ac\end{aligned}$$

Mathematica [A]
time = 0.04, size = 83, normalized size = 0.98

$$\frac{2x^{9/2}(25A(1547b^3+3213b^2cx^2+2457bc^2x^4+663c^3x^6)+9Bx^2(2975b^3+6825b^2cx^2+5525bc^2x^4+1547c^3x^6))}{348075}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(5/2),x]
```

```
[Out] (2*x^(9/2)*(25*A*(1547*b^3 + 3213*b^2*c*x^2 + 2457*b*c^2*x^4 + 663*c^3*x^6)
 + 9*B*x^2*(2975*b^3 + 6825*b^2*c*x^2 + 5525*b*c^2*x^4 + 1547*c^3*x^6)))/34
8075
```

Maple [A]
time = 0.35, size = 76, normalized size = 0.89

method	result
derivativedivides	$\frac{2B\,c^3x^{\frac{25}{2}}}{25}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{21}{2}}}{21}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{17}{2}}}{17}+\frac{2(3A\,b^2c+B\,b^3)x^{\frac{13}{2}}}{13}+\frac{2A\,b^3x^{\frac{9}{2}}}{9}$
default	$\frac{2B\,c^3x^{\frac{25}{2}}}{25}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{21}{2}}}{21}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{17}{2}}}{17}+\frac{2(3A\,b^2c+B\,b^3)x^{\frac{13}{2}}}{13}+\frac{2A\,b^3x^{\frac{9}{2}}}{9}$
gosper	$\frac{2x^{\frac{9}{2}}(13923B\,c^3x^8+16575A\,c^3x^6+49725x^6Bb\,c^2+61425Ab\,c^2x^4+61425x^4B\,b^2c+80325A\,b^2c\,x^2+26775x^2B\,b^3+38675A\,b}{348075}$
trager	$\frac{2x^{\frac{9}{2}}(13923B\,c^3x^8+16575A\,c^3x^6+49725x^6Bb\,c^2+61425Ab\,c^2x^4+61425x^4B\,b^2c+80325A\,b^2c\,x^2+26775x^2B\,b^3+38675A\,b}{348075}$
risch	$\frac{2x^{\frac{9}{2}}(13923B\,c^3x^8+16575A\,c^3x^6+49725x^6Bb\,c^2+61425Ab\,c^2x^4+61425x^4B\,b^2c+80325A\,b^2c\,x^2+26775x^2B\,b^3+38675A\,b}{348075}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/25*B*c^3*x^(25/2)+2/21*(A*c^3+3*B*b*c^2)*x^(21/2)+2/17*(3*A*b*c^2+3*B*b^2
*c)*x^(17/2)+2/13*(3*A*b^2*c+B*b^3)*x^(13/2)+2/9*A*b^3*x^(9/2)
```

Maxima [A]
time = 0.27, size = 73, normalized size = 0.86

$$\frac{2}{25}\,Bc^3x^{\frac{25}{2}}+\frac{2}{21}\left(3\,Bbc^2+Ac^3\right)x^{\frac{21}{2}}+\frac{6}{17}\left(Bb^2c+Abc^2\right)x^{\frac{17}{2}}+\frac{2}{9}\,Ab^3x^{\frac{9}{2}}+\frac{2}{13}\left(Bb^3+3\,Ab^2c\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/25*B*c^3*x^(25/2) + 2/21*(3*B*b*c^2 + A*c^3)*x^(21/2) + 6/17*(B*b^2*c + A
*b*c^2)*x^(17/2) + 2/9*A*b^3*x^(9/2) + 2/13*(B*b^3 + 3*A*b^2*c)*x^(13/2)
```

Fricas [A]
time = 1.57, size = 78, normalized size = 0.92

$$\frac{2}{348075}\left(13923\,Bc^3x^{12}+16575\left(3\,Bbc^2+Ac^3\right)x^{10}+61425\left(Bb^2c+Abc^2\right)x^8+38675\,Ab^3x^4+26775\left(Bb^3+3\,Ab^2c\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*B*c^3*x^12 + 16575*(3*B*b*c^2 + A*c^3)*x^10 + 61425*(B*b^2*
c + A*b*c^2)*x^8 + 38675*A*b^3*x^4 + 26775*(B*b^3 + 3*A*b^2*c)*x^6)*sqrt(x)
```

Sympy [A]
time = 2.02, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{9}{2}}}{9}+\frac{6Ab^2cx^{\frac{13}{2}}}{13}+\frac{6Abc^2x^{\frac{17}{2}}}{17}+\frac{2Ac^3x^{\frac{21}{2}}}{21}+\frac{2Bb^3x^{\frac{13}{2}}}{13}+\frac{6Bb^2cx^{\frac{17}{2}}}{17}+\frac{2Bbc^2x^{\frac{21}{2}}}{7}+\frac{2Bc^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(5/2),x)
```

```
[Out] 2*A*b**3*x**(9/2)/9 + 6*A*b**2*c*x**(13/2)/13 + 6*A*b*c**2*x**(17/2)/17 + 2
*A*c**3*x**(21/2)/21 + 2*B*b**3*x**(13/2)/13 + 6*B*b**2*c*x**(17/2)/17 + 2*
B*b*c**2*x**(21/2)/7 + 2*B*c**3*x**(25/2)/25
```

Giac [A]
time = 0.46, size = 77, normalized size = 0.91

$$\frac{2}{25}Bc^3x^{\frac{25}{2}}+\frac{2}{7}Bbc^2x^{\frac{21}{2}}+\frac{2}{21}Ac^3x^{\frac{21}{2}}+\frac{6}{17}Bb^2cx^{\frac{17}{2}}+\frac{6}{17}Abc^2x^{\frac{17}{2}}+\frac{2}{13}Bb^3x^{\frac{13}{2}}+\frac{6}{13}Ab^2cx^{\frac{13}{2}}+\frac{2}{9}Ab^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/25*B*c^3*x^(25/2) + 2/7*B*b*c^2*x^(21/2) + 2/21*A*c^3*x^(21/2) + 6/17*B*b
^2*c*x^(17/2) + 6/17*A*b*c^2*x^(17/2) + 2/13*B*b^3*x^(13/2) + 6/13*A*b^2*c*
x^(13/2) + 2/9*A*b^3*x^(9/2)
```

Mupad [B]
time = 0.04, size = 69, normalized size = 0.81

$$x^{13/2}\left(\frac{2\,B\,b^3}{13}+\frac{6\,A\,c\,b^2}{13}\right)+x^{21/2}\left(\frac{2\,A\,c^3}{21}+\frac{2\,B\,b\,c^2}{7}\right)+\frac{2\,A\,b^3\,x^{9/2}}{9}+\frac{2\,B\,c^3\,x^{25/2}}{25}+\frac{6\,b\,c\,x^{17/2}\,\left(A\,c+B\,b\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(5/2),x)
```

```
[Out] x^(13/2)*((2*B*b^3)/13 + (6*A*b^2*c)/13) + x^(21/2)*((2*A*c^3)/21 + (2*B*b*
c^2)/7) + (2*A*b^3*x^(9/2))/9 + (2*B*c^3*x^(25/2))/25 + (6*b*c*x^(17/2)*(A*
c + B*b))/17
```

3.182 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2}{7}Ab^3x^{7/2}+\frac{2}{11}b^2(bB+3Ac)x^{11/2}+\frac{2}{5}bc(bB+Ac)x^{15/2}+\frac{2}{19}c^2(3bB+Ac)x^{19/2}+\frac{2}{23}Bc^3x^{23/2}$$

```
[Out] 2/7*A*b^3*x^(7/2)+2/11*b^2*(3*A*c+B*b)*x^(11/2)+2/5*b*c*(A*c+B*b)*x^(15/2)+
2/19*c^2*(A*c+3*B*b)*x^(19/2)+2/23*B*c^3*x^(23/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 459}

$$\frac{2}{7}Ab^3x^{7/2}+\frac{2}{11}b^2x^{11/2}(3Ac+bB)+\frac{2}{19}c^2x^{19/2}(Ac+3bB)+\frac{2}{5}bcx^{15/2}(Ac+bB)+\frac{2}{23}Bc^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(7/2),x]
```

```
[Out] (2*A*b^3*x^(7/2))/7 + (2*b^2*(b*B + 3*A*c)*x^(11/2))/11 + (2*b*c*(b*B + A*c
)*x^(15/2))/5 + (2*c^2*(3*b*B + A*c)*x^(19/2))/19 + (2*B*c^3*x^(23/2))/23
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx &= \int x^{5/2}(A+Bx^2)\left(b+cx^2\right)^3\,dx\\ &= \int \left(Ab^3x^{5/2}+b^2(bB+3Ac)x^{9/2}+3bc(bB+Ac)x^{13/2}+c^2(3bB+Ac)x^{17/2}\right.\\ &= \frac{2}{7}Ab^3x^{7/2}+\frac{2}{11}b^2(bB+3Ac)x^{11/2}+\frac{2}{5}bc(bB+Ac)x^{15/2}+\frac{2}{19}c^2(3bB+Ac)x\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 83, normalized size = 0.98

$$\frac{2x^{7/2}(23A(1045b^3+1995b^2cx^2+1463bc^2x^4+385c^3x^6)+7Bx^2(2185b^3+4807b^2cx^2+3795bc^2x^4+1045c^3x^6))}{168245}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(7/2),x]
```

```
[Out] (2*x^(7/2)*(23*A*(1045*b^3 + 1995*b^2*c*x^2 + 1463*b*c^2*x^4 + 385*c^3*x^6)
 + 7*B*x^2*(2185*b^3 + 4807*b^2*c*x^2 + 3795*b*c^2*x^4 + 1045*c^3*x^6)))/16
8245
```

Maple [**A**]
time = 0.36, size = 76, normalized size = 0.89

method	result
derivativedivides	$\frac{2B\,c^3x^{\frac{23}{2}}}{23}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{19}{2}}}{19}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{15}{2}}}{15}+\frac{2(3A\,b^2c+B\,b^3)x^{\frac{11}{2}}}{11}+\frac{2A\,b^3x^{\frac{7}{2}}}{7}$
default	$\frac{2B\,c^3x^{\frac{23}{2}}}{23}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{19}{2}}}{19}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{15}{2}}}{15}+\frac{2(3A\,b^2c+B\,b^3)x^{\frac{11}{2}}}{11}+\frac{2A\,b^3x^{\frac{7}{2}}}{7}$
gosper	$\frac{2x^{\frac{7}{2}}(7315B\,c^3x^8+8855A\,c^3x^6+26565x^6Bb\,c^2+33649Ab\,c^2x^4+33649x^4B\,b^2c+45885A\,b^2c\,x^2+15295x^2B\,b^3+24035A\,b}{168245}$
trager	$\frac{2x^{\frac{7}{2}}(7315B\,c^3x^8+8855A\,c^3x^6+26565x^6Bb\,c^2+33649Ab\,c^2x^4+33649x^4B\,b^2c+45885A\,b^2c\,x^2+15295x^2B\,b^3+24035A\,b}{168245}$
risch	$\frac{2x^{\frac{7}{2}}(7315B\,c^3x^8+8855A\,c^3x^6+26565x^6Bb\,c^2+33649Ab\,c^2x^4+33649x^4B\,b^2c+45885A\,b^2c\,x^2+15295x^2B\,b^3+24035A\,b}{168245}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/23*B*c^3*x^(23/2)+2/19*(A*c^3+3*B*b*c^2)*x^(19/2)+2/15*(3*A*b*c^2+3*B*b^2
*c)*x^(15/2)+2/11*(3*A*b^2*c+B*b^3)*x^(11/2)+2/7*A*b^3*x^(7/2)
```

Maxima [**A**]
time = 0.30, size = 73, normalized size = 0.86

$$\frac{2}{23}\,Bc^3x^{\frac{23}{2}}+\frac{2}{19}\left(3\,Bbc^2+Ac^3\right)x^{\frac{19}{2}}+\frac{2}{5}\left(Bb^2c+Abc^2\right)x^{\frac{15}{2}}+\frac{2}{7}\,Ab^3x^{\frac{7}{2}}+\frac{2}{11}\left(Bb^3+3\,Ab^2c\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/23*B*c^3*x^(23/2) + 2/19*(3*B*b*c^2 + A*c^3)*x^(19/2) + 2/5*(B*b^2*c + A*
b*c^2)*x^(15/2) + 2/7*A*b^3*x^(7/2) + 2/11*(B*b^3 + 3*A*b^2*c)*x^(11/2)
```

Fricas [**A**]
time = 3.03, size = 78, normalized size = 0.92

$$\frac{2}{168245}\left(7315\,Bc^3x^{11}+8855\left(3\,Bbc^2+Ac^3\right)x^9+33649\left(Bb^2c+Abc^2\right)x^7+24035\,Ab^3x^3+15295\left(Bb^3+3\,Ab^2c\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*B*c^3*x^11 + 8855*(3*B*b*c^2 + A*c^3)*x^9 + 33649*(B*b^2*c +
 A*b*c^2)*x^7 + 24035*A*b^3*x^3 + 15295*(B*b^3 + 3*A*b^2*c)*x^5)*sqrt(x)
```

Sympy [A]
time = 2.39, size = 114, normalized size = 1.34

$$\frac{2Ab^3x^{\frac{7}{2}}}{7}+\frac{6Ab^2cx^{\frac{11}{2}}}{11}+\frac{2Abc^2x^{\frac{15}{2}}}{5}+\frac{2Ac^3x^{\frac{19}{2}}}{19}+\frac{2Bb^3x^{\frac{11}{2}}}{11}+\frac{2Bb^2cx^{\frac{15}{2}}}{5}+\frac{6Bbc^2x^{\frac{19}{2}}}{19}+\frac{2Bc^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(7/2),x)
```

```
[Out] 2*A*b**3*x**(7/2)/7 + 6*A*b**2*c*x**(11/2)/11 + 2*A*b*c**2*x**(15/2)/5 + 2*
A*c**3*x**(19/2)/19 + 2*B*b**3*x**(11/2)/11 + 2*B*b**2*c*x**(15/2)/5 + 6*B*
b*c**2*x**(19/2)/19 + 2*B*c**3*x**(23/2)/23
```

Giac [A]
time = 0.45, size = 77, normalized size = 0.91

$$\frac{2}{23}Bc^3x^{\frac{23}{2}}+\frac{6}{19}Bbc^2x^{\frac{19}{2}}+\frac{2}{19}Ac^3x^{\frac{19}{2}}+\frac{2}{5}Bb^2cx^{\frac{15}{2}}+\frac{2}{5}Abc^2x^{\frac{15}{2}}+\frac{2}{11}Bb^3x^{\frac{11}{2}}+\frac{6}{11}Ab^2cx^{\frac{11}{2}}+\frac{2}{7}Ab^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/23*B*c^3*x^(23/2) + 6/19*B*b*c^2*x^(19/2) + 2/19*A*c^3*x^(19/2) + 2/5*B*b
^2*c*x^(15/2) + 2/5*A*b*c^2*x^(15/2) + 2/11*B*b^3*x^(11/2) + 6/11*A*b^2*c*x
^(11/2) + 2/7*A*b^3*x^(7/2)
```

Mupad [B]
time = 0.03, size = 69, normalized size = 0.81

$$x^{11/2}\left(\frac{2\,B\,b^3}{11}+\frac{6\,A\,c\,b^2}{11}\right)+x^{19/2}\left(\frac{2\,A\,c^3}{19}+\frac{6\,B\,b\,c^2}{19}\right)+\frac{2\,A\,b^3\,x^{7/2}}{7}+\frac{2\,B\,c^3\,x^{23/2}}{23}+\frac{2\,b\,c\,x^{15/2}\,\left(A\,c+B\,b\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(7/2),x)
```

```
[Out] x^(11/2)*((2*B*b^3)/11 + (6*A*b^2*c)/11) + x^(19/2)*((2*A*c^3)/19 + (6*B*b*
c^2)/19) + (2*A*b^3*x^(7/2))/7 + (2*B*c^3*x^(23/2))/23 + (2*b*c*x^(15/2)*(A
*c + B*b))/5
```

3.183 $\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=278

$$\frac{2b(bB-Ac)x^{3/2}}{3c^3}-\frac{2(bB-Ac)x^{7/2}}{7c^2}+\frac{2Bx^{11/2}}{11c}+\frac{b^{7/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{15/4}}-\frac{b^{7/4}(bB-Ac)\,\mathrm{ta}}{}$$

```
[Out] 2/3*b*(-A*c+B*b)*x^(3/2)/c^3-2/7*(-A*c+B*b)*x^(7/2)/c^2+2/11*B*x^(11/2)/c+1
/2*b^(7/4)*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(15/4)*2^
(1/2)-1/2*b^(7/4)*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(1
5/4)*2^(1/2)-1/4*b^(7/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^
(1/2)*x^(1/2))/c^(15/4)*2^(1/2)+1/4*b^(7/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)
+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(15/4)*2^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 470, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{b^{7/4}(bB-Ac)\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{15/4}}-\frac{b^{7/4}(bB-Ac)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,c^{15/4}}-\frac{b^{7/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{15/4}}+\frac{b^{7/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{15/4}}+\frac{2bx^{3/2}(bB-Ac)}{3c^3}-\frac{2x^{7/2}(bB-Ac)}{7c^2}+\frac{2Bx^{11/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*(b*B - A*c)*x^(3/2))/(3*c^3) - (2*(b*B - A*c)*x^(7/2))/(7*c^2) + (2*B*
x^(11/2))/(11*c) + (b^(7/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(Sqrt[2]*c^(15/4)) - (b^(7/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(15/4)) - (b^(7/4)*(b*B - A*c)*Log[Sqr
t[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(15/4)) +
 (b^(7/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(2*Sqrt[2]*c^(15/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \int \frac{x^{9/2}(A+Bx^2)}{b+cx^2}\,dx$$
$$= \frac{2Bx^{11/2}}{11c} - \frac{\left(2\left(\frac{11bB}{2}-\frac{11Ac}{2}\right)\right)\int \frac{x^{9/2}}{b+cx^2}\,dx}{11c}$$
$$= -\frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} + \frac{(b(bB-Ac))\int \frac{x^{5/2}}{b+cx^2}\,dx}{c^2}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{(b^2(bB-Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{c^3}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{(2b^2(bB-Ac))\operatorname{Subst}\left(\int \frac{x^2}{b+cx^4}\,d\right.}{c^3}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} + \frac{(b^2(bB-Ac))\operatorname{Subst}\left(\int \frac{\sqrt{b}-\sqrt{}}{b+cx}\right.}{c^{7/2}}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{(b^2(bB-Ac))\operatorname{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}-}\right.}{2c^4}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{b^{7/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\right.}{2\sqrt{2}\,c^{15/4}}$$
$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} + \frac{b^{7/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{2}\,c^{15/4}}$$

Mathematica [A]

time = 0.22, size = 172, normalized size = 0.62

$$\frac{2x^{3/2}(77b^2B-77Abc-33bBcx^2+33Ac^2x^2+21Bc^2x^4)}{231c^3}+\frac{b^{7/4}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)}{\sqrt{2}\,c^{15/4}}+\frac{b^{7/4}(bB-Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,c^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2)*(77*b^2*B - 77*A*b*c - 33*b*B*c*x^2 + 33*A*c^2*x^2 + 21*B*c^2*x^
4))/(231*c^3) + (b^(7/4)*(b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*c^(15/4)) + (b^(7/4)*(b*B - A*c)*ArcTan
h[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*c^(15/
4))
```

Maple [A]

time = 0.45, size = 165, normalized size = 0.59

method	result
derivativedivides	$-\frac{2\left(-\frac{B\,x^{\frac{11}{2}}c^2}{11}+\frac{\left(-A\,c^2+bBc\right)x^{\frac{7}{2}}}{7}+\frac{\left(Abc-b^2B\right)x^{\frac{3}{2}}}{3}\right)}{c^3}+\frac{b^2(Ac-Bb)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\,\mathrm{arcta}\right.}{4c^4\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$-\frac{2\left(-\frac{B\,x^{\frac{11}{2}}c^2}{11}+\frac{\left(-A\,c^2+bBc\right)x^{\frac{7}{2}}}{7}+\frac{\left(Abc-b^2B\right)x^{\frac{3}{2}}}{3}\right)}{c^3}+\frac{b^2(Ac-Bb)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\,\mathrm{arcta}\right.}{4c^4\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
risch	$-\frac{2x^{\frac{3}{2}}\left(-21B\,c^2x^4-33A\,c^2x^2+33bB\,x^2c+77Abc-77b^2B\right)}{231c^3}+\frac{b^2\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2c^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\frac{b^2\sqrt{2}\,A\arctan}{2c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c^3*(-1/11*B*x^(11/2)*c^2+1/7*(-A*c^2+B*b*c)*x^(7/2)+1/3*(A*b*c-B*b^2)*x
^(3/2))+1/4*b^2*(A*c-B*b)/c^4/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2
)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arcta
n(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]

time = 0.49, size = 237, normalized size = 0.85

$$-\frac{(Bb^3-Ab^2c)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{4\,c^3}+\frac{2\left(21\,Bc^2x^{\frac{11}{2}}-33\,(Bbc-Ac^2)x^{\frac{7}{2}}+77\,(Bb^2-Abc)x^{\frac{3}{2}}\right)}{231\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(B*b^3 - A*b^2*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/
4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(
c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sq
rt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log
(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) +
 sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/
4)*c^(3/4)))/c^3 + 2/231*(21*B*c^2*x^(11/2) - 33*(B*b*c - A*c^2)*x^(7/2) +
77*(B*b^2 - A*b*c)*x^(3/2))/c^3
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(199) = 398$.
time = 2.49, size = 920, normalized size = 3.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/462*(924*c^3*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*
b^8*c^3 + A^4*b^7*c^4)/c^15)^(1/4)*arctan((sqrt((B^6*b^16 - 6*A*B^5*b^15*c
+ 15*A^2*B^4*b^14*c^2 - 20*A^3*B^3*b^13*c^3 + 15*A^4*B^2*b^12*c^4 - 6*A^5*B
*b^11*c^5 + A^6*b^10*c^6)*x - (B^4*b^11*c^7 - 4*A*B^3*b^10*c^8 + 6*A^2*B^2*
b^9*c^9 - 4*A^3*B*b^8*c^10 + A^4*b^7*c^11)*sqrt(-(B^4*b^11 - 4*A*B^3*b^10*c
 + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15))*c^4*(-(B^4*b^1
1 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^1
5)^(1/4) + (B^3*b^8*c^4 - 3*A*B^2*b^7*c^5 + 3*A^2*B*b^6*c^6 - A^3*b^5*c^7)*
sqrt(x)*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3
+ A^4*b^7*c^4)/c^15)^(1/4))/(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2
- 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)) - 231*c^3*(-(B^4*b^11 - 4*A*B^3*b^10*c +
6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(1/4)*log(c^11*(-(
B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c
^4)/c^15)^(3/4) - (B^3*b^8 - 3*A*B^2*b^7*c + 3*A^2*B*b^6*c^2 - A^3*b^5*c^3)
*sqrt(x)) + 231*c^3*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^
3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(1/4)*log(-c^11*(-(B^4*b^11 - 4*A*B^3*b^10
*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(3/4) - (B^3*
b^8 - 3*A*B^2*b^7*c + 3*A^2*B*b^6*c^2 - A^3*b^5*c^3)*sqrt(x)) - 4*(21*B*c^2
*x^5 - 33*(B*b*c - A*c^2)*x^3 + 77*(B*b^2 - A*b*c)*x)*sqrt(x))/c^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 298, normalized size = 1.07

$$-\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb^2-(bc^3)^{\frac{3}{4}}Abc\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^6}-\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb^2-(bc^3)^{\frac{3}{4}}Abc\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^6}+\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb^2-(bc^3)^{\frac{3}{4}}Abc\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^6}-\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb^2-(bc^3)^{\frac{3}{4}}Abc\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^6}+\frac{2\left(21Bc^{10}x^{\frac{11}{2}}-33Bbc^9x^{\frac{7}{2}}+33Ac^{10}x^{\frac{7}{2}}+77Bb^2c^8x^{\frac{3}{2}}-77Abc^9x^{\frac{3}{2}}\right)}{231c^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(3/4)*B*b^2 - (b*c^3)^(3/4)*A*b*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^6 - 1/2*sqrt(2)*((b*c^3)^
(3/4)*B*b^2 - (b*c^3)^(3/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 - 2*sqrt(x))/(b/c)^(1/4))/c^6 + 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b^2 - (b*c^3)
^(3/4)*A*b*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^6 - 1/4*sq
rt(2)*((b*c^3)^(3/4)*B*b^2 - (b*c^3)^(3/4)*A*b*c)*log(-sqrt(2)*sqrt(x)*(b/c
)^(1/4) + x + sqrt(b/c))/c^6 + 2/231*(21*B*c^10*x^(11/2) - 33*B*b*c^9*x^(7/
2) + 33*A*c^10*x^(7/2) + 77*B*b^2*c^8*x^(3/2) - 77*A*b*c^9*x^(3/2))/c^11
```

Mupad [B]
time = 0.23, size = 115, normalized size = 0.41

$$x^{7/2}\left(\frac{2A}{7c}-\frac{2Bb}{7c^2}\right)+\frac{2Bx^{11/2}}{11c}+\frac{(-b)^{7/4}\,\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(Ac-Bb)}{c^{15/4}}-\frac{bx^{3/2}\left(\frac{2A}{c}-\frac{2Bb}{c^2}\right)}{3c}+\frac{(-b)^{7/4}\,\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)(Ac-Bb)\,1\mathrm{i}}{c^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^(7/2)*((2*A)/(7*c) - (2*B*b)/(7*c^2)) + (2*B*x^(11/2))/(11*c) + ((-b)^(7/
4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/c^(15/4) + ((-b)^(7/4)*a
tan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(A*c - B*b)*1i)/c^(15/4) - (b*x^(3/2)*
((2*A)/c - (2*B*b)/c^2))/(3*c)
```

3.184 $\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=276

$$\frac{2b(bB-Ac)\sqrt{x}}{c^3}-\frac{2(bB-Ac)x^{5/2}}{5c^2}+\frac{2Bx^{9/2}}{9c}+\frac{b^{5/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{13/4}}-\frac{b^{5/4}(bB-Ac)\tan}{\sqrt{}}$$

```
[Out] -2/5*(-A*c+B*b)*x^(5/2)/c^2+2/9*B*x^(9/2)/c+1/2*b^(5/4)*(-A*c+B*b)*arctan(1
-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(13/4)*2^(1/2)-1/2*b^(5/4)*(-A*c+B*b)*a
rctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(13/4)*2^(1/2)+1/4*b^(5/4)*(-A*c
+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2
)-1/4*b^(5/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/
2))/c^(13/4)*2^(1/2)+2*b*(-A*c+B*b)*x^(1/2)/c^3
```

Rubi [A]
time = 0.17, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 470, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{b^{5/4}(bB-Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{13/4}}-\frac{b^{5/4}(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,c^{13/4}}+\frac{b^{5/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{13/4}}-\frac{b^{5/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{13/4}}+\frac{2b\sqrt{x}\,(bB-Ac)}{c^3}-\frac{2x^{5/2}(bB-Ac)}{5c^2}+\frac{2Bx^{9/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*(b*B - A*c)*Sqrt[x])/c^3 - (2*(b*B - A*c)*x^(5/2))/(5*c^2) + (2*B*x^(9
/2))/(9*c) + (b^(5/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(Sqrt[2]*c^(13/4)) - (b^(5/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(13/4)) + (b^(5/4)*(b*B - A*c)*Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(13/4)) - (b^(5
/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
/(2*Sqrt[2]*c^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \int \frac{x^{7/2}(A+Bx^2)}{b+cx^2}\,dx$$

$$= \frac{2Bx^{9/2}}{9c} - \frac{\left(2\left(\frac{9bB}{2}-\frac{9Ac}{2}\right)\right)\int \frac{x^{7/2}}{b+cx^2}\,dx}{9c}$$

$$= -\frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} + \frac{(b(bB-Ac))\int \frac{x^{3/2}}{b+cx^2}\,dx}{c^2}$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} - \frac{(b^2(bB-Ac))\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{c^3}$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} - \frac{(2b^2(bB-Ac))\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx \ldots\right.}{c^3}$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} - \frac{(b^{3/2}(bB-Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}-\ldots}{b+\ldots}\right.}{c^3}$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} - \frac{(b^{3/2}(bB-Ac))\,\text{Subst}\left(\int \frac{\ldots}{\frac{\sqrt{b}}{\sqrt{c}}-\ldots}\right.}{2c^{7/2}\ldots}$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} + \frac{b^{5/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{\ldots}\right.}{2\sqrt{2}\,c^{13/4}\ldots}$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} + \frac{b^{5/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{\ldots}}{\sqrt[4]{\ldots}}\right.}{\sqrt{2}\,c^{13/4}\ldots}$$

Mathematica [A]

time = 0.23, size = 173, normalized size = 0.63

$$\frac{2\sqrt{x}\,(45b^2B-45Abc-9bBcx^2+9Ac^2x^2+5Bc^2x^4)}{45c^3}+\frac{b^{5/4}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)}{\sqrt{2}\,c^{13/4}}-\frac{b^{5/4}(bB-Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[x]*(45*b^2*B - 45*A*b*c - 9*b*B*c*x^2 + 9*A*c^2*x^2 + 5*B*c^2*x^4))
/(45*c^3) + (b^(5/4)*(b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*c^(13/4)) - (b^(5/4)*(b*B - A*c)*ArcTanh[(S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*c^(13/4))
```

Maple [A]
time = 0.38, size = 164, normalized size = 0.59

method	result
derivativedivides	$-\frac{2\left(-\frac{B\,x^{\frac{9}{2}}c^2}{9}-\frac{A\,c^2x^{\frac{5}{2}}}{5}+\frac{Bbc\,x^{\frac{5}{2}}}{5}+Abc\sqrt{x}-b^2B\sqrt{x}\right)}{c^3}+\frac{b(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right.\right.}{}$
default	$-\frac{2\left(-\frac{B\,x^{\frac{9}{2}}c^2}{9}-\frac{A\,c^2x^{\frac{5}{2}}}{5}+\frac{Bbc\,x^{\frac{5}{2}}}{5}+Abc\sqrt{x}-b^2B\sqrt{x}\right)}{c^3}+\frac{b(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right.\right.}{}$
risch	$-\frac{2(-5B\,c^2x^4-9A\,c^2x^2+9bB\,x^2c+45Abc-45b^2B)\sqrt{x}}{45c^3}+\frac{b\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{4c^2}+\frac{b\left(\frac{b}{c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c^3*(-1/9*B*x^(9/2)*c^2-1/5*A*c^2*x^(5/2)+1/5*B*b*c*x^(5/2)+A*b*c*x^(1/2
)-b^2*B*x^(1/2))+1/4*b*(A*c-B*b)/c^3*(b/c)^(1/4)*2^(1/2)*(ln((x+(b/c)^(1/4)
*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+
2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)-1))
```

Maxima [A]
time = 0.49, size = 259, normalized size = 0.94

$$-\frac{\left(\frac{2\sqrt{2}\,(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,(Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,(Bb-Ac)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Bb-Ac)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)b^2}{4\,c^3}+\frac{2\left(5\,Bc^2x^{\frac{9}{2}}-9\,(Bbc-Ac^2)x^{\frac{5}{2}}+45\,(Bb^2-Abc)\sqrt{x}\right)}{45\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2
*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) +
2*sqrt(2)*(B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt
(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(
2)*(B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(
b^(3/4)*c^(1/4)) - sqrt(2)*(B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
 + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b^2/c^3 + 2/45*(5*B*c^2*x^(9/2)
- 9*(B*b*c - A*c^2)*x^(5/2) + 45*(B*b^2 - A*b*c)*sqrt(x))/c^3
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(199) = 398$.
time = 2.03, size = 714, normalized size = 2.59

$$\frac{180\,c^3\left(-\frac{B^4b^9-4\,AB^3b^8c+6\,A^2B^2b^7c^2-4\,A^3Bb^6c^3+A^4b^5c^4}{c^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^6\sqrt{-\frac{B^4b^9-4\,AB^3b^8c+6\,A^2B^2b^7c^2-4\,A^3Bb^6c^3+A^4b^5c^4}{c^{13}}}+\left(B^2b^4-2\,ABb^3c+A^2b^2c^2\right)x}\,c^{10}\left(-\frac{B^4b^9-4\,AB^3b^8c+6\,A^2B^2b^7c^2-4\,A^3Bb^6c^3+A^4b^5c^4}{c^{13}}\right)^{\frac{3}{4}}+\left(Bb^2c^{10}-Abc^{11}\right)\sqrt{x}\left(-\frac{B^4b^9-4\,AB^3b^8c+6\,A^2B^2b^7c^2-4\,A^3Bb^6c^3+A^4b^5c^4}{c^{13}}\right)^{\frac{3}{4}}}{B^4b^9-4\,AB^3b^8c+6\,A^2B^2b^7c^2-4\,A^3Bb^6c^3+A^4b^5c^4}\right)+45\,c^3\left(-\frac{B^4b^9-4\,AB^3b^8c+6\,A^2B^2b^7c^2-4\,A^3Bb^6c^3+A^4b^5c^4}{c^{13}}\right)^{\frac{1}{4}}\log\left(c^3\left(-\frac{B^4b^9-4\,AB^3b^8c+6\,A^2B^2b^7c^2-4\,A^3Bb^6c^3+A^4b^5c^4}{c^{13}}\right)^{\frac{1}{4}}-\left(Bb^2-Abc\right)\sqrt{x}\right)-45\,c^3\left(-\frac{B^4b^9-4\,AB^3b^8c+6\,A^2B^2b^7c^2-4\,A^3Bb^6c^3+A^4b^5c^4}{c^{13}}\right)^{\frac{1}{4}}\log\left(-c^3\left(-\frac{B^4b^9-4\,AB^3b^8c+6\,A^2B^2b^7c^2-4\,A^3Bb^6c^3+A^4b^5c^4}{c^{13}}\right)^{\frac{1}{4}}-\left(Bb^2-Abc\right)\sqrt{x}\right)+4\left(5\,Bc^2x^4+45\,Bb^2-45\,Abc-9\left(Bbc-Ac^2\right)x^2\right)\sqrt{x}}{90\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/90*(180*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*
c^3 + A^4*b^5*c^4)/c^13)^(1/4)*arctan((sqrt(c^6*sqrt(-(B^4*b^9 - 4*A*B^3*b^
8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13) + (B^2*b^4 -
 2*A*B*b^3*c + A^2*b^2*c^2)*x)*c^10*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*
b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(3/4) + (B*b^2*c^10 - A*b*c^
11)*sqrt(x)*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^
3 + A^4*b^5*c^4)/c^13)^(3/4))/(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2
- 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)) + 45*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A
^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4)*log(c^3*(-(B^4*
b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^
13)^(1/4) - (B*b^2 - A*b*c)*sqrt(x)) - 45*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c +
6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4)*log(-c^3*(-(
B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4
)/c^13)^(1/4) - (B*b^2 - A*b*c)*sqrt(x)) + 4*(5*B*c^2*x^4 + 45*B*b^2 - 45*A
*b*c - 9*(B*b*c - A*c^2)*x^2)*sqrt(x))/c^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

[Out] Timed out

Giac [A]
time = 0.44, size = 298, normalized size = 1.08

$$-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb^{2}-\left(bc^{3}\right)^{\frac{1}{4}}Abc\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{4}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb^{2}-\left(bc^{3}\right)^{\frac{1}{4}}Abc\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{4}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb^{2}-\left(bc^{3}\right)^{\frac{1}{4}}Abc\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^{4}}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb^{2}-\left(bc^{3}\right)^{\frac{1}{4}}Abc\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^{4}}+\frac{2\left(5Bc^{8}x^{\frac{9}{2}}-9Bbc^{7}x^{\frac{5}{2}}+9Ac^{8}x^{\frac{5}{2}}+45Bb^{2}c^{6}\sqrt{x}-45Abc^{7}\sqrt{x}\right)}{45c^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(1/4)*B*b^2 - (b*c^3)^(1/4)*A*b*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 - 1/2*sqrt(2)*((b*c^3)^
(1/4)*B*b^2 - (b*c^3)^(1/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 - 2*sqrt(x))/(b/c)^(1/4))/c^4 - 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b^2 - (b*c^3)
^(1/4)*A*b*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 1/4*sq
rt(2)*((b*c^3)^(1/4)*B*b^2 - (b*c^3)^(1/4)*A*b*c)*log(-sqrt(2)*sqrt(x)*(b/c
)^(1/4) + x + sqrt(b/c))/c^4 + 2/45*(5*B*c^8*x^(9/2) - 9*B*b*c^7*x^(5/2) +
9*A*c^8*x^(5/2) + 45*B*b^2*c^6*sqrt(x) - 45*A*b*c^7*sqrt(x))/c^9
```

Mupad [B]
time = 0.25, size = 788, normalized size = 2.86

$$x^{5/2}\left(\frac{2A}{5c}-\frac{2Bb}{5c^{2}}\right)+\frac{2Bx^{9/2}}{9c}-\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^{2}b^{4}c^{2}-2ABb^{5}c+B^{2}b^{6}\right)}{c^{3}}-\frac{(-b)^{5/4}(Ac-Bb)\left(32Bb^{4}-32Ab^{3}c\right)1\mathrm{i}}{2c^{13/4}}\right)(Ac-Bb)}{2c^{13/4}}+\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^{2}b^{4}c^{2}-2ABb^{5}c+B^{2}b^{6}\right)}{c^{3}}+\frac{(-b)^{5/4}(Ac-Bb)\left(32Bb^{4}-32Ab^{3}c\right)1\mathrm{i}}{2c^{13/4}}\right)(Ac-Bb)}{2c^{13/4}}}{\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^{2}b^{4}c^{2}-2ABb^{5}c+B^{2}b^{6}\right)}{c^{3}}-\frac{(-b)^{5/4}(Ac-Bb)\left(32Bb^{4}-32Ab^{3}c\right)1\mathrm{i}}{2c^{13/4}}\right)(Ac-Bb)\,1\mathrm{i}}{2c^{13/4}}-\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^{2}b^{4}c^{2}-2ABb^{5}c+B^{2}b^{6}\right)}{c^{3}}+\frac{(-b)^{5/4}(Ac-Bb)\left(32Bb^{4}-32Ab^{3}c\right)1\mathrm{i}}{2c^{13/4}}\right)(Ac-Bb)\,1\mathrm{i}}{2c^{13/4}}}\right)(Ac-Bb)}{c^{13/4}}-\frac{b\sqrt{x}\left(\frac{2A}{c}-\frac{2Bb}{c^{2}}\right)}{c}-\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^{2}b^{4}c^{2}-2ABb^{5}c+B^{2}b^{6}\right)}{c^{3}}-\frac{(-b)^{5/4}(Ac-Bb)\left(32Bb^{4}-32Ab^{3}c\right)}{2c^{13/4}}\right)(Ac-Bb)\,1\mathrm{i}}{2c^{13/4}}+\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^{2}b^{4}c^{2}-2ABb^{5}c+B^{2}b^{6}\right)}{c^{3}}+\frac{(-b)^{5/4}(Ac-Bb)\left(32Bb^{4}-32Ab^{3}c\right)}{2c^{13/4}}\right)(Ac-Bb)\,1\mathrm{i}}{2c^{13/4}}}{\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^{2}b^{4}c^{2}-2ABb^{5}c+B^{2}b^{6}\right)}{c^{3}}-\frac{(-b)^{5/4}(Ac-Bb)\left(32Bb^{4}-32Ab^{3}c\right)}{2c^{13/4}}\right)(Ac-Bb)}{2c^{13/4}}-\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^{2}b^{4}c^{2}-2ABb^{5}c+B^{2}b^{6}\right)}{c^{3}}+\frac{(-b)^{5/4}(Ac-Bb)\left(32Bb^{4}-32Ab^{3}c\right)}{2c^{13/4}}\right)(Ac-Bb)}{2c^{13/4}}}\right)(Ac-Bb)\,1\mathrm{i}}{c^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^(5/2)*((2*A)/(5*c) - (2*B*b)/(5*c^2)) + (2*B*x^(9/2))/(9*c) - ((-b)^(5/4)
*atan((((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3
- ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c))/(2*c^(13/4)))*(A*c - B*b
)*1i)/(2*c^(13/4)) + ((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*
B*b^5*c))/c^3 + ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c))/(2*c^(13/4
)))*(A*c - B*b)*1i)/(2*c^(13/4)))/(((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*
b^4*c^2 - 2*A*B*b^5*c))/c^3 - ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*
c))/(2*c^(13/4)))*(A*c - B*b))/(2*c^(13/4)) - ((-b)^(5/4)*((16*x^(1/2)*(B^2
*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 + ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4
- 32*A*b^3*c))/(2*c^(13/4)))*(A*c - B*b))/(2*c^(13/4))))*(A*c - B*b)*1i)/c^
(13/4) - ((-b)^(5/4)*atan((((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*b^4*c^2
- 2*A*B*b^5*c))/c^3 - ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c)*1i)/(
2*c^(13/4)))*(A*c - B*b))/(2*c^(13/4)) + ((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6
+ A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 + ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*
A*b^3*c)*1i)/(2*c^(13/4)))*(A*c - B*b))/(2*c^(13/4)))/(((-b)^(5/4)*((16*x^(
1/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 - ((-b)^(5/4)*(A*c - B*b)*(
32*B*b^4 - 32*A*b^3*c)*1i)/(2*c^(13/4)))*(A*c - B*b)*1i)/(2*c^(13/4)) - ((-
b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 + ((-b)^(5
/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c)*1i)/(2*c^(13/4)))*(A*c - B*b)*1i)/(
2*c^(13/4))))*(A*c - B*b))/c^(13/4) - (b*x^(1/2)*((2*A)/c - (2*B*b)/c^2))/c
```

3.185 $\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=257

$$-\frac{2(bB-Ac)x^{3/2}}{3c^2}+\frac{2Bx^{7/2}}{7c}-\frac{b^{3/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{11/4}}+\frac{b^{3/4}(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{\sqrt{2}\,c^{11/4}}$$

```
[Out] -2/3*(-A*c+B*b)*x^(3/2)/c^2+2/7*B*x^(7/2)/c-1/2*b^(3/4)*(-A*c+B*b)*arctan(1
-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(11/4)*2^(1/2)+1/2*b^(3/4)*(-A*c+B*b)*a
rctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(11/4)*2^(1/2)+1/4*b^(3/4)*(-A*c
+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)*2^(1/2
)-1/4*b^(3/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/
2))/c^(11/4)*2^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 470, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{b^{3/4}(bB-Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{11/4}}+\frac{b^{3/4}(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,c^{11/4}}+\frac{b^{3/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{11/4}}-\frac{b^{3/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{11/4}}-\frac{2x^{3/2}(bB-Ac)}{3c^2}+\frac{2Bx^{7/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*(b*B - A*c)*x^(3/2))/(3*c^2) + (2*B*x^(7/2))/(7*c) - (b^(3/4)*(b*B - A*
c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(
3/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^
(11/4)) + (b^(3/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4)) - (b^(3/4)*(b*B - A*c)*Log[Sqrt[b] + S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \int \frac{x^{5/2}(A+Bx^2)}{b+cx^2}\,dx$$

$$= \frac{2Bx^{7/2}}{7c} - \frac{\left(2\left(\frac{7bB}{2}-\frac{7Ac}{2}\right)\right)\int \frac{x^{5/2}}{b+cx^2}\,dx}{7c}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{(b(bB-Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{c^2}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{(2b(bB-Ac))\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} - \frac{(b(bB-Ac))\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{5/2}} + \frac{(b \cdots}{\cdots}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{(b(bB-Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x} \cdots\right.}{2c^3}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{b^{3/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{11/4}} \cdots$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} - \frac{b^{3/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{11/4}} + \frac{b^{3/4}(bB \cdots}{\cdots}$$

Mathematica [A]
time = 0.17, size = 152, normalized size = 0.59

$$\frac{2x^{3/2}(-7bB+7Ac+3Bcx^2)}{21c^2} - \frac{b^{3/4}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}\,c^{11/4}} - \frac{b^{3/4}(bB-Ac)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2)*(-7*b*B + 7*A*c + 3*B*c*x^2))/(21*c^2) - (b^(3/4)*(b*B - A*c)*Ar
cTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*c^(
11/4)) - (b^(3/4)*(b*B - A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sq
rt[b] + Sqrt[c]*x)])/(Sqrt[2]*c^(11/4))
```

Maple [A]
time = 0.42, size = 142, normalized size = 0.55

method	result
derivativedivides	$\frac{\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2(Ac-Bb)x^{\frac{3}{2}}}{3}}{c^{2}}-\frac{b(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{4c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2(Ac-Bb)x^{\frac{3}{2}}}{3}}{c^{2}}-\frac{b(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{4c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
risch	$\frac{2x^{\frac{3}{2}}\left(3Bc\,x^{2}+7Ac-7Bb\right)}{21c^{2}}-\frac{b\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{b\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{b\sqrt{2}\,A\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^2*(1/7*B*c*x^(7/2)+1/3*(A*c-B*b)*x^(3/2))-1/4*b*(A*c-B*b)/c^3/(b/c)^(1/
4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x
^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arct
an(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.53, size = 214, normalized size = 0.83

$$\frac{(Bb^{2}-Abc)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{4\,c^{2}}+\frac{2\left(3\,Bcx^{\frac{7}{2}}-7\,(Bb-Ac)x^{\frac{3}{2}}\right)}{21\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b^2 - A*b*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
+ 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(
x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sq
rt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*
c^(3/4)))/c^2 + 2/21*(3*B*c*x^(7/2) - 7*(B*b - A*c)*x^(3/2))/c^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(182) = 364$.
time = 2.84, size = 899, normalized size = 3.50

$$\frac{84\,c^{2}\left(-\frac{B^{4}b^{7}-4\,AB^{3}b^{6}c+6\,A^{2}B^{2}b^{5}c^{2}-4\,A^{3}Bb^{4}c^{3}+A^{4}b^{3}c^{4}}{c^{11}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{(B^{6}b^{10}-6\,AB^{5}b^{9}c+15\,A^{2}B^{4}b^{8}c^{2}-20\,A^{3}B^{3}b^{7}c^{3}+15\,A^{4}B^{2}b^{6}c^{4}-6\,A^{5}Bb^{5}c^{5}+A^{6}b^{4}c^{6})x-(B^{4}b^{7}c^{5}-4\,AB^{3}b^{6}c^{6}+6\,A^{2}B^{2}b^{5}c^{7}-4\,A^{3}Bb^{4}c^{8}+A^{4}b^{3}c^{9})\sqrt{-\frac{B^{4}b^{7}-4\,AB^{3}b^{6}c+6\,A^{2}B^{2}b^{5}c^{2}-4\,A^{3}Bb^{4}c^{3}+A^{4}b^{3}c^{4}}{c^{11}}}}\,c^{3}\left(-\frac{B^{4}b^{7}-4\,AB^{3}b^{6}c+6\,A^{2}B^{2}b^{5}c^{2}-4\,A^{3}Bb^{4}c^{3}+A^{4}b^{3}c^{4}}{c^{11}}\right)^{\frac{1}{4}}+(B^{3}b^{5}c^{3}-3\,AB^{2}b^{4}c^{4}+3\,A^{2}Bb^{3}c^{5}-A^{3}b^{2}c^{6})\sqrt{x}\left(-\frac{B^{4}b^{7}-4\,AB^{3}b^{6}c+6\,A^{2}B^{2}b^{5}c^{2}-4\,A^{3}Bb^{4}c^{3}+A^{4}b^{3}c^{4}}{c^{11}}\right)^{\frac{1}{4}}}{B^{4}b^{7}-4\,AB^{3}b^{6}c+6\,A^{2}B^{2}b^{5}c^{2}-4\,A^{3}Bb^{4}c^{3}+A^{4}b^{3}c^{4}}\right)-21\,c^{2}\left(-\frac{B^{4}b^{7}-4\,AB^{3}b^{6}c+6\,A^{2}B^{2}b^{5}c^{2}-4\,A^{3}Bb^{4}c^{3}+A^{4}b^{3}c^{4}}{c^{11}}\right)^{\frac{1}{4}}\log\left(c^{8}\left(-\frac{B^{4}b^{7}-4\,AB^{3}b^{6}c+6\,A^{2}B^{2}b^{5}c^{2}-4\,A^{3}Bb^{4}c^{3}+A^{4}b^{3}c^{4}}{c^{11}}\right)^{\frac{3}{4}}-(B^{3}b^{5}-3\,AB^{2}b^{4}c+3\,A^{2}Bb^{3}c^{2}-A^{3}b^{2}c^{3})\sqrt{x}\right)+21\,c^{2}\left(-\frac{B^{4}b^{7}-4\,AB^{3}b^{6}c+6\,A^{2}B^{2}b^{5}c^{2}-4\,A^{3}Bb^{4}c^{3}+A^{4}b^{3}c^{4}}{c^{11}}\right)^{\frac{1}{4}}\log\left(-c^{8}\left(-\frac{B^{4}b^{7}-4\,AB^{3}b^{6}c+6\,A^{2}B^{2}b^{5}c^{2}-4\,A^{3}Bb^{4}c^{3}+A^{4}b^{3}c^{4}}{c^{11}}\right)^{\frac{3}{4}}-(B^{3}b^{5}-3\,AB^{2}b^{4}c+3\,A^{2}Bb^{3}c^{2}-A^{3}b^{2}c^{3})\sqrt{x}\right)+4\,(3\,Bcx^{3}-7\,(Bb-Ac)x)\sqrt{x}}{42\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/42*(84*c^2*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c
^3 + A^4*b^3*c^4)/c^11)^(1/4)*arctan((sqrt((B^6*b^10 - 6*A*B^5*b^9*c + 15*A
^2*B^4*b^8*c^2 - 20*A^3*B^3*b^7*c^3 + 15*A^4*B^2*b^6*c^4 - 6*A^5*B*b^5*c^5
+ A^6*b^4*c^6)*x - (B^4*b^7*c^5 - 4*A*B^3*b^6*c^6 + 6*A^2*B^2*b^5*c^7 - 4*A
^3*B*b^4*c^8 + A^4*b^3*c^9)*sqrt(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*
c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11))*c^3*(-(B^4*b^7 - 4*A*B^3*b^6*c
+ 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(1/4) + (B^3*b^5
*c^3 - 3*A*B^2*b^4*c^4 + 3*A^2*B*b^3*c^5 - A^3*b^2*c^6)*sqrt(x)*(-(B^4*b^7
- 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^
(1/4))/(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4
*b^3*c^4)) - 21*c^2*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*
B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(1/4)*log(c^8*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6
*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(3/4) - (B^3*b^5 -
3*A*B^2*b^4*c + 3*A^2*B*b^3*c^2 - A^3*b^2*c^3)*sqrt(x)) + 21*c^2*(-(B^4*b^7
 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)
^(1/4)*log(-c^8*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^
4*c^3 + A^4*b^3*c^4)/c^11)^(3/4) - (B^3*b^5 - 3*A*B^2*b^4*c + 3*A^2*B*b^3*c
^2 - A^3*b^2*c^3)*sqrt(x)) + 4*(3*B*c*x^3 - 7*(B*b - A*c)*x)*sqrt(x))/c^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 264, normalized size = 1.03

$$\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{5}}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{5}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^{5}}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^{5}}+\frac{2\left(3Bc^{6}x^{\frac{7}{2}}-7Bbc^{5}x^{\frac{3}{2}}+7Ac^{6}x^{\frac{3}{2}}\right)}{21c^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 + 1/2*sqrt(2)*((b*c^3)^(3/4)
*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt
(x))/(b/c)^(1/4))/c^5 - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)
*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/4*sqrt(2)*((b*c^3
)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqr
t(b/c))/c^5 + 2/21*(3*B*c^6*x^(7/2) - 7*B*b*c^5*x^(3/2) + 7*A*c^6*x^(3/2))/
c^7
```

Mupad [B]
time = 0.24, size = 92, normalized size = 0.36

$$x^{3/2}\left(\frac{2A}{3c}-\frac{2Bb}{3c^{2}}\right)+\frac{2Bx^{7/2}}{7c}+\frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(Ac-Bb)}{c^{11/4}}+\frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)(Ac-Bb)\,1\mathrm{i}}{c^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^(3/2)*((2*A)/(3*c) - (2*B*b)/(3*c^2)) + (2*B*x^(7/2))/(7*c) + ((-b)^(3/4)
*atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/c^(11/4) + ((-b)^(3/4)*ata
n((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(A*c - B*b)*1i)/c^(11/4)
```

3.186 $\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=255

$$-\frac{2(bB-Ac)\sqrt{x}}{c^2}+\frac{2Bx^{5/2}}{5c}-\frac{\sqrt[4]{b}\,(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}}+\frac{\sqrt[4]{b}\,(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{\sqrt{2}\,c^{9/4}}$$

```
[Out] 2/5*B*x^(5/2)/c-1/2*b^(1/4)*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(
1/4))/c^(9/4)*2^(1/2)+1/2*b^(1/4)*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/
2)/b^(1/4))/c^(9/4)*2^(1/2)-1/4*b^(1/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(
1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)*2^(1/2)+1/4*b^(1/4)*(-A*c+B*b)*ln(b^(
1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)*2^(1/2)-2*(-A*c+B*b
)*x^(1/2)/c^2
```

Rubi [A]

time = 0.14, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 470, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\sqrt[4]{b}\,(bB-Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}}+\frac{\sqrt[4]{b}\,(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,c^{9/4}}-\frac{\sqrt[4]{b}\,(bB-Ac)\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}}+\frac{\sqrt[4]{b}\,(bB-Ac)\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}}-\frac{2\sqrt{x}\,(bB-Ac)}{c^2}+\frac{2Bx^{5/2}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*(b*B - A*c)*Sqrt[x])/c^2 + (2*B*x^(5/2))/(5*c) - (b^(1/4)*(b*B - A*c)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) + (b^(1/4)*
(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)
) - (b^(1/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x])/(2*Sqrt[2]*c^(9/4)) + (b^(1/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx &= \int \frac{x^{3/2}(A+Bx^2)}{b+cx^2}\,dx \\
&= \frac{2Bx^{5/2}}{5c} - \frac{\left(2\left(\frac{5bB}{2}-\frac{5Ac}{2}\right)\right)\int \frac{x^{3/2}}{b+cx^2}\,dx}{5c} \\
&= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} + \frac{(b(bB-Ac))\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{c^2} \\
&= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} + \frac{(2b(bB-Ac))\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2} \\
&= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} + \frac{\left(\sqrt{b}\,(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2} \\
&= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} + \frac{\left(\sqrt{b}\,(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x\right.}{2c^{5/2}} \\
&= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} - \frac{\sqrt[4]{b}\,(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right.}{2\sqrt{2}\,c^{9/4}} \\
&= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} - \frac{\sqrt[4]{b}\,(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}} + \frac{\sqrt[4]{b}\,(b}{}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 150, normalized size = 0.59

$$\frac{4\sqrt[4]{c}\sqrt{x}\left(-5bB+5Ac+Bcx^2\right)-5\sqrt{2}\sqrt[4]{b}\,(bB-Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+5\sqrt{2}\sqrt[4]{b}\,(bB-Ac)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{10c^{9/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]

[Out] (4*c^(1/4)*Sqrt[x]*(-5*b*B + 5*A*c + B*c*x^2) - 5*Sqrt[2]*b^(1/4)*(b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 5*Sqrt[2]*b^(1/4)*(b*B - A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(10*c^(9/4))

Maple [A]
time = 0.39, size = 141, normalized size = 0.55

method	result
derivativedivides	$\frac{\frac{2 B\, x^{\frac{5}{2}} c}{5}+2 A c \sqrt{x}-2 B b \sqrt{x}}{c^{2}}-\frac{\left(A c -B b \right) \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x +\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{b}{c}}}{x -\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right.\right.}{4 c^{2}}$
default	$\frac{\frac{2 B\, x^{\frac{5}{2}} c}{5}+2 A c \sqrt{x}-2 B b \sqrt{x}}{c^{2}}-\frac{\left(A c -B b \right) \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x +\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{b}{c}}}{x -\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right.\right.}{4 c^{2}}$
risch	$\frac{2 \left(B c \,x^{2}+5 A c -5 B b \right) \sqrt{x}}{5 c^{2}}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\, A \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2 c}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\, A \ln \left(\frac{x +\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{}}{x -\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{}}\right.}{4 c}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)

[Out] 2/c^2*(1/5*B*x^(5/2)*c+A*c*x^(1/2)-B*b*x^(1/2))-1/4*(A*c-B*b)/c^2*(b/c)^(1/4)*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))

Maxima [A]
time = 0.51, size = 235, normalized size = 0.92

$$\frac{\left(\frac{2\,\sqrt{2}\,(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,(Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,(Bb-Ac)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Bb-Ac)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)b}{4\,c^2}+\frac{2\left(Bcx^{\frac{5}{2}}-5\,(Bb-Ac)\sqrt{x}\right)}{5\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")

[Out] 1/4*(2*sqrt(2)*(B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2

```
)*(B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b
^(3/4)*c^(1/4)) - sqrt(2)*(B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
+ sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b/c^2 + 2/5*(B*c*x^(5/2) - 5*(B*b
 - A*c)*sqrt(x))/c^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(182) = 364$.
time = 2.49, size = 660, normalized size = 2.59

$$-\frac{20\,c^2\left(-\frac{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}{c^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^4\sqrt{-\frac{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}{c^9}}+(B^2b^2-2\,ABbc+A^2c^2)x}\,c^7\left(-\frac{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}{c^9}\right)^{\frac{3}{4}}+(Bbc^7-Ac^8)\sqrt{x}\left(-\frac{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}{c^9}\right)^{\frac{3}{4}}}{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}\right)+5\,c^2\left(-\frac{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}{c^9}\right)^{\frac{1}{4}}\log\left(c^2\left(-\frac{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}{c^9}\right)^{\frac{1}{4}}-(Bb-Ac)\sqrt{x}\right)-5\,c^2\left(-\frac{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}{c^9}\right)^{\frac{1}{4}}\log\left(-c^2\left(-\frac{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}{c^9}\right)^{\frac{1}{4}}-(Bb-Ac)\sqrt{x}\right)-4\,(Bcx^2-5\,Bb+5\,Ac)\sqrt{x}}{10\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*
c^3 + A^4*b*c^4)/c^9)^(1/4)*arctan((sqrt(c^4*sqrt(-(B^4*b^5 - 4*A*B^3*b^4*c
 + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9) + (B^2*b^2 - 2*A*B
*b*c + A^2*c^2)*x)*c^7*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A
^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(3/4) + (B*b*c^7 - A*c^8)*sqrt(x)*(-(B^4*b^5
 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(3
/4))/(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b
*c^4)) + 5*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2
*c^3 + A^4*b*c^4)/c^9)^(1/4)*log(c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2
*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(1/4) - (B*b - A*c)*sqrt(x)) -
 5*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A
^4*b*c^4)/c^9)^(1/4)*log(-c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^
2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(1/4) - (B*b - A*c)*sqrt(x)) - 4*(B*c
*x^2 - 5*B*b + 5*A*c)*sqrt(x))/c^2
```

Sympy [A]
time = 104.00, size = 275, normalized size = 1.08

$$\begin{cases} \tilde{\infty}\left(2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}\right) & \text{for}\: b=0 \wedge c=0 \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{9}{2}}}{9}}{b} & \text{for}\: c=0 \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}}{c} & \text{for}\: b=0 \\ \frac{2A\sqrt{x}}{c}+\frac{A\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2c}-\frac{A\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2c}-\frac{A\sqrt[4]{-\frac{b}{c}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{c}-\frac{2Bb\sqrt{x}}{c^2}-\frac{Bb\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2c^2}+\frac{Bb\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2c^2}+\frac{Bb\sqrt[4]{-\frac{b}{c}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{c^2}+\frac{2Bx^{\frac{5}{2}}}{5c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(5/2)/5), Eq(b, 0) & Eq(c, 0)), ((2*A*
x**(5/2)/5 + 2*B*x**(9/2)/9)/b, Eq(c, 0)), ((2*A*sqrt(x) + 2*B*x**(5/2)/5)/
c, Eq(b, 0)), (2*A*sqrt(x)/c + A*(-b/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/4))
/(2*c) - A*(-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*c) - A*(-b/c)**(1/
4)*atan(sqrt(x)/(-b/c)**(1/4))/c - 2*B*b*sqrt(x)/c**2 - B*b*(-b/c)**(1/4)*l
```

```
og(sqrt(x) - (-b/c)**(1/4))/(2*c**2) + B*b*(-b/c)**(1/4)*log(sqrt(x) + (-b/
c)**(1/4))/(2*c**2) + B*b*(-b/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/c**2 +
2*B*x**(5/2)/(5*c), True))
```

Giac [A]
time = 0.44, size = 263, normalized size = 1.03

$$\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{3}}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{3}}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^{3}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^{3}}+\frac{2\left(Bc^{4}x^{\frac{5}{2}}-5Bbc^{3}\sqrt{x}+5Ac^{4}\sqrt{x}\right)}{5c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^3 + 1/2*sqrt(2)*((b*c^3)^(1/4)
*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt
(x))/(b/c)^(1/4))/c^3 + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)
*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 - 1/4*sqrt(2)*((b*c^3
)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqr
t(b/c))/c^3 + 2/5*(B*c^4*x^(5/2) - 5*B*b*c^3*sqrt(x) + 5*A*c^4*sqrt(x))/c^5
```

Mupad [B]
time = 0.27, size = 789, normalized size = 3.09

$$\sqrt{x}\left(\frac{2A}{c}-\frac{2Bb}{c^2}\right)+\frac{2Bx^{5/2}}{5c}-\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{1/4}(Ac-Bb)\left(\frac{16\sqrt{x}\,(A^2b^2c^2-2ABb^3c+B^2b^4)}{c}-\frac{(-b)^{1/4}(32Ab^2c^2-32Bb^3c)(Ac-Bb)}{2c^{9/4}}\right)1i}{2c^{9/4}}+\frac{(-b)^{1/4}(Ac-Bb)\left(\frac{16\sqrt{x}\,(A^2b^2c^2-2ABb^3c+B^2b^4)}{c}+\frac{(-b)^{1/4}(32Ab^2c^2-32Bb^3c)(Ac-Bb)}{2c^{9/4}}\right)1i}{2c^{9/4}}}{\frac{(-b)^{1/4}(Ac-Bb)\left(\frac{16\sqrt{x}\,(A^2b^2c^2-2ABb^3c+B^2b^4)}{c}-\frac{(-b)^{1/4}(32Ab^2c^2-32Bb^3c)(Ac-Bb)}{2c^{9/4}}\right)}{2c^{9/4}}-\frac{(-b)^{1/4}(Ac-Bb)\left(\frac{16\sqrt{x}\,(A^2b^2c^2-2ABb^3c+B^2b^4)}{c}+\frac{(-b)^{1/4}(32Ab^2c^2-32Bb^3c)(Ac-Bb)}{2c^{9/4}}\right)}{2c^{9/4}}}\right)(Ac-Bb)\,1i}{c^{9/4}}-\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{1/4}(Ac-Bb)\left(\frac{16\sqrt{x}\,(A^2b^2c^2-2ABb^3c+B^2b^4)}{c}-\frac{(-b)^{1/4}(32Ab^2c^2-32Bb^3c)(Ac-Bb)\,1i}{2c^{9/4}}\right)}{2c^{9/4}}+\frac{(-b)^{1/4}(Ac-Bb)\left(\frac{16\sqrt{x}\,(A^2b^2c^2-2ABb^3c+B^2b^4)}{c}+\frac{(-b)^{1/4}(32Ab^2c^2-32Bb^3c)(Ac-Bb)\,1i}{2c^{9/4}}\right)}{2c^{9/4}}}{\frac{(-b)^{1/4}(Ac-Bb)\left(\frac{16\sqrt{x}\,(A^2b^2c^2-2ABb^3c+B^2b^4)}{c}-\frac{(-b)^{1/4}(32Ab^2c^2-32Bb^3c)(Ac-Bb)\,1i}{2c^{9/4}}\right)1i}{2c^{9/4}}-\frac{(-b)^{1/4}(Ac-Bb)\left(\frac{16\sqrt{x}\,(A^2b^2c^2-2ABb^3c+B^2b^4)}{c}+\frac{(-b)^{1/4}(32Ab^2c^2-32Bb^3c)(Ac-Bb)\,1i}{2c^{9/4}}\right)1i}{2c^{9/4}}}\right)(Ac-Bb)}{c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^(1/2)*((2*A)/c - (2*B*b)/c^2) + (2*B*x^(5/2))/(5*c) - ((-b)^(1/4)*atan(((
(-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/
c - ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*c - B*b))/(2*c^(9/4)))*1i)/(
2*c^(9/4)) + ((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 + A^2*b^2*c^2 -
2*A*B*b^3*c))/c + ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*c - B*b))/(2*c
^(9/4)))*1i)/(2*c^(9/4)))/(((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 +
A^2*b^2*c^2 - 2*A*B*b^3*c))/c - ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*
c - B*b))/(2*c^(9/4))))/(2*c^(9/4)) - ((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*
(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/c + ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B
*b^3*c)*(A*c - B*b))/(2*c^(9/4))))/(2*c^(9/4))))*(A*c - B*b)*1i)/c^(9/4) -
((-b)^(1/4)*atan((((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 + A^2*b^2*c
^2 - 2*A*B*b^3*c))/c - ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*c - B*b)*
1i)/(2*c^(9/4))))/(2*c^(9/4)) + ((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b
^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/c + ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c
)*(A*c - B*b)*1i)/(2*c^(9/4))))/(2*c^(9/4)))/(((-b)^(1/4)*(A*c - B*b)*((16*
x^(1/2)*(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/c - ((-b)^(1/4)*(32*A*b^2*c^
2 - 32*B*b^3*c)*(A*c - B*b)*1i)/(2*c^(9/4)))*1i)/(2*c^(9/4)) - ((-b)^(1/4)*
```

```
(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/c + ((-b)^(
1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*c - B*b)*1i)/(2*c^(9/4)))*1i)/(2*c^(9/4
))))*(A*c - B*b))/c^(9/4)
```

3.187 $\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=237

$$\frac{2Bx^{3/2}}{3c}+\frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{(bB-Ac)\log\left(\sqrt{b}\right.}{2\surd}$$

```
[Out] 2/3*B*x^(3/2)/c+1/2*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^
(1/4)/c^(7/4)*2^(1/2)-1/2*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/
4))/b^(1/4)/c^(7/4)*2^(1/2)-1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(
1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(7/4)*2^(1/2)+1/4*(-A*c+B*b)*ln(b^(1/2)+x*c
^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1598, 470, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{(bB-Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,c^{7/4}}+\frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,c^{7/4}}+\frac{2Bx^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*B*x^(3/2))/(3*c) + ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)])/(Sqrt[2]*b^(1/4)*c^(7/4)) - ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(1/4)*c^(7/4)) - ((b*B - A*c)*Log[Sqrt[b] -
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(1/4)*c^(7/4)) +
 ((b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(
2*Sqrt[2]*b^(1/4)*c^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \int \frac{\sqrt{x}\,(A+Bx^2)}{b+cx^2}\,dx$$

$$= \frac{2Bx^{3/2}}{3c} - \frac{\left(2\left(\frac{3bB}{2}-\frac{3Ac}{2}\right)\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{3c}$$

$$= \frac{2Bx^{3/2}}{3c} - \frac{\left(4\left(\frac{3bB}{2}-\frac{3Ac}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{3c}$$

$$= \frac{2Bx^{3/2}}{3c} + \frac{(bB-Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{3/2}} - \frac{(bB-Ac)\text{Subst}\left(\int \frac{\sqrt{b}+\cdots}{b+c\cdots}\right.}{c^{3/2}}$$

$$= \frac{2Bx^{3/2}}{3c} - \frac{(bB-Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^2} - \frac{(bB-Ac)\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{2Bx^{3/2}}{3c} - \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,\sqrt[4]{b}\,c^{7/4}} + \frac{(bB-Ac)\log\left(\sqrt{b}+\cdots\right.}{2\sqrt{2}\cdots}$$

$$= \frac{2Bx^{3/2}}{3c} + \frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,c^{7/4}} - \frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{\cdots}}{\sqrt[4]{b}}\right.}{\sqrt{2}\,\sqrt[4]{b}\,c^{7/4}}$$

Mathematica [A]
time = 0.15, size = 135, normalized size = 0.57

$$\frac{2Bx^{3/2}}{3c} + \frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,c^{7/4}} + \frac{(bB-Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,\sqrt[4]{b}\,c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*B*x^(3/2))/(3*c) + ((b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(1/4)*c^(7/4)) + ((b*B - A*c)*ArcTanh[(
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(1/4)*c
^(7/4))
```

Maple [A]
time = 0.40, size = 124, normalized size = 0.52

method	result
derivativedivides	$\frac{2Bx^{\frac{3}{2}}}{3c}+\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\right.}{4c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$\frac{2Bx^{\frac{3}{2}}}{3c}+\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\right.}{4c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
risch	$\frac{2Bx^{\frac{3}{2}}}{3c}+\frac{\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2c\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}}{(}\right.}{2c\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*B*x^(3/2)/c+1/4*(A*c-B*b)/c^2/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^
(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*a
rctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1
))
```

Maxima [A]
time = 0.52, size = 194, normalized size = 0.82

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,c}-\frac{(Bb-Ac)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/3*B*x^(3/2)/c - 1/4*(B*b - A*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^
(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqr
t(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2
*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) -
sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)
*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt
(b))/(b^(1/4)*c^(3/4)))/c
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(166) = 332$.
time = 2.08, size = 834, normalized size = 3.52

$$\frac{4 B x^{\frac{3}{2}} - 12 c \left(-\frac{B^{4} b^{4} - 4 A B^{3} b^{3} c + 6 A^{2} B^{2} b^{2} c^{2} - 4 A^{3} B b c^{3} + A^{4} c^{4}}{b c^{7}}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{(B^{6} b^{6} - 6 A B^{5} b^{5} c + 15 A^{2} B^{4} b^{4} c^{2} - 20 A^{3} B^{3} b^{3} c^{3} + 15 A^{4} B^{2} b^{2} c^{4} - 6 A^{5} B b c^{5} + A^{6} c^{6}) x - (B^{4} b^{5} c^{3} - 4 A B^{3} b^{4} c^{4} + 6 A^{2} B^{2} b^{3} c^{5} - 4 A^{3} B b^{2} c^{6} + A^{4} b c^{7}) \sqrt{-\frac{B^{4} b^{4} - 4 A B^{3} b^{3} c + 6 A^{2} B^{2} b^{2} c^{2} - 4 A^{3} B b c^{3} + A^{4} c^{4}}{b c^{7}}}}\, c^{2} \left(-\frac{B^{4} b^{4} - 4 A B^{3} b^{3} c + 6 A^{2} B^{2} b^{2} c^{2} - 4 A^{3} B b c^{3} + A^{4} c^{4}}{b c^{7}}\right)^{\frac{1}{4}} + (B^{3} b^{3} c^{2} - 3 A B^{2} b^{2} c^{3} + 3 A^{2} B b c^{4} - A^{3} c^{5}) \sqrt{x} \left(-\frac{B^{4} b^{4} - 4 A B^{3} b^{3} c + 6 A^{2} B^{2} b^{2} c^{2} - 4 A^{3} B b c^{3} + A^{4} c^{4}}{b c^{7}}\right)^{\frac{1}{4}}}{B^{4} b^{4} - 4 A B^{3} b^{3} c + 6 A^{2} B^{2} b^{2} c^{2} - 4 A^{3} B b c^{3} + A^{4} c^{4}}\right) + 3 c \left(-\frac{B^{4} b^{4} - 4 A B^{3} b^{3} c + 6 A^{2} B^{2} b^{2} c^{2} - 4 A^{3} B b c^{3} + A^{4} c^{4}}{b c^{7}}\right)^{\frac{1}{4}} \log\left(b c^{5} \left(-\frac{B^{4} b^{4} - 4 A B^{3} b^{3} c + 6 A^{2} B^{2} b^{2} c^{2} - 4 A^{3} B b c^{3} + A^{4} c^{4}}{b c^{7}}\right)^{\frac{3}{4}} - (B^{3} b^{3} - 3 A B^{2} b^{2} c + 3 A^{2} B b c^{2} - A^{3} c^{3}) \sqrt{x}\right) - 3 c \left(-\frac{B^{4} b^{4} - 4 A B^{3} b^{3} c + 6 A^{2} B^{2} b^{2} c^{2} - 4 A^{3} B b c^{3} + A^{4} c^{4}}{b c^{7}}\right)^{\frac{1}{4}} \log\left(-b c^{5} \left(-\frac{B^{4} b^{4} - 4 A B^{3} b^{3} c + 6 A^{2} B^{2} b^{2} c^{2} - 4 A^{3} B b c^{3} + A^{4} c^{4}}{b c^{7}}\right)^{\frac{3}{4}} - (B^{3} b^{3} - 3 A B^{2} b^{2} c + 3 A^{2} B b c^{2} - A^{3} c^{3}) \sqrt{x}\right)}{6 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*B*x^(3/2) - 12*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*
A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4)*arctan((sqrt((B^6*b^6 - 6*A*B^5*b^5*c
 + 15*A^2*B^4*b^4*c^2 - 20*A^3*B^3*b^3*c^3 + 15*A^4*B^2*b^2*c^4 - 6*A^5*B*b
*c^5 + A^6*c^6)*x - (B^4*b^5*c^3 - 4*A*B^3*b^4*c^4 + 6*A^2*B^2*b^3*c^5 - 4*
A^3*B*b^2*c^6 + A^4*b*c^7)*sqrt(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c
^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7)))*c^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*
A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4) + (B^3*b^3*c^2 -
3*A*B^2*b^2*c^3 + 3*A^2*B*b*c^4 - A^3*c^5)*sqrt(x)*(-(B^4*b^4 - 4*A*B^3*b^3
*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4))/(B^4*b^4
 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)) + 3*c*(-(B^
4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7
))^(1/4)*log(b*c^5*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B
*b*c^3 + A^4*c^4)/(b*c^7))^(3/4) - (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2
 - A^3*c^3)*sqrt(x)) - 3*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 -
 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4)*log(-b*c^5*(-(B^4*b^4 - 4*A*B^3*b^
3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(3/4) - (B^3*b^
3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)*sqrt(x)))/c
```

Sympy [**A**]
time = 38.97, size = 303, normalized size = 1.28

$$\begin{cases} \tilde{\infty} \left(- \frac{2 A}{\sqrt{x}} + \frac{2 B x^{\frac{3}{2}}}{3}\right) & \text{for}\: b = 0 \wedge c = 0 \\ \frac{- \frac{2 A}{\sqrt{x}} + \frac{2 B x^{\frac{3}{2}}}{3}}{c} & \text{for}\: b = 0 \\ \frac{\frac{2 A x^{\frac{3}{2}}}{3} + \frac{2 B x^{\frac{7}{2}}}{7}}{b} & \text{for}\: c = 0 \\ \frac{2 A \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{b}{c}}} \right)}}{c \sqrt[4]{- \frac{b}{c}}} - \frac{A \left(- \frac{b}{c}\right)^{\frac{3}{4}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{b}{c}} \right)}}{2 b} + \frac{A \left(- \frac{b}{c}\right)^{\frac{3}{4}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{b}{c}} \right)}}{2 b} + \frac{A \left(- \frac{b}{c}\right)^{\frac{3}{4}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{b}{c}}} \right)}}{b} - \frac{2 B b \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{b}{c}}} \right)}}{c^{2} \sqrt[4]{- \frac{b}{c}}} + \frac{2 B x^{\frac{3}{2}}}{3 c} + \frac{B \left(- \frac{b}{c}\right)^{\frac{3}{4}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{b}{c}} \right)}}{2 c} - \frac{B \left(- \frac{b}{c}\right)^{\frac{3}{4}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{b}{c}} \right)}}{2 c} - \frac{B \left(- \frac{b}{c}\right)^{\frac{3}{4}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{b}{c}}} \right)}}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*(-2*A/sqrt(x) + 2*B*x**(3/2)/3), Eq(b, 0) & Eq(c, 0)), ((-2*
A/sqrt(x) + 2*B*x**(3/2)/3)/c, Eq(b, 0)), ((2*A*x**(3/2)/3 + 2*B*x**(7/2)/7
)/b, Eq(c, 0)), (2*A*atan(sqrt(x)/(-b/c)**(1/4))/(c*(-b/c)**(1/4)) - A*(-b/
c)**(3/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b) + A*(-b/c)**(3/4)*log(sqrt(x)
+ (-b/c)**(1/4))/(2*b) + A*(-b/c)**(3/4)*atan(sqrt(x)/(-b/c)**(1/4))/b - 2*
B*b*atan(sqrt(x)/(-b/c)**(1/4))/(c**2*(-b/c)**(1/4)) + 2*B*x**(3/2)/(3*c) +
```

```
 B*(-b/c)**(3/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*c) - B*(-b/c)**(3/4)*log(s
qrt(x) + (-b/c)**(1/4))/(2*c) - B*(-b/c)**(3/4)*atan(sqrt(x)/(-b/c)**(1/4))
/c, True))
```

Giac [A]
time = 0.44, size = 251, normalized size = 1.06

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,c}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^{4}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^{4}}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^{4}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/c - 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arcta
n(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) - 1/2*
sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) + 1/4*sqrt(2)*((b*c^3)^(3/4
)*B*b - (b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/(b*c^4) - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)
*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^4)
```

Mupad [B]
time = 0.15, size = 71, normalized size = 0.30

$$\frac{2\,B\,x^{3/2}}{3\,c}+\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)\,\left(A\,c-B\,b\right)}{{\left(-b\right)}^{1/4}\,c^{7/4}}-\frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)\,\left(A\,c-B\,b\right)}{{\left(-b\right)}^{1/4}\,c^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] (2*B*x^(3/2))/(3*c) + (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/((-b
)^(1/4)*c^(7/4)) - (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/((-b)^
(1/4)*c^(7/4))
```

3.188 $\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=235

$$\frac{2B\sqrt{x}}{c}+\frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}c^{5/4}}-\frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{(bB-Ac)\log\left(\sqrt{b}\right.}{2}$$

```
[Out] 1/2*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(5/4)*2^
(1/2)-1/2*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(5
/4)*2^(1/2)+1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(
1/2))/b^(3/4)/c^(5/4)*2^(1/2)-1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c
^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(5/4)*2^(1/2)+2*B*x^(1/2)/c
```

Rubi [A]
time = 0.13, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$,
Rules used = {1598, 470, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{(bB-Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}c^{5/4}}-\frac{(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{(bB-Ac)\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}c^{5/4}}-\frac{(bB-Ac)\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{2B\sqrt{x}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*B*Sqrt[x])/c + ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
])/(Sqrt[2]*b^(3/4)*c^(5/4)) - ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[x])/b^(1/4)])/(Sqrt[2]*b^(3/4)*c^(5/4)) + ((b*B - A*c)*Log[Sqrt[b] - Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(3/4)*c^(5/4)) - ((b
*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sq
rt[2]*b^(3/4)*c^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \int \frac{A+Bx^2}{\sqrt{x}\,(b+cx^2)}\,dx$$
$$= \frac{2B\sqrt{x}}{c} - \frac{\left(2\left(\frac{bB}{2}-\frac{Ac}{2}\right)\right)\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{c}$$
$$= \frac{2B\sqrt{x}}{c} - \frac{\left(4\left(\frac{bB}{2}-\frac{Ac}{2}\right)\right)\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{c}$$
$$= \frac{2B\sqrt{x}}{c} - \frac{(bB-Ac)\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{\sqrt{b}\,c} - \frac{(bB-Ac)\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+c x^4}\,dx,x,\sqrt{x}\right)}{\sqrt{b}\,c}$$
$$= \frac{2B\sqrt{x}}{c} - \frac{(bB-Ac)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{2\sqrt{b}\,c^{3/2}} - \frac{(bB-Ac)\mathrm{Subst}\left(\dots\right)}{\dots}$$
$$= \frac{2B\sqrt{x}}{c} + \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}c^{5/4}} - \frac{(bB-Ac)\log\left(\sqrt{b}+\dots\right)}{2\sqrt{\dots}}$$
$$= \frac{2B\sqrt{x}}{c} + \frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}c^{5/4}} - \frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\dots}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}c^{5/4}}$$

Mathematica [A]
time = 0.15, size = 134, normalized size = 0.57

$$\frac{2B\sqrt{x}}{c} + \frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}\,b^{3/4}c^{5/4}} - \frac{(bB-Ac)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,b^{3/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*B*Sqrt[x])/c + ((b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(3/4)*c^(5/4)) - ((b*B - A*c)*ArcTanh[(Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(3/4)*c^(5/
4))
```

Maple [A]
time = 0.38, size = 127, normalized size = 0.54

method	result
derivativedivides	$\frac{2B\sqrt{x}}{c}+\frac{(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{b}{c}\right.}\right.\right.}{4cb}$
default	$\frac{2B\sqrt{x}}{c}+\frac{(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{b}{c}\right.}\right.\right.}{4cb}$
risch	$\frac{2B\sqrt{x}}{c}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2b}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\,\mathrm{l}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*B*x^(1/2)/c+1/4*(A*c-B*b)/c*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1
/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arc
tan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.52, size = 218, normalized size = 0.93

$$\frac{2\,B\sqrt{x}}{c}-\frac{\frac{2\,\sqrt{2}\,(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,(Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,(Bb-Ac)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Bb-Ac)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2*B*sqrt(x)/c - 1/4*(2*sqrt(2)*(B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1
/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(
b)*sqrt(c))) + 2*sqrt(2)*(B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c
^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sq
rt(c))) + sqrt(2)*(B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)
*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b - A*c)*log(-sqrt(2)*b^(1/4)*
c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(166) = 332.
time = 2.52, size = 645, normalized size = 2.74

$$\frac{4\,c\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^3c^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^2c^2\sqrt{-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^3c^5}}+(B^2b^2-2\,ABbc+A^2c^2)x}\,b^2c^4\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^3c^5}\right)^{\frac{3}{4}}+(Bb^3c^4-Ab^2c^5)\sqrt{x}\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^3c^5}\right)^{\frac{3}{4}}}{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}\right)+c\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^3c^5}\right)^{\frac{1}{4}}\log\left(bc\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^3c^5}\right)^{\frac{1}{4}}-(Bb-Ac)\sqrt{x}\right)-c\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^3c^5}\right)^{\frac{1}{4}}\log\left(-bc\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^3c^5}\right)^{\frac{1}{4}}-(Bb-Ac)\sqrt{x}\right)+4\,B\sqrt{x}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A
^4*c^4)/(b^3*c^5))^(1/4)*arctan((sqrt(b^2*c^2*sqrt(-(B^4*b^4 - 4*A*B^3*b^3*
c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5)) + (B^2*b^2 - 2*
A*B*b*c + A^2*c^2)*x)*b^2*c^4*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^
2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(3/4) + (B*b^3*c^4 - A*b^2*c^5)*sqr
t(x)*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c
^4)/(b^3*c^5))^(3/4))/(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*
B*b*c^3 + A^4*c^4)) + c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*
A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(1/4)*log(b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c
+ 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(1/4) - (B*b - A*
c)*sqrt(x)) - c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*
c^3 + A^4*c^4)/(b^3*c^5))^(1/4)*log(-b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2
*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(1/4) - (B*b - A*c)*sqrt
(x)) + 4*B*sqrt(x))/c
```

Sympy [A]
time = 16.40, size = 238, normalized size = 1.01

$$\begin{cases} \tilde{\infty} \left(- \frac{2A}{3x^{\frac{3}{2}}} + 2B\sqrt{x}\right) & \text{for}\: b = 0 \wedge c = 0 \\ \frac{- \frac{2A}{3x^{\frac{3}{2}}} + 2B\sqrt{x}}{c} & \text{for}\: b = 0 \\ \frac{2A\sqrt{x} + \frac{2Bx^{\frac{5}{2}}}{5}}{b} & \text{for}\: c = 0 \\ - \frac{A\sqrt[4]{- \frac{b}{c}} \log\left(\sqrt{x} - \sqrt[4]{- \frac{b}{c}}\right)}{2b} + \frac{A\sqrt[4]{- \frac{b}{c}} \log\left(\sqrt{x} + \sqrt[4]{- \frac{b}{c}}\right)}{2b} + \frac{A\sqrt[4]{- \frac{b}{c}} \operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{b}{c}}}\right)}{b} + \frac{2B\sqrt{x}}{c} + \frac{B\sqrt[4]{- \frac{b}{c}} \log\left(\sqrt{x} - \sqrt[4]{- \frac{b}{c}}\right)}{2c} - \frac{B\sqrt[4]{- \frac{b}{c}} \log\left(\sqrt{x} + \sqrt[4]{- \frac{b}{c}}\right)}{2c} - \frac{B\sqrt[4]{- \frac{b}{c}} \operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{b}{c}}}\right)}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*sqrt(x)), Eq(b, 0) & Eq(c, 0)), ((-
2*A/(3*x**(3/2)) + 2*B*sqrt(x))/c, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(5/2)/
5)/b, Eq(c, 0)), (-A*(-b/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b) + A*(
-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*b) + A*(-b/c)**(1/4)*atan(sqrt
(x)/(-b/c)**(1/4))/b + 2*B*sqrt(x)/c + B*(-b/c)**(1/4)*log(sqrt(x) - (-b/c)
**(1/4))/(2*c) - B*(-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*c) - B*(-b
/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/c, True))
```

Giac [A]
time = 0.43, size = 251, normalized size = 1.07

$$\frac{2\,B\sqrt{x}}{c} - \frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb - \left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^{2}} - \frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb - \left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^{2}} - \frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb - \left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{4\,bc^{2}} + \frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb - \left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{4\,bc^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/c - 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(
1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^2) - 1/2*sq
rt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*
(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^2) - 1/4*sqrt(2)*((b*c^3)^(1/4)*
B*b - (b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(
b*c^2) + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*s
qrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^2)
```

Mupad [B]
time = 0.29, size = 739, normalized size = 3.14

$$\frac{2\,B\,\sqrt{x}}{c}-\frac{\mathrm{atan}\left(\frac{\frac{(A\,c-B\,b)\left(\sqrt{x}\,\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)-\frac{\left(32\,B\,b^2\,c^2-32\,A\,b\,c^3\right)(A\,c-B\,b)}{2\,(-b)^{3/4}\,c^{5/4}}\right)1\mathrm{i}}{2\,(-b)^{3/4}\,c^{5/4}}+\frac{(A\,c-B\,b)\left(\sqrt{x}\,\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)+\frac{\left(32\,B\,b^2\,c^2-32\,A\,b\,c^3\right)(A\,c-B\,b)}{2\,(-b)^{3/4}\,c^{5/4}}\right)1\mathrm{i}}{2\,(-b)^{3/4}\,c^{5/4}}}{\frac{(A\,c-B\,b)\left(\sqrt{x}\,\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)-\frac{\left(32\,B\,b^2\,c^2-32\,A\,b\,c^3\right)(A\,c-B\,b)}{2\,(-b)^{3/4}\,c^{5/4}}\right)}{2\,(-b)^{3/4}\,c^{5/4}}-\frac{(A\,c-B\,b)\left(\sqrt{x}\,\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)+\frac{\left(32\,B\,b^2\,c^2-32\,A\,b\,c^3\right)(A\,c-B\,b)}{2\,(-b)^{3/4}\,c^{5/4}}\right)}{2\,(-b)^{3/4}\,c^{5/4}}}\right)(A\,c-B\,b)\,1\mathrm{i}}{(-b)^{3/4}\,c^{5/4}}-\frac{\mathrm{atan}\left(\frac{\frac{(A\,c-B\,b)\left(\sqrt{x}\,\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)-\frac{\left(32\,B\,b^2\,c^2-32\,A\,b\,c^3\right)(A\,c-B\,b)\,1\mathrm{i}}{2\,(-b)^{3/4}\,c^{5/4}}\right)}{2\,(-b)^{3/4}\,c^{5/4}}+\frac{(A\,c-B\,b)\left(\sqrt{x}\,\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)+\frac{\left(32\,B\,b^2\,c^2-32\,A\,b\,c^3\right)(A\,c-B\,b)\,1\mathrm{i}}{2\,(-b)^{3/4}\,c^{5/4}}\right)}{2\,(-b)^{3/4}\,c^{5/4}}}{\frac{(A\,c-B\,b)\left(\sqrt{x}\,\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)-\frac{\left(32\,B\,b^2\,c^2-32\,A\,b\,c^3\right)(A\,c-B\,b)\,1\mathrm{i}}{2\,(-b)^{3/4}\,c^{5/4}}\right)1\mathrm{i}}{2\,(-b)^{3/4}\,c^{5/4}}-\frac{(A\,c-B\,b)\left(\sqrt{x}\,\left(16\,A^2\,c^3-32\,A\,B\,b\,c^2+16\,B^2\,b^2\,c\right)+\frac{\left(32\,B\,b^2\,c^2-32\,A\,b\,c^3\right)(A\,c-B\,b)\,1\mathrm{i}}{2\,(-b)^{3/4}\,c^{5/4}}\right)1\mathrm{i}}{2\,(-b)^{3/4}\,c^{5/4}}}\right)(A\,c-B\,b)}{(-b)^{3/4}\,c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] (2*B*x^(1/2))/c - (atan((((A*c - B*b)*(x^(1/2)*(16*A^2*c^3 + 16*B^2*b^2*c -
 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b))/(2*(-b)^(3/4)*c^
(5/4)))*1i)/(2*(-b)^(3/4)*c^(5/4)) + ((A*c - B*b)*(x^(1/2)*(16*A^2*c^3 + 16
*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b))/(2*(
-b)^(3/4)*c^(5/4)))*1i)/(2*(-b)^(3/4)*c^(5/4)))/(((A*c - B*b)*(x^(1/2)*(16*
A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c
- B*b))/(2*(-b)^(3/4)*c^(5/4))))/(2*(-b)^(3/4)*c^(5/4)) - ((A*c - B*b)*(x^(
1/2)*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B*b^2*c^2 - 32*A*b*c
^3)*(A*c - B*b))/(2*(-b)^(3/4)*c^(5/4))))/(2*(-b)^(3/4)*c^(5/4))))*(A*c - B
*b)*1i)/((-b)^(3/4)*c^(5/4)) - (atan((((A*c - B*b)*(x^(1/2)*(16*A^2*c^3 + 1
6*B^2*b^2*c - 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b)*1i)/
(2*(-b)^(3/4)*c^(5/4))))/(2*(-b)^(3/4)*c^(5/4)) + ((A*c - B*b)*(x^(1/2)*(16
*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c
 - B*b)*1i)/(2*(-b)^(3/4)*c^(5/4))))/(2*(-b)^(3/4)*c^(5/4)))/(((A*c - B*b)*
(x^(1/2)*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A
*b*c^3)*(A*c - B*b)*1i)/(2*(-b)^(3/4)*c^(5/4)))*1i)/(2*(-b)^(3/4)*c^(5/4))
- ((A*c - B*b)*(x^(1/2)*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B
*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b)*1i)/(2*(-b)^(3/4)*c^(5/4)))*1i)/(2*(-b)^
(3/4)*c^(5/4))))*(A*c - B*b))/((-b)^(3/4)*c^(5/4))
```

3.189 $\int \frac{\sqrt{x}\,(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=235

$$-\frac{2A}{b\sqrt{x}}-\frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{(bB-Ac)\log\left(\sqrt{b}\right.}{2}$$

```
[Out] -1/2*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(3/4)*2
^(1/2)+1/2*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(
3/4)*2^(1/2)+1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^
(1/2))/b^(5/4)/c^(3/4)*2^(1/2)-1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(3/4)*2^(1/2)-2*A/b/x^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1598, 464, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{(bB-Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{(bB-Ac)\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}c^{3/4}}-\frac{(bB-Ac)\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}c^{3/4}}-\frac{2A}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*A)/(b*Sqrt[x]) - ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(Sqrt[2]*b^(5/4)*c^(3/4)) + ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*
Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(5/4)*c^(3/4)) + ((b*B - A*c)*Log[Sqrt[b] - S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4)*c^(3/4)) -
((b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2
*Sqrt[2]*b^(5/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}\,(A+Bx^2)}{bx^2+cx^4}\,dx = \int \frac{A+Bx^2}{x^{3/2}\,(b+cx^2)}\,dx$$

$$= -\frac{2A}{b\sqrt{x}} - \frac{\left(2\left(-\frac{bB}{2}+\frac{Ac}{2}\right)\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b}$$

$$= -\frac{2A}{b\sqrt{x}} - \frac{\left(4\left(-\frac{bB}{2}+\frac{Ac}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b}$$

$$= -\frac{2A}{b\sqrt{x}} - \frac{(bB-Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b\sqrt{c}} + \frac{(bB-Ac)\text{Subst}\left(\int \frac{\sqrt{b}+ \cdots}{b+c \cdots}\right.}{b\sqrt{c}}$$

$$= -\frac{2A}{b\sqrt{x}} + \frac{(bB-Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2bc} + \frac{(bB-Ac)\text{Subst}\left(\cdots \right.}{\cdots}$$

$$= -\frac{2A}{b\sqrt{x}} + \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}c^{3/4}} - \frac{(bB-Ac)\log\left(\sqrt{b} - \cdots\right.}{2\sqrt{\cdots}}$$

$$= -\frac{2A}{b\sqrt{x}} - \frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}c^{3/4}} + \frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{\cdots}}{\sqrt[4]{b}}\right.}{\sqrt{2}\,b^{5/4}c^{3/4}}$$

Mathematica [A]
time = 0.19, size = 135, normalized size = 0.57

$$-\frac{2A}{b\sqrt{x}} - \frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)}{\sqrt{2}\,b^{5/4}c^{3/4}} - \frac{(bB-Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,b^{5/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*A)/(b*Sqrt[x]) - ((b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(5/4)*c^(3/4)) - ((b*B - A*c)*ArcTanh[(S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(5/4)*c^
(3/4))
```

Maple [A]
time = 0.41, size = 127, normalized size = 0.54

method	result
derivativedivides	$-\frac{2A}{b\sqrt{x}}-\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4bc\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$-\frac{2A}{b\sqrt{x}}-\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4bc\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
risch	$-\frac{2A}{b\sqrt{x}}-\frac{\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2b\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{4b\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,A\arctan\left(\right.}{2b\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*A/b/x^(1/2)-1/4*(A*c-B*b)/b/c/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(
1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
)
```

Maxima [A]
time = 0.54, size = 194, normalized size = 0.83

$$\frac{(Bb-Ac)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{4b}-\frac{2A}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b - A*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*
sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/
sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2
)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2
)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3
/4)))/b - 2*A/(b*sqrt(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 843 vs. $2(166) = 332$.

time = 2.97, size = 843, normalized size = 3.59

$$\frac{4bx\left(-\frac{B^4b^4-4AB^3b^3c+6A^2B^2b^2c^2-4A^3Bbc^3+A^4c^4}{b^5c^3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(B^6b^6-6AB^5b^5c+15A^2B^4b^4c^2-20A^3B^3b^3c^3+15A^4B^2b^2c^4-6A^5Bbc^5+A^6c^6\right)x-\left(B^4b^7c-4AB^3b^6c^2+6A^2B^2b^5c^3-4A^3Bb^4c^4+A^4b^3c^5\right)\sqrt{-\frac{B^4b^4-4AB^3b^3c+6A^2B^2b^2c^2-4A^3Bbc^3+A^4c^4}{b^5c^3}}}\,bc\left(-\frac{B^4b^4-4AB^3b^3c+6A^2B^2b^2c^2-4A^3Bbc^3+A^4c^4}{b^5c^3}\right)^{\frac{1}{4}}+\left(B^3b^4c-3AB^2b^3c^2+3A^2Bb^2c^3-A^3bc^4\right)\sqrt{x}\left(-\frac{B^4b^4-4AB^3b^3c+6A^2B^2b^2c^2-4A^3Bbc^3+A^4c^4}{b^5c^3}\right)^{\frac{1}{4}}}{B^4b^4-4AB^3b^3c+6A^2B^2b^2c^2-4A^3Bbc^3+A^4c^4}\right)-bx\left(-\frac{B^4b^4-4AB^3b^3c+6A^2B^2b^2c^2-4A^3Bbc^3+A^4c^4}{b^5c^3}\right)^{\frac{1}{4}}\log\left(b^4c^2\left(-\frac{B^4b^4-4AB^3b^3c+6A^2B^2b^2c^2-4A^3Bbc^3+A^4c^4}{b^5c^3}\right)^{\frac{3}{4}}-\left(B^3b^3-3AB^2b^2c+3A^2Bbc^2-A^3c^3\right)\sqrt{x}\right)+bx\left(-\frac{B^4b^4-4AB^3b^3c+6A^2B^2b^2c^2-4A^3Bbc^3+A^4c^4}{b^5c^3}\right)^{\frac{1}{4}}\log\left(-b^4c^2\left(-\frac{B^4b^4-4AB^3b^3c+6A^2B^2b^2c^2-4A^3Bbc^3+A^4c^4}{b^5c^3}\right)^{\frac{3}{4}}-\left(B^3b^3-3AB^2b^2c+3A^2Bbc^2-A^3c^3\right)\sqrt{x}\right)-4A\sqrt{x}}{2bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*x*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 +
 A^4*c^4)/(b^5*c^3))^(1/4)*arctan((sqrt((B^6*b^6 - 6*A*B^5*b^5*c + 15*A^2*B
^4*b^4*c^2 - 20*A^3*B^3*b^3*c^3 + 15*A^4*B^2*b^2*c^4 - 6*A^5*B*b*c^5 + A^6*
c^6)*x - (B^4*b^7*c - 4*A*B^3*b^6*c^2 + 6*A^2*B^2*b^5*c^3 - 4*A^3*B*b^4*c^4
 + A^4*b^3*c^5)*sqrt(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*
B*b*c^3 + A^4*c^4)/(b^5*c^3)))*b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b
^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(1/4) + (B^3*b^4*c - 3*A*B^2*b
^3*c^2 + 3*A^2*B*b^2*c^3 - A^3*b*c^4)*sqrt(x)*(-(B^4*b^4 - 4*A*B^3*b^3*c +
6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(1/4))/(B^4*b^4 - 4
*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)) - b*x*(-(B^4*b
^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3)
)^(1/4)*log(b^4*c^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*
B*b*c^3 + A^4*c^4)/(b^5*c^3))^(3/4) - (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*
c^2 - A^3*c^3)*sqrt(x)) + b*x*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^
2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(1/4)*log(-b^4*c^2*(-(B^4*b^4 - 4*A
*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(3/4)
- (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)*sqrt(x)) - 4*A*sqrt(x
))/(b*x)
```

Sympy [A]

time = 8.92, size = 309, normalized size = 1.31

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{5x^{\frac{5}{2}}}-\frac{2B}{\sqrt{x}}\right) & \text{for}\: b=0\wedge c=0 \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}}-\frac{2B}{\sqrt{x}}}{c} & \text{for}\: b=0 \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{3}{2}}}{3}}{b} & \text{for}\: c=0 \\ -\frac{2A\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b\sqrt[4]{-\frac{b}{c}}}-\frac{2A}{b\sqrt{x}}+\frac{Ac\left(-\frac{b}{c}\right)^{\frac{3}{4}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^2}-\frac{Ac\left(-\frac{b}{c}\right)^{\frac{3}{4}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^2}-\frac{Ac\left(-\frac{b}{c}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b^2}+\frac{2B\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{c\sqrt[4]{-\frac{b}{c}}}-\frac{B\left(-\frac{b}{c}\right)^{\frac{3}{4}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b}+\frac{B\left(-\frac{b}{c}\right)^{\frac{3}{4}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b}+\frac{B\left(-\frac{b}{c}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(5*x**(5/2)) - 2*B/sqrt(x)), Eq(b, 0) & Eq(c, 0)), ((-
2*A/(5*x**(5/2)) - 2*B/sqrt(x))/c, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*x**(3/2)
/3)/b, Eq(c, 0)), (-2*A*atan(sqrt(x)/(-b/c)**(1/4))/(b*(-b/c)**(1/4)) - 2*A
/(b*sqrt(x)) + A*c*(-b/c)**(3/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b**2) - A*
c*(-b/c)**(3/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*b**2) - A*c*(-b/c)**(3/4)*a
```

```
tan(sqrt(x)/(-b/c)**(1/4))/b**2 + 2*B*atan(sqrt(x)/(-b/c)**(1/4))/(c*(-b/c)
**(1/4)) - B*(-b/c)**(3/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b) + B*(-b/c)**(
3/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*b) + B*(-b/c)**(3/4)*atan(sqrt(x)/(-b/
c)**(1/4))/b, True))
```

Giac [A]
time = 0.44, size = 251, normalized size = 1.07

$$-\frac{2\,A}{b\sqrt{x}}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{2}c^{3}}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{2}c^{3}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^{2}c^{3}}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{3}{4}}Bb-\left(bc^{3}\right)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -2*A/(b*sqrt(x)) + 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arct
an(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1
/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) - 1/4*sqrt(2)*((b*c^3)
^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(
b/c))/(b^2*c^3) + 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-
sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^3)
```

Mupad [B]
time = 0.22, size = 71, normalized size = 0.30

$$\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)\,\left(A\,c-B\,b\right)}{{\left(-b\right)}^{5/4}\,c^{3/4}}-\frac{2\,A}{b\,\sqrt{x}}-\frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)\,\left(A\,c-B\,b\right)}{{\left(-b\right)}^{5/4}\,c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/((-b)^(5/4)*c^(3/4)) - (2*
A)/(b*x^(1/2)) - (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/((-b)^(5
/4)*c^(3/4))
```

3.190 $\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)}\,dx$

Optimal. Leaf size=237

$$-\frac{2A}{3bx^{3/2}}-\frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}\sqrt[4]{c}}-\frac{(bB-Ac)\log\left(\sqrt{b}\right.}{2}$$

```
[Out] -2/3*A/b/x^(3/2)-1/2*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b
^(7/4)/c^(1/4)*2^(1/2)+1/2*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1
/4))/b^(7/4)/c^(1/4)*2^(1/2)-1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^
(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(1/4)*2^(1/2)+1/4*(-A*c+B*b)*ln(b^(1/2)+x*
c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(1/4)*2^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1598, 464, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{(bB-Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{7/4}\sqrt[4]{c}}-\frac{(bB-Ac)\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{(bB-Ac)\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}\sqrt[4]{c}}-\frac{2A}{3bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(3*b*x^(3/2)) - ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)])/(Sqrt[2]*b^(7/4)*c^(1/4)) + ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(7/4)*c^(1/4)) - ((b*B - A*c)*Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4)*c^(1/4))
+ ((b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/
(2*Sqrt[2]*b^(7/4)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)}\,dx = \int \frac{A+Bx^2}{x^{5/2}\,(b+cx^2)}\,dx$$

$$= -\frac{2A}{3bx^{3/2}} - \frac{\left(2\left(-\frac{3bB}{2}+\frac{3Ac}{2}\right)\right)\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{3b}$$

$$= -\frac{2A}{3bx^{3/2}} - \frac{\left(4\left(-\frac{3bB}{2}+\frac{3Ac}{2}\right)\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{3b}$$

$$= -\frac{2A}{3bx^{3/2}} + \frac{(bB-Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}} + \frac{(bB-Ac)\text{Subst}\left(\int \frac{\sqrt{b}}{b}\right.}{b^{3/2}}$$

$$= -\frac{2A}{3bx^{3/2}} + \frac{(bB-Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}\sqrt{c}} + \frac{(bB-Ac)\text{Subst}}{}$$

$$= -\frac{2A}{3bx^{3/2}} - \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}\sqrt[4]{c}} + \frac{(bB-Ac)\log\left(\sqrt{b}\right.}{2}$$

$$= -\frac{2A}{3bx^{3/2}} - \frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}\sqrt[4]{c}} + \frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{\sqrt{2}\,b^{7/4}\sqrt[4]{c}}$$

Mathematica [A]
time = 0.20, size = 136, normalized size = 0.57

$$-\frac{2A}{3bx^{3/2}} - \frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}\,b^{7/4}\sqrt[4]{c}} + \frac{(bB-Ac)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,b^{7/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(3*b*x^(3/2)) - ((b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(7/4)*c^(1/4)) + ((b*B - A*c)*ArcTanh[
(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(7/4)*
c^(1/4))
```

Maple [A]
time = 0.40, size = 124, normalized size = 0.52

method	result
derivativedivides	$-\frac{2A}{3b\,x^{\frac{3}{2}}}+\frac{(-Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{(}\right.\right.}{4b^{2}}$
default	$-\frac{2A}{3b\,x^{\frac{3}{2}}}+\frac{(-Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{(}\right.\right.}{4b^{2}}$
risch	$-\frac{2A}{3b\,x^{\frac{3}{2}}}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)c}{2b^{2}}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)c}{2b^{2}}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*A/b/x^(3/2)+1/4*(-A*c+B*b)/b^2*(b/c)^(1/4)*2^(1/2)*(ln((x+(b/c)^(1/4)*
x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)
-1))
```

Maxima [A]
time = 0.50, size = 218, normalized size = 0.92

$$\frac{\frac{2\sqrt{2}\,(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,(Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,(Bb-Ac)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Bb-Ac)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{4\,b}-\frac{2\,A}{3\,bx^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*
sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2
*sqrt(2)*(B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(
c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2
)*(B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b
^(3/4)*c^(1/4)) - sqrt(2)*(B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
+ sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b - 2/3*A/(b*x^(3/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(166) = 332$.

time = 2.94, size = 653, normalized size = 2.76

$$-\frac{12\,bx^{2}\left(-\frac{B^{4}b^{4}-4\,AB^{3}b^{3}c+6\,A^{2}B^{2}b^{2}c^{2}-4\,A^{3}Bbc^{3}+A^{4}c^{4}}{b^{7}c}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^{4}\sqrt{-\frac{B^{4}b^{4}-4\,AB^{3}b^{3}c+6\,A^{2}B^{2}b^{2}c^{2}-4\,A^{3}Bbc^{3}+A^{4}c^{4}}{b^{7}c}}+\left(B^{2}b^{2}-2\,ABbc+A^{2}c^{2}\right)x}\,b^{5}c\left(-\frac{B^{4}b^{4}-4\,AB^{3}b^{3}c+6\,A^{2}B^{2}b^{2}c^{2}-4\,A^{3}Bbc^{3}+A^{4}c^{4}}{b^{7}c}\right)^{\frac{3}{4}}+\left(Bb^{6}c-Ab^{5}c^{2}\right)\sqrt{x}\left(-\frac{B^{4}b^{4}-4\,AB^{3}b^{3}c+6\,A^{2}B^{2}b^{2}c^{2}-4\,A^{3}Bbc^{3}+A^{4}c^{4}}{b^{7}c}\right)^{\frac{3}{4}}}{B^{4}b^{4}-4\,AB^{3}b^{3}c+6\,A^{2}B^{2}b^{2}c^{2}-4\,A^{3}Bbc^{3}+A^{4}c^{4}}\right)+3\,bx^{2}\left(-\frac{B^{4}b^{4}-4\,AB^{3}b^{3}c+6\,A^{2}B^{2}b^{2}c^{2}-4\,A^{3}Bbc^{3}+A^{4}c^{4}}{b^{7}c}\right)^{\frac{1}{4}}\log\left(b^{2}\left(-\frac{B^{4}b^{4}-4\,AB^{3}b^{3}c+6\,A^{2}B^{2}b^{2}c^{2}-4\,A^{3}Bbc^{3}+A^{4}c^{4}}{b^{7}c}\right)^{\frac{1}{4}}-\left(Bb-Ac\right)\sqrt{x}\right)-3\,bx^{2}\left(-\frac{B^{4}b^{4}-4\,AB^{3}b^{3}c+6\,A^{2}B^{2}b^{2}c^{2}-4\,A^{3}Bbc^{3}+A^{4}c^{4}}{b^{7}c}\right)^{\frac{1}{4}}\log\left(-b^{2}\left(-\frac{B^{4}b^{4}-4\,AB^{3}b^{3}c+6\,A^{2}B^{2}b^{2}c^{2}-4\,A^{3}Bbc^{3}+A^{4}c^{4}}{b^{7}c}\right)^{\frac{1}{4}}-\left(Bb-Ac\right)\sqrt{x}\right)+4\,A\sqrt{x}}{6\,bx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(12*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c
^3 + A^4*c^4)/(b^7*c))^(1/4)*arctan((sqrt(b^4*sqrt(-(B^4*b^4 - 4*A*B^3*b^3*
c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c)) + (B^2*b^2 - 2*A*
B*b*c + A^2*c^2)*x)*b^5*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 -
4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(3/4) + (B*b^6*c - A*b^5*c^2)*sqrt(x)*(-(
B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7
*c))^(3/4))/(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 +
A^4*c^4)) + 3*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*
B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4)*log(b^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^
2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4) - (B*b - A*c)*sqrt(
x)) - 3*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^
3 + A^4*c^4)/(b^7*c))^(1/4)*log(-b^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2
*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4) - (B*b - A*c)*sqrt(x)) +
 4*A*sqrt(x))/(b*x^2)
```

Sympy [A]
time = 14.20, size = 257, normalized size = 1.08

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) & \text{for}\: b=0\wedge c=0 \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{c} & \text{for}\: b=0 \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{b} & \text{for}\: c=0 \\ -\frac{2A}{3bx^{\frac{3}{2}}}+\frac{Ac\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^{2}}-\frac{Ac\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^{2}}-\frac{Ac\sqrt[4]{-\frac{b}{c}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b^{2}}-\frac{B\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b}+\frac{B\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b}+\frac{B\sqrt[4]{-\frac{b}{c}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2))), Eq(b, 0) & Eq(c, 0))
, ((-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2)))/c, Eq(b, 0)), ((-2*A/(3*x**(3/2))
 + 2*B*sqrt(x))/b, Eq(c, 0)), (-2*A/(3*b*x**(3/2)) + A*c*(-b/c)**(1/4)*log(
sqrt(x) - (-b/c)**(1/4))/(2*b**2) - A*c*(-b/c)**(1/4)*log(sqrt(x) + (-b/c)*
*(1/4))/(2*b**2) - A*c*(-b/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/b**2 - B*(
-b/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b) + B*(-b/c)**(1/4)*log(sqrt(
x) + (-b/c)**(1/4))/(2*b) + B*(-b/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/b,
True))
```

Giac [A]
time = 0.43, size = 251, normalized size = 1.06

$$\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^{2}c}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^{2}c}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^{2}c}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^{2}c}-\frac{2A}{3bx^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c) + 1/2*sqrt(2)*((b*c^3)^(
1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*
sqrt(x))/(b/c)^(1/4))/(b^2*c) + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1
/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c) - 1/4*sqr
t(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/
4) + x + sqrt(b/c))/(b^2*c) - 2/3*A/(b*x^(3/2))
```

Mupad [B]
time = 0.30, size = 811, normalized size = 3.42

$$-\frac{2A}{3bx^{3/2}}-\frac{\mathrm{atan}\left(\frac{\frac{(Ac-Bb)\left(\sqrt{x}\,\left(16A^2b^3c^5-32ABb^4c^4+16B^2b^5c^3\right)-\frac{(Ac-Bb)\left(32Ab^5c^4-32Bb^6c^3\right)}{2(-b)^{7/4}c^{1/4}}\right)1i}{2(-b)^{7/4}c^{1/4}}+\frac{(Ac-Bb)\left(\sqrt{x}\,\left(16A^2b^3c^5-32ABb^4c^4+16B^2b^5c^3\right)+\frac{(Ac-Bb)\left(32Ab^5c^4-32Bb^6c^3\right)}{2(-b)^{7/4}c^{1/4}}\right)1i}{2(-b)^{7/4}c^{1/4}}}{\frac{(Ac-Bb)\left(\sqrt{x}\,\left(16A^2b^3c^5-32ABb^4c^4+16B^2b^5c^3\right)-\frac{(Ac-Bb)\left(32Ab^5c^4-32Bb^6c^3\right)}{2(-b)^{7/4}c^{1/4}}\right)}{2(-b)^{7/4}c^{1/4}}-\frac{(Ac-Bb)\left(\sqrt{x}\,\left(16A^2b^3c^5-32ABb^4c^4+16B^2b^5c^3\right)+\frac{(Ac-Bb)\left(32Ab^5c^4-32Bb^6c^3\right)}{2(-b)^{7/4}c^{1/4}}\right)}{2(-b)^{7/4}c^{1/4}}}\right)(Ac-Bb)\,1i}{(-b)^{7/4}c^{1/4}}-\frac{\mathrm{atan}\left(\frac{\frac{(Ac-Bb)\left(\sqrt{x}\,\left(16A^2b^3c^5-32ABb^4c^4+16B^2b^5c^3\right)-\frac{(Ac-Bb)\left(32Ab^5c^4-32Bb^6c^3\right)1i}{2(-b)^{7/4}c^{1/4}}\right)}{2(-b)^{7/4}c^{1/4}}+\frac{(Ac-Bb)\left(\sqrt{x}\,\left(16A^2b^3c^5-32ABb^4c^4+16B^2b^5c^3\right)+\frac{(Ac-Bb)\left(32Ab^5c^4-32Bb^6c^3\right)1i}{2(-b)^{7/4}c^{1/4}}\right)}{2(-b)^{7/4}c^{1/4}}}{\frac{(Ac-Bb)\left(\sqrt{x}\,\left(16A^2b^3c^5-32ABb^4c^4+16B^2b^5c^3\right)-\frac{(Ac-Bb)\left(32Ab^5c^4-32Bb^6c^3\right)1i}{2(-b)^{7/4}c^{1/4}}\right)1i}{2(-b)^{7/4}c^{1/4}}-\frac{(Ac-Bb)\left(\sqrt{x}\,\left(16A^2b^3c^5-32ABb^4c^4+16B^2b^5c^3\right)+\frac{(Ac-Bb)\left(32Ab^5c^4-32Bb^6c^3\right)1i}{2(-b)^{7/4}c^{1/4}}\right)1i}{2(-b)^{7/4}c^{1/4}}}\right)(Ac-Bb)}{(-b)^{7/4}c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] - (2*A)/(3*b*x^(3/2)) - (atan((((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B
^2*b^5*c^3 - 32*A*B*b^4*c^4) - ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3))/
(2*(-b)^(7/4)*c^(1/4)))*1i)/(2*(-b)^(7/4)*c^(1/4)) + ((A*c - B*b)*(x^(1/2)*
(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4) + ((A*c - B*b)*(32*A*b^5
*c^4 - 32*B*b^6*c^3))/(2*(-b)^(7/4)*c^(1/4)))*1i)/(2*(-b)^(7/4)*c^(1/4)))/(
((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4) -
((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3))/(2*(-b)^(7/4)*c^(1/4))))/(2*(-b
)^(7/4)*c^(1/4)) - ((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 -
 32*A*B*b^4*c^4) + ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3))/(2*(-b)^(7/4
)*c^(1/4))))/(2*(-b)^(7/4)*c^(1/4))))*(A*c - B*b)*1i)/((-b)^(7/4)*c^(1/4))
- (atan((((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^
4*c^4) - ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)*1i)/(2*(-b)^(7/4)*c^(1/
4))))/(2*(-b)^(7/4)*c^(1/4)) + ((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B
^2*b^5*c^3 - 32*A*B*b^4*c^4) + ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)*1
i)/(2*(-b)^(7/4)*c^(1/4))))/(2*(-b)^(7/4)*c^(1/4)))/(((A*c - B*b)*(x^(1/2)*
(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4) - ((A*c - B*b)*(32*A*b^5
*c^4 - 32*B*b^6*c^3)*1i)/(2*(-b)^(7/4)*c^(1/4)))*1i)/(2*(-b)^(7/4)*c^(1/4))
 - ((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4)
 + ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)*1i)/(2*(-b)^(7/4)*c^(1/4)))*1
i)/(2*(-b)^(7/4)*c^(1/4))))*(A*c - B*b))/((-b)^(7/4)*c^(1/4))
```

3.191 $\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=255

$$-\frac{2A}{5bx^{5/2}}-\frac{2(bB-Ac)}{b^2\sqrt{x}}+\frac{\sqrt[4]{c}\,(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}}-\frac{\sqrt[4]{c}\,(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}}$$

```
[Out] -2/5*A/b/x^(5/2)+1/2*c^(1/4)*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^
(1/4))/b^(9/4)*2^(1/2)-1/2*c^(1/4)*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1
/2)/b^(1/4))/b^(9/4)*2^(1/2)-1/4*c^(1/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^
(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)*2^(1/2)+1/4*c^(1/4)*(-A*c+B*b)*ln(b^
(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)*2^(1/2)-2*(-A*c+B*
b)/b^2/x^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 464, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt[4]{c}\,(bB-Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}}-\frac{\sqrt[4]{c}\,(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{9/4}}-\frac{\sqrt[4]{c}\,(bB-Ac)\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}}+\frac{\sqrt[4]{c}\,(bB-Ac)\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}}-\frac{2(bB-Ac)}{b^2\sqrt{x}}-\frac{2A}{5bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(5*b*x^(5/2)) - (2*(b*B - A*c))/(b^2*Sqrt[x]) + (c^(1/4)*(b*B - A*c)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) - (c^(1/4
)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/
4)) - (c^(1/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4)) + (c^(1/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)}\,dx = \int \frac{A+Bx^2}{x^{7/2}\left(b+cx^2\right)}\,dx$$
$$= -\frac{2A}{5bx^{5/2}} - \frac{\left(2\left(-\frac{5bB}{2}+\frac{5Ac}{2}\right)\right)\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{5b}$$
$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} - \frac{(c(bB-Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b^2}$$
$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} - \frac{(2c(bB-Ac))\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} + \frac{\left(\sqrt{c}\,(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2} - \frac{(\sqrt{}}{}$$
$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} - \frac{(bB-Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} - \frac{(b}{}$$
$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} - \frac{\sqrt[4]{c}\,(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}} + \frac{}{}$$
$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} + \frac{\sqrt[4]{c}\,(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}} - \frac{\sqrt[4]{c}\,(bB-A}{}$$

Mathematica [A]

time = 0.20, size = 152, normalized size = 0.60

$$-\frac{2(Ab+5bBx^2-5Acx^2)}{5b^2x^{5/2}}+\frac{\sqrt[4]{c}\,(bB-Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)}{\sqrt{2}\,b^{9/4}}+\frac{\sqrt[4]{c}\,(bB-Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*(A*b + 5*b*B*x^2 - 5*A*c*x^2))/(5*b^2*x^(5/2)) + (c^(1/4)*(b*B - A*c)*A
rcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^
(9/4)) + (c^(1/4)*(b*B - A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sq
rt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(9/4))
```

Maple [A]
time = 0.40, size = 140, normalized size = 0.55

method	result
derivativedivides	$\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{}{5}$
default	$\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{}{5}$
risch	$-\frac{2(-5Acx^2+5bBx^2+Ab)}{5b^2x^{\frac{5}{2}}}+\frac{\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)c}{4b^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)c}{2b^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(A*c-B*b)/b^2/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b
/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))-2/5*A/b/x^(5/
2)-2*(-A*c+B*b)/b^2/x^(1/2)
```

Maxima [A]
time = 0.53, size = 213, normalized size = 0.84

$$-\frac{(Bbc-Ac^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{4b^2}-\frac{2\left(5(Bb-Ac)x^2+Ab\right)}{5b^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(B*b*c - A*c^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
 + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt
(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(s
qrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + s
qrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)
*c^(3/4)))/b^2 - 2/5*(5*(B*b - A*c)*x^2 + A*b)/(b^2*x^(5/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 883 vs. $2(182) = 364$.
time = 2.50, size = 883, normalized size = 3.46

$$-\frac{20\,b^2x^3\left(-\frac{B^4b^4c-4\,AB^3b^3c^2+6\,A^2B^2b^2c^3-4\,A^3Bbc^4+A^4c^5}{b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{(B^6b^6c^2-6\,AB^5b^5c^3+15\,A^2B^4b^4c^4-20\,A^3B^3b^3c^5+15\,A^4B^2b^2c^6-6\,A^5Bbc^7+A^6c^8)x-(B^4b^9c-4\,AB^3b^8c^2+6\,A^2B^2b^7c^3-4\,A^3Bb^6c^4+A^4b^5c^5)\sqrt{-\frac{B^4b^4c-4\,AB^3b^3c^2+6\,A^2B^2b^2c^3-4\,A^3Bbc^4+A^4c^5}{b^9}}}\,b^2\left(-\frac{B^4b^4c-4\,AB^3b^3c^2+6\,A^2B^2b^2c^3-4\,A^3Bbc^4+A^4c^5}{b^9}\right)^{\frac{1}{4}}+(B^3b^5c-3\,AB^2b^4c^2+3\,A^2Bb^3c^3-A^3b^2c^4)\sqrt{x}\left(-\frac{B^4b^4c-4\,AB^3b^3c^2+6\,A^2B^2b^2c^3-4\,A^3Bbc^4+A^4c^5}{b^9}\right)^{\frac{1}{4}}}{B^4b^4c-4\,AB^3b^3c^2+6\,A^2B^2b^2c^3-4\,A^3Bbc^4+A^4c^5}\right)-5\,b^2x^3\left(-\frac{B^4b^4c-4\,AB^3b^3c^2+6\,A^2B^2b^2c^3-4\,A^3Bbc^4+A^4c^5}{b^9}\right)^{\frac{1}{4}}\log\left(b^7\left(-\frac{B^4b^4c-4\,AB^3b^3c^2+6\,A^2B^2b^2c^3-4\,A^3Bbc^4+A^4c^5}{b^9}\right)^{\frac{3}{4}}-(B^3b^3c-3\,AB^2b^2c^2+3\,A^2Bbc^3-A^3c^4)\sqrt{x}\right)+5\,b^2x^3\left(-\frac{B^4b^4c-4\,AB^3b^3c^2+6\,A^2B^2b^2c^3-4\,A^3Bbc^4+A^4c^5}{b^9}\right)^{\frac{1}{4}}\log\left(-b^7\left(-\frac{B^4b^4c-4\,AB^3b^3c^2+6\,A^2B^2b^2c^3-4\,A^3Bbc^4+A^4c^5}{b^9}\right)^{\frac{3}{4}}-(B^3b^3c-3\,AB^2b^2c^2+3\,A^2Bbc^3-A^3c^4)\sqrt{x}\right)+4\,(5\,(Bb-Ac)x^2+Ab)\sqrt{x}}{10\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*b^2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^
3*B*b*c^4 + A^4*c^5)/b^9)^(1/4)*arctan((sqrt((B^6*b^6*c^2 - 6*A*B^5*b^5*c^3
 + 15*A^2*B^4*b^4*c^4 - 20*A^3*B^3*b^3*c^5 + 15*A^4*B^2*b^2*c^6 - 6*A^5*B*b
*c^7 + A^6*c^8)*x - (B^4*b^9*c - 4*A*B^3*b^8*c^2 + 6*A^2*B^2*b^7*c^3 - 4*A^
3*B*b^6*c^4 + A^4*b^5*c^5)*sqrt(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b
^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9))*b^2*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2
+ 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4) + (B^3*b^5*c - 3*
A*B^2*b^4*c^2 + 3*A^2*B*b^3*c^3 - A^3*b^2*c^4)*sqrt(x)*(-(B^4*b^4*c - 4*A*B
^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4))/(B^4*
b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)) - 5
*b^2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4
 + A^4*c^5)/b^9)^(1/4)*log(b^7*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b
^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(3/4) - (B^3*b^3*c - 3*A*B^2*b^2*c^2
 + 3*A^2*B*b*c^3 - A^3*c^4)*sqrt(x)) + 5*b^2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3
*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4)*log(-b^7*(-(
B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/
b^9)^(3/4) - (B^3*b^3*c - 3*A*B^2*b^2*c^2 + 3*A^2*B*b*c^3 - A^3*c^4)*sqrt(x
)) + 4*(5*(B*b - A*c)*x^2 + A*b)*sqrt(x))/(b^2*x^3)
```

Sympy [A]
time = 71.03, size = 258, normalized size = 1.01

$$A\left(\begin{cases}\frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: b=0\wedge c=0\\-\frac{2}{9cx^{\frac{9}{2}}} & \text{for}\: b=0\\-\frac{2}{5bx^{\frac{5}{2}}} & \text{for}\: c=0\\-\frac{2}{5bx^{\frac{5}{2}}}+\frac{c\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^2\sqrt[4]{-\frac{b}{c}}}-\frac{c\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^2\sqrt[4]{-\frac{b}{c}}}+\frac{c\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b^2\sqrt[4]{-\frac{b}{c}}}+\frac{2c}{b^2\sqrt{x}} & \text{otherwise}\end{cases}\right)+B\left(\begin{cases}\frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: b=0\wedge c=0\\-\frac{2}{5cx^{\frac{5}{2}}} & \text{for}\: b=0\\-\frac{2}{b\sqrt{x}} & \text{for}\: c=0\\-\frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b\sqrt[4]{-\frac{b}{c}}}+\frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b\sqrt[4]{-\frac{b}{c}}}-\frac{\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b\sqrt[4]{-\frac{b}{c}}}-\frac{2}{b\sqrt{x}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2),x)
```

```
[Out] A*Piecewise((zoo/x**(9/2), Eq(b, 0) & Eq(c, 0)), (-2/(9*c*x**(9/2)), Eq(b,
0)), (-2/(5*b*x**(5/2)), Eq(c, 0)), (-2/(5*b*x**(5/2)) + c*log(sqrt(x) - (-
b/c)**(1/4))/(2*b**2*(-b/c)**(1/4)) - c*log(sqrt(x) + (-b/c)**(1/4))/(2*b**
2*(-b/c)**(1/4)) + c*atan(sqrt(x)/(-b/c)**(1/4))/(b**2*(-b/c)**(1/4)) + 2*c
/(b**2*sqrt(x)), True)) + B*Piecewise((zoo/x**(5/2), Eq(b, 0) & Eq(c, 0)),
(-2/(5*c*x**(5/2)), Eq(b, 0)), (-2/(b*sqrt(x)), Eq(c, 0)), (-log(sqrt(x) -
(-b/c)**(1/4))/(2*b*(-b/c)**(1/4)) + log(sqrt(x) + (-b/c)**(1/4))/(2*b*(-b/
c)**(1/4)) - atan(sqrt(x)/(-b/c)**(1/4))/(b*(-b/c)**(1/4)) - 2/(b*sqrt(x)),
 True))
```

Giac [A]
time = 0.51, size = 268, normalized size = 1.05

$$-\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3c^2}-\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3c^2}+\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3c^2}-\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3c^2}-\frac{2\,(5\,Bbx^2-5\,Acx^2+Ab)}{5\,b^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) - 1/2*sqrt(2)*((b*c^3
)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) -
 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^
3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) -
1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b
/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) - 2/5*(5*B*b*x^2 - 5*A*c*x^2 + A*b)/(b
^2*x^(5/2))
```

Mupad [B]
time = 0.23, size = 90, normalized size = 0.35

$$\frac{(-c)^{1/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(A\,c-B\,b)}{b^{9/4}}-\frac{\frac{2\,A}{5\,b}-\frac{2\,x^2\,(A\,c-B\,b)}{b^2}}{x^{5/2}}-\frac{(-c)^{1/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(A\,c-B\,b)}{b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(1/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(A*c - B*b))/b^(9/4) - ((2*A
)/(5*b) - (2*x^2*(A*c - B*b))/b^2)/x^(5/2) - ((-c)^(1/4)*atanh(((-c)^(1/4)*
x^(1/2))/b^(1/4))*(A*c - B*b))/b^(9/4)
```

3.192 $\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=257

$$-\frac{2A}{7bx^{7/2}}-\frac{2(bB-Ac)}{3b^2x^{3/2}}+\frac{c^{3/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}}-\frac{c^{3/4}(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}}$$

```
[Out] -2/7*A/b/x^(7/2)-2/3*(-A*c+B*b)/b^2/x^(3/2)+1/2*c^(3/4)*(-A*c+B*b)*arctan(1
-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)*2^(1/2)-1/2*c^(3/4)*(-A*c+B*b)*a
rctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)*2^(1/2)+1/4*c^(3/4)*(-A*c
+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)*2^(1/2
)-1/4*c^(3/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/
2))/b^(11/4)*2^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 464, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{c^{3/4}(bB-Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}}-\frac{c^{3/4}(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{11/4}}+\frac{c^{3/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}-\frac{c^{3/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}-\frac{2(bB-Ac)}{3b^2x^{3/2}}-\frac{2A}{7bx^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(7*b*x^(7/2)) - (2*(b*B - A*c))/(3*b^2*x^(3/2)) + (c^(3/4)*(b*B - A*
c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) - (c^(
3/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^
(11/4)) + (c^(3/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4)) - (c^(3/4)*(b*B - A*c)*Log[Sqrt[b] + S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)}\,dx = \int \frac{A+Bx^2}{x^{9/2}\left(b+cx^2\right)}\,dx$$
$$= -\frac{2A}{7bx^{7/2}} - \frac{\left(2\left(-\frac{7bB}{2}+\frac{7Ac}{2}\right)\right)\int \frac{1}{x^{5/2}(b+cx^2)}\,dx}{7b}$$
$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{(c(bB-Ac))\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{b^2}$$
$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{(2c(bB-Ac))\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{(c(bB-Ac))\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{5/2}} - \frac{(c(bB\cdots}{\cdots}$$
$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{\left(\sqrt{c}\,(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{5/2}} \cdots$$
$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} + \frac{c^{3/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}} - \frac{c\cdots}{\cdots}$$
$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} + \frac{c^{3/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}} - \frac{c^{3/4}(bB-A\cdots}{\cdots}$$

Mathematica [A]

time = 0.21, size = 154, normalized size = 0.60

$$-\frac{2(3Ab+7bBx^2-7Acx^2)}{21b^2x^{7/2}} + \frac{c^{3/4}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}\,b^{11/4}} - \frac{c^{3/4}(bB-Ac)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*(3*A*b + 7*b*B*x^2 - 7*A*c*x^2))/(21*b^2*x^(7/2)) + (c^(3/4)*(b*B - A*c
)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]
*b^(11/4)) - (c^(3/4)*(b*B - A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])
/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(11/4))
```

Maple [A]
time = 0.40, size = 141, normalized size = 0.55

method	result
derivativedivides	$\frac{c(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right.}{4b^3}$
default	$\frac{c(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right.}{4b^3}$
risch	$-\frac{2\left(-7Ac\,x^{2}+7bB\,x^{2}+3Ab\right)}{21b^{2}x^{\frac{7}{2}}}+\frac{c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2b^{3}}+\frac{c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{}}\right.}{4b^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c*(A*c-B*b)/b^3*(b/c)^(1/4)*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+
(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/
(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))-2/7*A/b/x^(
7/2)-2/3*(-A*c+B*b)/b^2/x^(3/2)
```

Maxima [A]
time = 0.49, size = 247, normalized size = 0.96

$$-\frac{\frac{2\sqrt{2}\,(Bbc-Ac^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,(Bbc-Ac^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,(Bbc-Ac^2)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Bbc-Ac^2)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{4\,b^2}-\frac{2\,(7\,(Bb-Ac)x^2+3\,Ab)}{21\,b^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(B*b*c - A*c^2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
 + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))
```

```
) + 2*sqrt(2)*(B*b*c - A*c^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
- 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c)))
 + sqrt(2)*(B*b*c - A*c^2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x
+ sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b*c - A*c^2)*log(-sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^2 - 2/21*(7*(B
*b - A*c)*x^2 + 3*A*b)/(b^2*x^(7/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(182) = 364$.
time = 2.66, size = 707, normalized size = 2.75

$$\frac{84\,b^{2}x^{4}\left(-\frac{B^{4}b^{4}c^{3}-4\,AB^{3}b^{3}c^{4}+6\,A^{2}B^{2}b^{2}c^{5}-4\,A^{3}Bbc^{6}+A^{4}c^{7}}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^{6}\sqrt{-\frac{B^{4}b^{4}c^{3}-4\,AB^{3}b^{3}c^{4}+6\,A^{2}B^{2}b^{2}c^{5}-4\,A^{3}Bbc^{6}+A^{4}c^{7}}{b^{11}}}+(B^{2}b^{2}c^{2}-2\,ABbc^{3}+A^{2}c^{4})x}\,b^{8}\left(-\frac{B^{4}b^{4}c^{3}-4\,AB^{3}b^{3}c^{4}+6\,A^{2}B^{2}b^{2}c^{5}-4\,A^{3}Bbc^{6}+A^{4}c^{7}}{b^{11}}\right)^{\frac{3}{4}}+(Bb^{9}c-Ab^{8}c^{2})\sqrt{x}\left(-\frac{B^{4}b^{4}c^{3}-4\,AB^{3}b^{3}c^{4}+6\,A^{2}B^{2}b^{2}c^{5}-4\,A^{3}Bbc^{6}+A^{4}c^{7}}{b^{11}}\right)^{\frac{3}{4}}}{B^{4}b^{4}c^{3}-4\,AB^{3}b^{3}c^{4}+6\,A^{2}B^{2}b^{2}c^{5}-4\,A^{3}Bbc^{6}+A^{4}c^{7}}\right)+21\,b^{2}x^{4}\left(-\frac{B^{4}b^{4}c^{3}-4\,AB^{3}b^{3}c^{4}+6\,A^{2}B^{2}b^{2}c^{5}-4\,A^{3}Bbc^{6}+A^{4}c^{7}}{b^{11}}\right)^{\frac{1}{4}}\log\left(b^{3}\left(-\frac{B^{4}b^{4}c^{3}-4\,AB^{3}b^{3}c^{4}+6\,A^{2}B^{2}b^{2}c^{5}-4\,A^{3}Bbc^{6}+A^{4}c^{7}}{b^{11}}\right)^{\frac{1}{4}}-(Bbc-Ac^{2})\sqrt{x}\right)-21\,b^{2}x^{4}\left(-\frac{B^{4}b^{4}c^{3}-4\,AB^{3}b^{3}c^{4}+6\,A^{2}B^{2}b^{2}c^{5}-4\,A^{3}Bbc^{6}+A^{4}c^{7}}{b^{11}}\right)^{\frac{1}{4}}\log\left(-b^{3}\left(-\frac{B^{4}b^{4}c^{3}-4\,AB^{3}b^{3}c^{4}+6\,A^{2}B^{2}b^{2}c^{5}-4\,A^{3}Bbc^{6}+A^{4}c^{7}}{b^{11}}\right)^{\frac{1}{4}}-(Bbc-Ac^{2})\sqrt{x}\right)-4\,(7\,(Bb-Ac)x^{2}+3\,Ab)\sqrt{x}}{42\,b^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/42*(84*b^2*x^4*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A
^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*arctan((sqrt(b^6*sqrt(-(B^4*b^4*c^3 - 4*A
*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11) + (B^2*b^
2*c^2 - 2*A*B*b*c^3 + A^2*c^4)*x)*b^8*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*
A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(3/4) + (B*b^9*c - A*b^8*c
^2)*sqrt(x)*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*
b*c^6 + A^4*c^7)/b^11)^(3/4))/(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^
2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)) + 21*b^2*x^4*(-(B^4*b^4*c^3 - 4*A*B^3*b^3
*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*log(b^3*(-(
B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7
)/b^11)^(1/4) - (B*b*c - A*c^2)*sqrt(x)) - 21*b^2*x^4*(-(B^4*b^4*c^3 - 4*A*
B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*log(
-b^3*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 +
 A^4*c^7)/b^11)^(1/4) - (B*b*c - A*c^2)*sqrt(x)) - 4*(7*(B*b - A*c)*x^2 + 3
*A*b)*sqrt(x))/(b^2*x^4)
```

Sympy [A]
time = 70.48, size = 303, normalized size = 1.18

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{11x^{\frac{11}{2}}}-\frac{2B}{7x^{\frac{7}{2}}}\right) & \text{for}\: b=0 \wedge c=0 \\ \frac{-\frac{2A}{11x^{\frac{11}{2}}}-\frac{2B}{7x^{\frac{7}{2}}}}{c} & \text{for}\: b=0 \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b} & \text{for}\: c=0 \\ -\frac{2A}{7bx^{\frac{7}{2}}}+\frac{2Ac}{3b^{2}x^{\frac{3}{2}}}-\frac{Ac^{2}\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^{3}}+\frac{Ac^{2}\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^{3}}+\frac{Ac^{2}\sqrt[4]{-\frac{b}{c}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b^{3}}-\frac{2B}{3bx^{\frac{3}{2}}}+\frac{Bc\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^{2}}-\frac{Bc\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^{2}}-\frac{Bc\sqrt[4]{-\frac{b}{c}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(7*x**(7/2))), Eq(b, 0) & Eq(c, 0
)), ((-2*A/(11*x**(11/2)) - 2*B/(7*x**(7/2)))/c, Eq(b, 0)), ((-2*A/(7*x**(7
```

```
/2)) - 2*B/(3*x**(3/2)))/b, Eq(c, 0)), (-2*A/(7*b*x**(7/2)) + 2*A*c/(3*b**2
*x**(3/2)) - A*c**2*(-b/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b**3) + A
*c**2*(-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*b**3) + A*c**2*(-b/c)**
(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/b**3 - 2*B/(3*b*x**(3/2)) + B*c*(-b/c)**(
1/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b**2) - B*c*(-b/c)**(1/4)*log(sqrt(x)
+ (-b/c)**(1/4))/(2*b**2) - B*c*(-b/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/b
**2, True))
```

Giac [A]
time = 0.52, size = 257, normalized size = 1.00

$$-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{3}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{3}}-\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^{3}}+\frac{\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb-\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^{3}}-\frac{2\left(7\,Bbx^{2}-7\,Acx^{2}+3\,Ab\right)}{21\,b^{2}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^3 - 1/2*sqrt(2)*((b*c^3)^(1/4
)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqr
t(x))/(b/c)^(1/4))/b^3 - 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c
)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 + 1/4*sqrt(2)*((b*c^
3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sq
rt(b/c))/b^3 - 2/21*(7*B*b*x^2 - 7*A*c*x^2 + 3*A*b)/(b^2*x^(7/2))
```

Mupad [B]
time = 0.32, size = 555, normalized size = 2.16

$$-\frac{\frac{2\,A}{7\,b}-\frac{2\,x^2\,\left(A\,c-B\,b\right)}{3\,b^2}}{x^{7/2}}+\frac{{\left(-c\right)}^{3/4}\,\mathrm{atan}\left(\frac{\frac{{\left(-c\right)}^{3/4}\,\left(A\,c-B\,b\right)\,\left(\sqrt{x}\,\left(16\,A^2\,b^6\,c^7-32\,A\,B\,b^7\,c^6+16\,B^2\,b^8\,c^5\right)-\frac{{\left(-c\right)}^{3/4}\,\left(A\,c-B\,b\right)\,\left(32\,A\,b^9\,c^5-32\,B\,b^{10}\,c^4\right)\,1{}\mathrm{i}}{2\,b^{11/4}}\right)}{2\,b^{11/4}}+\frac{{\left(-c\right)}^{3/4}\,\left(A\,c-B\,b\right)\,\left(\sqrt{x}\,\left(16\,A^2\,b^6\,c^7-32\,A\,B\,b^7\,c^6+16\,B^2\,b^8\,c^5\right)+\frac{{\left(-c\right)}^{3/4}\,\left(A\,c-B\,b\right)\,\left(32\,A\,b^9\,c^5-32\,B\,b^{10}\,c^4\right)\,1{}\mathrm{i}}{2\,b^{11/4}}\right)}{2\,b^{11/4}}}{\frac{{\left(-c\right)}^{3/4}\,\left(A\,c-B\,b\right)\,\left(\sqrt{x}\,\left(16\,A^2\,b^6\,c^7-32\,A\,B\,b^7\,c^6+16\,B^2\,b^8\,c^5\right)-\frac{{\left(-c\right)}^{3/4}\,\left(A\,c-B\,b\right)\,\left(32\,A\,b^9\,c^5-32\,B\,b^{10}\,c^4\right)\,1{}\mathrm{i}}{2\,b^{11/4}}\right)\,1{}\mathrm{i}}{2\,b^{11/4}}-\frac{{\left(-c\right)}^{3/4}\,\left(A\,c-B\,b\right)\,\left(\sqrt{x}\,\left(16\,A^2\,b^6\,c^7-32\,A\,B\,b^7\,c^6+16\,B^2\,b^8\,c^5\right)+\frac{{\left(-c\right)}^{3/4}\,\left(A\,c-B\,b\right)\,\left(32\,A\,b^9\,c^5-32\,B\,b^{10}\,c^4\right)\,1{}\mathrm{i}}{2\,b^{11/4}}\right)\,1{}\mathrm{i}}{2\,b^{11/4}}}\right)\,\left(A\,c-B\,b\right)}{b^{11/4}}-\frac{{\left(-c\right)}^{3/4}\,\mathrm{atan}\left(\frac{A^3\,c^8\,\sqrt{x}\,1{}\mathrm{i}-B^3\,b^3\,c^5\,\sqrt{x}\,1{}\mathrm{i}-A^2\,B\,b\,c^7\,\sqrt{x}\,3{}\mathrm{i}+A\,B^2\,b^2\,c^6\,\sqrt{x}\,3{}\mathrm{i}}{b^{1/4}\,{\left(-c\right)}^{19/4}\,\left(c\,\left(c\,\left(A^3\,c-3\,A^2\,B\,b\right)+3\,A\,B^2\,b^2\right)-B^3\,b^3\right)}\right)\,\left(A\,c-B\,b\right)\,1{}\mathrm{i}}{b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(3/4)*atan((((-c)^(3/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^6*c^7 + 16*B^2
*b^8*c^5 - 32*A*B*b^7*c^6) - ((-c)^(3/4)*(A*c - B*b)*(32*A*b^9*c^5 - 32*B*b
^10*c^4)*1i)/(2*b^(11/4))))/(2*b^(11/4)) + ((-c)^(3/4)*(A*c - B*b)*(x^(1/2)
*(16*A^2*b^6*c^7 + 16*B^2*b^8*c^5 - 32*A*B*b^7*c^6) + ((-c)^(3/4)*(A*c - B*
b)*(32*A*b^9*c^5 - 32*B*b^10*c^4)*1i)/(2*b^(11/4))))/(2*b^(11/4)))/(((-c)^(
3/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^6*c^7 + 16*B^2*b^8*c^5 - 32*A*B*b^7*c^6
) - ((-c)^(3/4)*(A*c - B*b)*(32*A*b^9*c^5 - 32*B*b^10*c^4)*1i)/(2*b^(11/4))
)*1i)/(2*b^(11/4)) - ((-c)^(3/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^6*c^7 + 16*
B^2*b^8*c^5 - 32*A*B*b^7*c^6) + ((-c)^(3/4)*(A*c - B*b)*(32*A*b^9*c^5 - 32*
B*b^10*c^4)*1i)/(2*b^(11/4)))*1i)/(2*b^(11/4))))*(A*c - B*b))/b^(11/4) - ((
2*A)/(7*b) - (2*x^2*(A*c - B*b))/(3*b^2))/x^(7/2) - ((-c)^(3/4)*atan((A^3*c
^8*x^(1/2)*1i - B^3*b^3*c^5*x^(1/2)*1i - A^2*B*b*c^7*x^(1/2)*3i + A*B^2*b^2
*c^6*x^(1/2)*3i)/(b^(1/4)*(-c)^(19/4)*(c*(c*(A^3*c - 3*A^2*B*b) + 3*A*B^2*b
^2) - B^3*b^3)))*(A*c - B*b)*1i)/b^(11/4)
```

3.193 $\int \frac{A+Bx^2}{x^{7/2}\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=276

$$-\frac{2A}{9bx^{9/2}}-\frac{2(bB-Ac)}{5b^2x^{5/2}}+\frac{2c(bB-Ac)}{b^3\sqrt{x}}-\frac{c^{5/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{13/4}}+\frac{c^{5/4}(bB-Ac)\tan^{-1}\left(1+\right.}{\sqrt{2}\,b^{13/4}}$$

```
[Out] -2/9*A/b/x^(9/2)-2/5*(-A*c+B*b)/b^2/x^(5/2)-1/2*c^(5/4)*(-A*c+B*b)*arctan(1
-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^(1/2)+1/2*c^(5/4)*(-A*c+B*b)*a
rctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^(1/2)+1/4*c^(5/4)*(-A*c
+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2
)-1/4*c^(5/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/
2))/b^(13/4)*2^(1/2)+2*c*(-A*c+B*b)/b^3/x^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 464, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{c^{5/4}(bB-Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{13/4}}+\frac{c^{5/4}(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{13/4}}+\frac{c^{5/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}}-\frac{c^{5/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}}+\frac{2c(bB-Ac)}{b^3\sqrt{x}}-\frac{2(bB-Ac)}{5b^2x^{5/2}}-\frac{2A}{9bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(9*b*x^(9/2)) - (2*(b*B - A*c))/(5*b^2*x^(5/2)) + (2*c*(b*B - A*c))/
(b^3*Sqrt[x]) - (c^(5/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)])/(Sqrt[2]*b^(13/4)) + (c^(5/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(13/4)) + (c^(5/4)*(b*B - A*c)*Log[Sqrt[b
] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(13/4)) - (c
^(5/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x])/(2*Sqrt[2]*b^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)}\,dx = \int \frac{A+Bx^2}{x^{11/2}(b+cx^2)}\,dx$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{\left(2\left(-\frac{9bB}{2}+\frac{9Ac}{2}\right)\right)\int \frac{1}{x^{7/2}(b+cx^2)}\,dx}{9b}$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} - \frac{(c(bB-Ac))\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{b^2}$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} + \frac{(c^2(bB-Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b^3}$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} + \frac{(2c^2(bB-Ac))\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right.}{b^3}$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} - \frac{(c^{3/2}(bB-Ac))\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx\right.}{b^3}$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} + \frac{(c(bB-Ac))\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x\right.}{2b^3}$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} + \frac{c^{5/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{\right.}}{2\sqrt{2}\,b^{13/4}}$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} - \frac{c^{5/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{13/4}}$$

Mathematica [A]

time = 0.25, size = 174, normalized size = 0.63

$$\frac{-\frac{4\sqrt[4]{b}\,(9bBx^2(b-5cx^2)+A(5b^2-9bcx^2+45c^2x^4))}{x^{9/2}}+45\sqrt{2}\,c^{5/4}(-bB+Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)+45\sqrt{2}\,c^{5/4}(-bB+Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{90b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] ((-4*b^(1/4)*(9*b*B*x^2*(b - 5*c*x^2) + A*(5*b^2 - 9*b*c*x^2 + 45*c^2*x^4))
)/x^(9/2) + 45*Sqrt[2]*c^(5/4)*(-(b*B) + A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/
(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 45*Sqrt[2]*c^(5/4)*(-(b*B) + A*c)*ArcT
anh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(90*b^(13/4))
```

Maple [A]

time = 0.40, size = 158, normalized size = 0.57

method	result
derivativedivides	$-\frac{2A}{9b\,x^{\frac{9}{2}}}-\frac{2(-Ac+Bb)}{5b^2x^{\frac{5}{2}}}-\frac{2c(Ac-Bb)}{b^3\sqrt{x}}-\frac{c(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\ \ldots}{\left(\frac{b}{c}\right)\ldots}\right.\right.}{4b^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$-\frac{2A}{9b\,x^{\frac{9}{2}}}-\frac{2(-Ac+Bb)}{5b^2x^{\frac{5}{2}}}-\frac{2c(Ac-Bb)}{b^3\sqrt{x}}-\frac{c(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\ \ldots}{\left(\frac{b}{c}\right)\ldots}\right.\right.}{4b^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
risch	$-\frac{2\left(45A\,c^2x^4-45x^4bBc-9Abc\,x^2+9b^2B\,x^2+5b^2A\right)}{45b^3x^{\frac{9}{2}}}-\frac{c^2\sqrt{2}\ A\arctan\left(\frac{\sqrt{2}\ \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{c^2\sqrt{2}\ A\arctan\left(\ldots\right.}{2b^3\left(\frac{b}{c}\right)\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*A/b/x^(9/2)-2/5*(-A*c+B*b)/b^2/x^(5/2)-2*c*(A*c-B*b)/b^3/x^(1/2)-1/4*c
*(A*c-B*b)/b^3/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)
^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)
^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]

time = 0.51, size = 237, normalized size = 0.86

$$\frac{(Bbc^2-Ac^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{4\,b^3}+\frac{2\left(45\,(Bbc-Ac^2)x^4-5\,Ab^2-9\,(Bb^2-Abc)x^2\right)}{45\,b^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b*c^2 - A*c^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4
) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c
)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqr
t(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(
sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) +
sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4
)*c^(3/4)))/b^3 + 2/45*(45*(B*b*c - A*c^2)*x^4 - 5*A*b^2 - 9*(B*b^2 - A*b*c
)*x^2)/(b^3*x^(9/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(199) = 398$.
time = 2.46, size = 931, normalized size = 3.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/90*(180*b^3*x^5*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*
A^3*B*b*c^8 + A^4*c^9)/b^13)^(1/4)*arctan((sqrt((B^6*b^6*c^8 - 6*A*B^5*b^5*
c^9 + 15*A^2*B^4*b^4*c^10 - 20*A^3*B^3*b^3*c^11 + 15*A^4*B^2*b^2*c^12 - 6*A
^5*B*b*c^13 + A^6*c^14)*x - (B^4*b^11*c^5 - 4*A*B^3*b^10*c^6 + 6*A^2*B^2*b^
9*c^7 - 4*A^3*B*b^8*c^8 + A^4*b^7*c^9)*sqrt(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6
 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13))*b^3*(-(B^4*b^4*c^5 -
 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(1/4)
 + (B^3*b^6*c^4 - 3*A*B^2*b^5*c^5 + 3*A^2*B*b^4*c^6 - A^3*b^3*c^7)*sqrt(x)*
(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*
c^9)/b^13)^(1/4))/(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^
3*B*b*c^8 + A^4*c^9)) - 45*b^3*x^5*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2
*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(1/4)*log(b^10*(-(B^4*b^4*c^5
 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(3/
4) - (B^3*b^3*c^4 - 3*A*B^2*b^2*c^5 + 3*A^2*B*b*c^6 - A^3*c^7)*sqrt(x)) + 4
5*b^3*x^5*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*
c^8 + A^4*c^9)/b^13)^(1/4)*log(-b^10*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A
^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(3/4) - (B^3*b^3*c^4 - 3*A*
B^2*b^2*c^5 + 3*A^2*B*b*c^6 - A^3*c^7)*sqrt(x)) + 4*(45*(B*b*c - A*c^2)*x^4
 - 5*A*b^2 - 9*(B*b^2 - A*b*c)*x^2)*sqrt(x))/(b^3*x^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(7/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 291, normalized size = 1.05

$$\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^4c}+\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^4c}-\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^4c}+\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^4c}+\frac{2\,(45\,Bbcx^4-45\,Ac^2x^4-9\,Bb^2x^2+9\,Abcx^2-5\,Ab^2)}{45\,b^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 1/2*sqrt(2)*((b*c^3)^(
3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*
sqrt(x))/(b/c)^(1/4))/(b^4*c) - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3
/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c) + 1/4*sqr
t(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/
4) + x + sqrt(b/c))/(b^4*c) + 2/45*(45*B*b*c*x^4 - 45*A*c^2*x^4 - 9*B*b^2*x
^2 + 9*A*b*c*x^2 - 5*A*b^2)/(b^3*x^(9/2))
```

Mupad [B]
time = 0.23, size = 107, normalized size = 0.39

$$\frac{(-c)^{5/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(A\,c-B\,b)}{b^{13/4}}-\frac{\frac{2\,A}{9\,b}-\frac{2\,x^2\,(A\,c-B\,b)}{5\,b^2}+\frac{2\,c\,x^4\,(A\,c-B\,b)}{b^3}}{x^{9/2}}-\frac{(-c)^{5/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(A\,c-B\,b)}{b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(5/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(A*c - B*b))/b^(13/4) - ((2*
A)/(9*b) - (2*x^2*(A*c - B*b))/(5*b^2) + (2*c*x^4*(A*c - B*b))/b^3)/x^(9/2)
 - ((-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4))*(A*c - B*b))/b^(13/4)
```

3.194 $\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx$

Optimal. Leaf size=278

$$-\frac{2A}{11bx^{11/2}}-\frac{2(bB-Ac)}{7b^2x^{7/2}}+\frac{2c(bB-Ac)}{3b^3x^{3/2}}-\frac{c^{7/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{15/4}}+\frac{c^{7/4}(bB-Ac)\tan^{-1}\left(1\right.}{\sqrt{2}\,b^{15/}}$$

```
[Out] -2/11*A/b/x^(11/2)-2/7*(-A*c+B*b)/b^2/x^(7/2)+2/3*c*(-A*c+B*b)/b^3/x^(3/2)-
1/2*c^(7/4)*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2
^(1/2)+1/2*c^(7/4)*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(
15/4)*2^(1/2)-1/4*c^(7/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2
^(1/2)*x^(1/2))/b^(15/4)*2^(1/2)+1/4*c^(7/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2
)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/4)*2^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 464, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{c^{7/4}(bB-Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{15/4}}+\frac{c^{7/4}(bB-Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{15/4}}-\frac{c^{7/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{15/4}}+\frac{c^{7/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{15/4}}+\frac{2c(bB-Ac)}{3b^3x^{3/2}}-\frac{2(bB-Ac)}{7b^2x^{7/2}}-\frac{2A}{11bx^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(11*b*x^(11/2)) - (2*(b*B - A*c))/(7*b^2*x^(7/2)) + (2*c*(b*B - A*c)
)/(3*b^3*x^(3/2)) - (c^(7/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x
])/b^(1/4)])/(Sqrt[2]*b^(15/4)) + (c^(7/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(15/4)) - (c^(7/4)*(b*B - A*c)*Log[Sq
rt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(15/4))
 + (c^(7/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt
[c]*x])/(2*Sqrt[2]*b^(15/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{9/2}\left(bx^2+cx^4\right)}\,dx = \int \frac{A+Bx^2}{x^{13/2}\left(b+cx^2\right)}\,dx$$
$$= -\frac{2A}{11bx^{11/2}} - \frac{\left(2\left(-\frac{11bB}{2}+\frac{11Ac}{2}\right)\right)\int \frac{1}{x^{9/2}(b+cx^2)}\,dx}{11b}$$
$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} - \frac{(c(bB-Ac))\int \frac{1}{x^{5/2}(b+cx^2)}\,dx}{b^2}$$
$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} + \frac{(c^2(bB-Ac))\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{b^3}$$
$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} + \frac{(2c^2(bB-Ac))\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{\right.}{b^3}$$
$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} + \frac{(c^2(bB-Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx\right.}{b^{7/2}}$$
$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} + \frac{\left(c^{3/2}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{c}}}\right.}{2b^{7/2}}$$
$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} - \frac{c^{7/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\right.}{2\sqrt{2}\,b^{15/4}}$$
$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} - \frac{c^{7/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt{2}\,b^{15/4}}$$

Mathematica [A]

time = 0.25, size = 176, normalized size = 0.63

$$-\frac{2(21Ab^2+33b^2Bx^2-33Abcx^2-77bBcx^4+77Ac^2x^4)}{231b^3x^{11/2}}-\frac{c^{7/4}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)}{\sqrt{2}\,b^{15/4}}+\frac{c^{7/4}(bB-Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,b^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*(21*A*b^2 + 33*b^2*B*x^2 - 33*A*b*c*x^2 - 77*b*B*c*x^4 + 77*A*c^2*x^4))
/(231*b^3*x^(11/2)) - (c^(7/4)*(b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(15/4)) + (c^(7/4)*(b*B - A*c)*
ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*
b^(15/4))
```

Maple [A]
time = 0.40, size = 160, normalized size = 0.58

method	result
derivativedivides	$-\frac{c^2(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\right.\right.}{4b^4}$
default	$-\frac{c^2(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\right.\right.}{4b^4}$
risch	$-\frac{2(77Ac^2x^4-77x^4bBc-33Abcx^2+33b^2Bx^2+21b^2A)}{231b^3x^{\frac{11}{2}}}-\frac{c^3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)}{2b^4}-\frac{c^3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*c^2*(A*c-B*b)/b^4*(b/c)^(1/4)*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/
2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/
2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))-2/11*A/b
/x^(11/2)-2/7*(-A*c+B*b)/b^2/x^(7/2)-2/3*c*(A*c-B*b)/b^3/x^(3/2)
```

Maxima [A]
time = 0.55, size = 276, normalized size = 0.99

$$\frac{\frac{2\sqrt{2}\,(Bbc^2-Ac^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,(Bbc^2-Ac^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,(Bbc^2-Ac^3)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Bbc^2-Ac^3)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{4\,b^3}+\frac{2\,(77\,(Bbc-Ac^2)x^4-21\,Ab^2-33\,(Bb^2-Abc)x^2)}{231\,b^3x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(B*b*c^2 - A*c^3)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4
) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c)
)) + 2*sqrt(2)*(B*b*c^2 - A*c^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/
4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c
))) + sqrt(2)*(B*b*c^2 - A*c^3)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(
c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b*c^2 - A*c^3)*log(-sqrt(2)*
b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^3 + 2/2
31*(77*(B*b*c - A*c^2)*x^4 - 21*A*b^2 - 33*(B*b^2 - A*b*c)*x^2)/(b^3*x^(11/
2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 734 vs. $2(199) = 398$.
time = 2.72, size = 734, normalized size = 2.64

$$-\frac{924\,b^{3}x^{6}\left(-\frac{B^{4}b^{4}c^{7}-4\,AB^{3}b^{3}c^{8}+6\,A^{2}B^{2}b^{2}c^{9}-4\,A^{3}Bbc^{10}+A^{4}c^{11}}{b^{15}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^{8}\sqrt{-\frac{B^{4}b^{4}c^{7}-4\,AB^{3}b^{3}c^{8}+6\,A^{2}B^{2}b^{2}c^{9}-4\,A^{3}Bbc^{10}+A^{4}c^{11}}{b^{15}}}+\left(B^{2}b^{2}c^{4}-2\,ABbc^{5}+A^{2}c^{6}\right)x}\,b^{11}\left(-\frac{B^{4}b^{4}c^{7}-4\,AB^{3}b^{3}c^{8}+6\,A^{2}B^{2}b^{2}c^{9}-4\,A^{3}Bbc^{10}+A^{4}c^{11}}{b^{15}}\right)^{\frac{3}{4}}+\left(Bb^{12}c^{2}-Ab^{11}c^{3}\right)\sqrt{x}\left(-\frac{B^{4}b^{4}c^{7}-4\,AB^{3}b^{3}c^{8}+6\,A^{2}B^{2}b^{2}c^{9}-4\,A^{3}Bbc^{10}+A^{4}c^{11}}{b^{15}}\right)^{\frac{3}{4}}}{B^{4}b^{4}c^{7}-4\,AB^{3}b^{3}c^{8}+6\,A^{2}B^{2}b^{2}c^{9}-4\,A^{3}Bbc^{10}+A^{4}c^{11}}\right)+231\,b^{3}x^{6}\left(-\frac{B^{4}b^{4}c^{7}-4\,AB^{3}b^{3}c^{8}+6\,A^{2}B^{2}b^{2}c^{9}-4\,A^{3}Bbc^{10}+A^{4}c^{11}}{b^{15}}\right)^{\frac{1}{4}}\log\left(b^{4}\left(-\frac{B^{4}b^{4}c^{7}-4\,AB^{3}b^{3}c^{8}+6\,A^{2}B^{2}b^{2}c^{9}-4\,A^{3}Bbc^{10}+A^{4}c^{11}}{b^{15}}\right)^{\frac{1}{4}}-\left(Bbc^{2}-Ac^{3}\right)\sqrt{x}\right)-231\,b^{3}x^{6}\left(-\frac{B^{4}b^{4}c^{7}-4\,AB^{3}b^{3}c^{8}+6\,A^{2}B^{2}b^{2}c^{9}-4\,A^{3}Bbc^{10}+A^{4}c^{11}}{b^{15}}\right)^{\frac{1}{4}}\log\left(-b^{4}\left(-\frac{B^{4}b^{4}c^{7}-4\,AB^{3}b^{3}c^{8}+6\,A^{2}B^{2}b^{2}c^{9}-4\,A^{3}Bbc^{10}+A^{4}c^{11}}{b^{15}}\right)^{\frac{1}{4}}-\left(Bbc^{2}-Ac^{3}\right)\sqrt{x}\right)-4\left(77\left(Bbc-Ac^{2}\right)x^{4}-21\,Ab^{2}-33\left(Bb^{2}-Abc\right)x^{2}\right)\sqrt{x}}{462\,b^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/462*(924*b^3*x^6*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 -
4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4)*arctan((sqrt(b^8*sqrt(-(B^4*b^4*c^7
- 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15) +
(B^2*b^2*c^4 - 2*A*B*b*c^5 + A^2*c^6)*x)*b^11*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*
c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(3/4) + (B*b^12*
c^2 - A*b^11*c^3)*sqrt(x)*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*
c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(3/4))/(B^4*b^4*c^7 - 4*A*B^3*b^3*c^
8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)) + 231*b^3*x^6*(-(B^4*b^
4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^
15)^(1/4)*log(b^4*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*
A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4) - (B*b*c^2 - A*c^3)*sqrt(x)) - 231*b^3
*x^6*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10
+ A^4*c^11)/b^15)^(1/4)*log(-b^4*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B
^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4) - (B*b*c^2 - A*c^3)*sqr
t(x)) - 4*(77*(B*b*c - A*c^2)*x^4 - 21*A*b^2 - 33*(B*b^2 - A*b*c)*x^2)*sqrt
(x))/(b^3*x^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(9/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.60, size = 291, normalized size = 1.05

$$\frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bbc-(bc^3)^{\frac{1}{4}}Ac^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4}+\frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bbc-(bc^3)^{\frac{1}{4}}Ac^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4}+\frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bbc-(bc^3)^{\frac{1}{4}}Ac^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^4}-\frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bbc-(bc^3)^{\frac{1}{4}}Ac^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^4}+\frac{2\left(77\,Bbcx^4-77\,Ac^2x^4-33\,Bb^2x^2+33\,Abcx^2-21\,Ab^2\right)}{231\,b^3x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b*c - (b*c^3)^(1/4)*A*c^2)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^4 + 1/2*sqrt(2)*((b*c^3)^(
1/4)*B*b*c - (b*c^3)^(1/4)*A*c^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
- 2*sqrt(x))/(b/c)^(1/4))/b^4 + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b*c - (b*c^3)^
(1/4)*A*c^2)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/4*sqr
t(2)*((b*c^3)^(1/4)*B*b*c - (b*c^3)^(1/4)*A*c^2)*log(-sqrt(2)*sqrt(x)*(b/c)
^(1/4) + x + sqrt(b/c))/b^4 + 2/231*(77*B*b*c*x^4 - 77*A*c^2*x^4 - 33*B*b^2
*x^2 + 33*A*b*c*x^2 - 21*A*b^2)/(b^3*x^(11/2))
```

Mupad [B]
time = 0.35, size = 563, normalized size = 2.03

$$\frac{(-c)^{7/4}\,\mathrm{atan}\left(\frac{A^3c^{10}\sqrt{x}-B^3b^3c^7\sqrt{x}-3A^2Bbc^9\sqrt{x}+3AB^2b^2c^8\sqrt{x}}{b^{1/4}(-c)^{27/4}\left(c\left(c\left(A^3c-3A^2Bb\right)+3AB^2b^2\right)-B^3b^3\right)}\right)(Ac-Bb)}{b^{15/4}}-\frac{\frac{2A}{11b}-\frac{2x^2(Ac-Bb)}{7b^2}+\frac{2cx^4(Ac-Bb)}{3b^3}}{x^{11/2}}-\frac{(-c)^{7/4}\,\mathrm{atan}\left(\frac{\frac{(-c)^{7/4}(Ac-Bb)\left(\sqrt{x}\left(16A^2b^9c^9-32ABb^{10}c^8+16B^2b^{11}c^7\right)-\frac{(-c)^{7/4}(Ac-Bb)\left(32Ab^{13}c^6-32Bb^{14}c^5\right)}{2b^{15/4}}\right)1i}{2b^{15/4}}+\frac{(-c)^{7/4}(Ac-Bb)\left(\sqrt{x}\left(16A^2b^9c^9-32ABb^{10}c^8+16B^2b^{11}c^7\right)+\frac{(-c)^{7/4}(Ac-Bb)\left(32Ab^{13}c^6-32Bb^{14}c^5\right)}{2b^{15/4}}\right)1i}{2b^{15/4}}}{\frac{(-c)^{7/4}(Ac-Bb)\left(\sqrt{x}\left(16A^2b^9c^9-32ABb^{10}c^8+16B^2b^{11}c^7\right)-\frac{(-c)^{7/4}(Ac-Bb)\left(32Ab^{13}c^6-32Bb^{14}c^5\right)}{2b^{15/4}}\right)}{2b^{15/4}}-\frac{(-c)^{7/4}(Ac-Bb)\left(\sqrt{x}\left(16A^2b^9c^9-32ABb^{10}c^8+16B^2b^{11}c^7\right)+\frac{(-c)^{7/4}(Ac-Bb)\left(32Ab^{13}c^6-32Bb^{14}c^5\right)}{2b^{15/4}}\right)}{2b^{15/4}}}\right)(Ac-Bb)\,1i}{b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(7/4)*atan((A^3*c^10*x^(1/2) - B^3*b^3*c^7*x^(1/2) - 3*A^2*B*b*c^9*x^
(1/2) + 3*A*B^2*b^2*c^8*x^(1/2))/(b^(1/4)*(-c)^(27/4)*(c*(c*(A^3*c - 3*A^2*
B*b) + 3*A*B^2*b^2) - B^3*b^3)))*(A*c - B*b))/b^(15/4) - ((-c)^(7/4)*atan((
((-c)^(7/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^9*c^9 + 16*B^2*b^11*c^7 - 32*A*B
*b^10*c^8) - ((-c)^(7/4)*(A*c - B*b)*(32*A*b^13*c^6 - 32*B*b^14*c^5))/(2*b^
(15/4)))*1i)/(2*b^(15/4)) + ((-c)^(7/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^9*c^
9 + 16*B^2*b^11*c^7 - 32*A*B*b^10*c^8) + ((-c)^(7/4)*(A*c - B*b)*(32*A*b^13
*c^6 - 32*B*b^14*c^5))/(2*b^(15/4)))*1i)/(2*b^(15/4)))/(((-c)^(7/4)*(A*c -
B*b)*(x^(1/2)*(16*A^2*b^9*c^9 + 16*B^2*b^11*c^7 - 32*A*B*b^10*c^8) - ((-c)^
(7/4)*(A*c - B*b)*(32*A*b^13*c^6 - 32*B*b^14*c^5))/(2*b^(15/4))))/(2*b^(15/
4)) - ((-c)^(7/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^9*c^9 + 16*B^2*b^11*c^7 -
32*A*B*b^10*c^8) + ((-c)^(7/4)*(A*c - B*b)*(32*A*b^13*c^6 - 32*B*b^14*c^5))
/(2*b^(15/4))))/(2*b^(15/4))))*(A*c - B*b)*1i)/b^(15/4) - ((2*A)/(11*b) - (
2*x^2*(A*c - B*b))/(7*b^2) + (2*c*x^4*(A*c - B*b))/(3*b^3))/x^(11/2)
```

3.195 $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=332

$$\frac{b(13bB-9Ac)\sqrt{x}}{2c^4}-\frac{(13bB-9Ac)x^{5/2}}{10c^3}+\frac{(13bB-9Ac)x^{9/2}}{18bc^2}-\frac{(bB-Ac)x^{13/2}}{2bc\,(b+cx^2)}+\frac{b^{5/4}(13bB-9Ac)\tan^{-1}\Big(1-}{4\sqrt{2}\,c^{17/4}}$$

```
[Out] -1/10*(-9*A*c+13*B*b)*x^(5/2)/c^3+1/18*(-9*A*c+13*B*b)*x^(9/2)/b/c^2-1/2*(-
A*c+B*b)*x^(13/2)/b/c/(c*x^2+b)+1/8*b^(5/4)*(-9*A*c+13*B*b)*arctan(1-c^(1/4
)*2^(1/2)*x^(1/2)/b^(1/4))/c^(17/4)*2^(1/2)-1/8*b^(5/4)*(-9*A*c+13*B*b)*arc
tan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(17/4)*2^(1/2)+1/16*b^(5/4)*(-9*A*
c+13*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(17/4)*2^
(1/2)-1/16*b^(5/4)*(-9*A*c+13*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(
1/2)*x^(1/2))/c^(17/4)*2^(1/2)+1/2*b*(-9*A*c+13*B*b)*x^(1/2)/c^4
```

Rubi [A]
time = 0.18, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{b^{5/4}(13bB-9Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{17/4}}-\frac{b^{5/4}(13bB-9Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,c^{17/4}}+\frac{b^{5/4}(13bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{17/4}}-\frac{b^{5/4}(13bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{17/4}}+\frac{b\sqrt{x}\,(13bB-9Ac)}{2c^4}-\frac{x^{5/2}(13bB-9Ac)}{10c^3}+\frac{x^{9/2}(13bB-9Ac)}{18bc^2}-\frac{x^{13/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*(13*b*B - 9*A*c)*Sqrt[x])/(2*c^4) - ((13*b*B - 9*A*c)*x^(5/2))/(10*c^3)
+ ((13*b*B - 9*A*c)*x^(9/2))/(18*b*c^2) - ((b*B - A*c)*x^(13/2))/(2*b*c*(b
+ c*x^2)) + (b^(5/4)*(13*b*B - 9*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)])/(4*Sqrt[2]*c^(17/4)) - (b^(5/4)*(13*b*B - 9*A*c)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(17/4)) + (b^(5/4)*(13*b*B - 9*
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]
*c^(17/4)) - (b^(5/4)*(13*b*B - 9*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{x^{11/2}(A+Bx^2)}{(b+cx^2)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{13/2}}{2bc\,(b+cx^2)} + \frac{\left(\frac{13bB}{2}-\frac{9Ac}{2}\right)\int \frac{x^{11/2}}{b+cx^2}\,dx}{2bc}$$

$$= \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\,(b+cx^2)} - \frac{(13bB-9Ac)\int \frac{x^{7/2}}{b+cx^2}\,dx}{4c^2}$$

$$= -\frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\,(b+cx^2)} + \frac{(b(13bB-9Ac))\int}{4c^3}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\,(b+cx^2)}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\,(b+cx^2)}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\,(b+cx^2)}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\,(b+cx^2)}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\,(b+cx^2)}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc\,(b+cx^2)}$$

<u>Mathematica</u> [A]

time = 0.54, size = 206, normalized size = 0.62

$$\frac{\frac{4\sqrt[4]{c}\sqrt{x}\left(585b^3B-9b^2c(45A-52Bx^2)+4c^3x^4(9A+5Bx^2)-4bc^2x^2(81A+13Bx^2)\right)}{b+cx^2}+45\sqrt{2}\,b^{5/4}(13bB-9Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-45\sqrt{2}\,b^{5/4}(13bB-9Ac)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{360c^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*c^(1/4)*Sqrt[x]*(585*b^3*B - 9*b^2*c*(45*A - 52*B*x^2) + 4*c^3*x^4*(9*A
 + 5*B*x^2) - 4*b*c^2*x^2*(81*A + 13*B*x^2)))/(b + c*x^2) + 45*Sqrt[2]*b^(5
```

```
/4)*(13*b*B - 9*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*
Sqrt[x])] - 45*Sqrt[2]*b^(5/4)*(13*b*B - 9*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(360*c^(17/4))
```

Maple [A]
time = 0.55, size = 196, normalized size = 0.59

method	result
derivativedivides	$-\frac{2\left(-\frac{B x^{\frac{9}{2}} c^{2}}{9}-\frac{A c^{2} x^{\frac{5}{2}}}{5}+\frac{2 B b c x^{\frac{5}{2}}}{5}+2 A b c \sqrt{x}-3 b^{2} B \sqrt{x}\right)}{c^{4}}+\frac{2 b^{2}\left(\frac{\left(-\frac{A c}{4}+\frac{B b}{4}\right) \sqrt{x}}{c x^{2}+b}+\frac{(9 A c-13 B b)\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\left(\ln\left(\cdots\right)\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
default	$-\frac{2\left(-\frac{B x^{\frac{9}{2}} c^{2}}{9}-\frac{A c^{2} x^{\frac{5}{2}}}{5}+\frac{2 B b c x^{\frac{5}{2}}}{5}+2 A b c \sqrt{x}-3 b^{2} B \sqrt{x}\right)}{c^{4}}+\frac{2 b^{2}\left(\frac{\left(-\frac{A c}{4}+\frac{B b}{4}\right) \sqrt{x}}{c x^{2}+b}+\frac{(9 A c-13 B b)\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\left(\ln\left(\cdots\right)\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
risch	$-\frac{2\left(-5 B c^{2} x^{4}-9 A c^{2} x^{2}+18 b B x^{2} c+90 A b c-135 b^{2} B\right) \sqrt{x}}{45 c^{4}}-\frac{b^{2} \sqrt{x}\, A}{2 c^{3}\left(c x^{2}+b\right)}+\frac{b^{3} \sqrt{x}\, B}{2 c^{4}\left(c x^{2}+b\right)}+\frac{9 b\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\, A \arctan\cdots}{8 c^{3}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/c^4*(-1/9*B*x^(9/2)*c^2-1/5*A*c^2*x^(5/2)+2/5*B*b*c*x^(5/2)+2*A*b*c*x^(1
/2)-3*b^2*B*x^(1/2))+2*b^2/c^4*((-1/4*A*c+1/4*B*b)*x^(1/2)/(c*x^2+b)+1/32*(
9*A*c-13*B*b)*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c
)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c
)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.52, size = 298, normalized size = 0.90

$$\frac{(Bb^3 - Ab^2c)\sqrt{x}}{2\,(c^5x^2 + bc^4)} - \frac{\left(\frac{2\sqrt{2}\,(13\,Bb-9\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}\,(13\,Bb-9\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}\,(13\,Bb-9\,Ac)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}\,(13\,Bb-9\,Ac)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)b^2}{16\,c^4} + \frac{2\left(5\,Bc^2x^{\frac{9}{2}} - 9\,(2\,Bbc - Ac^2)x^{\frac{5}{2}} + 45\,(3\,Bb^2 - 2\,Abc)\sqrt{x}\right)}{45\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b^3 - A*b^2*c)*sqrt(x)/(c^5*x^2 + b*c^4) - 1/16*(2*sqrt(2)*(13*B*b -
 9*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sq
rt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(13*B*b -
9*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sq
rt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(13*B*b - 9*
A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^
(1/4)) - sqrt(2)*(13*B*b - 9*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sq
rt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b^2/c^4 + 2/45*(5*B*c^2*x^(9/2) - 9*(
2*B*b*c - A*c^2)*x^(5/2) + 45*(3*B*b^2 - 2*A*b*c)*sqrt(x))/c^4
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(244) = 488$.
time = 2.63, size = 804, normalized size = 2.42

$$\frac{180\,(c^5x^2+bc^4)\left(-\frac{28561\,B^4b^9-79092\,AB^3b^8c+82134\,A^2B^2b^7c^2-37908\,A^3Bb^6c^3+6561\,A^4b^5c^4}{c^{17}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^8\sqrt{-\frac{28561\,B^4b^9-79092\,AB^3b^8c+82134\,A^2B^2b^7c^2-37908\,A^3Bb^6c^3+6561\,A^4b^5c^4}{c^{17}}}+(169\,B^2b^4-234\,ABb^3c+81\,A^2b^2c^2)x}\,c^{13}\left(-\frac{28561\,B^4b^9-79092\,AB^3b^8c+82134\,A^2B^2b^7c^2-37908\,A^3Bb^6c^3+6561\,A^4b^5c^4}{c^{17}}\right)^{\frac{3}{4}}+(13\,Bb^2c^{13}-9\,Abc^{14})\sqrt{x}\left(-\frac{28561\,B^4b^9-79092\,AB^3b^8c+82134\,A^2B^2b^7c^2-37908\,A^3Bb^6c^3+6561\,A^4b^5c^4}{c^{17}}\right)^{\frac{3}{4}}}{28561\,B^4b^9-79092\,AB^3b^8c+82134\,A^2B^2b^7c^2-37908\,A^3Bb^6c^3+6561\,A^4b^5c^4}\right)+45\,(c^5x^2+bc^4)\left(-\frac{28561\,B^4b^9-79092\,AB^3b^8c+82134\,A^2B^2b^7c^2-37908\,A^3Bb^6c^3+6561\,A^4b^5c^4}{c^{17}}\right)^{\frac{1}{4}}\log\left(c^4\left(-\frac{28561\,B^4b^9-79092\,AB^3b^8c+82134\,A^2B^2b^7c^2-37908\,A^3Bb^6c^3+6561\,A^4b^5c^4}{c^{17}}\right)^{\frac{1}{4}}-(13\,Bb^2-9\,Abc)\sqrt{x}\right)-45\,(c^5x^2+bc^4)\left(-\frac{28561\,B^4b^9-79092\,AB^3b^8c+82134\,A^2B^2b^7c^2-37908\,A^3Bb^6c^3+6561\,A^4b^5c^4}{c^{17}}\right)^{\frac{1}{4}}\log\left(-c^4\left(-\frac{28561\,B^4b^9-79092\,AB^3b^8c+82134\,A^2B^2b^7c^2-37908\,A^3Bb^6c^3+6561\,A^4b^5c^4}{c^{17}}\right)^{\frac{1}{4}}-(13\,Bb^2-9\,Abc)\sqrt{x}\right)+4\,(20\,Bc^3x^6-4\,(13\,Bbc^2-9\,Ac^3)x^4+585\,Bb^3-405\,Ab^2c+36\,(13\,Bb^2c-9\,Abc^2)x^2)\sqrt{x}}{360\,(c^5x^2+bc^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/360*(180*(c^5*x^2 + b*c^4)*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A
^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4)*arctan
((sqrt(c^8*sqrt(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2
 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17) + (169*B^2*b^4 - 234*A*B*b
^3*c + 81*A^2*b^2*c^2)*x)*c^13*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134
*A^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(3/4) + (1
3*B*b^2*c^13 - 9*A*b*c^14)*sqrt(x)*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 8
2134*A^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(3/4))
/(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2 - 37908*A^3*B*b
^6*c^3 + 6561*A^4*b^5*c^4)) + 45*(c^5*x^2 + b*c^4)*(-(28561*B^4*b^9 - 79092
*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c
^4)/c^17)^(1/4)*log(c^4*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^
2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4) - (13*B*b^2
 - 9*A*b*c)*sqrt(x)) - 45*(c^5*x^2 + b*c^4)*(-(28561*B^4*b^9 - 79092*A*B^3*
b^8*c + 82134*A^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^1
7)^(1/4)*log(-c^4*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*
c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4) - (13*B*b^2 - 9*A
*b*c)*sqrt(x)) + 4*(20*B*c^3*x^6 - 4*(13*B*b*c^2 - 9*A*c^3)*x^4 + 585*B*b^3
 - 405*A*b^2*c + 36*(13*B*b^2*c - 9*A*b*c^2)*x^2)*sqrt(x))/(c^5*x^2 + b*c^4
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 335, normalized size = 1.01

$$-\frac{\sqrt{2}\left(13\left(bc^{3}\right)^{\frac{1}{4}}Bb^{2}-9\left(bc^{3}\right)^{\frac{1}{4}}Abc\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^{5}}-\frac{\sqrt{2}\left(13\left(bc^{3}\right)^{\frac{1}{4}}Bb^{2}-9\left(bc^{3}\right)^{\frac{1}{4}}Abc\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^{5}}-\frac{\sqrt{2}\left(13\left(bc^{3}\right)^{\frac{1}{4}}Bb^{2}-9\left(bc^{3}\right)^{\frac{1}{4}}Abc\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16c^{5}}+\frac{\sqrt{2}\left(13\left(bc^{3}\right)^{\frac{1}{4}}Bb^{2}-9\left(bc^{3}\right)^{\frac{1}{4}}Abc\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16c^{5}}+\frac{Bb^{3}\sqrt{x}-Ab^{2}c\sqrt{x}}{2\left(cx^{2}+b\right)c^{4}}+\frac{2\left(5Bc^{16}x^{\frac{9}{2}}-18Bbc^{15}x^{\frac{5}{2}}+9Ac^{16}x^{\frac{5}{2}}+135Bb^{2}c^{14}\sqrt{x}-90Abc^{15}\sqrt{x}\right)}{45c^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(13*(b*c^3)^(1/4)*B*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*arctan(1/2*sq
rt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 - 1/8*sqrt(2)*(13*
(b*c^3)^(1/4)*B*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^5 - 1/16*sqrt(2)*(13*(b*c^3)^(1/4)*B
*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/
c))/c^5 + 1/16*sqrt(2)*(13*(b*c^3)^(1/4)*B*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*log
(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/2*(B*b^3*sqrt(x) - A
*b^2*c*sqrt(x))/((c*x^2 + b)*c^4) + 2/45*(5*B*c^16*x^(9/2) - 18*B*b*c^15*x^
(5/2) + 9*A*c^16*x^(5/2) + 135*B*b^2*c^14*sqrt(x) - 90*A*b*c^15*sqrt(x))/c^
18
```

Mupad [B]
time = 0.22, size = 857, normalized size = 2.58

$$x^{5/2}\left(\frac{2A}{5c^2}-\frac{4Bb}{5c^3}\right)-\sqrt{x}\left(\frac{2b\left(\frac{2A}{c^2}-\frac{4Bb}{c^3}\right)}{c}+\frac{2Bb^2}{c^4}\right)+\frac{2Bx^{9/2}}{9c^2}+\frac{\sqrt{x}\left(\frac{Bb^3}{2}-\frac{Ab^2c}{2}\right)}{c^5x^2+bc^4}+\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81A^2b^4c^2-234ABb^5c+169B^2b^6\right)}{c^5}+\frac{(-b)^{5/4}(9Ac-13Bb)\left(13Bb^4-9Ab^3c\right)}{c^{21/4}}\right)(9Ac-13Bb)\,1i}{8c^{17/4}}+\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81A^2b^4c^2-234ABb^5c+169B^2b^6\right)}{c^5}-\frac{(-b)^{5/4}(9Ac-13Bb)\left(13Bb^4-9Ab^3c\right)}{c^{21/4}}\right)(9Ac-13Bb)\,1i}{8c^{17/4}}}{\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81A^2b^4c^2-234ABb^5c+169B^2b^6\right)}{c^5}+\frac{(-b)^{5/4}(9Ac-13Bb)\left(13Bb^4-9Ab^3c\right)}{c^{21/4}}\right)(9Ac-13Bb)}{8c^{17/4}}-\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81A^2b^4c^2-234ABb^5c+169B^2b^6\right)}{c^5}-\frac{(-b)^{5/4}(9Ac-13Bb)\left(13Bb^4-9Ab^3c\right)}{c^{21/4}}\right)(9Ac-13Bb)}{8c^{17/4}}}\right)(9Ac-13Bb)\,1i}{4c^{17/4}}-\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81A^2b^4c^2-234ABb^5c+169B^2b^6\right)}{c^5}-\frac{(-b)^{5/4}(9Ac-13Bb)\left(13Bb^4-9Ab^3c\right)1i}{c^{21/4}}\right)(9Ac-13Bb)}{8c^{17/4}}+\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81A^2b^4c^2-234ABb^5c+169B^2b^6\right)}{c^5}+\frac{(-b)^{5/4}(9Ac-13Bb)\left(13Bb^4-9Ab^3c\right)1i}{c^{21/4}}\right)(9Ac-13Bb)}{8c^{17/4}}}{\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81A^2b^4c^2-234ABb^5c+169B^2b^6\right)}{c^5}-\frac{(-b)^{5/4}(9Ac-13Bb)\left(13Bb^4-9Ab^3c\right)1i}{c^{21/4}}\right)(9Ac-13Bb)\,1i}{8c^{17/4}}-\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81A^2b^4c^2-234ABb^5c+169B^2b^6\right)}{c^5}+\frac{(-b)^{5/4}(9Ac-13Bb)\left(13Bb^4-9Ab^3c\right)1i}{c^{21/4}}\right)(9Ac-13Bb)\,1i}{8c^{17/4}}}\right)(9Ac-13Bb)}{4c^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(5/2)*((2*A)/(5*c^2) - (4*B*b)/(5*c^3)) - x^(1/2)*((2*b*((2*A)/c^2 - (4*B
*b)/c^3))/c + (2*B*b^2)/c^4) + (2*B*x^(9/2))/(9*c^2) + (x^(1/2)*((B*b^3)/2
- (A*b^2*c)/2))/(b*c^4 + c^5*x^2) + ((-b)^(5/4)*atan((((-b)^(5/4)*((x^(1/2)
*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 + ((-b)^(5/4)*(9*A*c -
 13*B*b)*(13*B*b^4 - 9*A*b^3*c))/c^(21/4))*(9*A*c - 13*B*b)*1i)/(8*c^(17/4)
) + ((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c
^5 - ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^3*c))/c^(21/4))*(9*A*c
- 13*B*b)*1i)/(8*c^(17/4)))/(((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^
4*c^2 - 234*A*B*b^5*c))/c^5 + ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*
b^3*c))/c^(21/4))*(9*A*c - 13*B*b))/(8*c^(17/4)) - ((-b)^(5/4)*((x^(1/2)*(1
69*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 - ((-b)^(5/4)*(9*A*c - 13
*B*b)*(13*B*b^4 - 9*A*b^3*c))/c^(21/4))*(9*A*c - 13*B*b))/(8*c^(17/4))))*(9
*A*c - 13*B*b)*1i)/(4*c^(17/4)) - ((-b)^(5/4)*atan((((-b)^(5/4)*((x^(1/2)*(
169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 - ((-b)^(5/4)*(9*A*c - 1
3*B*b)*(13*B*b^4 - 9*A*b^3*c)*1i)/c^(21/4))*(9*A*c - 13*B*b))/(8*c^(17/4))
```

```
+ ((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5
 + ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^3*c)*1i)/c^(21/4))*(9*A*c
 - 13*B*b))/(8*c^(17/4)))/(((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*
c^2 - 234*A*B*b^5*c))/c^5 - ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^
3*c)*1i)/c^(21/4))*(9*A*c - 13*B*b)*1i)/(8*c^(17/4)) - ((-b)^(5/4)*((x^(1/2
)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 + ((-b)^(5/4)*(9*A*c
- 13*B*b)*(13*B*b^4 - 9*A*b^3*c)*1i)/c^(21/4))*(9*A*c - 13*B*b)*1i)/(8*c^(1
7/4))))*(9*A*c - 13*B*b))/(4*c^(17/4))
```

3.196 $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=310

$$-\frac{(11bB-7Ac)x^{3/2}}{6c^3}+\frac{(11bB-7Ac)x^{7/2}}{14bc^2}-\frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)}-\frac{b^{3/4}(11bB-7Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{15/4}}+\frac{b^3}{}$$

```
[Out] -1/6*(-7*A*c+11*B*b)*x^(3/2)/c^3+1/14*(-7*A*c+11*B*b)*x^(7/2)/b/c^2-1/2*(-A
*c+B*b)*x^(11/2)/b/c/(c*x^2+b)-1/8*b^(3/4)*(-7*A*c+11*B*b)*arctan(1-c^(1/4)
*2^(1/2)*x^(1/2)/b^(1/4))/c^(15/4)*2^(1/2)+1/8*b^(3/4)*(-7*A*c+11*B*b)*arct
an(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(15/4)*2^(1/2)+1/16*b^(3/4)*(-7*A*c
+11*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(15/4)*2^(
1/2)-1/16*b^(3/4)*(-7*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1
/2)*x^(1/2))/c^(15/4)*2^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{b^{3/4}(11bB-7Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{15/4}}+\frac{b^{3/4}(11bB-7Ac)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,c^{15/4}}+\frac{b^{3/4}(11bB-7Ac)\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{15/4}}-\frac{b^{3/4}(11bB-7Ac)\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{15/4}}-\frac{x^{3/2}(11bB-7Ac)}{6c^3}+\frac{x^{7/2}(11bB-7Ac)}{14bc^2}-\frac{x^{11/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/6*((11*b*B - 7*A*c)*x^(3/2))/c^3 + ((11*b*B - 7*A*c)*x^(7/2))/(14*b*c^2)
 - ((b*B - A*c)*x^(11/2))/(2*b*c*(b + c*x^2)) - (b^(3/4)*(11*b*B - 7*A*c)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(15/4)) + (b^(3/
4)*(11*b*B - 7*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[
2]*c^(15/4)) + (b^(3/4)*(11*b*B - 7*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(15/4)) - (b^(3/4)*(11*b*B - 7*A*c)*
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(1
5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{x^{9/2}(A+Bx^2)}{(b+cx^2)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)} + \frac{\left(\frac{11bB}{2}-\frac{7Ac}{2}\right)\int \frac{x^{9/2}}{b+cx^2}\,dx}{2bc}$$

$$= \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)} - \frac{(11bB-7Ac)\int \frac{x^{5/2}}{b+cx^2}\,dx}{4c^2}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)} + \frac{(b(11bB-7Ac))\int}{4c^3}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)} + \frac{(b(11bB-7Ac))\text{Su}}{}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)} - \frac{(b(11bB-7Ac))\text{Su}}{}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)} + \frac{(b(11bB-7Ac))\text{Su}}{}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)} + \frac{b^{3/4}(11bB-7Ac)\,\text{l}}{}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)} - \frac{b^{3/4}(11bB-7Ac)\,\text{t}}{4}$$

Mathematica [A]
time = 0.51, size = 184, normalized size = 0.59

$$\frac{\frac{4c^{3/4}x^{3/2}(-77b^2B+bc(49A-44Bx^2)+4c^2x^2(7A+3Bx^2))}{b+cx^2} - 21\sqrt{2}\,b^{3/4}(11bB-7Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - 21\sqrt{2}\,b^{3/4}(11bB-7Ac)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{168c^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*c^(3/4)*x^(3/2)*(-77*b^2*B + b*c*(49*A - 44*B*x^2) + 4*c^2*x^2*(7*A + 3
*B*x^2)))/(b + c*x^2) - 21*Sqrt[2]*b^(3/4)*(11*b*B - 7*A*c)*ArcTan[(Sqrt[b]
 - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 21*Sqrt[2]*b^(3/4)*(11*b
```

```
*B - 7*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)
])/(168*c^(15/4))
```

Maple [A]
time = 0.50, size = 171, normalized size = 0.55

method	result
derivativedivides	$\frac{\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2(Ac-2Bb)x^{\frac{3}{2}}}{3}}{c^{3}}-\frac{2b\left(\frac{\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^{2}+b}+\frac{\left(\frac{7Ac}{4}-\frac{11Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{b}{c}\right.}\right.\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{c^{3}}$
default	$\frac{\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2(Ac-2Bb)x^{\frac{3}{2}}}{3}}{c^{3}}-\frac{2b\left(\frac{\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^{2}+b}+\frac{\left(\frac{7Ac}{4}-\frac{11Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{b}{c}\right.}\right.\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{c^{3}}$
risch	$\frac{2x^{\frac{3}{2}}\left(3Bc\,x^{2}+7Ac-14Bb\right)}{21c^{3}}+\frac{b\,x^{\frac{3}{2}}A}{2c^{2}\left(c\,x^{2}+b\right)}-\frac{b^{2}x^{\frac{3}{2}}B}{2c^{3}\left(c\,x^{2}+b\right)}-\frac{7b\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{16c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{7b\sqrt{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^3*(1/7*B*c*x^(7/2)+1/3*(A*c-2*B*b)*x^(3/2))-2*b/c^3*((-1/4*A*c+1/4*B*b)
*x^(3/2)/(c*x^2+b)+1/8*(7/4*A*c-11/4*B*b)/c/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c
)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(
1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)
*x^(1/2)-1)))
```

Maxima [A]
time = 0.49, size = 247, normalized size = 0.80

$$-\frac{(Bb^2-Abc)x^{\frac{3}{2}}}{2\left(c^4x^2+bc^3\right)}+\frac{\left(11\,Bb^2-7\,Abc\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{16\,c^3}+\frac{2\left(3\,Bcx^{\frac{7}{2}}-7\,(2\,Bb-Ac)x^{\frac{3}{2}}\right)}{21\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b^2 - A*b*c)*x^(3/2)/(c^4*x^2 + b*c^3) + 1/16*(11*B*b^2 - 7*A*b*c)*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))
/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqr
t(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4
)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b
^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c^3 + 2/21
*(3*B*c*x^(7/2) - 7*(2*B*b - A*c)*x^(3/2))/c^3
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 989 vs. $2(226) = 452$.
time = 2.79, size = 989, normalized size = 3.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(84*(c^4*x^2 + b*c^3)*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^
2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4)*arctan(
(sqrt((1771561*B^6*b^10 - 6764142*A*B^5*b^9*c + 10761135*A^2*B^4*b^8*c^2 -
9130660*A^3*B^3*b^7*c^3 + 4357815*A^4*B^2*b^6*c^4 - 1109262*A^5*B*b^5*c^5 +
 117649*A^6*b^4*c^6)*x - (14641*B^4*b^7*c^7 - 37268*A*B^3*b^6*c^8 + 35574*A
^2*B^2*b^5*c^9 - 15092*A^3*B*b^4*c^10 + 2401*A^4*b^3*c^11)*sqrt(-(14641*B^4
*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 24
01*A^4*b^3*c^4)/c^15))*c^4*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2
*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4) + (1331*
B^3*b^5*c^4 - 2541*A*B^2*b^4*c^5 + 1617*A^2*B*b^3*c^6 - 343*A^3*b^2*c^7)*sq
rt(x)*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*
A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4))/(14641*B^4*b^7 - 37268*A*B^3
*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)) -
 21*(c^4*x^2 + b*c^3)*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*
b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4)*log(c^11*(-(1
4641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*
c^3 + 2401*A^4*b^3*c^4)/c^15)^(3/4) - (1331*B^3*b^5 - 2541*A*B^2*b^4*c + 16
17*A^2*B*b^3*c^2 - 343*A^3*b^2*c^3)*sqrt(x)) + 21*(c^4*x^2 + b*c^3)*(-(1464
1*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3
 + 2401*A^4*b^3*c^4)/c^15)^(1/4)*log(-c^11*(-(14641*B^4*b^7 - 37268*A*B^3*b
^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15
)^(3/4) - (1331*B^3*b^5 - 2541*A*B^2*b^4*c + 1617*A^2*B*b^3*c^2 - 343*A^3*b
^2*c^3)*sqrt(x)) + 4*(12*B*c^2*x^5 - 4*(11*B*b*c - 7*A*c^2)*x^3 - 7*(11*B*b
^2 - 7*A*b*c)*x)*sqrt(x))/(c^4*x^2 + b*c^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 299, normalized size = 0.96

$$-\frac{Bb^{2}x^{\frac{3}{2}}-Abcx^{\frac{3}{2}}}{2\,(cx^{2}+b)c^{3}}+\frac{\sqrt{2}\left(11\,(bc^{3})^{\frac{3}{4}}Bb-7\,(bc^{3})^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^{6}}+\frac{\sqrt{2}\left(11\,(bc^{3})^{\frac{3}{4}}Bb-7\,(bc^{3})^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^{6}}-\frac{\sqrt{2}\left(11\,(bc^{3})^{\frac{3}{4}}Bb-7\,(bc^{3})^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^{6}}+\frac{\sqrt{2}\left(11\,(bc^{3})^{\frac{3}{4}}Bb-7\,(bc^{3})^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^{6}}+\frac{2\left(3\,Bc^{12}x^{\frac{7}{2}}-14\,Bbc^{11}x^{\frac{3}{2}}+7\,Ac^{12}x^{\frac{3}{2}}\right)}{21\,c^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^2*x^(3/2) - A*b*c*x^(3/2))/((c*x^2 + b)*c^3) + 1/8*sqrt(2)*(11*(b
*c^3)^(3/4)*B*b - 7*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) + 2*sqrt(x))/(b/c)^(1/4))/c^6 + 1/8*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 7*(
b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/
c)^(1/4))/c^6 - 1/16*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 7*(b*c^3)^(3/4)*A*c)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^6 + 1/16*sqrt(2)*(11*(b*c
^3)^(3/4)*B*b - 7*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/c^6 + 2/21*(3*B*c^12*x^(7/2) - 14*B*b*c^11*x^(3/2) + 7*A*c^12*x
^(3/2))/c^14
```

Mupad [B]
time = 0.17, size = 127, normalized size = 0.41

$$x^{3/2}\left(\frac{2\,A}{3\,c^2}-\frac{4\,B\,b}{3\,c^3}\right)+\frac{2\,B\,x^{7/2}}{7\,c^2}-\frac{x^{3/2}\left(\frac{B\,b^2}{2}-\frac{A\,b\,c}{2}\right)}{c^4\,x^2+b\,c^3}+\frac{{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)\,\left(7\,A\,c-11\,B\,b\right)}{4\,c^{15/4}}+\frac{{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-b\right)}^{1/4}}\right)\,\left(7\,A\,c-11\,B\,b\right)\,1{}\mathrm{i}}{4\,c^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(3/2)*((2*A)/(3*c^2) - (4*B*b)/(3*c^3)) + (2*B*x^(7/2))/(7*c^2) - (x^(3/2
)*((B*b^2)/2 - (A*b*c)/2))/(b*c^3 + c^4*x^2) + ((-b)^(3/4)*atan((c^(1/4)*x^
(1/2))/(-b)^(1/4))*(7*A*c - 11*B*b))/(4*c^(15/4)) + ((-b)^(3/4)*atan((c^(1/
4)*x^(1/2)*1i)/(-b)^(1/4))*(7*A*c - 11*B*b)*1i)/(4*c^(15/4))
```

3.197 $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=310

$$-\frac{(9bB-5Ac)\sqrt{x}}{2c^3}+\frac{(9bB-5Ac)x^{5/2}}{10bc^2}-\frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)}-\frac{\sqrt[4]{b}\,(9bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{13/4}}+\frac{\sqrt[4]{b}}{}$$

```
[Out] 1/10*(-5*A*c+9*B*b)*x^(5/2)/b/c^2-1/2*(-A*c+B*b)*x^(9/2)/b/c/(c*x^2+b)-1/8*
b^(1/4)*(-5*A*c+9*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(13/4)*2
^(1/2)+1/8*b^(1/4)*(-5*A*c+9*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))
/c^(13/4)*2^(1/2)-1/16*b^(1/4)*(-5*A*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2)+1/16*b^(1/4)*(-5*A*c+9*B*b)*ln(b^
(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2)-1/2*(-5*A
*c+9*B*b)*x^(1/2)/c^3
```

Rubi [A]
time = 0.16, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\sqrt[4]{b}\,(9bB-5Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{13/4}}+\frac{\sqrt[4]{b}\,(9bB-5Ac)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,c^{13/4}}-\frac{\sqrt[4]{b}\,(9bB-5Ac)\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{13/4}}+\frac{\sqrt[4]{b}\,(9bB-5Ac)\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{13/4}}-\frac{\sqrt{x}\,(9bB-5Ac)}{2c^3}+\frac{x^{5/2}(9bB-5Ac)}{10bc^2}-\frac{x^{9/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((9*b*B - 5*A*c)*Sqrt[x])/c^3 + ((9*b*B - 5*A*c)*x^(5/2))/(10*b*c^2) -
 ((b*B - A*c)*x^(9/2))/(2*b*c*(b + c*x^2)) - (b^(1/4)*(9*b*B - 5*A*c)*ArcTa
n[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(13/4)) + (b^(1/4)*(
9*b*B - 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^
(13/4)) - (b^(1/4)*(9*b*B - 5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sq
rt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4)) + (b^(1/4)*(9*b*B - 5*A*c)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{x^{7/2}(A+Bx^2)}{(b+cx^2)^2}\,dx$$
$$= -\frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)} + \frac{\left(\frac{9bB}{2}-\frac{5Ac}{2}\right)\int \frac{x^{7/2}}{b+cx^2}\,dx}{2bc}$$
$$= \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)} - \frac{(9bB-5Ac)\int \frac{x^{3/2}}{b+cx^2}\,dx}{4c^2}$$
$$= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3} + \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)} + \frac{(b(9bB-5Ac))\int \frac{}{\sqrt{x}\,(}}{4c^3}$$
$$= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3} + \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)} + \frac{(b(9bB-5Ac))\mathrm{Subst}(}{2c^3}$$
$$= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3} + \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)} + \frac{\left(\sqrt{b}\,(9bB-5Ac)\right)\mathrm{Su}}{}$$
$$= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3} + \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)} + \frac{\left(\sqrt{b}\,(9bB-5Ac)\right)\mathrm{Su}}{}$$
$$= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3} + \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)} - \frac{\sqrt[4]{b}\,(9bB-5Ac)\log\left(\right.}{8}$$
$$= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3} + \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)} - \frac{\sqrt[4]{b}\,(9bB-5Ac)\tan^{-1}}{4\sqrt{2}\,c}$$

Mathematica [A]
time = 0.51, size = 183, normalized size = 0.59

$$\frac{\frac{4\sqrt[4]{c}\,\sqrt{x}\,(-45b^2B+bc(25A-36Bx^2)+4c^2x^2(5A+Bx^2))}{b+cx^2} - 5\sqrt{2}\,\sqrt[4]{b}\,(9bB-5Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right) + 5\sqrt{2}\,\sqrt[4]{b}\,(9bB-5Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{40c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*c^(1/4)*Sqrt[x]*(-45*b^2*B + b*c*(25*A - 36*B*x^2) + 4*c^2*x^2*(5*A + B
*x^2)))/(b + c*x^2) - 5*Sqrt[2]*b^(1/4)*(9*b*B - 5*A*c)*ArcTan[(Sqrt[b] - S
qrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 5*Sqrt[2]*b^(1/4)*(9*b*B - 5
*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(40
*c^(13/4))
```

Maple [A]
time = 0.41, size = 171, normalized size = 0.55

method	result
derivativedivides	$\frac{\frac{2Bx^{\frac{5}{2}}c}{5}+2Ac\sqrt{x}-4Bb\sqrt{x}}{c^3}-\frac{2b\left(\frac{\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{cx^2+b}+\frac{(5Ac-9Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+\right.}{32b}\right.}{c^3}$
default	$\frac{\frac{2Bx^{\frac{5}{2}}c}{5}+2Ac\sqrt{x}-4Bb\sqrt{x}}{c^3}-\frac{2b\left(\frac{\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{cx^2+b}+\frac{(5Ac-9Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+\right.}{32b}\right.}{c^3}$
risch	$\frac{2(Bcx^2+5Ac-10Bb)\sqrt{x}}{5c^3}+\frac{b\sqrt{x}\,A}{2c^2(cx^2+b)}-\frac{b^2\sqrt{x}\,B}{2c^3(cx^2+b)}-\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8c^2}-\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^3*(1/5*B*x^(5/2)*c+A*c*x^(1/2)-2*B*b*x^(1/2))-2*b/c^3*((-1/4*A*c+1/4*B*
b)*x^(1/2)/(c*x^2+b)+1/32*(5*A*c-9*B*b)*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)^
(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/
2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)-1)))
```

Maxima [A]
time = 0.54, size = 271, normalized size = 0.87

$$-\frac{(Bb^2-Abc)\sqrt{x}}{2(c^4x^2+bc^3)}+\frac{\left(\frac{2\sqrt{2}(9Bb-5Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(9Bb-5Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(9Bb-5Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}(9Bb-5Ac)\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)b}{16c^3}+\frac{2\left(Bcx^{\frac{5}{2}}-5(2Bb-Ac)\sqrt{x}\right)}{5c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b^2 - A*b*c)*sqrt(x)/(c^4*x^2 + b*c^3) + 1/16*(2*sqrt(2)*(9*B*b - 5
*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt
(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(9*B*b - 5*A
```

```
*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(
sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(9*B*b - 5*A*c)
*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4
)) - sqrt(2)*(9*B*b - 5*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)
*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b/c^3 + 2/5*(B*c*x^(5/2) - 5*(2*B*b - A*c)
*sqrt(x))/c^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 748 vs. $2(226) = 452$.
time = 2.04, size = 748, normalized size = 2.41

$$-\frac{20\,(c^4x^2+bc^3)\left(-\frac{6561B^4b^5-14580AB^3b^4c+12150A^2B^2b^3c^2-4500A^3Bb^2c^3+625A^4bc^4}{c^{13}}\right)^{\frac14}\arctan\left(\frac{\sqrt{c^6\sqrt{-\frac{6561B^4b^5-14580AB^3b^4c+12150A^2B^2b^3c^2-4500A^3Bb^2c^3+625A^4bc^4}{c^{13}}}+(81B^2b^2-90ABbc+25A^2c^2)x}\,c^{10}\left(-\frac{6561B^4b^5-14580AB^3b^4c+12150A^2B^2b^3c^2-4500A^3Bb^2c^3+625A^4bc^4}{c^{13}}\right)^{\frac34}+(9Bbc^{10}-5Ac^{11})\sqrt{x}\left(-\frac{6561B^4b^5-14580AB^3b^4c+12150A^2B^2b^3c^2-4500A^3Bb^2c^3+625A^4bc^4}{c^{13}}\right)^{\frac34}}{6561B^4b^5-14580AB^3b^4c+12150A^2B^2b^3c^2-4500A^3Bb^2c^3+625A^4bc^4}\right)+5\,(c^4x^2+bc^3)\left(-\frac{6561B^4b^5-14580AB^3b^4c+12150A^2B^2b^3c^2-4500A^3Bb^2c^3+625A^4bc^4}{c^{13}}\right)^{\frac14}\log\left(c^3\left(-\frac{6561B^4b^5-14580AB^3b^4c+12150A^2B^2b^3c^2-4500A^3Bb^2c^3+625A^4bc^4}{c^{13}}\right)^{\frac14}-(9Bb-5Ac)\sqrt{x}\right)-5\,(c^4x^2+bc^3)\left(-\frac{6561B^4b^5-14580AB^3b^4c+12150A^2B^2b^3c^2-4500A^3Bb^2c^3+625A^4bc^4}{c^{13}}\right)^{\frac14}\log\left(-c^3\left(-\frac{6561B^4b^5-14580AB^3b^4c+12150A^2B^2b^3c^2-4500A^3Bb^2c^3+625A^4bc^4}{c^{13}}\right)^{\frac14}-(9Bb-5Ac)\sqrt{x}\right)-4\,(4Bc^2x^4-45Bb^2+25Abc-4\,(9Bbc-5Ac^2)x^2)\sqrt{x}}{40\,(c^4x^2+bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*(c^4*x^2 + b*c^3)*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2
*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4)*arctan((sqrt
(c^6*sqrt(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500
*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13) + (81*B^2*b^2 - 90*A*B*b*c + 25*A^2*c
^2)*x)*c^10*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4
500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(3/4) + (9*B*b*c^10 - 5*A*c^11)*sq
rt(x)*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^
3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(3/4))/(6561*B^4*b^5 - 14580*A*B^3*b^4*c
 + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)) + 5*(c^4*x^
2 + b*c^3)*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 45
00*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4)*log(c^3*(-(6561*B^4*b^5 - 145
80*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4
)/c^13)^(1/4) - (9*B*b - 5*A*c)*sqrt(x)) - 5*(c^4*x^2 + b*c^3)*(-(6561*B^4*
b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*
A^4*b*c^4)/c^13)^(1/4)*log(-c^3*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150
*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4) - (9*B*b
 - 5*A*c)*sqrt(x)) - 4*(4*B*c^2*x^4 - 45*B*b^2 + 25*A*b*c - 4*(9*B*b*c - 5*
A*c^2)*x^2)*sqrt(x))/(c^4*x^2 + b*c^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 298, normalized size = 0.96

$$\frac{\sqrt{2}\left(9\left(bc^{3}\right)^{\frac{1}{4}}Bb-5\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^{4}}+\frac{\sqrt{2}\left(9\left(bc^{3}\right)^{\frac{1}{4}}Bb-5\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^{4}}+\frac{\sqrt{2}\left(9\left(bc^{3}\right)^{\frac{1}{4}}Bb-5\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16c^{4}}-\frac{\sqrt{2}\left(9\left(bc^{3}\right)^{\frac{1}{4}}Bb-5\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16c^{4}}-\frac{Bb^{2}\sqrt{x}-Abc\sqrt{x}}{2\left(cx^{2}+b\right)c^{3}}+\frac{2\left(Bc^{8}x^{\frac{5}{2}}-10Bbc^{7}\sqrt{x}+5Ac^{8}\sqrt{x}\right)}{5c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 + 1/8*sqrt(2)*(9*(b*c^3)
^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
- 2*sqrt(x))/(b/c)^(1/4))/c^4 + 1/16*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - 5*(b*c^
3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/16*s
qrt(2)*(9*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/
c)^(1/4) + x + sqrt(b/c))/c^4 - 1/2*(B*b^2*sqrt(x) - A*b*c*sqrt(x))/((c*x^2
 + b)*c^3) + 2/5*(B*c^8*x^(5/2) - 10*B*b*c^7*sqrt(x) + 5*A*c^8*sqrt(x))/c^1
0
```

Mupad [B]
time = 0.26, size = 823, normalized size = 2.65

$$\sqrt{x}\left(\frac{2A}{c^2}-\frac{4Bb}{c^3}\right)+\frac{2Bx^{5/2}}{5c^2}-\frac{\sqrt{x}\left(\frac{Bb^2}{2}-\frac{Abc}{2}\right)}{c^4x^2+bc^3}+\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2b^2c^2-90ABb^3c+81B^2b^4\right)}{c^3}-\frac{(-b)^{1/4}(5Ac-9Bb)\left(72Bb^3-40Ab^2c\right)}{8c^{13/4}}\right)(5Ac-9Bb)\,1i}{8c^{13/4}}+\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2b^2c^2-90ABb^3c+81B^2b^4\right)}{c^3}+\frac{(-b)^{1/4}(5Ac-9Bb)\left(72Bb^3-40Ab^2c\right)}{8c^{13/4}}\right)(5Ac-9Bb)\,1i}{8c^{13/4}}}{\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2b^2c^2-90ABb^3c+81B^2b^4\right)}{c^3}-\frac{(-b)^{1/4}(5Ac-9Bb)\left(72Bb^3-40Ab^2c\right)}{8c^{13/4}}\right)(5Ac-9Bb)}{8c^{13/4}}-\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2b^2c^2-90ABb^3c+81B^2b^4\right)}{c^3}+\frac{(-b)^{1/4}(5Ac-9Bb)\left(72Bb^3-40Ab^2c\right)}{8c^{13/4}}\right)(5Ac-9Bb)}{8c^{13/4}}}\right)(5Ac-9Bb)\,1i}{4c^{13/4}}+\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2b^2c^2-90ABb^3c+81B^2b^4\right)}{c^3}-\frac{(-b)^{1/4}(5Ac-9Bb)\left(72Bb^3-40Ab^2c\right)1i}{8c^{13/4}}\right)(5Ac-9Bb)}{8c^{13/4}}+\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2b^2c^2-90ABb^3c+81B^2b^4\right)}{c^3}+\frac{(-b)^{1/4}(5Ac-9Bb)\left(72Bb^3-40Ab^2c\right)1i}{8c^{13/4}}\right)(5Ac-9Bb)}{8c^{13/4}}}{\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2b^2c^2-90ABb^3c+81B^2b^4\right)}{c^3}-\frac{(-b)^{1/4}(5Ac-9Bb)\left(72Bb^3-40Ab^2c\right)1i}{8c^{13/4}}\right)(5Ac-9Bb)\,1i}{8c^{13/4}}-\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25A^2b^2c^2-90ABb^3c+81B^2b^4\right)}{c^3}+\frac{(-b)^{1/4}(5Ac-9Bb)\left(72Bb^3-40Ab^2c\right)1i}{8c^{13/4}}\right)(5Ac-9Bb)\,1i}{8c^{13/4}}}\right)(5Ac-9Bb)}{4c^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(1/2)*((2*A)/c^2 - (4*B*b)/c^3) + (2*B*x^(5/2))/(5*c^2) - (x^(1/2)*((B*b^
2)/2 - (A*b*c)/2))/(b*c^3 + c^4*x^2) + ((-b)^(1/4)*atan((((-b)^(1/4)*((x^(1
/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 - ((-b)^(1/4)*(5*A*c
- 9*B*b)*(72*B*b^3 - 40*A*b^2*c))/(8*c^(13/4)))*(5*A*c - 9*B*b)*1i)/(8*c^(1
3/4)) + ((-b)^(1/4)*((x^(1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))
/c^3 + ((-b)^(1/4)*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c))/(8*c^(13/4)))*(
5*A*c - 9*B*b)*1i)/(8*c^(13/4)))/(((-b)^(1/4)*((x^(1/2)*(81*B^2*b^4 + 25*A^
2*b^2*c^2 - 90*A*B*b^3*c))/c^3 - ((-b)^(1/4)*(5*A*c - 9*B*b)*(72*B*b^3 - 40
*A*b^2*c))/(8*c^(13/4)))*(5*A*c - 9*B*b))/(8*c^(13/4)) - ((-b)^(1/4)*((x^(1
/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 + ((-b)^(1/4)*(5*A*c
- 9*B*b)*(72*B*b^3 - 40*A*b^2*c))/(8*c^(13/4)))*(5*A*c - 9*B*b))/(8*c^(13/4
))))*(5*A*c - 9*B*b)*1i)/(4*c^(13/4)) + ((-b)^(1/4)*atan((((-b)^(1/4)*((x^(
1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 - ((-b)^(1/4)*(5*A*c
 - 9*B*b)*(72*B*b^3 - 40*A*b^2*c)*1i)/(8*c^(13/4)))*(5*A*c - 9*B*b))/(8*c^(
13/4)) + ((-b)^(1/4)*((x^(1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c)
)/c^3 + ((-b)^(1/4)*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c)*1i)/(8*c^(13/4)
))*(5*A*c - 9*B*b))/(8*c^(13/4)))/(((-b)^(1/4)*((x^(1/2)*(81*B^2*b^4 + 25*A
^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 - ((-b)^(1/4)*(5*A*c - 9*B*b)*(72*B*b^3 - 4
```

```
0*A*b^2*c)*1i)/(8*c^(13/4)))*(5*A*c - 9*B*b)*1i)/(8*c^(13/4)) - ((-b)^(1/4)
*((x^(1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 + ((-b)^(1/4)*
(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c)*1i)/(8*c^(13/4)))*(5*A*c - 9*B*b)*1
i)/(8*c^(13/4))))*(5*A*c - 9*B*b))/(4*c^(13/4))
```

3.198 $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=289

$$\frac{(7bB-3Ac)x^{3/2}}{6bc^2}-\frac{(bB-Ac)x^{7/2}}{2bc\,(b+cx^2)}+\frac{(7bB-3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,\sqrt[4]{b}\,c^{11/4}}-\frac{(7bB-3Ac)\tan^{-1}\left(1+\sqrt{2}\right.}{4\sqrt{2}\,\sqrt[4]{b}\,c^{11/4}}$$

```
[Out] 1/6*(-3*A*c+7*B*b)*x^(3/2)/b/c^2-1/2*(-A*c+B*b)*x^(7/2)/b/c/(c*x^2+b)+1/8*(
-3*A*c+7*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(11/4)*2^
(1/2)-1/8*(-3*A*c+7*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/
c^(11/4)*2^(1/2)-1/16*(-3*A*c+7*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2
^(1/2)*x^(1/2))/b^(1/4)/c^(11/4)*2^(1/2)+1/16*(-3*A*c+7*B*b)*ln(b^(1/2)+x*c
^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(11/4)*2^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{(7bB-3Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{11/4}}-\frac{(7bB-3Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{11/4}}-\frac{(7bB-3Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{11/4}}+\frac{(7bB-3Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{11/4}}+\frac{x^{3/2}(7bB-3Ac)}{6bc^2}-\frac{x^{7/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((7*b*B - 3*A*c)*x^(3/2))/(6*b*c^2) - ((b*B - A*c)*x^(7/2))/(2*b*c*(b + c*x
^2)) + ((7*b*B - 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*S
qrt[2]*b^(1/4)*c^(11/4)) - ((7*b*B - 3*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[x])/b^(1/4)])/(4*Sqrt[2]*b^(1/4)*c^(11/4)) - ((7*b*B - 3*A*c)*Log[Sqrt[b]
 - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(11/4
)) + ((7*b*B - 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(8*Sqrt[2]*b^(1/4)*c^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{x^{5/2}(A+Bx^2)}{(b+cx^2)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{7/2}}{2bc\,(b+cx^2)} + \frac{\left(\frac{7bB}{2}-\frac{3Ac}{2}\right)\int \frac{x^{5/2}}{b+cx^2}\,dx}{2bc}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\,(b+cx^2)} - \frac{(7bB-3Ac)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4c^2}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\,(b+cx^2)} - \frac{(7bB-3Ac)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^2}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\,(b+cx^2)} + \frac{(7bB-3Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right.}{4c^{5/2}}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\,(b+cx^2)} - \frac{(7bB-3Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx\right.}{8c^3}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\,(b+cx^2)} - \frac{(7bB-3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{8\sqrt{2}\sqrt[4]{b}\,c^{11/4}}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\,(b+cx^2)} + \frac{(7bB-3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{11/4}} - \cdots$$

Mathematica [A]

time = 0.51, size = 162, normalized size = 0.56

$$\frac{\frac{4c^{3/4}x^{3/2}(7bB-3Ac+4Bcx^2)}{b+cx^2}+\frac{3\sqrt{2}\,(7bB-3Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)}{\sqrt[4]{b}}+\frac{3\sqrt{2}\,(7bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt[4]{b}}}{24c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*c^(3/4)*x^(3/2)*(7*b*B - 3*A*c + 4*B*c*x^2))/(b + c*x^2) + (3*Sqrt[2]*(
7*b*B - 3*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
])])/b^(1/4) + (3*Sqrt[2]*(7*b*B - 3*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*
Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/b^(1/4))/(24*c^(11/4))
```

Maple [A]

time = 0.41, size = 153, normalized size = 0.53

method	result
derivativedivides	$\frac{2Bx^{\frac{3}{2}}}{3c^2}+\frac{\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)x^{\frac{3}{2}}}{cx^2+b}+\frac{\left(-\frac{7Bb}{4}+\frac{3Ac}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^2}$
default	$\frac{2Bx^{\frac{3}{2}}}{3c^2}+\frac{\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)x^{\frac{3}{2}}}{cx^2+b}+\frac{\left(-\frac{7Bb}{4}+\frac{3Ac}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^2}$
risch	$\frac{2Bx^{\frac{3}{2}}}{3c^2}-\frac{x^{\frac{3}{2}}A}{2c(cx^2+b)}+\frac{x^{\frac{3}{2}}Bb}{2c^2(cx^2+b)}-\frac{7\sqrt{2}\,Bb\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{16c^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{7\sqrt{2}\,Bb\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{8c^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*B*x^(3/2)/c^2+2/c^2*((-1/4*A*c+1/4*B*b)*x^(3/2)/(c*x^2+b)+1/8*(-7/4*B*b
+3/4*A*c)/c/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1
/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1
/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.49, size = 223, normalized size = 0.77

$$\frac{(Bb-Ac)x^{\frac{3}{2}}}{2\,(c^3x^2+bc^2)}+\frac{2\,Bx^{\frac{3}{2}}}{3\,c^2}-\frac{(7\,Bb-3\,Ac)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{16\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b - A*c)*x^(3/2)/(c^3*x^2 + b*c^2) + 2/3*B*x^(3/2)/c^2 - 1/16*(7*B*b
 - 3*A*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c
)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(s
qrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1
/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(
-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/
c^2
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(209) = 418$.
time = 1.69, size = 925, normalized size = 3.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(c^3*x^2 + b*c^2)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B
^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4)*arctan((sqrt((11
7649*B^6*b^6 - 302526*A*B^5*b^5*c + 324135*A^2*B^4*b^4*c^2 - 185220*A^3*B^3
*b^3*c^3 + 59535*A^4*B^2*b^2*c^4 - 10206*A^5*B*b*c^5 + 729*A^6*c^6)*x - (24
01*B^4*b^5*c^5 - 4116*A*B^3*b^4*c^6 + 2646*A^2*B^2*b^3*c^7 - 756*A^3*B*b^2*
c^8 + 81*A^4*b*c^9)*sqrt(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b
^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11)))*c^3*(-(2401*B^4*b^4 - 411
6*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^1
1))^(1/4) + (343*B^3*b^3*c^3 - 441*A*B^2*b^2*c^4 + 189*A^2*B*b*c^5 - 27*A^3
*c^6)*sqrt(x)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 7
56*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4))/(2401*B^4*b^4 - 4116*A*B^3*b^
3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)) - 3*(c^3*x^2 +
b*c^2)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*
B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4)*log(b*c^8*(-(2401*B^4*b^4 - 4116*A*B^
3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(3
/4) - (343*B^3*b^3 - 441*A*B^2*b^2*c + 189*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt(x
)) + 3*(c^3*x^2 + b*c^2)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*
b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4)*log(-b*c^8*(-(2401*
```

```
B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^
4*c^4)/(b*c^11))^(3/4) - (343*B^3*b^3 - 441*A*B^2*b^2*c + 189*A^2*B*b*c^2 -
 27*A^3*c^3)*sqrt(x)) - 4*(4*B*c*x^3 + (7*B*b - 3*A*c)*x)*sqrt(x))/(c^3*x^2
 + b*c^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 283, normalized size = 0.98

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,c^2}+\frac{Bbx^{\frac{3}{2}}-Acx^{\frac{3}{2}}}{2\,(cx^2+b)c^2}-\frac{\sqrt{2}\left(7\,(bc^3)^{\frac{3}{4}}\,Bb-3\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^5}-\frac{\sqrt{2}\left(7\,(bc^3)^{\frac{3}{4}}\,Bb-3\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^5}+\frac{\sqrt{2}\left(7\,(bc^3)^{\frac{3}{4}}\,Bb-3\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^5}-\frac{\sqrt{2}\left(7\,(bc^3)^{\frac{3}{4}}\,Bb-3\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/c^2 + 1/2*(B*b*x^(3/2) - A*c*x^(3/2))/((c*x^2 + b)*c^2) - 1/8
*sqrt(2)*(7*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^5) - 1/8*sqrt(2)*(7*(b*c^3
)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 - 2*sqrt(x))/(b/c)^(1/4))/(b*c^5) + 1/16*sqrt(2)*(7*(b*c^3)^(3/4)*B*b - 3*
(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5)
 - 1/16*sqrt(2)*(7*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sq
rt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5)
```

Mupad [**B**]
time = 0.25, size = 106, normalized size = 0.37

$$\frac{2\,B\,x^{3/2}}{3\,c^2}-\frac{x^{3/2}\,\left(\frac{A\,c}{2}-\frac{B\,b}{2}\right)}{c^3\,x^2+b\,c^2}+\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)\,\left(3\,A\,c-7\,B\,b\right)}{4\,{\left(-b\right)}^{1/4}\,c^{11/4}}+\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-b\right)}^{1/4}}\right)\,\left(3\,A\,c-7\,B\,b\right)\,1{}\mathrm{i}}{4\,{\left(-b\right)}^{1/4}\,c^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (2*B*x^(3/2))/(3*c^2) - (x^(3/2)*((A*c)/2 - (B*b)/2))/(b*c^2 + c^3*x^2) + (
atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(3*A*c - 7*B*b))/(4*(-b)^(1/4)*c^(11/4))
 + (atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(3*A*c - 7*B*b)*1i)/(4*(-b)^(1/4)
*c^(11/4))
```

3.199 $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=289

$$\frac{(5bB-Ac)\sqrt{x}}{2bc^2}-\frac{(bB-Ac)x^{5/2}}{2bc\,(b+cx^2)}+\frac{(5bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{9/4}}-\frac{(5bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{4\sqrt{2}\,b^{3/4}c^{9/4}}$$

```
[Out] -1/2*(-A*c+B*b)*x^(5/2)/b/c/(c*x^2+b)+1/8*(-A*c+5*B*b)*arctan(1-c^(1/4)*2^(
1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(9/4)*2^(1/2)-1/8*(-A*c+5*B*b)*arctan(1+c^(
1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(9/4)*2^(1/2)+1/16*(-A*c+5*B*b)*ln(
b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(9/4)*2^(1/2)-
1/16*(-A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(
3/4)/c^(9/4)*2^(1/2)+1/2*(-A*c+5*B*b)*x^(1/2)/b/c^2
```

Rubi [A]
time = 0.15, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{(5bB-Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{9/4}}-\frac{(5bB-Ac)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{3/4}c^{9/4}}+\frac{(5bB-Ac)\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{9/4}}-\frac{(5bB-Ac)\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{9/4}}+\frac{\sqrt{x}\,(5bB-Ac)}{2bc^2}-\frac{x^{5/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((5*b*B - A*c)*Sqrt[x])/(2*b*c^2) - ((b*B - A*c)*x^(5/2))/(2*b*c*(b + c*x^2
)) + ((5*b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[
2]*b^(3/4)*c^(9/4)) - ((5*b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)])/(4*Sqrt[2]*b^(3/4)*c^(9/4)) + ((5*b*B - A*c)*Log[Sqrt[b] - Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(3/4)*c^(9/4)) - ((5*b*
B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqr
t[2]*b^(3/4)*c^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{x^{3/2}(A+Bx^2)}{(b+cx^2)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{5/2}}{2bc\,(b+cx^2)} + \frac{\left(\frac{5bB}{2}-\frac{Ac}{2}\right)\int \frac{x^{3/2}}{b+cx^2}\,dx}{2bc}$$

$$= \frac{(5bB-Ac)\sqrt{x}}{2bc^2} - \frac{(bB-Ac)x^{5/2}}{2bc\,(b+cx^2)} - \frac{(5bB-Ac)\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{4c^2}$$

$$= \frac{(5bB-Ac)\sqrt{x}}{2bc^2} - \frac{(bB-Ac)x^{5/2}}{2bc\,(b+cx^2)} - \frac{(5bB-Ac)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^2}$$

$$= \frac{(5bB-Ac)\sqrt{x}}{2bc^2} - \frac{(bB-Ac)x^{5/2}}{2bc\,(b+cx^2)} - \frac{(5bB-Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4\sqrt{b}\,c^2}$$

$$= \frac{(5bB-Ac)\sqrt{x}}{2bc^2} - \frac{(bB-Ac)x^{5/2}}{2bc\,(b+cx^2)} - \frac{(5bB-Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x\right.}{8\sqrt{b}\,c^{5/2}}$$

$$= \frac{(5bB-Ac)\sqrt{x}}{2bc^2} - \frac{(bB-Ac)x^{5/2}}{2bc\,(b+cx^2)} + \frac{(5bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{}\right.}{8\sqrt{2}\,b^{3/4}c^{9/4}}$$

$$= \frac{(5bB-Ac)\sqrt{x}}{2bc^2} - \frac{(bB-Ac)x^{5/2}}{2bc\,(b+cx^2)} + \frac{(5bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{9/4}} - \frac{(5b}{}$$

Mathematica [A]

time = 0.50, size = 161, normalized size = 0.56

$$\frac{\frac{4\sqrt[4]{c}\,\sqrt{x}\,(5bB-Ac+4Bcx^2)}{b+cx^2}+\frac{\sqrt{2}\,(5bB-Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)}{b^{3/4}}-\frac{\sqrt{2}\,(5bB-Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{b^{3/4}}}{8c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*c^(1/4)*Sqrt[x]*(5*b*B - A*c + 4*B*c*x^2))/(b + c*x^2) + (Sqrt[2]*(5*b*
B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/b
^(3/4) - (Sqrt[2]*(5*b*B - A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(
Sqrt[b] + Sqrt[c]*x)])/b^(3/4))/(8*c^(9/4))
```

Maple [A]
time = 0.41, size = 152, normalized size = 0.53

method	result
derivativedivides	$\frac{2B\sqrt{x}}{c^2}+\frac{\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(Ac-5Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2}{16b}}{c^2}$
default	$\frac{2B\sqrt{x}}{c^2}+\frac{\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(Ac-5Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2}{16b}}{c^2}$
risch	$\frac{2B\sqrt{x}}{c^2}-\frac{\sqrt{x}\,A}{2c(c\,x^2+b)}+\frac{\sqrt{x}\,Bb}{2c^2(c\,x^2+b)}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8cb}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}}{\left(\frac{b}{c}\right)}\right.}{8cb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*B/c^2*x^(1/2)+2/c^2*((-1/4*A*c+1/4*B*b)*x^(1/2)/(c*x^2+b)+1/32*(A*c-5*B*b
)*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-
(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1
/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.53, size = 250, normalized size = 0.87

$$\frac{(Bb-Ac)\sqrt{x}}{2\,(c^3x^2+bc^2)}+\frac{2\,B\sqrt{x}}{c^2}-\frac{\frac{2\sqrt{2}\,(5\,Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,(5\,Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,(5\,Bb-Ac)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,(5\,Bb-Ac)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{16\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b - A*c)*sqrt(x)/(c^3*x^2 + b*c^2) + 2*B*sqrt(x)/c^2 - 1/16*(2*sqrt(
2)*(5*B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sq
rt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(
5*B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(
x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(5*B*b
 - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)
*c^(1/4)) - sqrt(2)*(5*B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sq
rt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/c^2
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(209) = 418$.
time = 1.92, size = 725, normalized size = 2.51

$$\frac{4\left(c^{3}x^{2}+bc^{2}\right)\left(-\frac{625B^{4}b^{4}-500AB^{3}b^{3}c+150A^{2}B^{2}b^{2}c^{2}-20A^{3}Bbc^{3}+A^{4}c^{4}}{b^{3}c^{9}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^{2}c^{4}\sqrt{-\frac{625B^{4}b^{4}-500AB^{3}b^{3}c+150A^{2}B^{2}b^{2}c^{2}-20A^{3}Bbc^{3}+A^{4}c^{4}}{b^{3}c^{9}}}+\left(25B^{2}b^{2}-10ABbc+A^{2}c^{2}\right)x}\,b^{2}c^{7}\left(-\frac{625B^{4}b^{4}-500AB^{3}b^{3}c+150A^{2}B^{2}b^{2}c^{2}-20A^{3}Bbc^{3}+A^{4}c^{4}}{b^{3}c^{9}}\right)^{\frac{3}{4}}+\left(5Bb^{3}c^{7}-Ab^{2}c^{8}\right)\sqrt{x}\left(-\frac{625B^{4}b^{4}-500AB^{3}b^{3}c+150A^{2}B^{2}b^{2}c^{2}-20A^{3}Bbc^{3}+A^{4}c^{4}}{b^{3}c^{9}}\right)^{\frac{3}{4}}}{625B^{4}b^{4}-500AB^{3}b^{3}c+150A^{2}B^{2}b^{2}c^{2}-20A^{3}Bbc^{3}+A^{4}c^{4}}\right)+\left(c^{3}x^{2}+bc^{2}\right)\left(-\frac{625B^{4}b^{4}-500AB^{3}b^{3}c+150A^{2}B^{2}b^{2}c^{2}-20A^{3}Bbc^{3}+A^{4}c^{4}}{b^{3}c^{9}}\right)^{\frac{1}{4}}\log\left(bc^{2}\left(-\frac{625B^{4}b^{4}-500AB^{3}b^{3}c+150A^{2}B^{2}b^{2}c^{2}-20A^{3}Bbc^{3}+A^{4}c^{4}}{b^{3}c^{9}}\right)^{\frac{1}{4}}-\left(5Bb-Ac\right)\sqrt{x}\right)-\left(c^{3}x^{2}+bc^{2}\right)\left(-\frac{625B^{4}b^{4}-500AB^{3}b^{3}c+150A^{2}B^{2}b^{2}c^{2}-20A^{3}Bbc^{3}+A^{4}c^{4}}{b^{3}c^{9}}\right)^{\frac{1}{4}}\log\left(-bc^{2}\left(-\frac{625B^{4}b^{4}-500AB^{3}b^{3}c+150A^{2}B^{2}b^{2}c^{2}-20A^{3}Bbc^{3}+A^{4}c^{4}}{b^{3}c^{9}}\right)^{\frac{1}{4}}-\left(5Bb-Ac\right)\sqrt{x}\right)+4\left(4Bcx^{2}+5Bb-Ac\right)\sqrt{x}}{8\left(c^{3}x^{2}+bc^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(c^3*x^2 + b*c^2)*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2
*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4)*arctan((sqrt(b^2*c^4*sqrt
(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A
^4*c^4)/(b^3*c^9)) + (25*B^2*b^2 - 10*A*B*b*c + A^2*c^2)*x)*b^2*c^7*(-(625*
B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)
/(b^3*c^9))^(3/4) + (5*B*b^3*c^7 - A*b^2*c^8)*sqrt(x)*(-(625*B^4*b^4 - 500*
A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(3
/4))/(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3
+ A^4*c^4)) + (c^3*x^2 + b*c^2)*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*
B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4)*log(b*c^2*(-(625*B
^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/
(b^3*c^9))^(1/4) - (5*B*b - A*c)*sqrt(x)) - (c^3*x^2 + b*c^2)*(-(625*B^4*b^
4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*
c^9))^(1/4)*log(-b*c^2*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c
^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4) - (5*B*b - A*c)*sqrt(x)) +
4*(4*B*c*x^2 + 5*B*b - A*c)*sqrt(x))/(c^3*x^2 + b*c^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

[Out] Timed out

Giac [A]

time = 0.45, size = 283, normalized size = 0.98

$$\frac{2\,B\sqrt{x}}{c^2}-\frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^3}-\frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^3}-\frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^3}+\frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^3}+\frac{Bb\sqrt{x}-Ac\sqrt{x}}{2\,(cx^2+b)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/c^2 - 1/8*sqrt(2)*(5*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arc
tan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) - 1/
8*sqrt(2)*(5*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) - 1/16*sqrt(2)*(5*(b*c^
3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqr
t(b/c))/(b*c^3) + 1/16*sqrt(2)*(5*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*lo
g(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3) + 1/2*(B*b*sqrt(x)
- A*c*sqrt(x))/((c*x^2 + b)*c^2)
```

Mupad [B]

time = 0.21, size = 744, normalized size = 2.57

$$\frac{2\,B\,\sqrt{x}}{c^2}-\frac{\sqrt{x}\,\left(\frac{A\,c}{2}-\frac{B\,b}{2}\right)}{c^3\,x^2+b\,c^2}+\frac{\mathrm{atan}\left(\frac{\frac{(A\,c-5\,B\,b)\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}-\frac{(A\,c-5\,B\,b)\left(8\,A\,b\,c^2-40\,B\,b^2\,c\right)}{8\,(-b)^{3/4}\,c^{9/4}}\right)1\mathrm{i}}{8\,(-b)^{3/4}\,c^{9/4}}+\frac{(A\,c-5\,B\,b)\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}+\frac{(A\,c-5\,B\,b)\left(8\,A\,b\,c^2-40\,B\,b^2\,c\right)}{8\,(-b)^{3/4}\,c^{9/4}}\right)1\mathrm{i}}{8\,(-b)^{3/4}\,c^{9/4}}}{\frac{(A\,c-5\,B\,b)\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}-\frac{(A\,c-5\,B\,b)\left(8\,A\,b\,c^2-40\,B\,b^2\,c\right)}{8\,(-b)^{3/4}\,c^{9/4}}\right)}{8\,(-b)^{3/4}\,c^{9/4}}-\frac{(A\,c-5\,B\,b)\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}+\frac{(A\,c-5\,B\,b)\left(8\,A\,b\,c^2-40\,B\,b^2\,c\right)}{8\,(-b)^{3/4}\,c^{9/4}}\right)}{8\,(-b)^{3/4}\,c^{9/4}}}\right)(A\,c-5\,B\,b)\,1\mathrm{i}}{4\,(-b)^{3/4}\,c^{9/4}}+\frac{\mathrm{atan}\left(\frac{\frac{(A\,c-5\,B\,b)\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}-\frac{(A\,c-5\,B\,b)\left(8\,A\,b\,c^2-40\,B\,b^2\,c\right)1\mathrm{i}}{8\,(-b)^{3/4}\,c^{9/4}}\right)}{8\,(-b)^{3/4}\,c^{9/4}}+\frac{(A\,c-5\,B\,b)\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}+\frac{(A\,c-5\,B\,b)\left(8\,A\,b\,c^2-40\,B\,b^2\,c\right)1\mathrm{i}}{8\,(-b)^{3/4}\,c^{9/4}}\right)}{8\,(-b)^{3/4}\,c^{9/4}}}{\frac{(A\,c-5\,B\,b)\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}-\frac{(A\,c-5\,B\,b)\left(8\,A\,b\,c^2-40\,B\,b^2\,c\right)1\mathrm{i}}{8\,(-b)^{3/4}\,c^{9/4}}\right)1\mathrm{i}}{8\,(-b)^{3/4}\,c^{9/4}}-\frac{(A\,c-5\,B\,b)\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}+\frac{(A\,c-5\,B\,b)\left(8\,A\,b\,c^2-40\,B\,b^2\,c\right)1\mathrm{i}}{8\,(-b)^{3/4}\,c^{9/4}}\right)1\mathrm{i}}{8\,(-b)^{3/4}\,c^{9/4}}}\right)(A\,c-5\,B\,b)}{4\,(-b)^{3/4}\,c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (2*B*x^(1/2))/c^2 - (x^(1/2)*((A*c)/2 - (B*b)/2))/(b*c^2 + c^3*x^2) + (atan
((((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A*B*b*c))/c - ((A*c -
 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c))/(8*(-b)^(3/4)*c^(9/4)))*1i)/(8*(-b)^(3/4)
*c^(9/4)) + ((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A*B*b*c))/c
 + ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c))/(8*(-b)^(3/4)*c^(9/4)))*1i)/(8*
(-b)^(3/4)*c^(9/4)))/(((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A
*B*b*c))/c - ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c))/(8*(-b)^(3/4)*c^(9/4)
)))/(8*(-b)^(3/4)*c^(9/4)) - ((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2
 - 10*A*B*b*c))/c + ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c))/(8*(-b)^(3/4)*
c^(9/4))))/(8*(-b)^(3/4)*c^(9/4))))*(A*c - 5*B*b)*1i)/(4*(-b)^(3/4)*c^(9/4)
) + (atan((((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A*B*b*c))/c
- ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c)*1i)/(8*(-b)^(3/4)*c^(9/4))))/(8*(
-b)^(3/4)*c^(9/4)) + ((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A*
B*b*c))/c + ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c)*1i)/(8*(-b)^(3/4)*c^(9/
4))))/(8*(-b)^(3/4)*c^(9/4)))/(((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b
^2 - 10*A*B*b*c))/c - ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c)*1i)/(8*(-b)^(
3/4)*c^(9/4)))*1i)/(8*(-b)^(3/4)*c^(9/4)) - ((A*c - 5*B*b)*((x^(1/2)*(A^2*c
^2 + 25*B^2*b^2 - 10*A*B*b*c))/c + ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c)*
1i)/(8*(-b)^(3/4)*c^(9/4)))*1i)/(8*(-b)^(3/4)*c^(9/4))))*(A*c - 5*B*b))/(4*
(-b)^(3/4)*c^(9/4))
```

3.200 $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=261

$$-\frac{(bB-Ac)x^{3/2}}{2bc\,(b+cx^2)}-\frac{(3bB+Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{7/4}}+\frac{(3bB+Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{7/4}}+\frac{(3bB+}{}$$

```
[Out] -1/2*(-A*c+B*b)*x^(3/2)/b/c/(c*x^2+b)-1/8*(A*c+3*B*b)*arctan(1-c^(1/4)*2^(1
/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(7/4)*2^(1/2)+1/8*(A*c+3*B*b)*arctan(1+c^(1/
4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(7/4)*2^(1/2)+1/16*(A*c+3*B*b)*ln(b^(
1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(7/4)*2^(1/2)-1/1
6*(A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)
/c^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1598, 468, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{(Ac+3bB)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{7/4}}+\frac{(Ac+3bB)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{5/4}c^{7/4}}+\frac{(Ac+3bB)\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{7/4}}-\frac{(Ac+3bB)\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{7/4}}-\frac{x^{3/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((b*B - A*c)*x^(3/2))/(b*c*(b + c*x^2)) - ((3*b*B + A*c)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(5/4)*c^(7/4)) + ((3*b*B + A
*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(5/4)*c^(7/
4)) + ((3*b*B + A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(8*Sqrt[2]*b^(5/4)*c^(7/4)) - ((3*b*B + A*c)*Log[Sqrt[b] + Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(5/4)*c^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{\sqrt{x}\,(A+Bx^2)}{(b+cx^2)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\,(b+cx^2)} + \frac{\left(\frac{3bB}{2}+\frac{Ac}{2}\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{2bc}$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\,(b+cx^2)} + \frac{\left(\frac{3bB}{2}+\frac{Ac}{2}\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{bc}$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\,(b+cx^2)} - \frac{(3bB+Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4bc^{3/2}} + \frac{(3bB+Ac)\text{Sul}}{}$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\,(b+cx^2)} + \frac{(3bB+Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8bc^2} + \frac{(3bB+}{}$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\,(b+cx^2)} + \frac{(3bB+Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{7/4}} - \frac{(3bB+A}{}$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\,(b+cx^2)} - \frac{(3bB+Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{7/4}} + \frac{(3bB+Ac)\tan^{-1}\left(1}{4\sqrt{2}\,b^{5/4}}$$

Mathematica [A]
time = 0.47, size = 152, normalized size = 0.58

$$\frac{\frac{4\sqrt[4]{b}\,c^{3/4}(-bB+Ac)x^{3/2}}{b+cx^2}-\sqrt{2}\,(3bB+Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-\sqrt{2}\,(3bB+Ac)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{8b^{5/4}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
[Out] ((4*b^(1/4)*c^(3/4)*(-(b*B) + A*c)*x^(3/2))/(b + c*x^2) - Sqrt[2]*(3*b*B +
A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - Sqrt
```

```
[2]*(3*b*B + A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt
[c]*x)])/(8*b^(5/4)*c^(7/4))
```

Maple [A]
time = 0.39, size = 146, normalized size = 0.56

method	result
derivativedivides	$\frac{(Ac-Bb)x^{\frac{3}{2}}}{2bc(c\,x^2+b)}+\frac{(Ac+3Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{16b\,c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$\frac{(Ac-Bb)x^{\frac{3}{2}}}{2bc(c\,x^2+b)}+\frac{(Ac+3Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{16b\,c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(A*c-B*b)/b/c*x^(3/2)/(c*x^2+b)+1/16*(A*c+3*B*b)/b/c^2/(b/c)^(1/4)*2^(1
/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*
2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1
/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.55, size = 217, normalized size = 0.83

$$-\frac{(Bb-Ac)x^{\frac{3}{2}}}{2\,(bc^2x^2+b^2c)}+\frac{(3\,Bb+Ac)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{16\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b - A*c)*x^(3/2)/(b*c^2*x^2 + b^2*c) + 1/16*(3*B*b + A*c)*(2*sqrt(2
)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(s
qrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
 + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^
(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(185) = 370$.

time = 1.90, size = 912, normalized size = 3.49

$$-\frac{4\,(Bb-Ac)x^{\frac{3}{2}}+4\,(bc^2x^2+b^2c)\left(-\frac{81\,B^4b^4+108\,AB^3b^3c+54\,A^2B^2b^2c^2+12\,A^3Bbc^3+A^4c^4}{b^5c^7}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{(729\,B^6b^6+1458\,AB^5b^5c+1215\,A^2B^4b^4c^2+540\,A^3B^3b^3c^3+135\,A^4B^2b^2c^4+18\,A^5Bbc^5+A^6c^6)x-(81\,B^4b^7c^3+108\,AB^3b^6c^4+54\,A^2B^2b^5c^5+12\,A^3Bb^4c^6+A^4b^3c^7)\sqrt{-\frac{81\,B^4b^4+108\,AB^3b^3c+54\,A^2B^2b^2c^2+12\,A^3Bbc^3+A^4c^4}{b^5c^7}}}\,bc^2\left(-\frac{81\,B^4b^4+108\,AB^3b^3c+54\,A^2B^2b^2c^2+12\,A^3Bbc^3+A^4c^4}{b^5c^7}\right)^{\frac{1}{4}}-(27\,B^3b^4c^2+27\,AB^2b^3c^3+9\,A^2Bb^2c^4+A^3bc^5)\sqrt{x}\left(-\frac{81\,B^4b^4+108\,AB^3b^3c+54\,A^2B^2b^2c^2+12\,A^3Bbc^3+A^4c^4}{b^5c^7}\right)^{\frac{1}{4}}}{81\,B^4b^4+108\,AB^3b^3c+54\,A^2B^2b^2c^2+12\,A^3Bbc^3+A^4c^4}\right)-(bc^2x^2+b^2c)\left(-\frac{81\,B^4b^4+108\,AB^3b^3c+54\,A^2B^2b^2c^2+12\,A^3Bbc^3+A^4c^4}{b^5c^7}\right)^{\frac{1}{4}}\log\left(b^4c^5\left(-\frac{81\,B^4b^4+108\,AB^3b^3c+54\,A^2B^2b^2c^2+12\,A^3Bbc^3+A^4c^4}{b^5c^7}\right)^{\frac{3}{4}}+(27\,B^3b^3+27\,AB^2b^2c+9\,A^2Bbc^2+A^3c^3)\sqrt{x}\right)+(bc^2x^2+b^2c)\left(-\frac{81\,B^4b^4+108\,AB^3b^3c+54\,A^2B^2b^2c^2+12\,A^3Bbc^3+A^4c^4}{b^5c^7}\right)^{\frac{1}{4}}\log\left(-b^4c^5\left(-\frac{81\,B^4b^4+108\,AB^3b^3c+54\,A^2B^2b^2c^2+12\,A^3Bbc^3+A^4c^4}{b^5c^7}\right)^{\frac{3}{4}}+(27\,B^3b^3+27\,AB^2b^2c+9\,A^2Bbc^2+A^3c^3)\sqrt{x}\right)}{8\,(bc^2x^2+b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(B*b - A*c)*x^(3/2) + 4*(b*c^2*x^2 + b^2*c)*(-(81*B^4*b^4 + 108*A*B
^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/4)*
arctan((sqrt((729*B^6*b^6 + 1458*A*B^5*b^5*c + 1215*A^2*B^4*b^4*c^2 + 540*A
^3*B^3*b^3*c^3 + 135*A^4*B^2*b^2*c^4 + 18*A^5*B*b*c^5 + A^6*c^6)*x - (81*B^
4*b^7*c^3 + 108*A*B^3*b^6*c^4 + 54*A^2*B^2*b^5*c^5 + 12*A^3*B*b^4*c^6 + A^4
*b^3*c^7)*sqrt(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3
*B*b*c^3 + A^4*c^4)/(b^5*c^7)))*b*c^2*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*
A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/4) - (27*B^3*b^4*
c^2 + 27*A*B^2*b^3*c^3 + 9*A^2*B*b^2*c^4 + A^3*b*c^5)*sqrt(x)*(-(81*B^4*b^4
 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^
7))^(1/4))/(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*
c^3 + A^4*c^4)) - (b*c^2*x^2 + b^2*c)*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*
A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/4)*log(b^4*c^5*(-
(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c
^4)/(b^5*c^7))^(3/4) + (27*B^3*b^3 + 27*A*B^2*b^2*c + 9*A^2*B*b*c^2 + A^3*c
^3)*sqrt(x)) + (b*c^2*x^2 + b^2*c)*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2
*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/4)*log(-b^4*c^5*(-(8
1*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4
)/(b^5*c^7))^(3/4) + (27*B^3*b^3 + 27*A*B^2*b^2*c + 9*A^2*B*b*c^2 + A^3*c^3
)*sqrt(x)))/(b*c^2*x^2 + b^2*c)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.50, size = 273, normalized size = 1.05

$$-\frac{Bbx^{\frac{3}{2}}-Acx^{\frac{3}{2}}}{2\,(cx^2+b)bc}+\frac{\sqrt{2}\left(3\,(bc^3)^{\frac{3}{4}}Bb+(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^4}+\frac{\sqrt{2}\left(3\,(bc^3)^{\frac{3}{4}}Bb+(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^4}-\frac{\sqrt{2}\left(3\,(bc^3)^{\frac{3}{4}}Bb+(bc^3)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c^4}+\frac{\sqrt{2}\left(3\,(bc^3)^{\frac{3}{4}}Bb+(bc^3)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b*x^(3/2) - A*c*x^(3/2))/((c*x^2 + b)*b*c) + 1/8*sqrt(2)*(3*(b*c^3)
^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2
*sqrt(x))/(b/c)^(1/4))/(b^2*c^4) + 1/8*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + (b*c^
3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(
1/4))/(b^2*c^4) - 1/16*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*lo
g(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^4) + 1/16*sqrt(2)*(3*
(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
 + sqrt(b/c))/(b^2*c^4)
```

Mupad [B]
time = 0.23, size = 91, normalized size = 0.35

$$\frac{\operatorname{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(A\,c+3\,B\,b)}{4\,(-b)^{5/4}\,c^{7/4}}-\frac{\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(A\,c+3\,B\,b)}{4\,(-b)^{5/4}\,c^{7/4}}+\frac{x^{3/2}\,(A\,c-B\,b)}{2\,b\,c\,(c\,x^2+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c + 3*B*b))/(4*(-b)^(5/4)*c^(7/4))
- (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c + 3*B*b))/(4*(-b)^(5/4)*c^(7/4))
 + (x^(3/2)*(A*c - B*b))/(2*b*c*(b + c*x^2))
```

3.201 $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=261

$$-\frac{(bB-Ac)\sqrt{x}}{2bc\,(b+cx^2)}-\frac{(bB+3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{(bB+3Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}c^{5/4}}-\frac{(bB+}{}$$

```
[Out] -1/8*(3*A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(5/4)*
2^(1/2)+1/8*(3*A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c
^(5/4)*2^(1/2)-1/16*(3*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2
)*x^(1/2))/b^(7/4)/c^(5/4)*2^(1/2)+1/16*(3*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^
(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(5/4)*2^(1/2)-1/2*(-A*c+B*b)*x^(1/
2)/b/c/(c*x^2+b)
```

Rubi [A]
time = 0.13, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1598, 468, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{(3Ac+bB)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{(3Ac+bB)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{7/4}c^{5/4}}-\frac{(3Ac+bB)\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{(3Ac+bB)\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}c^{5/4}}-\frac{\sqrt{x}\,(bB-Ac)}{2bc\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((b*B - A*c)*Sqrt[x])/(b*c*(b + c*x^2)) - ((b*B + 3*A*c)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(5/4)) + ((b*B + 3*A
*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(5/
4)) - ((b*B + 3*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(8*Sqrt[2]*b^(7/4)*c^(5/4)) + ((b*B + 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{A+Bx^2}{\sqrt{x}\,(b+cx^2)^2}\,dx$$

$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\,(b+cx^2)} + \frac{\left(\frac{bB}{2}+\frac{3Ac}{2}\right)\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{2bc}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\,(b+cx^2)} + \frac{\left(\frac{bB}{2}+\frac{3Ac}{2}\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{bc}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\,(b+cx^2)} + \frac{(bB+3Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{3/2}c} + \frac{(bB+3Ac)\text{Su}}{}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\,(b+cx^2)} + \frac{(bB+3Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{3/2}c^{3/2}} + \frac{(bB+}{}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\,(b+cx^2)} - \frac{(bB+3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}c^{5/4}} + \frac{(bB+3A}{}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\,(b+cx^2)} - \frac{(bB+3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}c^{5/4}} + \frac{(bB+3Ac)\tan^{-1}\left(1\right.}{4\sqrt{2}\,b^{7/}}$$

Mathematica [A]
time = 0.46, size = 151, normalized size = 0.58

$$\frac{\frac{4b^{3/4}\sqrt[4]{c}\,(-bB+Ac)\sqrt{x}}{b+cx^2}-\sqrt{2}\,(bB+3Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+\sqrt{2}\,(bB+3Ac)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{8b^{7/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
[Out] ((4*b^(3/4)*c^(1/4)*(-(b*B) + A*c)*Sqrt[x])/(b + c*x^2) - Sqrt[2]*(b*B + 3*
A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + Sqrt
```

```
[2]*(b*B + 3*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt
[c]*x)])/(8*b^(7/4)*c^(5/4))
```

Maple [A]
time = 0.37, size = 146, normalized size = 0.56

method	result
derivativedivides	$\frac{(Ac-Bb)\sqrt{x}}{2bc(c\,x^2+b)}+\frac{(3Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{16b^2c}$
default	$\frac{(Ac-Bb)\sqrt{x}}{2bc(c\,x^2+b)}+\frac{(3Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{16b^2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(A*c-B*b)/b/c*x^(1/2)/(c*x^2+b)+1/16*(3*A*c+B*b)/b^2/c*(b/c)^(1/4)*2^(1
/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*
2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1
/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.52, size = 241, normalized size = 0.92

$$-\frac{(Bb-Ac)\sqrt{x}}{2\,(bc^2x^2+b^2c)}+\frac{\frac{2\sqrt{2}\,(Bb+3\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,(Bb+3\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,(Bb+3\,Ac)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Bb+3\,Ac)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{16\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b - A*c)*sqrt(x)/(b*c^2*x^2 + b^2*c) + 1/16*(2*sqrt(2)*(B*b + 3*A*c
)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(B*b + 3*A*c)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(B*b + 3*A*c)*log(sqr
t(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqr
t(2)*(B*b + 3*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(
b))/(b^(3/4)*c^(1/4)))/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 717 vs. $2(185) = 370$.
time = 2.06, size = 717, normalized size = 2.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*c^2*x^2 + b^2*c)*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2
 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4)*arctan((sqrt(b^4*c^2*sqrt
(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4
*c^4)/(b^7*c^5)) + (B^2*b^2 + 6*A*B*b*c + 9*A^2*c^2)*x)*b^5*c^4*(-(B^4*b^4
+ 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*
c^5))^(3/4) - (B*b^6*c^4 + 3*A*b^5*c^5)*sqrt(x)*(-(B^4*b^4 + 12*A*B^3*b^3*c
 + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(3/4))/(B^
4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)
) + (b*c^2*x^2 + b^2*c)*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 +
108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4)*log(b^2*c*(-(B^4*b^4 + 12*A*
B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(
1/4) + (B*b + 3*A*c)*sqrt(x)) - (b*c^2*x^2 + b^2*c)*(-(B^4*b^4 + 12*A*B^3*b
^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4)*
log(-b^2*c*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c
^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4) + (B*b + 3*A*c)*sqrt(x)) - 4*(B*b - A*c)*
sqrt(x))/(b*c^2*x^2 + b^2*c)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 273, normalized size = 1.05

$$\frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+3(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^2c^2}+\frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+3(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^2c^2}+\frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+3(bc^3)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16b^2c^2}-\frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+3(bc^3)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16b^2c^2}-\frac{Bb\sqrt{x}-Ac\sqrt{x}}{2(cx^2+b)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*((b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) + 1/8*sqrt(2)*((b*c^
3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4
) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) + 1/16*sqrt(2)*((b*c^3)^(1/4)*B*b + 3
*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c
^2) - 1/16*sqrt(2)*((b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*s
```

```
qrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^2) - 1/2*(B*b*sqrt(x) - A*c*sqrt
(x))/((c*x^2 + b)*b*c)
```

Mupad [B]
time = 0.33, size = 750, normalized size = 2.87

$$\frac{\mathrm{atan}\left(\frac{\frac{(3\,A\,c+B\,b)\,\left(\frac{\sqrt{x}\,\left(9\,A^2\,c^3+6\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{b^2}-\frac{(3\,A\,c+B\,b)\,\left(24\,A\,c^3+8\,B\,b\,c^2\right)\,1{}\mathrm{i}}{8\,{(-b)}^{7/4}\,c^{5/4}}\right)}{8\,{(-b)}^{7/4}\,c^{5/4}}+\frac{(3\,A\,c+B\,b)\,\left(\frac{\sqrt{x}\,\left(9\,A^2\,c^3+6\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{b^2}+\frac{(3\,A\,c+B\,b)\,\left(24\,A\,c^3+8\,B\,b\,c^2\right)\,1{}\mathrm{i}}{8\,{(-b)}^{7/4}\,c^{5/4}}\right)}{8\,{(-b)}^{7/4}\,c^{5/4}}}{\frac{(3\,A\,c+B\,b)\,\left(\frac{\sqrt{x}\,\left(9\,A^2\,c^3+6\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{b^2}-\frac{(3\,A\,c+B\,b)\,\left(24\,A\,c^3+8\,B\,b\,c^2\right)\,1{}\mathrm{i}}{8\,{(-b)}^{7/4}\,c^{5/4}}\right)\,1{}\mathrm{i}}{8\,{(-b)}^{7/4}\,c^{5/4}}-\frac{(3\,A\,c+B\,b)\,\left(\frac{\sqrt{x}\,\left(9\,A^2\,c^3+6\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{b^2}+\frac{(3\,A\,c+B\,b)\,\left(24\,A\,c^3+8\,B\,b\,c^2\right)\,1{}\mathrm{i}}{8\,{(-b)}^{7/4}\,c^{5/4}}\right)\,1{}\mathrm{i}}{8\,{(-b)}^{7/4}\,c^{5/4}}}\right)\,(3\,A\,c+B\,b)}{4\,{(-b)}^{7/4}\,c^{5/4}}+\frac{\sqrt{x}\,(A\,c-B\,b)}{2\,b\,c\,\left(c\,x^2+b\right)}+\frac{\mathrm{atan}\left(\frac{\frac{(3\,A\,c+B\,b)\,\left(\frac{\sqrt{x}\,\left(9\,A^2\,c^3+6\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{b^2}-\frac{(3\,A\,c+B\,b)\,\left(24\,A\,c^3+8\,B\,b\,c^2\right)}{8\,{(-b)}^{7/4}\,c^{5/4}}\right)\,1{}\mathrm{i}}{8\,{(-b)}^{7/4}\,c^{5/4}}+\frac{(3\,A\,c+B\,b)\,\left(\frac{\sqrt{x}\,\left(9\,A^2\,c^3+6\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{b^2}+\frac{(3\,A\,c+B\,b)\,\left(24\,A\,c^3+8\,B\,b\,c^2\right)}{8\,{(-b)}^{7/4}\,c^{5/4}}\right)\,1{}\mathrm{i}}{8\,{(-b)}^{7/4}\,c^{5/4}}}{\frac{(3\,A\,c+B\,b)\,\left(\frac{\sqrt{x}\,\left(9\,A^2\,c^3+6\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{b^2}-\frac{(3\,A\,c+B\,b)\,\left(24\,A\,c^3+8\,B\,b\,c^2\right)}{8\,{(-b)}^{7/4}\,c^{5/4}}\right)}{8\,{(-b)}^{7/4}\,c^{5/4}}-\frac{(3\,A\,c+B\,b)\,\left(\frac{\sqrt{x}\,\left(9\,A^2\,c^3+6\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{b^2}+\frac{(3\,A\,c+B\,b)\,\left(24\,A\,c^3+8\,B\,b\,c^2\right)}{8\,{(-b)}^{7/4}\,c^{5/4}}\right)}{8\,{(-b)}^{7/4}\,c^{5/4}}}\right)\,(3\,A\,c+B\,b)\,1{}\mathrm{i}}{4\,{(-b)}^{7/4}\,c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (atan((((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^2*c + 6*A*B*b*c^2))/b^2
- ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2))/(8*(-b)^(7/4)*c^(5/4)))*1i)/(8*(-b
)^(7/4)*c^(5/4)) + ((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^2*c + 6*A*B*
b*c^2))/b^2 + ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2))/(8*(-b)^(7/4)*c^(5/4))
)*1i)/(8*(-b)^(7/4)*c^(5/4)))/(((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^
2*c + 6*A*B*b*c^2))/b^2 - ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2))/(8*(-b)^(7
/4)*c^(5/4))))/(8*(-b)^(7/4)*c^(5/4)) - ((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3
 + B^2*b^2*c + 6*A*B*b*c^2))/b^2 + ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2))/(
8*(-b)^(7/4)*c^(5/4))))/(8*(-b)^(7/4)*c^(5/4))))*(3*A*c + B*b)*1i)/(4*(-b)^
(7/4)*c^(5/4)) + (atan((((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^2*c + 6
*A*B*b*c^2))/b^2 - ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2)*1i)/(8*(-b)^(7/4)*
c^(5/4))))/(8*(-b)^(7/4)*c^(5/4)) + ((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B
^2*b^2*c + 6*A*B*b*c^2))/b^2 + ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2)*1i)/(8
*(-b)^(7/4)*c^(5/4))))/(8*(-b)^(7/4)*c^(5/4)))/(((3*A*c + B*b)*((x^(1/2)*(9
*A^2*c^3 + B^2*b^2*c + 6*A*B*b*c^2))/b^2 - ((3*A*c + B*b)*(24*A*c^3 + 8*B*b
*c^2)*1i)/(8*(-b)^(7/4)*c^(5/4)))*1i)/(8*(-b)^(7/4)*c^(5/4)) - ((3*A*c + B*
b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^2*c + 6*A*B*b*c^2))/b^2 + ((3*A*c + B*b)*(2
4*A*c^3 + 8*B*b*c^2)*1i)/(8*(-b)^(7/4)*c^(5/4)))*1i)/(8*(-b)^(7/4)*c^(5/4))
))*(3*A*c + B*b))/(4*(-b)^(7/4)*c^(5/4)) + (x^(1/2)*(A*c - B*b))/(2*b*c*(b
+ c*x^2))
```

3.202 $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=284

$$\frac{bB-5Ac}{2b^2c\sqrt{x}}-\frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)}-\frac{(bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}c^{3/4}}+\frac{(bB-5Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}c^{3/4}}$$

```
[Out] -1/8*(-5*A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4)/c^(3/4)
*2^(1/2)+1/8*(-5*A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4)
/c^(3/4)*2^(1/2)+1/16*(-5*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(
1/2)*x^(1/2))/b^(9/4)/c^(3/4)*2^(1/2)-1/16*(-5*A*c+B*b)*ln(b^(1/2)+x*c^(1/2
)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)/c^(3/4)*2^(1/2)+1/2*(-5*A*c+B*b)
/b^2/c/x^(1/2)+1/2*(A*c-B*b)/b/c/(c*x^2+b)/x^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{(bB-5Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}c^{3/4}}+\frac{(bB-5Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{9/4}c^{3/4}}+\frac{(bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}c^{3/4}}-\frac{(bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}c^{3/4}}+\frac{bB-5Ac}{2b^2c\sqrt{x}}-\frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*B - 5*A*c)/(2*b^2*c*Sqrt[x]) - (b*B - A*c)/(2*b*c*Sqrt[x]*(b + c*x^2)) -
 ((b*B - 5*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b
^(9/4)*c^(3/4)) + ((b*B - 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)])/(4*Sqrt[2]*b^(9/4)*c^(3/4)) + ((b*B - 5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(9/4)*c^(3/4)) - ((b*B - 5*
A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]
*b^(9/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{A+Bx^2}{x^{3/2}\,(b+cx^2)^2}\,dx$$

$$= -\frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)} + \frac{\left(-\frac{bB}{2}+\frac{5Ac}{2}\right)\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{2bc}$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)} + \frac{(bB-5Ac)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^2}$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)} + \frac{(bB-5Ac)\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)} - \frac{(bB-5Ac)\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^2\sqrt{c}} + \cdots$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)} + \frac{(bB-5Ac)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x} \cdots\right)}{8b^2c}$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)} + \frac{(bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}c^{3/4}} \cdots$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)} - \frac{(bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}c^{3/4}} + \frac{(bB-5\cdots}{\cdots}$$

Mathematica [A]

time = 0.51, size = 162, normalized size = 0.57

$$\frac{\frac{4\sqrt[4]{b}\,(-4Ab+bBx^2-5Acx^2)}{\sqrt{x}\,(b+cx^2)}+\frac{\sqrt{2}\,(-bB+5Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)}{c^{3/4}}+\frac{\sqrt{2}\,(-bB+5Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{c^{3/4}}}{8b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*b^(1/4)*(-4*A*b + b*B*x^2 - 5*A*c*x^2))/(Sqrt[x]*(b + c*x^2)) + (Sqrt[2
]*(-(b*B) + 5*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sq
rt[x])])/c^(3/4) + (Sqrt[2]*(-(b*B) + 5*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/c^(3/4))/(8*b^(9/4))
```

Maple [A]
time = 0.40, size = 153, normalized size = 0.54

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{5Ac}{4}-\frac{Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{b^2}$
default	$-\frac{2\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{5Ac}{4}-\frac{Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{b^2}$
risch	$-\frac{2A}{b^2\sqrt{x}}-\frac{x^{\frac{3}{2}}Ac}{2b^2(c\,x^2+b)}+\frac{x^{\frac{3}{2}}B}{2b(c\,x^2+b)}-\frac{5\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{16b^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{5\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}}{\left(\frac{b}{c}\right)}\right.}{8b^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^2*((1/4*A*c-1/4*B*b)*x^(3/2)/(c*x^2+b)+1/8*(5/4*A*c-1/4*B*b)/c/(b/c)^(
1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)
```

```
*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))-2*A/b^2/x^(1/2)
```

Maxima [A]
time = 0.54, size = 222, normalized size = 0.78

$$\frac{(Bb-5Ac)x^2-4Ab}{2\left(b^2cx^{\frac{5}{2}}+b^3\sqrt{x}\right)}+\frac{(Bb-5Ac)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b - 5*A*c)*x^2 - 4*A*b)/(b^2*c*x^(5/2) + b^3*sqrt(x)) + 1/16*(B*b -
 5*A*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-s
qrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^
2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(204) = 408$.
time = 1.76, size = 920, normalized size = 3.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^2*c*x^3 + b^3*x)*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^
2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4)*arctan((sqrt((B^6*b^6 -
 30*A*B^5*b^5*c + 375*A^2*B^4*b^4*c^2 - 2500*A^3*B^3*b^3*c^3 + 9375*A^4*B^2
*b^2*c^4 - 18750*A^5*B*b*c^5 + 15625*A^6*c^6)*x - (B^4*b^9*c - 20*A*B^3*b^8
*c^2 + 150*A^2*B^2*b^7*c^3 - 500*A^3*B*b^6*c^4 + 625*A^4*b^5*c^5)*sqrt(-(B^
4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^
4)/(b^9*c^3)))*b^2*c*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 50
0*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4) + (B^3*b^5*c - 15*A*B^2*b^4*c
^2 + 75*A^2*B*b^3*c^3 - 125*A^3*b^2*c^4)*sqrt(x)*(-(B^4*b^4 - 20*A*B^3*b^3*
c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4))/
(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4
*c^4)) - (b^2*c*x^3 + b^3*x)*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*
c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4)*log(b^7*c^2*(-(B^4*b^
4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(
```

```
b^9*c^3))^(3/4) - (B^3*b^3 - 15*A*B^2*b^2*c + 75*A^2*B*b*c^2 - 125*A^3*c^3)
*sqrt(x)) + (b^2*c*x^3 + b^3*x)*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b
^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4)*log(-b^7*c^2*(-(B^
4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^
4)/(b^9*c^3))^(3/4) - (B^3*b^3 - 15*A*B^2*b^2*c + 75*A^2*B*b*c^2 - 125*A^3*
c^3)*sqrt(x)) + 4*((B*b - 5*A*c)*x^2 - 4*A*b)*sqrt(x))/(b^2*c*x^3 + b^3*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 278, normalized size = 0.98

$$\frac{Bbx^2 - 5\,Acx^2 - 4\,Ab}{2\left(cx^{\frac{5}{2}} + b\sqrt{x}\right)b^2} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb - 5\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^3} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb - 5\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^3} - \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb - 5\left(bc^3\right)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{16\,b^3c^3} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb - 5\left(bc^3\right)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{16\,b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b*x^2 - 5*A*c*x^2 - 4*A*b)/((c*x^(5/2) + b*sqrt(x))*b^2) + 1/8*sqrt(
2)*((b*c^3)^(3/4)*B*b - 5*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b
/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^3) + 1/8*sqrt(2)*((b*c^3)^(3/4)*
B*b - 5*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqr
t(x))/(b/c)^(1/4))/(b^3*c^3) - 1/16*sqrt(2)*((b*c^3)^(3/4)*B*b - 5*(b*c^3)^
(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3) + 1/1
6*sqrt(2)*((b*c^3)^(3/4)*B*b - 5*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b
/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3)
```

Mupad [B]
time = 0.23, size = 104, normalized size = 0.37

$$\frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)\,\left(5\,A\,c-B\,b\right)}{4\,{\left(-b\right)}^{9/4}\,c^{3/4}}-\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)\,\left(5\,A\,c-B\,b\right)}{4\,{\left(-b\right)}^{9/4}\,c^{3/4}}-\frac{\frac{2\,A}{b}+\frac{x^2\,\left(5\,A\,c-B\,b\right)}{2\,b^2}}{b\,\sqrt{x}+c\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(5*A*c - B*b))/(4*(-b)^(9/4)*c^(3/4))
- (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(5*A*c - B*b))/(4*(-b)^(9/4)*c^(3/4))
 - ((2*A)/b + (x^2*(5*A*c - B*b))/(2*b^2))/(b*x^(1/2) + c*x^(5/2))
```

3.203 $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=289

$$\frac{3bB-7Ac}{6b^2cx^{3/2}}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}-\frac{(3bB-7Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{(3bB-7Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}\sqrt[4]{c}}$$

```
[Out] 1/6*(-7*A*c+3*B*b)/b^2/c/x^(3/2)+1/2*(A*c-B*b)/b/c/x^(3/2)/(c*x^2+b)-1/8*(-
7*A*c+3*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)/c^(1/4)*2^(
1/2)+1/8*(-7*A*c+3*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)/
c^(1/4)*2^(1/2)-1/16*(-7*A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^
(1/2)*x^(1/2))/b^(11/4)/c^(1/4)*2^(1/2)+1/16*(-7*A*c+3*B*b)*ln(b^(1/2)+x*c^
(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)/c^(1/4)*2^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{(3bB-7Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{(3bB-7Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{11/4}\sqrt[4]{c}}-\frac{(3bB-7Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{(3bB-7Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{3bB-7Ac}{6b^2cx^{3/2}}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (3*b*B - 7*A*c)/(6*b^2*c*x^(3/2)) - (b*B - A*c)/(2*b*c*x^(3/2)*(b + c*x^2))
 - ((3*b*B - 7*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[
2]*b^(11/4)*c^(1/4)) + ((3*b*B - 7*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(4*Sqrt[2]*b^(11/4)*c^(1/4)) - ((3*b*B - 7*A*c)*Log[Sqrt[b] - S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(11/4)*c^(1/4)) +
 ((3*b*B - 7*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x
])/(8*Sqrt[2]*b^(11/4)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{A+Bx^2}{x^{5/2}\,(b+cx^2)^2}\,dx$$

$$= -\frac{bB-Ac}{2bcx^{3/2}\,(b+cx^2)} + \frac{\left(-\frac{3bB}{2}+\frac{7Ac}{2}\right)\int \frac{1}{x^{5/2}(b+cx^2)}\,dx}{2bc}$$

$$= \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}\,(b+cx^2)} + \frac{(3bB-7Ac)\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{4b^2}$$

$$= \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}\,(b+cx^2)} + \frac{(3bB-7Ac)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}\,(b+cx^2)} + \frac{(3bB-7Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{5/2}} +$$

$$= \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}\,(b+cx^2)} + \frac{(3bB-7Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \right.}{8b^{5/2}\sqrt{c}}$$

$$= \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}\,(b+cx^2)} - \frac{(3bB-7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\right.}{8\sqrt{2}\,b^{11/4}\sqrt[4]{c}}$$

$$= \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}\,(b+cx^2)} - \frac{(3bB-7Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}\sqrt[4]{c}} + \frac{(3bB}{}$$

Mathematica [**A**]

time = 0.51, size = 165, normalized size = 0.57

$$\frac{\frac{4b^{3/4}(-4Ab+3bBx^2-7Acx^2)}{x^{3/2}(b+cx^2)}+\frac{3\sqrt{2}\,(-3bB+7Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)}{\sqrt[4]{c}}+\frac{3\sqrt{2}\,(3bB-7Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt[4]{c}}}{24b^{11/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]

[Out] ((4*b^(3/4)*(-4*A*b + 3*b*B*x^2 - 7*A*c*x^2))/(x^(3/2)*(b + c*x^2)) + (3*Sqrt[2]*(-3*b*B + 7*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/c^(1/4) + (3*Sqrt[2]*(3*b*B - 7*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/c^(1/4))/(24*b^(11/4))

Maple [A]
time = 0.42, size = 153, normalized size = 0.53

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(7Ac-3Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{32b}\right.}{b^2}$
default	$-\frac{2\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(7Ac-3Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{32b}\right.}{b^2}$
risch	$-\frac{2A}{3b^2x^{\frac{3}{2}}}-\frac{\sqrt{x}\,Ac}{2b^2(c\,x^2+b)}+\frac{\sqrt{x}\,B}{2b(c\,x^2+b)}-\frac{7\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)c}{8b^3}-\frac{7\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\right.}{8b^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)

[Out] -2/b^2*((1/4*A*c-1/4*B*b)*x^(1/2)/(c*x^2+b)+1/32*(7*A*c-3*B*b)*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))-2/3*A/b^2/x^(3/2)

Maxima [A]
time = 0.52, size = 251, normalized size = 0.87

$$\frac{(3\,Bb-7\,Ac)x^2-4\,Ab}{6\left(b^2cx^{\frac{7}{2}}+b^3x^{\frac{3}{2}}\right)}+\frac{\frac{2\sqrt{2}\,(3\,Bb-7\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,(3\,Bb-7\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,(3\,Bb-7\,Ac)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,(3\,Bb-7\,Ac)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*((3*B*b - 7*A*c)*x^2 - 4*A*b)/(b^2*c*x^(7/2) + b^3*x^(3/2)) + 1/16*(2*s
qrt(2)*(3*B*b - 7*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt
(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqr
t(2)*(3*B*b - 7*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(
c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2
)*(3*B*b - 7*A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b)
)/(b^(3/4)*c^(1/4)) - sqrt(2)*(3*B*b - 7*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*
sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(209) = 418$.
time = 2.18, size = 741, normalized size = 2.56

$$-\frac{12\,(b^2cx^4+b^3x^2)\left(-\frac{81\,B^4b^4-756\,AB^3b^3c+2646\,A^2B^2b^2c^2-4116\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^6\sqrt{-\frac{81\,B^4b^4-756\,AB^3b^3c+2646\,A^2B^2b^2c^2-4116\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c}}+(9\,B^2b^2-42\,ABbc+49\,A^2c^2)x}\;b^8c\left(-\frac{81\,B^4b^4-756\,AB^3b^3c+2646\,A^2B^2b^2c^2-4116\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c}\right)^{\frac{3}{4}}+(3\,Bb^9c-7\,Ab^8c^2)\sqrt{x}\left(-\frac{81\,B^4b^4-756\,AB^3b^3c+2646\,A^2B^2b^2c^2-4116\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c}\right)^{\frac{3}{4}}}{81\,B^4b^4-756\,AB^3b^3c+2646\,A^2B^2b^2c^2-4116\,A^3Bbc^3+2401\,A^4c^4}\right)+3\,(b^2cx^4+b^3x^2)\left(-\frac{81\,B^4b^4-756\,AB^3b^3c+2646\,A^2B^2b^2c^2-4116\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c}\right)^{\frac{1}{4}}\log\left(b^3\left(-\frac{81\,B^4b^4-756\,AB^3b^3c+2646\,A^2B^2b^2c^2-4116\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c}\right)^{\frac{1}{4}}-(3\,Bb-7\,Ac)\sqrt{x}\right)-3\,(b^2cx^4+b^3x^2)\left(-\frac{81\,B^4b^4-756\,AB^3b^3c+2646\,A^2B^2b^2c^2-4116\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c}\right)^{\frac{1}{4}}\log\left(-b^3\left(-\frac{81\,B^4b^4-756\,AB^3b^3c+2646\,A^2B^2b^2c^2-4116\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c}\right)^{\frac{1}{4}}-(3\,Bb-7\,Ac)\sqrt{x}\right)-4\,((3\,Bb-7\,Ac)x^2-4\,Ab)\sqrt{x}}{24\,(b^2cx^4+b^3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(b^2*c*x^4 + b^3*x^2)*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*
B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4)*arctan((sqrt
(b^6*sqrt(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*
B*b*c^3 + 2401*A^4*c^4)/(b^11*c)) + (9*B^2*b^2 - 42*A*B*b*c + 49*A^2*c^2)*x
)*b^8*c*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B
*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(3/4) + (3*B*b^9*c - 7*A*b^8*c^2)*sqrt(x)*
(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 +
 2401*A^4*c^4)/(b^11*c))^(3/4))/(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^
2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)) + 3*(b^2*c*x^4 + b^3*x^2)*(-(
81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 24
01*A^4*c^4)/(b^11*c))^(1/4)*log(b^3*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*
A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4) - (3*B*b
 - 7*A*c)*sqrt(x)) - 3*(b^2*c*x^4 + b^3*x^2)*(-(81*B^4*b^4 - 756*A*B^3*b^3*
c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4)
*log(-b^3*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3
*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4) - (3*B*b - 7*A*c)*sqrt(x)) - 4*((3
*B*b - 7*A*c)*x^2 - 4*A*b)*sqrt(x))/(b^2*c*x^4 + b^3*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 283, normalized size = 0.98

$$\frac{\sqrt{2}\left(3\left(bc^{3}\right)^{\frac{1}{4}}Bb-7\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^{3}c}+\frac{\sqrt{2}\left(3\left(bc^{3}\right)^{\frac{1}{4}}Bb-7\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8b^{3}c}+\frac{\sqrt{2}\left(3\left(bc^{3}\right)^{\frac{1}{4}}Bb-7\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16b^{3}c}-\frac{\sqrt{2}\left(3\left(bc^{3}\right)^{\frac{1}{4}}Bb-7\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16b^{3}c}+\frac{Bb\sqrt{x}-Ac\sqrt{x}}{2\left(cx^{2}+b\right)b^{2}}-\frac{2A}{3b^{2}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 7*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 1/8*sqrt(2)*(3*(b*
c^3)^(1/4)*B*b - 7*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 1/16*sqrt(2)*(3*(b*c^3)^(1/4)*B*b -
 7*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3
*c) - 1/16*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 7*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)
*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) + 1/2*(B*b*sqrt(x) - A*c*sqrt
(x))/((c*x^2 + b)*b^2) - 2/3*A/(b^2*x^(3/2))
```

Mupad [**B**]
time = 0.37, size = 859, normalized size = 2.97

$$-\frac{\frac{2A}{3b}+\frac{x^{2}\left(7Ac-3Bb\right)}{6b^{2}}}{bx^{3/2}+cx^{7/2}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(7Ac-3Bb\right)\left(\sqrt{x}\left(1568A^{2}b^{6}c^{5}-1344ABb^{7}c^{4}+288B^{2}b^{8}c^{3}\right)-\frac{\left(7Ac-3Bb\right)\left(1792Ab^{9}c^{4}-768Bb^{10}c^{3}\right)}{8\left(-b\right)^{11/4}c^{1/4}}\right)1i}{8\left(-b\right)^{11/4}c^{1/4}}+\frac{\left(7Ac-3Bb\right)\left(\sqrt{x}\left(1568A^{2}b^{6}c^{5}-1344ABb^{7}c^{4}+288B^{2}b^{8}c^{3}\right)+\frac{\left(7Ac-3Bb\right)\left(1792Ab^{9}c^{4}-768Bb^{10}c^{3}\right)}{8\left(-b\right)^{11/4}c^{1/4}}\right)1i}{8\left(-b\right)^{11/4}c^{1/4}}}{\frac{\left(7Ac-3Bb\right)\left(\sqrt{x}\left(1568A^{2}b^{6}c^{5}-1344ABb^{7}c^{4}+288B^{2}b^{8}c^{3}\right)-\frac{\left(7Ac-3Bb\right)\left(1792Ab^{9}c^{4}-768Bb^{10}c^{3}\right)}{8\left(-b\right)^{11/4}c^{1/4}}\right)}{8\left(-b\right)^{11/4}c^{1/4}}-\frac{\left(7Ac-3Bb\right)\left(\sqrt{x}\left(1568A^{2}b^{6}c^{5}-1344ABb^{7}c^{4}+288B^{2}b^{8}c^{3}\right)+\frac{\left(7Ac-3Bb\right)\left(1792Ab^{9}c^{4}-768Bb^{10}c^{3}\right)}{8\left(-b\right)^{11/4}c^{1/4}}\right)}{8\left(-b\right)^{11/4}c^{1/4}}}\right)\left(7Ac-3Bb\right)1i}{4\left(-b\right)^{11/4}c^{1/4}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(7Ac-3Bb\right)\left(\sqrt{x}\left(1568A^{2}b^{6}c^{5}-1344ABb^{7}c^{4}+288B^{2}b^{8}c^{3}\right)-\frac{\left(7Ac-3Bb\right)\left(1792Ab^{9}c^{4}-768Bb^{10}c^{3}\right)1i}{8\left(-b\right)^{11/4}c^{1/4}}\right)}{8\left(-b\right)^{11/4}c^{1/4}}+\frac{\left(7Ac-3Bb\right)\left(\sqrt{x}\left(1568A^{2}b^{6}c^{5}-1344ABb^{7}c^{4}+288B^{2}b^{8}c^{3}\right)+\frac{\left(7Ac-3Bb\right)\left(1792Ab^{9}c^{4}-768Bb^{10}c^{3}\right)1i}{8\left(-b\right)^{11/4}c^{1/4}}\right)}{8\left(-b\right)^{11/4}c^{1/4}}}{\frac{\left(7Ac-3Bb\right)\left(\sqrt{x}\left(1568A^{2}b^{6}c^{5}-1344ABb^{7}c^{4}+288B^{2}b^{8}c^{3}\right)-\frac{\left(7Ac-3Bb\right)\left(1792Ab^{9}c^{4}-768Bb^{10}c^{3}\right)1i}{8\left(-b\right)^{11/4}c^{1/4}}\right)1i}{8\left(-b\right)^{11/4}c^{1/4}}-\frac{\left(7Ac-3Bb\right)\left(\sqrt{x}\left(1568A^{2}b^{6}c^{5}-1344ABb^{7}c^{4}+288B^{2}b^{8}c^{3}\right)+\frac{\left(7Ac-3Bb\right)\left(1792Ab^{9}c^{4}-768Bb^{10}c^{3}\right)1i}{8\left(-b\right)^{11/4}c^{1/4}}\right)1i}{8\left(-b\right)^{11/4}c^{1/4}}}\right)\left(7Ac-3Bb\right)}{4\left(-b\right)^{11/4}c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((2*A)/(3*b) + (x^2*(7*A*c - 3*B*b))/(6*b^2))/(b*x^(3/2) + c*x^(7/2)) - (
atan((((7*A*c - 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*
A*B*b^7*c^4) - ((7*A*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10*c^3))/(8*(-b)^
(11/4)*c^(1/4)))*1i)/(8*(-b)^(11/4)*c^(1/4)) + ((7*A*c - 3*B*b)*(x^(1/2)*(1
568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) + ((7*A*c - 3*B*b)*(1
792*A*b^9*c^4 - 768*B*b^10*c^3))/(8*(-b)^(11/4)*c^(1/4)))*1i)/(8*(-b)^(11/4
)*c^(1/4)))/(((7*A*c - 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3
- 1344*A*B*b^7*c^4) - ((7*A*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10*c^3))/(
8*(-b)^(11/4)*c^(1/4))))/(8*(-b)^(11/4)*c^(1/4)) - ((7*A*c - 3*B*b)*(x^(1/2
)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) + ((7*A*c - 3*B*b
```

```
)*(1792*A*b^9*c^4 - 768*B*b^10*c^3))/(8*(-b)^(11/4)*c^(1/4))))/(8*(-b)^(11/
4)*c^(1/4))))*(7*A*c - 3*B*b)*1i)/(4*(-b)^(11/4)*c^(1/4)) - (atan((((7*A*c
- 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) -
 ((7*A*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10*c^3)*1i)/(8*(-b)^(11/4)*c^(1
/4))))/(8*(-b)^(11/4)*c^(1/4)) + ((7*A*c - 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^
5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) + ((7*A*c - 3*B*b)*(1792*A*b^9*c^4
- 768*B*b^10*c^3)*1i)/(8*(-b)^(11/4)*c^(1/4))))/(8*(-b)^(11/4)*c^(1/4)))/((
(7*A*c - 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7
*c^4) - ((7*A*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10*c^3)*1i)/(8*(-b)^(11/
4)*c^(1/4)))*1i)/(8*(-b)^(11/4)*c^(1/4)) - ((7*A*c - 3*B*b)*(x^(1/2)*(1568*
A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) + ((7*A*c - 3*B*b)*(1792*
A*b^9*c^4 - 768*B*b^10*c^3)*1i)/(8*(-b)^(11/4)*c^(1/4)))*1i)/(8*(-b)^(11/4)
*c^(1/4))))*(7*A*c - 3*B*b))/(4*(-b)^(11/4)*c^(1/4))
```

3.204 $\int \frac{\sqrt{x}\,(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=310

$$\frac{5bB-9Ac}{10b^2cx^{5/2}}-\frac{5bB-9Ac}{2b^3\sqrt{x}}-\frac{bB-Ac}{2bcx^{5/2}\,(b+cx^2)}+\frac{\sqrt[4]{c}\,(5bB-9Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{13/4}}-\frac{\sqrt[4]{c}\,(5bB-9Ac)}{}$$

```
[Out] 1/10*(-9*A*c+5*B*b)/b^2/c/x^(5/2)+1/2*(A*c-B*b)/b/c/x^(5/2)/(c*x^2+b)+1/8*c
^(1/4)*(-9*A*c+5*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^
(1/2)-1/8*c^(1/4)*(-9*A*c+5*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/
b^(13/4)*2^(1/2)-1/16*c^(1/4)*(-9*A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c
^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)+1/16*c^(1/4)*(-9*A*c+5*B*b)*ln(b^(
1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)+1/2*(9*A*c
-5*B*b)/b^3/x^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt[4]{c}\,(5bB-9Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{13/4}}-\frac{\sqrt[4]{c}\,(5bB-9Ac)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{13/4}}-\frac{\sqrt[4]{c}\,(5bB-9Ac)\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{13/4}}+\frac{\sqrt[4]{c}\,(5bB-9Ac)\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{13/4}}-\frac{5bB-9Ac}{2b^3\sqrt{x}}+\frac{5bB-9Ac}{10b^2cx^{5/2}}-\frac{bB-Ac}{2bcx^{5/2}\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (5*b*B - 9*A*c)/(10*b^2*c*x^(5/2)) - (5*b*B - 9*A*c)/(2*b^3*Sqrt[x]) - (b*B
 - A*c)/(2*b*c*x^(5/2)*(b + c*x^2)) + (c^(1/4)*(5*b*B - 9*A*c)*ArcTan[1 - (
Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13/4)) - (c^(1/4)*(5*b*B -
 9*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13/4))
 - (c^(1/4)*(5*b*B - 9*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4)) + (c^(1/4)*(5*b*B - 9*A*c)*Log[Sqrt[b] +
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(bx^2+cx^4\right)^2}\,dx = \int \frac{A+Bx^2}{x^{7/2}\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} + \frac{\left(-\frac{5bB}{2}+\frac{9Ac}{2}\right)\int \frac{1}{x^{7/2}(b+cx^2)}\,dx}{2bc}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} + \frac{(5bB-9Ac)\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{4b^2}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} - \frac{(c(5bB-9Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^3}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} - \frac{(c(5bB-9Ac))\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx,\right.}{2b^3}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} + \frac{\left(\sqrt{c}\,(5bB-9Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-}{b-}\right.}{4b^3}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} - \frac{(5bB-9Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}}{\sqrt[4]{}}}\right.}{8b^3}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} - \frac{\sqrt[4]{c}\,(5bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\right.}{8\sqrt{2}\,b^{13/4}}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} + \frac{\sqrt[4]{c}\,(5bB-9Ac)\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{4\sqrt{2}\,b^{13/4}}$$

Mathematica [A]
time = 0.52, size = 187, normalized size = 0.60

$$\frac{-\frac{4\sqrt[4]{b}\,(5bBx^2(4b+5cx^2)+A(4b^2-36bcx^2-45c^2x^4))}{x^{5/2}(b+cx^2)}+5\sqrt{2}\sqrt[4]{c}\,(5bB-9Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+5\sqrt{2}\sqrt[4]{c}\,(5bB-9Ac)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{40b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*b^(1/4)*(5*b*B*x^2*(4*b + 5*c*x^2) + A*(4*b^2 - 36*b*c*x^2 - 45*c^2*x^
4)))/(x^(5/2)*(b + c*x^2)) + 5*Sqrt[2]*c^(1/4)*(5*b*B - 9*A*c)*ArcTan[(Sqrt
[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 5*Sqrt[2]*c^(1/4)*(5*
```

```
b*B - 9*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x
)])/(40*b^(13/4))
```

Maple [A]

time = 0.40, size = 170, normalized size = 0.55

method	result
derivativedivides	$\frac{2c\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^{2}+b}+\frac{\left(\frac{9Ac}{4}-\frac{5Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}\cdots}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots\right)\right)}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^{3}}\cdots$
default	$\frac{2c\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^{2}+b}+\frac{\left(\frac{9Ac}{4}-\frac{5Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}\cdots}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots\right)\right)}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^{3}}\cdots$
risch	$-\frac{2\left(-10Ac\,x^{2}+5bB\,x^{2}+Ab\right)}{5b^{3}x^{\frac{5}{2}}}+\frac{c^{2}x^{\frac{3}{2}}A}{2b^{3}\left(c\,x^{2}+b\right)}-\frac{c\,x^{\frac{3}{2}}B}{2b^{2}\left(c\,x^{2}+b\right)}+\frac{9c\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{16b^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\frac{9c\sqrt{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*c*((1/4*A*c-1/4*B*b)*x^(3/2)/(c*x^2+b)+1/8*(9/4*A*c-5/4*B*b)/c/(b/c)^
(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4
)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*a
rctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))-2/5*A/b^2/x^(5/2)-2*(-2*A*c+B*b)/b^3
/x^(1/2)
```

Maxima [A]

time = 0.53, size = 250, normalized size = 0.81

$$-\frac{5\,(5\,Bbc-9\,Ac^{2})x^{4}+4\,Ab^{2}+4\,(5\,Bb^{2}-9\,Abc)x^{2}}{10\left(b^{3}cx^{\frac{9}{2}}+b^{4}x^{\frac{5}{2}}\right)}-\frac{(5\,Bbc-9\,Ac^{2})\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{16\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/10*(5*(5*B*b*c - 9*A*c^2)*x^4 + 4*A*b^2 + 4*(5*B*b^2 - 9*A*b*c)*x^2)/(b^
3*c*x^(9/2) + b^4*x^(5/2)) - 1/16*(5*B*b*c - 9*A*c^2)*(2*sqrt(2)*arctan(1/2
*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)
))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)
*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*
sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x
 + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x
) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^3
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(226) = 452$.

time = 2.29, size = 974, normalized size = 3.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*(b^3*c*x^5 + b^4*x^3)*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 121
50*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(1/4)*arctan((
sqrt((15625*B^6*b^6*c^2 - 168750*A*B^5*b^5*c^3 + 759375*A^2*B^4*b^4*c^4 - 1
822500*A^3*B^3*b^3*c^5 + 2460375*A^4*B^2*b^2*c^6 - 1771470*A^5*B*b*c^7 + 53
1441*A^6*c^8)*x - (625*B^4*b^11*c - 4500*A*B^3*b^10*c^2 + 12150*A^2*B^2*b^9
*c^3 - 14580*A^3*B*b^8*c^4 + 6561*A^4*b^7*c^5)*sqrt(-(625*B^4*b^4*c - 4500*
A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b
^13))*b^3*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 1
4580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(1/4) + (125*B^3*b^6*c - 675*A*B^2*b
^5*c^2 + 1215*A^2*B*b^4*c^3 - 729*A^3*b^3*c^4)*sqrt(x)*(-(625*B^4*b^4*c - 4
500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^
5)/b^13)^(1/4))/(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3
 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)) - 5*(b^3*c*x^5 + b^4*x^3)*(-(625*B^4*
b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 65
61*A^4*c^5)/b^13)^(1/4)*log(b^10*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12
150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(3/4) - (125*
B^3*b^3*c - 675*A*B^2*b^2*c^2 + 1215*A^2*B*b*c^3 - 729*A^3*c^4)*sqrt(x)) +
5*(b^3*c*x^5 + b^4*x^3)*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B
^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(1/4)*log(-b^10*(-(625
*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4
 + 6561*A^4*c^5)/b^13)^(3/4) - (125*B^3*b^3*c - 675*A*B^2*b^2*c^2 + 1215*A^
2*B*b*c^3 - 729*A^3*c^4)*sqrt(x)) + 4*(5*(5*B*b*c - 9*A*c^2)*x^4 + 4*A*b^2
+ 4*(5*B*b^2 - 9*A*b*c)*x^2)*sqrt(x))/(b^3*c*x^5 + b^4*x^3)
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 303, normalized size = 0.98

$$-\frac{Bbcx^{\frac{3}{2}} - Ac^2x^{\frac{3}{2}}}{2\,(cx^2+b)b^3} - \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{3}{4}}\,Bb - 9\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4c^2} - \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{3}{4}}\,Bb - 9\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4c^2} + \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{3}{4}}\,Bb - 9\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4c^2} - \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{3}{4}}\,Bb - 9\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4c^2} - \frac{2\,(5\,Bbx^2 - 10\,Acx^2 + Ab)}{5\,b^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b*c*x^(3/2) - A*c^2*x^(3/2))/((c*x^2 + b)*b^3) - 1/8*sqrt(2)*(5*(b*
c^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c^2) - 1/8*sqrt(2)*(5*(b*c^3)^(3/4)*B*b -
 9*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))
/(b/c)^(1/4))/(b^4*c^2) + 1/16*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 9*(b*c^3)^(3/
4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c^2) - 1/16*s
qrt(2)*(5*(b*c^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/
c)^(1/4) + x + sqrt(b/c))/(b^4*c^2) - 2/5*(5*B*b*x^2 - 10*A*c*x^2 + A*b)/(b
^3*x^(5/2))
```

Mupad [B]
time = 0.25, size = 121, normalized size = 0.39

$$\frac{\frac{2\,x^2\,(9\,A\,c-5\,B\,b)}{5\,b^2}-\frac{2\,A}{5\,b}+\frac{c\,x^4\,(9\,A\,c-5\,B\,b)}{2\,b^3}}{b\,x^{5/2}+c\,x^{9/2}}+\frac{{(-c)}^{1/4}\,\mathrm{atan}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(9\,A\,c-5\,B\,b)}{4\,b^{13/4}}-\frac{{(-c)}^{1/4}\,\mathrm{atanh}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(9\,A\,c-5\,B\,b)}{4\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] ((2*x^2*(9*A*c - 5*B*b))/(5*b^2) - (2*A)/(5*b) + (c*x^4*(9*A*c - 5*B*b))/(2
*b^3))/(b*x^(5/2) + c*x^(9/2)) + ((-c)^(1/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1
/4))*(9*A*c - 5*B*b))/(4*b^(13/4)) - ((-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))
/b^(1/4))*(9*A*c - 5*B*b))/(4*b^(13/4))
```

3.205 $\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=310

$$\frac{7bB-11Ac}{14b^2cx^{7/2}}-\frac{7bB-11Ac}{6b^3x^{3/2}}-\frac{bB-Ac}{2bcx^{7/2}\,(b+cx^2)}+\frac{c^{3/4}(7bB-11Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{15/4}}-\frac{c^{3/4}(7bB-1}{}$$

```
[Out] 1/14*(-11*A*c+7*B*b)/b^2/c/x^(7/2)+1/6*(11*A*c-7*B*b)/b^3/x^(3/2)+1/2*(A*c-
B*b)/b/c/x^(7/2)/(c*x^2+b)+1/8*c^(3/4)*(-11*A*c+7*B*b)*arctan(1-c^(1/4)*2^(
1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2^(1/2)-1/8*c^(3/4)*(-11*A*c+7*B*b)*arctan(1
+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2^(1/2)+1/16*c^(3/4)*(-11*A*c+7*
B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/4)*2^(1/2)
-1/16*c^(3/4)*(-11*A*c+7*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*
x^(1/2))/b^(15/4)*2^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{c^{3/4}(7bB-11Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{15/4}}-\frac{c^{3/4}(7bB-11Ac)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{15/4}}+\frac{c^{3/4}(7bB-11Ac)\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{15/4}}-\frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{15/4}}-\frac{7bB-11Ac}{6b^3x^{3/2}}+\frac{7bB-11Ac}{14b^2cx^{7/2}}-\frac{bB-Ac}{2bcx^{7/2}\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (7*b*B - 11*A*c)/(14*b^2*c*x^(7/2)) - (7*b*B - 11*A*c)/(6*b^3*x^(3/2)) - (b
*B - A*c)/(2*b*c*x^(7/2)*(b + c*x^2)) + (c^(3/4)*(7*b*B - 11*A*c)*ArcTan[1
- (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(15/4)) - (c^(3/4)*(7*b*
B - 11*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(15
/4)) + (c^(3/4)*(7*b*B - 11*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4)) - (c^(3/4)*(7*b*B - 11*A*c)*Log[Sqrt
[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\,\left(bx^2+cx^4\right)^2}\,dx = \int \frac{A+Bx^2}{x^{9/2}\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} + \frac{\left(-\frac{7bB}{2}+\frac{11Ac}{2}\right)\int \frac{1}{x^{9/2}(b+cx^2)}\,dx}{2bc}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} + \frac{(7bB-11Ac)\int \frac{1}{x^{5/2}(b+cx^2)}\,dx}{4b^2}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} - \frac{(c(7bB-11Ac))\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,d}{4b^3}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} - \frac{(c(7bB-11Ac))\text{Subst}\left(\int \frac{1}{b+cx^4}\right.}{2b^3}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} - \frac{(c(7bB-11Ac))\text{Subst}\left(\int \frac{\sqrt{b}}{b}\right.}{4b^{7/2}}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} - \frac{\left(\sqrt{c}\,(7bB-11Ac)\right)\text{Subst}\left(\int\right.}{8b}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} + \frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{b}-\right.}{8\sqrt{2}\,b^{15}}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} + \frac{c^{3/4}(7bB-11Ac)\tan^{-1}\left(1-\right.}{4\sqrt{2}\,b^{15/4}}$$

Mathematica [A]
time = 0.53, size = 187, normalized size = 0.60

$$\frac{-\frac{4b^{3/4}\left(7bBx^2\left(4b+7cx^2\right)+A\left(12b^2-44bcx^2-77c^2x^4\right)\right)}{x^{7/2}\left(b+cx^2\right)}+21\sqrt{2}\,c^{3/4}(7bB-11Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)+21\sqrt{2}\,c^{3/4}(-7bB+11Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{168b^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*b^(3/4)*(7*b*B*x^2*(4*b + 7*c*x^2) + A*(12*b^2 - 44*b*c*x^2 - 77*c^2*x
^4)))/(x^(7/2)*(b + c*x^2)) + 21*Sqrt[2]*c^(3/4)*(7*b*B - 11*A*c)*ArcTan[(S
qrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 21*Sqrt[2]*c^(3/4)
*(-7*b*B + 11*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqr
t[c]*x)])/(168*b^(15/4))
```

Maple [A]
time = 0.40, size = 170, normalized size = 0.55

method	result
derivativedivides	$-\frac{2A}{7b^2x^{\frac{7}{2}}}-\frac{2(-2Ac+Bb)}{3b^3x^{\frac{3}{2}}}+\frac{2c\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(11Ac-7Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\,\mathrm{arc}\right.}{32b}\right.}{b^3}$
default	$-\frac{2A}{7b^2x^{\frac{7}{2}}}-\frac{2(-2Ac+Bb)}{3b^3x^{\frac{3}{2}}}+\frac{2c\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(11Ac-7Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\,\mathrm{arc}\right.}{32b}\right.}{b^3}$
risch	$-\frac{2(-14Ac\,x^2+7bB\,x^2+3Ab)}{21b^3x^{\frac{7}{2}}}+\frac{c^2\sqrt{x}\,A}{2b^3(c\,x^2+b)}-\frac{c\sqrt{x}\,B}{2b^2(c\,x^2+b)}+\frac{11c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8b^4}+ -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*A/b^2/x^(7/2)-2/3*(-2*A*c+B*b)/b^3/x^(3/2)+2/b^3*c*((1/4*A*c-1/4*B*b)*
x^(1/2)/(c*x^2+b)+1/32*(11*A*c-7*B*b)*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)^(1
/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)
))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(
1/2)-1)))
```

Maxima [A]
time = 0.52, size = 286, normalized size = 0.92

$$-\frac{7\,(7\,Bbc-11\,Ac^2)x^4+12\,Ab^2+4\,(7\,Bb^2-11\,Abc)x^2}{42\,\left(b^3cx^{\frac{11}{2}}+b^4x^{\frac{7}{2}}\right)}-\frac{\frac{2\sqrt{2}\,(7\,Bbc-11\,Ac^2)\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,(7\,Bbc-11\,Ac^2)\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,(7\,Bbc-11\,Ac^2)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,(7\,Bbc-11\,Ac^2)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{16\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/42*(7*(7*B*b*c - 11*A*c^2)*x^4 + 12*A*b^2 + 4*(7*B*b^2 - 11*A*b*c)*x^2)/
(b^3*c*x^(11/2) + b^4*x^(7/2)) - 1/16*(2*sqrt(2)*(7*B*b*c - 11*A*c^2)*arcta
n(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sq
```

```
rt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(7*B*b*c - 11*A*c^2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(7*B*b*c - 11*A*c^2)*
log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)
) - sqrt(2)*(7*B*b*c - 11*A*c^2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqr
t(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^3
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(226) = 452$.
time = 2.09, size = 795, normalized size = 2.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/168*(84*(b^3*c*x^6 + b^4*x^4)*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 +
 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4)*arc
tan((sqrt(b^8*sqrt(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2
*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15) + (49*B^2*b^2*c^2 - 154
*A*B*b*c^3 + 121*A^2*c^4)*x)*b^11*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4
 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(3/4) +
 (7*B*b^12*c - 11*A*b^11*c^2)*sqrt(x)*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3
*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(3/
4))/(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268
*A^3*B*b*c^6 + 14641*A^4*c^7)) + 21*(b^3*c*x^6 + b^4*x^4)*(-(2401*B^4*b^4*c
^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 1464
1*A^4*c^7)/b^15)^(1/4)*log(b^4*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 +
35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4) - (7
*B*b*c - 11*A*c^2)*sqrt(x)) - 21*(b^3*c*x^6 + b^4*x^4)*(-(2401*B^4*b^4*c^3
- 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A
^4*c^7)/b^15)^(1/4)*log(-b^4*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35
574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4) - (7*B
*b*c - 11*A*c^2)*sqrt(x)) - 4*(7*(7*B*b*c - 11*A*c^2)*x^4 + 12*A*b^2 + 4*(7
*B*b^2 - 11*A*b*c)*x^2)*sqrt(x))/(b^3*c*x^6 + b^4*x^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 292, normalized size = 0.94

$$-\frac{\sqrt{2}\left(7\,(bc^3)^{\frac{1}{4}}Bb-11\,(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4}-\frac{\sqrt{2}\left(7\,(bc^3)^{\frac{1}{4}}Bb-11\,(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4}-\frac{\sqrt{2}\left(7\,(bc^3)^{\frac{1}{4}}Bb-11\,(bc^3)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4}+\frac{\sqrt{2}\left(7\,(bc^3)^{\frac{1}{4}}Bb-11\,(bc^3)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4}-\frac{Bbc\sqrt{x}-Ac^2\sqrt{x}}{2\,(cx^2+b)b^3}-\frac{2\,(7\,Bbx^2-14\,Acx^2+3\,Ab)}{21\,b^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^4 - 1/8*sqrt(2)*(7*(b*c^
3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 - 1/16*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 11*(
b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 + 1/
16*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x
)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/2*(B*b*c*sqrt(x) - A*c^2*sqrt(x))/((
c*x^2 + b)*b^3) - 2/21*(7*B*b*x^2 - 14*A*c*x^2 + 3*A*b)/(b^3*x^(7/2))
```

Mupad [B]
time = 0.40, size = 595, normalized size = 1.92

$$\frac{\frac{2\,x^2\,(11\,A\,c-7\,B\,b)}{21\,b^2}-\frac{2\,A}{7\,b}+\frac{c\,x^4\,(11\,A\,c-7\,B\,b)}{6\,b^3}}{b\,x^{7/2}+c\,x^{11/2}}+\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{\frac{(-c)^{3/4}(11\,A\,c-7\,B\,b)\left(\sqrt{x}\,(3872\,A^2b^9c^7-4928\,A\,B\,b^{10}c^6+1568\,B^2b^{11}c^5)-\frac{(-c)^{3/4}(11\,A\,c-7\,B\,b)(2816\,A\,b^{13}c^5-1792\,B\,b^{14}c^4)\,1i}{8\,b^{15/4}}\right)}{8\,b^{15/4}}+\frac{(-c)^{3/4}(11\,A\,c-7\,B\,b)\left(\sqrt{x}\,(3872\,A^2b^9c^7-4928\,A\,B\,b^{10}c^6+1568\,B^2b^{11}c^5)+\frac{(-c)^{3/4}(11\,A\,c-7\,B\,b)(2816\,A\,b^{13}c^5-1792\,B\,b^{14}c^4)\,1i}{8\,b^{15/4}}\right)}{8\,b^{15/4}}}{\frac{(-c)^{3/4}(11\,A\,c-7\,B\,b)\left(\sqrt{x}\,(3872\,A^2b^9c^7-4928\,A\,B\,b^{10}c^6+1568\,B^2b^{11}c^5)-\frac{(-c)^{3/4}(11\,A\,c-7\,B\,b)(2816\,A\,b^{13}c^5-1792\,B\,b^{14}c^4)\,1i}{8\,b^{15/4}}\right)1i}{8\,b^{15/4}}-\frac{(-c)^{3/4}(11\,A\,c-7\,B\,b)\left(\sqrt{x}\,(3872\,A^2b^9c^7-4928\,A\,B\,b^{10}c^6+1568\,B^2b^{11}c^5)+\frac{(-c)^{3/4}(11\,A\,c-7\,B\,b)(2816\,A\,b^{13}c^5-1792\,B\,b^{14}c^4)\,1i}{8\,b^{15/4}}\right)1i}{8\,b^{15/4}}}\right)(11\,A\,c-7\,B\,b)}{4\,b^{15/4}}-\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{A^3c^8\sqrt{x}\,1331{}i-B^3b^3c^5\sqrt{x}\,343{}i-A^2B\,b\,c^7\sqrt{x}\,2541{}i+A\,B^2b^2c^6\sqrt{x}\,1617{}i}{b^{1/4}(-c)^{19/4}\left(c\,(c\,(1331\,A^3c-2541\,A^2B\,b)+1617\,A\,B^2b^2)-343\,B^3b^3\right)}\right)(11\,A\,c-7\,B\,b)\,1i}{4\,b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^2),x)
```

```
[Out] ((2*x^2*(11*A*c - 7*B*b))/(21*b^2) - (2*A)/(7*b) + (c*x^4*(11*A*c - 7*B*b))
/(6*b^3))/(b*x^(7/2) + c*x^(11/2)) + ((-c)^(3/4)*atan((((-c)^(3/4)*(11*A*c
- 7*B*b)*(x^(1/2)*(3872*A^2*b^9*c^7 + 1568*B^2*b^11*c^5 - 4928*A*B*b^10*c^6
) - ((-c)^(3/4)*(11*A*c - 7*B*b)*(2816*A*b^13*c^5 - 1792*B*b^14*c^4)*1i)/(8
*b^(15/4))))/(8*b^(15/4)) + ((-c)^(3/4)*(11*A*c - 7*B*b)*(x^(1/2)*(3872*A^2
*b^9*c^7 + 1568*B^2*b^11*c^5 - 4928*A*B*b^10*c^6) + ((-c)^(3/4)*(11*A*c - 7
*B*b)*(2816*A*b^13*c^5 - 1792*B*b^14*c^4)*1i)/(8*b^(15/4))))/(8*b^(15/4)))/
(((-c)^(3/4)*(11*A*c - 7*B*b)*(x^(1/2)*(3872*A^2*b^9*c^7 + 1568*B^2*b^11*c^
5 - 4928*A*B*b^10*c^6) - ((-c)^(3/4)*(11*A*c - 7*B*b)*(2816*A*b^13*c^5 - 17
92*B*b^14*c^4)*1i)/(8*b^(15/4)))*1i)/(8*b^(15/4)) - ((-c)^(3/4)*(11*A*c - 7
*B*b)*(x^(1/2)*(3872*A^2*b^9*c^7 + 1568*B^2*b^11*c^5 - 4928*A*B*b^10*c^6) +
 ((-c)^(3/4)*(11*A*c - 7*B*b)*(2816*A*b^13*c^5 - 1792*B*b^14*c^4)*1i)/(8*b^
(15/4)))*1i)/(8*b^(15/4))))*(11*A*c - 7*B*b))/(4*b^(15/4)) - ((-c)^(3/4)*at
an((A^3*c^8*x^(1/2)*1331i - B^3*b^3*c^5*x^(1/2)*343i - A^2*B*b*c^7*x^(1/2)*
2541i + A*B^2*b^2*c^6*x^(1/2)*1617i)/(b^(1/4)*(-c)^(19/4)*(c*(c*(1331*A^3*c
 - 2541*A^2*B*b) + 1617*A*B^2*b^2) - 343*B^3*b^3)))*(11*A*c - 7*B*b)*1i)/(4
*b^(15/4))
```

3.206 $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^2} dx$

Optimal. Leaf size=332

$$\frac{9bB-13Ac}{18b^2cx^{9/2}}-\frac{9bB-13Ac}{10b^3x^{5/2}}+\frac{c(9bB-13Ac)}{2b^4\sqrt{x}}-\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)}-\frac{c^{5/4}(9bB-13Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{4\sqrt{2}\,b^{17/4}}$$

```
[Out] 1/18*(-13*A*c+9*B*b)/b^2/c/x^(9/2)+1/10*(13*A*c-9*B*b)/b^3/x^(5/2)+1/2*(A*c
-B*b)/b/c/x^(9/2)/(c*x^2+b)-1/8*c^(5/4)*(-13*A*c+9*B*b)*arctan(1-c^(1/4)*2^
(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)+1/8*c^(5/4)*(-13*A*c+9*B*b)*arctan(
1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)+1/16*c^(5/4)*(-13*A*c+9
*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(17/4)*2^(1/2
)-1/16*c^(5/4)*(-13*A*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)
*x^(1/2))/b^(17/4)*2^(1/2)+1/2*c*(-13*A*c+9*B*b)/b^4/x^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{c^{5/4}(9bB-13Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{17/4}}+\frac{c^{5/4}(9bB-13Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{17/4}}+\frac{c^{5/4}(9bB-13Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{17/4}}-\frac{c^{5/4}(9bB-13Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{17/4}}+\frac{c(9bB-13Ac)}{2b^4\sqrt{x}}-\frac{9bB-13Ac}{10b^3x^{5/2}}+\frac{9bB-13Ac}{18b^2cx^{9/2}}-\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (9*b*B - 13*A*c)/(18*b^2*c*x^(9/2)) - (9*b*B - 13*A*c)/(10*b^3*x^(5/2)) + (
c*(9*b*B - 13*A*c))/(2*b^4*Sqrt[x]) - (b*B - A*c)/(2*b*c*x^(9/2)*(b + c*x^2
)) - (c^(5/4)*(9*b*B - 13*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
])/(4*Sqrt[2]*b^(17/4)) + (c^(5/4)*(9*b*B - 13*A*c)*ArcTan[1 + (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) + (c^(5/4)*(9*b*B - 13*A*c)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/
4)) - (c^(5/4)*(9*b*B - 13*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{A+Bx^2}{x^{11/2}\left(b+cx^2\right)^2}\,dx \\
&= -\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{\left(-\frac{9bB}{2}+\frac{13Ac}{2}\right)\int \frac{1}{x^{11/2}(b+cx^2)}\,dx}{2bc} \\
&= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{(9bB-13Ac)\int \frac{1}{x^{7/2}(b+cx^2)}\,dx}{4b^2} \\
&= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} - \frac{(c(9bB-13Ac))\int \frac{1}{x^{3/2}(b+cx^2)}\,\cdots}{4b^3} \\
&= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{(c^2(9bB-\cdots}{\cdots} \\
&= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{(c^2(9bB-\cdots}{\cdots} \\
&= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} - \frac{(c^{3/2}(9bB-\cdots}{\cdots} \\
&= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{(c(9bB-1\cdots}{\cdots} \\
&= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{c^{5/4}(9bB-\cdots}{\cdots} \\
&= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} - \frac{c^{5/4}(9bB-\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A]

time = 0.58, size = 209, normalized size = 0.63

$$
\frac{-\frac{4\sqrt[4]{b}\left(9bBx^2\left(4b^2-36bcx^2-45c^2x^4\right)+A\left(20b^3-52b^2cx^2+468bc^2x^4+585c^3x^6\right)\right)}{x^{9/2}(b+cx^2)}+45\sqrt{2}\,c^{5/4}(-9bB+13Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+45\sqrt{2}\,c^{5/4}(-9bB+13Ac)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{360b^{17/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
[Out] ((-4*b^(1/4)*(9*b*B*x^2*(4*b^2 - 36*b*c*x^2 - 45*c^2*x^4) + A*(20*b^3 - 52*
b^2*c*x^2 + 468*b*c^2*x^4 + 585*c^3*x^6)))/(x^(9/2)*(b + c*x^2)) + 45*Sqrt[
```

```
2]*c^(5/4)*(-9*b*B + 13*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x])] + 45*Sqrt[2]*c^(5/4)*(-9*b*B + 13*A*c)*ArcTanh[(Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(360*b^(17/4))
```

Maple [A]

time = 0.41, size = 190, normalized size = 0.57

method	result
derivativedivides	$-\frac{2c^2\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{13Ac}{4}-\frac{9Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\right.}\right.\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^4}$
default	$-\frac{2c^2\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{13Ac}{4}-\frac{9Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\right.}\right.\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^4}$
risch	$-\frac{2(135A\,c^2x^4-90x^4bBc-18Abc\,x^2+9b^2B\,x^2+5b^2A)}{45b^4x^{\frac{9}{2}}}-\frac{c^3x^{\frac{3}{2}}A}{2b^4(c\,x^2+b)}+\frac{c^2x^{\frac{3}{2}}B}{2b^3(c\,x^2+b)}-\frac{13c^2\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{}}\right.}{16b^4\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^4*c^2*((1/4*A*c-1/4*B*b)*x^(3/2)/(c*x^2+b)+1/8*(13/4*A*c-9/4*B*b)/c/(b
/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^
(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)
+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))-2/9*A/b^2/x^(9/2)-2/5*(-2*A*c+B*
b)/b^3/x^(5/2)-2*c*(3*A*c-2*B*b)/b^4/x^(1/2)
```

Maxima [A]

time = 0.51, size = 276, normalized size = 0.83

$$\frac{45\,(9\,Bbc^2-13\,Ac^3)x^6+36\,(9\,Bb^2c-13\,Abc^2)x^4-20\,Ab^3-4\,(9\,Bb^3-13\,Ab^2c)x^2}{90\left(b^4cx^{\frac{13}{2}}+b^5x^{\frac{9}{2}}\right)}+\frac{(9\,Bbc^2-13\,Ac^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{16\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/90*(45*(9*B*b*c^2 - 13*A*c^3)*x^6 + 36*(9*B*b^2*c - 13*A*b*c^2)*x^4 - 20*
A*b^3 - 4*(9*B*b^3 - 13*A*b^2*c)*x^2)/(b^4*c*x^(13/2) + b^5*x^(9/2)) + 1/16
*(9*B*b*c^2 - 13*A*c^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1
/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt
(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*s
qrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*lo
g(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4))
+ sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1
/4)*c^(3/4)))/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1024 vs. $2(244) = 488$.
time = 2.70, size = 1024, normalized size = 3.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/360*(180*(b^4*c*x^7 + b^5*x^5)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6
+ 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(1/4)*ar
ctan((sqrt((531441*B^6*b^6*c^8 - 4605822*A*B^5*b^5*c^9 + 16632135*A^2*B^4*b
^4*c^10 - 32032260*A^3*B^3*b^3*c^11 + 34701615*A^4*B^2*b^2*c^12 - 20049822*
A^5*B*b*c^13 + 4826809*A^6*c^14)*x - (6561*B^4*b^13*c^5 - 37908*A*B^3*b^12*
c^6 + 82134*A^2*B^2*b^11*c^7 - 79092*A^3*B*b^10*c^8 + 28561*A^4*b^9*c^9)*sq
rt(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092
*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17))*b^4*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*
b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^
(1/4) + (729*B^3*b^7*c^4 - 3159*A*B^2*b^6*c^5 + 4563*A^2*B*b^5*c^6 - 2197*A
^3*b^4*c^7)*sqrt(x)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B
^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(1/4))/(6561*B^4*b^4*
c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 285
61*A^4*c^9)) - 45*(b^4*c*x^7 + b^5*x^5)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b
^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(
1/4)*log(b^13*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2
*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(3/4) - (729*B^3*b^3*c^4 -
3159*A*B^2*b^2*c^5 + 4563*A^2*B*b*c^6 - 2197*A^3*c^7)*sqrt(x)) + 45*(b^4*c*
x^7 + b^5*x^5)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^
2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(1/4)*log(-b^13*(-(6561*B^
4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8
 + 28561*A^4*c^9)/b^17)^(3/4) - (729*B^3*b^3*c^4 - 3159*A*B^2*b^2*c^5 + 456
3*A^2*B*b*c^6 - 2197*A^3*c^7)*sqrt(x)) + 4*(45*(9*B*b*c^2 - 13*A*c^3)*x^6 +
 36*(9*B*b^2*c - 13*A*b*c^2)*x^4 - 20*A*b^3 - 4*(9*B*b^3 - 13*A*b^2*c)*x^2)
*sqrt(x))/(b^4*c*x^7 + b^5*x^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 328, normalized size = 0.99

$$\frac{\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}\,Bb-13\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^5c}+\frac{\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}\,Bb-13\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^5c}-\frac{\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}\,Bb-13\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^5c}+\frac{\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}\,Bb-13\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^5c}+\frac{Bbc^2x^{\frac{3}{2}}-Ac^3x^{\frac{3}{2}}}{2\,(cx^2+b)b^4}+\frac{2\,(90\,Bbcx^4-135\,Ac^2x^4-9\,Bb^2x^2+18\,Abcx^2-5\,Ab^2)}{45\,b^4x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) + 1/8*sqrt(2)*(9*(b
*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^
(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) - 1/16*sqrt(2)*(9*(b*c^3)^(3/4)*B*b
 - 13*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(
b^5*c) + 1/16*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*log(-sqr
t(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c) + 1/2*(B*b*c^2*x^(3/2) -
A*c^3*x^(3/2))/((c*x^2 + b)*b^4) + 2/45*(90*B*b*c*x^4 - 135*A*c^2*x^4 - 9*B
*b^2*x^2 + 18*A*b*c*x^2 - 5*A*b^2)/(b^4*x^(9/2))
```

Mupad [B]
time = 0.20, size = 142, normalized size = 0.43

$$\frac{(-c)^{5/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(13\,A\,c-9\,B\,b)}{4\,b^{17/4}}-\frac{\frac{2\,A}{9\,b}-\frac{2\,x^2\,(13\,A\,c-9\,B\,b)}{45\,b^2}+\frac{c^2\,x^6\,(13\,A\,c-9\,B\,b)}{2\,b^4}+\frac{2\,c\,x^4\,(13\,A\,c-9\,B\,b)}{5\,b^3}}{b\,x^{9/2}+c\,x^{13/2}}-\frac{(-c)^{5/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(13\,A\,c-9\,B\,b)}{4\,b^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^2),x)
```

```
[Out] ((-c)^(5/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(13*A*c - 9*B*b))/(4*b^(17/4
)) - ((2*A)/(9*b) - (2*x^2*(13*A*c - 9*B*b))/(45*b^2) + (c^2*x^6*(13*A*c -
9*B*b))/(2*b^4) + (2*c*x^4*(13*A*c - 9*B*b))/(5*b^3))/(b*x^(9/2) + c*x^(13/
2)) - ((-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4))*(13*A*c - 9*B*b))/(4*
b^(17/4))
```

3.207 $\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=343

$$-\frac{9(13bB-5Ac)\sqrt{x}}{16c^4}+\frac{9(13bB-5Ac)x^{5/2}}{80bc^3}-\frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2}-\frac{(13bB-5Ac)x^{9/2}}{16bc^2\,(b+cx^2)}-\frac{9\sqrt[4]{b}\,(13bB-5Ac)\tan^{-}}{32\sqrt{2}}$$

```
[Out] 9/80*(-5*A*c+13*B*b)*x^(5/2)/b/c^3-1/4*(-A*c+B*b)*x^(13/2)/b/c/(c*x^2+b)^2-
1/16*(-5*A*c+13*B*b)*x^(9/2)/b/c^2/(c*x^2+b)-9/64*b^(1/4)*(-5*A*c+13*B*b)*a
rctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(17/4)*2^(1/2)+9/64*b^(1/4)*(-5*
A*c+13*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(17/4)*2^(1/2)-9/12
8*b^(1/4)*(-5*A*c+13*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1
/2))/c^(17/4)*2^(1/2)+9/128*b^(1/4)*(-5*A*c+13*B*b)*ln(b^(1/2)+x*c^(1/2)+b^
(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(17/4)*2^(1/2)-9/16*(-5*A*c+13*B*b)*x^(1/2
)/c^4
```

Rubi [A]
time = 0.19, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1598, 468, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{9\sqrt[4]{b}\,(13bB-5Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,c^{17/4}}+\frac{9\sqrt[4]{b}\,(13bB-5Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,c^{17/4}}-\frac{9\sqrt[4]{b}\,(13bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,c^{17/4}}+\frac{9\sqrt[4]{b}\,(13bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,c^{17/4}}-\frac{9\sqrt{x}\,(13bB-5Ac)}{16c^4}+\frac{9x^{5/2}(13bB-5Ac)}{80bc^3}-\frac{x^{9/2}(13bB-5Ac)}{16bc^2\,(b+cx^2)}-\frac{x^{13/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(23/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-9*(13*b*B - 5*A*c)*Sqrt[x])/(16*c^4) + (9*(13*b*B - 5*A*c)*x^(5/2))/(80*b
*c^3) - ((b*B - A*c)*x^(13/2))/(4*b*c*(b + c*x^2)^2) - ((13*b*B - 5*A*c)*x^
(9/2))/(16*b*c^2*(b + c*x^2)) - (9*b^(1/4)*(13*b*B - 5*A*c)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*c^(17/4)) + (9*b^(1/4)*(13*b*B
- 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*c^(17/4
)) - (9*b^(1/4)*(13*b*B - 5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(17/4)) + (9*b^(1/4)*(13*b*B - 5*A*c)*Log[S
qrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(17/4)
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \int \frac{x^{11/2}(A+Bx^2)}{(b+cx^2)^3}\,dx$$

$$= -\frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2} + \frac{\left(\frac{13bB}{2}-\frac{5Ac}{2}\right)\int \frac{x^{11/2}}{(b+cx^2)^2}\,dx}{4bc}$$

$$= -\frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2} - \frac{(13bB-5Ac)x^{9/2}}{16bc^2\,(b+cx^2)} + \frac{(9(13bB-5Ac))\int \frac{x^{7/2}}{b+cx^2}\,dx}{32bc^2}$$

$$= \frac{9(13bB-5Ac)x^{5/2}}{80bc^3} - \frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2} - \frac{(13bB-5Ac)x^{9/2}}{16bc^2\,(b+cx^2)} - \frac{(9(13bB-5Ac))\int}{32c^3}$$

$$= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB-5Ac)x^{5/2}}{80bc^3} - \frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2} - \frac{(13bB-5Ac)x^{9/2}}{16bc^2\,(b+cx^2)}$$

$$= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB-5Ac)x^{5/2}}{80bc^3} - \frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2} - \frac{(13bB-5Ac)x^{9/2}}{16bc^2\,(b+cx^2)}$$

$$= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB-5Ac)x^{5/2}}{80bc^3} - \frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2} - \frac{(13bB-5Ac)x^{9/2}}{16bc^2\,(b+cx^2)}$$

$$= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB-5Ac)x^{5/2}}{80bc^3} - \frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2} - \frac{(13bB-5Ac)x^{9/2}}{16bc^2\,(b+cx^2)}$$

$$= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB-5Ac)x^{5/2}}{80bc^3} - \frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2} - \frac{(13bB-5Ac)x^{9/2}}{16bc^2\,(b+cx^2)}$$

$$= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB-5Ac)x^{5/2}}{80bc^3} - \frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2} - \frac{(13bB-5Ac)x^{9/2}}{16bc^2\,(b+cx^2)}$$

Mathematica [A]
time = 0.53, size = 204, normalized size = 0.59

$$\frac{\frac{4\sqrt[4]{c}\sqrt{x}\left(-585b^3B+bc^2x^2(405A-416Bx^2)+9b^2c(25A-117Bx^2)+32c^3x^4(5A+Bx^2)\right)}{(b+cx^2)^2} - 45\sqrt{2}\sqrt[4]{b}\,(13bB-5Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) + 45\sqrt{2}\sqrt[4]{b}\,(13bB-5Ac)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{320c^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(23/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*c^(1/4)*Sqrt[x]*(-585*b^3*B + b*c^2*x^2*(405*A - 416*B*x^2) + 9*b^2*c*(
25*A - 117*B*x^2) + 32*c^3*x^4*(5*A + B*x^2)))/(b + c*x^2)^2 - 45*Sqrt[2]*b
```

```
^(1/4)*(13*b*B - 5*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x])] + 45*Sqrt[2]*b^(1/4)*(13*b*B - 5*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(320*c^(17/4))
```

Maple [A]
time = 0.41, size = 191, normalized size = 0.56

method	result
derivativedivides	$\frac{\frac{2 B x^{\frac{5}{2}} c}{5}+2 A c \sqrt{x}-6 B b \sqrt{x}}{c^{4}}-\frac{2 b \left(\frac{\left(-\frac{17}{32} A c^{2}+\frac{25}{32} b B c\right) x^{\frac{5}{2}}-\frac{b (13 A c-21 B b) \sqrt{x}}{32}}{\left(c x^{2}+b\right)^{2}}+\frac{9 (5 A c-13 B b) \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\left(\ln\left(\frac{x+(\cdots}{x-(\cdots}\right.\right.\cdots}{\cdots}\right)}{c\cdots}$
default	$\frac{\frac{2 B x^{\frac{5}{2}} c}{5}+2 A c \sqrt{x}-6 B b \sqrt{x}}{c^{4}}-\frac{2 b \left(\frac{\left(-\frac{17}{32} A c^{2}+\frac{25}{32} b B c\right) x^{\frac{5}{2}}-\frac{b (13 A c-21 B b) \sqrt{x}}{32}}{\left(c x^{2}+b\right)^{2}}+\frac{9 (5 A c-13 B b) \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{2}\left(\ln\left(\frac{x+(\cdots}{x-(\cdots}\right.\right.\cdots}{\cdots}\right)}{c\cdots}$
risch	$\frac{2 \left(B c x^{2}+5 A c-15 B b\right) \sqrt{x}}{5 c^{4}}+\frac{17 b x^{\frac{5}{2}} A}{16 c^{2} \left(c x^{2}+b\right)^{2}}-\frac{25 b^{2} x^{\frac{5}{2}} B}{16 c^{3} \left(c x^{2}+b\right)^{2}}+\frac{13 b^{2} A \sqrt{x}}{16 c^{3} \left(c x^{2}+b\right)^{2}}-\frac{21 b^{3} B \sqrt{x}}{16 c^{4} \left(c x^{2}+b\right)^{2}}-\frac{45 \left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^4*(1/5*B*x^(5/2)*c+A*c*x^(1/2)-3*B*b*x^(1/2))-2*b/c^4*(((-17/32*A*c^2+2
5/32*b*B*c)*x^(5/2)-1/32*b*(13*A*c-21*B*b)*x^(1/2))/(c*x^2+b)^2+9/256*(5*A*
c-13*B*b)*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1
/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1
/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.55, size = 306, normalized size = 0.89

$$-\frac{\left(25 \, B b^{2} c - 17 \, A b c^{2}\right) x^{\frac{5}{2}} + \left(21 \, B b^{3} - 13 \, A b^{2} c\right) \sqrt{x}}{16 \, \left(c^{6} x^{4} + 2 \, b c^{5} x^{2} + b^{2} c^{4}\right)} + \frac{9 \, \left(\frac{2 \, \sqrt{2} \left(13 \, B b - 5 \, A c\right) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} + 2 \, \sqrt{c} \sqrt{x}\right)}{2 \, \sqrt{\sqrt{b} \sqrt{c}}}\right)}{\sqrt{b} \sqrt{\sqrt{b} \sqrt{c}}} + \frac{2 \, \sqrt{2} \left(13 \, B b - 5 \, A c\right) \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} - 2 \, \sqrt{c} \sqrt{x}\right)}{2 \, \sqrt{\sqrt{b} \sqrt{c}}}\right)}{\sqrt{b} \sqrt{\sqrt{b} \sqrt{c}}} + \frac{\sqrt{2} \left(13 \, B b - 5 \, A c\right) \log\left(\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x} + \sqrt{c} x + \sqrt{b}\right)}{b^{\frac{3}{4}} c^{\frac{1}{4}}} - \frac{\sqrt{2} \left(13 \, B b - 5 \, A c\right) \log\left(-\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x} + \sqrt{c} x + \sqrt{b}\right)}{b^{\frac{3}{4}} c^{\frac{1}{4}}}\right)} b}{128 \, c^{4}} + \frac{2 \, \left(B c x^{\frac{5}{2}} - 5 \, \left(3 \, B b - A c\right) \sqrt{x}\right)}{5 \, c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((25*B*b^2*c - 17*A*b*c^2)*x^(5/2) + (21*B*b^3 - 13*A*b^2*c)*sqrt(x))
/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4) + 9/128*(2*sqrt(2)*(13*B*b - 5*A*c)*arct
an(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*s
qrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(13*B*b - 5*A*c)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*s
qrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(13*B*b - 5*A*c)*log(sqr
t(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqr
t(2)*(13*B*b - 5*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sq
rt(b))/(b^(3/4)*c^(1/4)))*b/c^4 + 2/5*(B*c*x^(5/2) - 5*(3*B*b - A*c)*sqrt(x
))/c^4
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(255) = 510$.
time = 2.47, size = 817, normalized size = 2.38

$$[illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(180*(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)*(-(28561*B^4*b^5 - 43940*A*B^
3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)
^(1/4)*arctan((sqrt(c^8*sqrt(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^
2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17) + (169*B^2*b^2 -
130*A*B*b*c + 25*A^2*c^2)*x)*c^13*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25
350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(3/4) + (13
*B*b*c^13 - 5*A*c^14)*sqrt(x)*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*
A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(3/4))/(28561*B
^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 6
25*A^4*b*c^4)) + 45*(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)*(-(28561*B^4*b^5 - 43
940*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^
4)/c^17)^(1/4)*log(9*c^4*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^2*B
^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(1/4) - 9*(13*B*b -
5*A*c)*sqrt(x)) - 45*(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)*(-(28561*B^4*b^5 - 4
3940*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c
^4)/c^17)^(1/4)*log(-9*c^4*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^2
*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(1/4) - 9*(13*B*b
- 5*A*c)*sqrt(x)) - 4*(32*B*c^3*x^6 - 32*(13*B*b*c^2 - 5*A*c^3)*x^4 - 585*B
*b^3 + 225*A*b^2*c - 81*(13*B*b^2*c - 5*A*b*c^2)*x^2)*sqrt(x))/(c^6*x^4 + 2
*b*c^5*x^2 + b^2*c^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(23/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 321, normalized size = 0.94

$$\frac{9\sqrt{2}\left(13\left(bc^3\right)^{\frac{1}{4}}Bb-5\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64c^5}+\frac{9\sqrt{2}\left(13\left(bc^3\right)^{\frac{1}{4}}Bb-5\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64c^5}+\frac{9\sqrt{2}\left(13\left(bc^3\right)^{\frac{1}{4}}Bb-5\left(bc^3\right)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128c^5}-\frac{9\sqrt{2}\left(13\left(bc^3\right)^{\frac{1}{4}}Bb-5\left(bc^3\right)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128c^5}-\frac{25Bb^2cx^{\frac{5}{2}}-17Abc^2x^{\frac{5}{2}}+21Bb^3\sqrt{x}-13Ab^2c\sqrt{x}}{16\left(cx^2+b\right)^2c^4}+\frac{2\left(Bc^{12}x^{\frac{5}{2}}-15Bbc^{11}\sqrt{x}+5Ac^{12}\sqrt{x}\right)}{5c^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 9/64*sqrt(2)*(13*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 + 9/64*sqrt(2)*(13*(b*
c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) - 2*sqrt(x))/(b/c)^(1/4))/c^5 + 9/128*sqrt(2)*(13*(b*c^3)^(1/4)*B*b - 5
*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 -
9/128*sqrt(2)*(13*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqr
t(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 - 1/16*(25*B*b^2*c*x^(5/2) - 17*A*b*c
^2*x^(5/2) + 21*B*b^3*sqrt(x) - 13*A*b^2*c*sqrt(x))/((c*x^2 + b)^2*c^4) + 2
/5*(B*c^12*x^(5/2) - 15*B*b*c^11*sqrt(x) + 5*A*c^12*sqrt(x))/c^15
```

Mupad [B]
time = 0.25, size = 865, normalized size = 2.52

$$\frac{x^{5/2}\left(\frac{17Abc^2}{16}-\frac{25Bb^2c}{16}\right)-\sqrt{x}\left(\frac{21Bb^3}{16}-\frac{13Ab^2c}{16}\right)}{b^2c^4+2bc^5x^2+c^6x^4}+\sqrt{x}\left(\frac{2A}{c^3}-\frac{6Bb}{c^4}\right)+\frac{2Bx^{5/2}}{5c^3}+\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{1/4}\left(\frac{81\sqrt{x}\left(25A^2b^2c^2-130ABb^3c+169B^2b^4\right)}{64c^5}-\frac{81(-b)^{1/4}(5Ac-13Bb)\left(13Bb^3-5Ab^2c\right)}{64c^{21/4}}\right)(5Ac-13Bb)\,9\mathrm{i}}{64c^{17/4}}+\frac{(-b)^{1/4}\left(\frac{81\sqrt{x}\left(25A^2b^2c^2-130ABb^3c+169B^2b^4\right)}{64c^5}+\frac{81(-b)^{1/4}(5Ac-13Bb)\left(13Bb^3-5Ab^2c\right)}{64c^{21/4}}\right)(5Ac-13Bb)\,9\mathrm{i}}{64c^{17/4}}}{\frac{9(-b)^{1/4}\left(\frac{81\sqrt{x}\left(25A^2b^2c^2-130ABb^3c+169B^2b^4\right)}{64c^5}-\frac{81(-b)^{1/4}(5Ac-13Bb)\left(13Bb^3-5Ab^2c\right)}{64c^{21/4}}\right)(5Ac-13Bb)}{64c^{17/4}}-\frac{9(-b)^{1/4}\left(\frac{81\sqrt{x}\left(25A^2b^2c^2-130ABb^3c+169B^2b^4\right)}{64c^5}+\frac{81(-b)^{1/4}(5Ac-13Bb)\left(13Bb^3-5Ab^2c\right)}{64c^{21/4}}\right)(5Ac-13Bb)}{64c^{17/4}}}\right)(5Ac-13Bb)\,9\mathrm{i}}{32c^{17/4}}+\frac{9(-b)^{1/4}\,\mathrm{atan}\left(\frac{\frac{9(-b)^{1/4}\left(\frac{81\sqrt{x}\left(25A^2b^2c^2-130ABb^3c+169B^2b^4\right)}{64c^5}-\frac{(-b)^{1/4}(5Ac-13Bb)\left(13Bb^3-5Ab^2c\right)81\mathrm{i}}{64c^{21/4}}\right)(5Ac-13Bb)}{64c^{17/4}}+\frac{9(-b)^{1/4}\left(\frac{81\sqrt{x}\left(25A^2b^2c^2-130ABb^3c+169B^2b^4\right)}{64c^5}+\frac{(-b)^{1/4}(5Ac-13Bb)\left(13Bb^3-5Ab^2c\right)81\mathrm{i}}{64c^{21/4}}\right)(5Ac-13Bb)}{64c^{17/4}}}{\frac{(-b)^{1/4}\left(\frac{81\sqrt{x}\left(25A^2b^2c^2-130ABb^3c+169B^2b^4\right)}{64c^5}-\frac{(-b)^{1/4}(5Ac-13Bb)\left(13Bb^3-5Ab^2c\right)81\mathrm{i}}{64c^{21/4}}\right)(5Ac-13Bb)\,9\mathrm{i}}{64c^{17/4}}-\frac{(-b)^{1/4}\left(\frac{81\sqrt{x}\left(25A^2b^2c^2-130ABb^3c+169B^2b^4\right)}{64c^5}+\frac{(-b)^{1/4}(5Ac-13Bb)\left(13Bb^3-5Ab^2c\right)81\mathrm{i}}{64c^{21/4}}\right)(5Ac-13Bb)\,9\mathrm{i}}{64c^{17/4}}}\right)(5Ac-13Bb)}{32c^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(23/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^(5/2)*((17*A*b*c^2)/16 - (25*B*b^2*c)/16) - x^(1/2)*((21*B*b^3)/16 - (13
*A*b^2*c)/16))/(b^2*c^4 + c^6*x^4 + 2*b*c^5*x^2) + x^(1/2)*((2*A)/c^3 - (6*
B*b)/c^4) + (2*B*x^(5/2))/(5*c^3) + ((-b)^(1/4)*atan((((-b)^(1/4)*((81*x^(1
/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) - (81*(-b)^(1/
4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c))/(64*c^(21/4)))*(5*A*c - 13*B*b)
*9i)/(64*c^(17/4)) + ((-b)^(1/4)*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2
 - 130*A*B*b^3*c))/(64*c^5) + (81*(-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5
*A*b^2*c))/(64*c^(21/4)))*(5*A*c - 13*B*b)*9i)/(64*c^(17/4)))/((9*(-b)^(1/4
)*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) - (
81*(-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c))/(64*c^(21/4)))*(5*A*
c - 13*B*b))/(64*c^(17/4)) - (9*(-b)^(1/4)*((81*x^(1/2)*(169*B^2*b^4 + 25*A
^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) + (81*(-b)^(1/4)*(5*A*c - 13*B*b)*(13
*B*b^3 - 5*A*b^2*c))/(64*c^(21/4)))*(5*A*c - 13*B*b))/(64*c^(17/4))))*(5*A*
c - 13*B*b)*9i)/(32*c^(17/4)) + (9*(-b)^(1/4)*atan(((9*(-b)^(1/4)*((81*x^(1
/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) - ((-b)^(1/4)*
(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c)*81i)/(64*c^(21/4)))*(5*A*c - 13*B*b
```

```
))/(64*c^(17/4)) + (9*(-b)^(1/4)*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2
 - 130*A*B*b^3*c))/(64*c^5) + ((-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*
b^2*c)*81i)/(64*c^(21/4)))*(5*A*c - 13*B*b))/(64*c^(17/4)))/(((-b)^(1/4)*((
81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) - ((-b)
^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c)*81i)/(64*c^(21/4)))*(5*A*c -
 13*B*b)*9i)/(64*c^(17/4)) - ((-b)^(1/4)*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2
*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) + ((-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*b^
3 - 5*A*b^2*c)*81i)/(64*c^(21/4)))*(5*A*c - 13*B*b)*9i)/(64*c^(17/4))))*(5*
A*c - 13*B*b))/(32*c^(17/4))
```

3.208 $\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=322

$$\frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)}+\frac{7(11bB-3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}\,c^{15/4}}-\frac{7(1}{}$$

```
[Out] 7/48*(-3*A*c+11*B*b)*x^(3/2)/b/c^3-1/4*(-A*c+B*b)*x^(11/2)/b/c/(c*x^2+b)^2-
1/16*(-3*A*c+11*B*b)*x^(7/2)/b/c^2/(c*x^2+b)+7/64*(-3*A*c+11*B*b)*arctan(1-
c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(15/4)*2^(1/2)-7/64*(-3*A*c+11*B
*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(15/4)*2^(1/2)-7/12
8*(-3*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(
1/4)/c^(15/4)*2^(1/2)+7/128*(-3*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^
(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(15/4)*2^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1598, 468, 294, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{7(11bB-3Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}\,c^{15/4}}-\frac{7(11bB-3Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\sqrt[4]{b}\,c^{15/4}}-\frac{7(11bB-3Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\sqrt[4]{b}\,c^{15/4}}+\frac{7(11bB-3Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\sqrt[4]{b}\,c^{15/4}}+\frac{7x^{3/2}(11bB-3Ac)}{48bc^3}-\frac{x^{7/2}(11bB-3Ac)}{16bc^2\left(b+cx^2\right)}-\frac{x^{11/2}(bB-Ac)}{4bc\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(21/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (7*(11*b*B - 3*A*c)*x^(3/2))/(48*b*c^3) - ((b*B - A*c)*x^(11/2))/(4*b*c*(b
+ c*x^2)^2) - ((11*b*B - 3*A*c)*x^(7/2))/(16*b*c^2*(b + c*x^2)) + (7*(11*b*
B - 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(1/
4)*c^(15/4)) - (7*(11*b*B - 3*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)])/(32*Sqrt[2]*b^(1/4)*c^(15/4)) - (7*(11*b*B - 3*A*c)*Log[Sqrt[b] - Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(1/4)*c^(15/4)) +
 (7*(11*b*B - 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(64*Sqrt[2]*b^(1/4)*c^(15/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= \int \frac{x^{9/2}(A+Bx^2)}{(b+cx^2)^3}\,dx \\
&= -\frac{(bB-Ac)x^{11/2}}{4bc\,(b+cx^2)^2}+\frac{\left(\frac{11bB}{2}-\frac{3Ac}{2}\right)\int \frac{x^{9/2}}{(b+cx^2)^2}\,dx}{4bc} \\
&= -\frac{(bB-Ac)x^{11/2}}{4bc\,(b+cx^2)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\,(b+cx^2)}+\frac{(7(11bB-3Ac))\int \frac{x^{5/2}}{b+cx^2}\,dx}{32bc^2} \\
&= \frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\,(b+cx^2)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\,(b+cx^2)}-\frac{(7(11bB-3Ac))\int \frac{\sqrt{\;}}{b\cdots}}{32c^3} \\
&= \frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\,(b+cx^2)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\,(b+cx^2)}-\frac{(7(11bB-3Ac))\text{Sub}\cdots}{1\cdots} \\
&= \frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\,(b+cx^2)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\,(b+cx^2)}+\frac{(7(11bB-3Ac))\text{Sub}\cdots}{\cdots} \\
&= \frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\,(b+cx^2)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\,(b+cx^2)}-\frac{(7(11bB-3Ac))\text{Sub}\cdots}{\cdots} \\
&= \frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\,(b+cx^2)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\,(b+cx^2)}-\frac{7(11bB-3Ac)\log\left(\cdots\right.}{6\cdots} \\
&= \frac{7(11bB-3Ac)x^{3/2}}{48bc^3}-\frac{(bB-Ac)x^{11/2}}{4bc\,(b+cx^2)^2}-\frac{(11bB-3Ac)x^{7/2}}{16bc^2\,(b+cx^2)}+\frac{7(11bB-3Ac)\tan^{-1}\cdots}{32\sqrt{2}\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.58, size = 184, normalized size = 0.57

$$\frac{\frac{4c^{3/4}x^{3/2}(77b^2B-21Abc+121bBcx^2-33Ac^2x^2+32Bc^2x^4)}{(b+cx^2)^2}+\frac{21\sqrt{2}\,(11bB-3Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)}{\sqrt[4]{b}}+\frac{21\sqrt{2}\,(11bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt[4]{b}}}{192c^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(21/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*c^(3/4)*x^(3/2)*(77*b^2*B - 21*A*b*c + 121*b*B*c*x^2 - 33*A*c^2*x^2 + 3
2*B*c^2*x^4))/(b + c*x^2)^2 + (21*Sqrt[2]*(11*b*B - 3*A*c)*ArcTan[(Sqrt[b]
```

```
- Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/b^(1/4) + (21*Sqrt[2]*(11*
b*B - 3*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x
)])/b^(1/4))/(192*c^(15/4))
```

Maple [A]

time = 0.53, size = 173, normalized size = 0.54

method	result
derivativedivides	$\frac{2 B x^{\frac{3}{2}}}{3 c^{3}}+\frac{\frac{2 \left(-\frac{c (11 A c -19 B b) x^{\frac{7}{2}}}{32}+\left(-\frac{7}{32} A b c +\frac{15}{32} b^{2} B \right) x^{\frac{3}{2}}\right)}{\left(c \,x^{2}+b \right)^{2}}+\frac{\left(\frac{21 A c}{32}-\frac{77 B b}{32}\right) \sqrt{2}\, \left(\ln \left(\frac{x -\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{b}{c}}}{x +\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2 \mathrm{a}}{4 c \left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^{3}}$
default	$\frac{2 B x^{\frac{3}{2}}}{3 c^{3}}+\frac{\frac{2 \left(-\frac{c (11 A c -19 B b) x^{\frac{7}{2}}}{32}+\left(-\frac{7}{32} A b c +\frac{15}{32} b^{2} B \right) x^{\frac{3}{2}}\right)}{\left(c \,x^{2}+b \right)^{2}}+\frac{\left(\frac{21 A c}{32}-\frac{77 B b}{32}\right) \sqrt{2}\, \left(\ln \left(\frac{x -\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{b}{c}}}{x +\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2 \mathrm{a}}{4 c \left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^{3}}$
risch	$\frac{2 B \,x^{\frac{3}{2}}}{3 c^{3}}-\frac{11 A \,x^{\frac{7}{2}}}{16 c \left(c \,x^{2}+b \right)^{2}}+\frac{19 B \,x^{\frac{7}{2}} b}{16 c^{2} \left(c \,x^{2}+b \right)^{2}}-\frac{7 x^{\frac{3}{2}} A b}{16 c^{2} \left(c \,x^{2}+b \right)^{2}}+\frac{15 x^{\frac{3}{2}} b^{2} B}{16 c^{3} \left(c \,x^{2}+b \right)^{2}}+\frac{21 \sqrt{2}\, A \ln \left(\frac{x -\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}}{x +\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}}\right.}{128 c^{3} \left(\frac{b}{c}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*B*x^(3/2)/c^3+2/c^3*((-1/32*c*(11*A*c-19*B*b)*x^(7/2)+(-7/32*A*b*c+15/3
2*b^2*B)*x^(3/2))/(c*x^2+b)^2+1/8*(21/32*A*c-77/32*B*b)/c/(b/c)^(1/4)*2^(1/
2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2
^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/
2)/(b/c)^(1/4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.49, size = 256, normalized size = 0.80

$$\frac{(19\,Bbc-11\,Ac^2)x^{\frac{7}{2}}+(15\,Bb^2-7\,Abc)x^{\frac{3}{2}}}{16\,(c^5x^4+2\,bc^4x^2+b^2c^3)}+\frac{2\,Bx^{\frac{3}{2}}}{3\,c^3}-\frac{7\,(11\,Bb-3\,Ac)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{128\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((19*B*b*c - 11*A*c^2)*x^(7/2) + (15*B*b^2 - 7*A*b*c)*x^(3/2))/(c^5*x^
4 + 2*b*c^4*x^2 + b^2*c^3) + 2/3*B*x^(3/2)/c^3 - 7/128*(11*B*b - 3*A*c)*(2*
```

```
sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sq
rt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c
)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*s
qrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1
/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c^3
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 993 vs. $2(238) = 476$.
time = 2.44, size = 993, normalized size = 3.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(84*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(14641*B^4*b^4 - 15972*A*B^3
*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(1
/4)*arctan((sqrt((1771561*B^6*b^6 - 2898918*A*B^5*b^5*c + 1976535*A^2*B^4*b
^4*c^2 - 718740*A^3*B^3*b^3*c^3 + 147015*A^4*B^2*b^2*c^4 - 16038*A^5*B*b*c^
5 + 729*A^6*c^6)*x - (14641*B^4*b^5*c^7 - 15972*A*B^3*b^4*c^8 + 6534*A^2*B^
2*b^3*c^9 - 1188*A^3*B*b^2*c^10 + 81*A^4*b*c^11)*sqrt(-(14641*B^4*b^4 - 159
72*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c
^15)))*c^4*(-(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 11
88*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(1/4) + (1331*B^3*b^3*c^4 - 1089*A*B
^2*b^2*c^5 + 297*A^2*B*b*c^6 - 27*A^3*c^7)*sqrt(x)*(-(14641*B^4*b^4 - 15972
*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^1
5))^(1/4))/(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188
*A^3*B*b*c^3 + 81*A^4*c^4)) - 21*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(14641
*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81
*A^4*c^4)/(b*c^15))^(1/4)*log(343*b*c^11*(-(14641*B^4*b^4 - 15972*A*B^3*b^3
*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(3/4)
- 343*(1331*B^3*b^3 - 1089*A*B^2*b^2*c + 297*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt
(x)) + 21*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(14641*B^4*b^4 - 15972*A*B^3*
b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(1/
4)*log(-343*b*c^11*(-(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*
c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(3/4) - 343*(1331*B^3*b^3 -
1089*A*B^2*b^2*c + 297*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt(x)) - 4*(32*B*c^2*x^5
 + 11*(11*B*b*c - 3*A*c^2)*x^3 + 7*(11*B*b^2 - 3*A*b*c)*x)*sqrt(x))/(c^5*x^
4 + 2*b*c^4*x^2 + b^2*c^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(21/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 304, normalized size = 0.94

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,c^3}+\frac{19\,Bbcx^{\frac{7}{2}}-11\,Ac^2x^{\frac{7}{2}}+15\,Bb^2x^{\frac{3}{2}}-7\,Abcx^{\frac{3}{2}}}{16\,(cx^2+b)^2c^3}-\frac{7\,\sqrt{2}\left(11\,(bc^3)^{\frac{3}{4}}\,Bb-3\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^6}-\frac{7\,\sqrt{2}\left(11\,(bc^3)^{\frac{3}{4}}\,Bb-3\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^6}+\frac{7\,\sqrt{2}\left(11\,(bc^3)^{\frac{3}{4}}\,Bb-3\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^6}-\frac{7\,\sqrt{2}\left(11\,(bc^3)^{\frac{3}{4}}\,Bb-3\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/c^3 + 1/16*(19*B*b*c*x^(7/2) - 11*A*c^2*x^(7/2) + 15*B*b^2*x^
(3/2) - 7*A*b*c*x^(3/2))/((c*x^2 + b)^2*c^3) - 7/64*sqrt(2)*(11*(b*c^3)^(3/
4)*B*b - 3*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*s
qrt(x))/(b/c)^(1/4))/(b*c^6) - 7/64*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 3*(b*c^
3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(
1/4))/(b*c^6) + 7/128*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*
log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^6) - 7/128*sqrt(2)*(1
1*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4)
 + x + sqrt(b/c))/(b*c^6)
```

Mupad [B]
time = 0.26, size = 138, normalized size = 0.43

$$\frac{x^{3/2}\left(\frac{15\,B\,b^2}{16}-\frac{7\,A\,b\,c}{16}\right)-x^{7/2}\left(\frac{11\,A\,c^2}{16}-\frac{19\,B\,b\,c}{16}\right)}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}+\frac{2\,B\,x^{3/2}}{3\,c^3}+\frac{7\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)\,\left(3\,A\,c-11\,B\,b\right)}{32\,{(-b)}^{1/4}\,c^{15/4}}+\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{(-b)}^{1/4}}\right)\,\left(3\,A\,c-11\,B\,b\right)\,7{}\mathrm{i}}{32\,{(-b)}^{1/4}\,c^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(21/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^(3/2)*((15*B*b^2)/16 - (7*A*b*c)/16) - x^(7/2)*((11*A*c^2)/16 - (19*B*b*
c)/16))/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (2*B*x^(3/2))/(3*c^3) + (7*atan
((c^(1/4)*x^(1/2))/(-b)^(1/4))*(3*A*c - 11*B*b))/(32*(-b)^(1/4)*c^(15/4)) +
 (atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(3*A*c - 11*B*b)*7i)/(32*(-b)^(1/4)
*c^(15/4))
```

3.209 $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=322

$$\frac{5(9bB-Ac)\sqrt{x}}{16bc^3}-\frac{(bB-Ac)x^{9/2}}{4bc\,(b+cx^2)^2}-\frac{(9bB-Ac)x^{5/2}}{16bc^2\,(b+cx^2)}+\frac{5(9bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{3/4}c^{13/4}}-\frac{5(9bB-Ac)}{}$$

```
[Out] -1/4*(-A*c+B*b)*x^(9/2)/b/c/(c*x^2+b)^2-1/16*(-A*c+9*B*b)*x^(5/2)/b/c^2/(c*
x^2+b)+5/64*(-A*c+9*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/
c^(13/4)*2^(1/2)-5/64*(-A*c+9*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4)
)/b^(3/4)/c^(13/4)*2^(1/2)+5/128*(-A*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(13/4)*2^(1/2)-5/128*(-A*c+9*B*b)*ln(b^(
1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(13/4)*2^(1/2)+5/
16*(-A*c+9*B*b)*x^(1/2)/b/c^3
```

Rubi [A]
time = 0.18, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1598, 468, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{5(9bB-Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{3/4}c^{13/4}}-\frac{5(9bB-Ac)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{3/4}c^{13/4}}+\frac{5(9bB-Ac)\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{3/4}c^{13/4}}-\frac{5(9bB-Ac)\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{3/4}c^{13/4}}+\frac{5\sqrt{x}\,(9bB-Ac)}{16bc^3}-\frac{x^{5/2}(9bB-Ac)}{16bc^2\,(b+cx^2)}-\frac{x^{9/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (5*(9*b*B - A*c)*Sqrt[x])/(16*b*c^3) - ((b*B - A*c)*x^(9/2))/(4*b*c*(b + c*
x^2)^2) - ((9*b*B - A*c)*x^(5/2))/(16*b*c^2*(b + c*x^2)) + (5*(9*b*B - A*c)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(3/4)*c^(13/4
)) - (5*(9*b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sq
rt[2]*b^(3/4)*c^(13/4)) + (5*(9*b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(3/4)*c^(13/4)) - (5*(9*b*B - A*c
)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b
^(3/4)*c^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \int \frac{x^{7/2}(A+Bx^2)}{(b+cx^2)^3}\,dx$$

$$= -\frac{(bB-Ac)x^{9/2}}{4bc\,(b+cx^2)^2} + \frac{\left(\frac{9bB}{2}-\frac{Ac}{2}\right)\int \frac{x^{7/2}}{(b+cx^2)^2}\,dx}{4bc}$$

$$= -\frac{(bB-Ac)x^{9/2}}{4bc\,(b+cx^2)^2} - \frac{(9bB-Ac)x^{5/2}}{16bc^2\,(b+cx^2)} + \frac{(5(9bB-Ac))\int \frac{x^{3/2}}{b+cx^2}\,dx}{32bc^2}$$

$$= \frac{5(9bB-Ac)\sqrt{x}}{16bc^3} - \frac{(bB-Ac)x^{9/2}}{4bc\,(b+cx^2)^2} - \frac{(9bB-Ac)x^{5/2}}{16bc^2\,(b+cx^2)} - \frac{(5(9bB-Ac))\int \frac{1}{\sqrt{x}\,(b+\cdots}}{32c^3}$$

$$= \frac{5(9bB-Ac)\sqrt{x}}{16bc^3} - \frac{(bB-Ac)x^{9/2}}{4bc\,(b+cx^2)^2} - \frac{(9bB-Ac)x^{5/2}}{16bc^2\,(b+cx^2)} - \frac{(5(9bB-Ac))\text{Subst}\left(\int \cdots\right.}{16c^3}$$

$$= \frac{5(9bB-Ac)\sqrt{x}}{16bc^3} - \frac{(bB-Ac)x^{9/2}}{4bc\,(b+cx^2)^2} - \frac{(9bB-Ac)x^{5/2}}{16bc^2\,(b+cx^2)} - \frac{(5(9bB-Ac))\text{Subst}\left(\int \cdots\right.}{32\sqrt{\cdots}}$$

$$= \frac{5(9bB-Ac)\sqrt{x}}{16bc^3} - \frac{(bB-Ac)x^{9/2}}{4bc\,(b+cx^2)^2} - \frac{(9bB-Ac)x^{5/2}}{16bc^2\,(b+cx^2)} - \frac{(5(9bB-Ac))\text{Subst}\left(\int \cdots\right.}{6\cdots}$$

$$= \frac{5(9bB-Ac)\sqrt{x}}{16bc^3} - \frac{(bB-Ac)x^{9/2}}{4bc\,(b+cx^2)^2} - \frac{(9bB-Ac)x^{5/2}}{16bc^2\,(b+cx^2)} + \frac{5(9bB-Ac)\log\left(\sqrt{b}-\cdots\right.}{64\sqrt{2}\cdots}$$

$$= \frac{5(9bB-Ac)\sqrt{x}}{16bc^3} - \frac{(bB-Ac)x^{9/2}}{4bc\,(b+cx^2)^2} - \frac{(9bB-Ac)x^{5/2}}{16bc^2\,(b+cx^2)} + \frac{5(9bB-Ac)\tan^{-1}\left(1-\cdots\right.}{32\sqrt{2}\,b^{3/4}c^{1\cdots}}$$

Mathematica [A]
time = 0.57, size = 184, normalized size = 0.57

$$\frac{\frac{4\sqrt[4]{c}\,\sqrt{x}\,(45b^2B-5Abc+81bBcx^2-9Ac^2x^2+32Bc^2x^4)}{(b+cx^2)^2} + \frac{5\sqrt{2}\,(9bB-Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)}{b^{3/4}} - \frac{5\sqrt{2}\,(9bB-Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{b^{3/4}}}{64c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*c^(1/4)*Sqrt[x]*(45*b^2*B - 5*A*b*c + 81*b*B*c*x^2 - 9*A*c^2*x^2 + 32*B
*c^2*x^4))/(b + c*x^2)^2 + (5*Sqrt[2]*(9*b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[
```

```
c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/b^(3/4) - (5*Sqrt[2]*(9*b*B - A*c
)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/b^(3/4)
)/(64*c^(13/4))
```

Maple [A]
time = 0.47, size = 172, normalized size = 0.53

method	result
derivativedivides	$\frac{2B\sqrt{x}}{c^{3}}+\frac{\frac{2\left(\left(-\frac{9}{32}A\,c^{2}+\frac{17}{32}bBc\right)x^{\frac{5}{2}}-\frac{b(5Ac-13Bb)\sqrt{x}}{32}\right)}{\left(c\,x^{2}+b\right)^{2}}+\frac{5(Ac-9Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+\cdots\right.}{128b\cdots}}{c^{3}}$
default	$\frac{2B\sqrt{x}}{c^{3}}+\frac{\frac{2\left(\left(-\frac{9}{32}A\,c^{2}+\frac{17}{32}bBc\right)x^{\frac{5}{2}}-\frac{b(5Ac-13Bb)\sqrt{x}}{32}\right)}{\left(c\,x^{2}+b\right)^{2}}+\frac{5(Ac-9Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+\cdots\right.}{128b\cdots}}{c^{3}}$
risch	$\frac{2B\sqrt{x}}{c^{3}}-\frac{9x^{\frac{5}{2}}A}{16c\left(c\,x^{2}+b\right)^{2}}+\frac{17x^{\frac{5}{2}}bB}{16c^{2}\left(c\,x^{2}+b\right)^{2}}-\frac{5A\sqrt{x}\,b}{16c^{2}\left(c\,x^{2}+b\right)^{2}}+\frac{13B\sqrt{x}\,b^{2}}{16c^{3}\left(c\,x^{2}+b\right)^{2}}+\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{\cdots}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}\cdots}\right.}{64c^{2}b\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*B/c^3*x^(1/2)+2/c^3*(((-9/32*A*c^2+17/32*b*B*c)*x^(5/2)-1/32*b*(5*A*c-13*
B*b)*x^(1/2))/(c*x^2+b)^2+5/256*(A*c-9*B*b)*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b
/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)
^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/
4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.51, size = 283, normalized size = 0.88

$$\frac{(17\,Bbc-9\,Ac^{2})x^{\frac{5}{2}}+(13\,Bb^{2}-5\,Abc)\sqrt{x}}{16\,(c^{5}x^{4}+2\,bc^{4}x^{2}+b^{2}c^{3})}+\frac{2\,B\sqrt{x}}{c^{3}}-\frac{5\left(\frac{2\sqrt{2}\,(9\,Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\,(9\,Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\,(9\,Bb-Ac)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,(9\,Bb-Ac)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{128\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((17*B*b*c - 9*A*c^2)*x^(5/2) + (13*B*b^2 - 5*A*b*c)*sqrt(x))/(c^5*x^4
 + 2*b*c^4*x^2 + b^2*c^3) + 2*B*sqrt(x)/c^3 - 5/128*(2*sqrt(2)*(9*B*b - A*c
)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(9*B*b - A*c)*ar
```

```
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(9*B*b - A*c)*log(sqr
t(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqr
t(2)*(9*B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(
b))/(b^(3/4)*c^(1/4)))/c^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(238) = 476$.
time = 2.56, size = 793, normalized size = 2.46

$$\frac{20\,(c^5x^4+2\,bc^4x^2+b^2c^3)\left(-\frac{6561\,B^4b^4-2916\,AB^3b^3c+486\,A^2B^2b^2c^2-36\,A^3Bbc^3+A^4c^4}{b^3c^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^2c^6\sqrt{-\frac{6561\,B^4b^4-2916\,AB^3b^3c+486\,A^2B^2b^2c^2-36\,A^3Bbc^3+A^4c^4}{b^3c^{13}}}+(81\,B^2b^2-18\,ABbc+A^2c^2)x}\,b^2c^{10}\left(-\frac{6561\,B^4b^4-2916\,AB^3b^3c+486\,A^2B^2b^2c^2-36\,A^3Bbc^3+A^4c^4}{b^3c^{13}}\right)^{\frac{3}{4}}+(9\,Bb^3c^{10}-Ab^2c^{11})\sqrt{x}\left(-\frac{6561\,B^4b^4-2916\,AB^3b^3c+486\,A^2B^2b^2c^2-36\,A^3Bbc^3+A^4c^4}{b^3c^{13}}\right)^{\frac{3}{4}}}{6561\,B^4b^4-2916\,AB^3b^3c+486\,A^2B^2b^2c^2-36\,A^3Bbc^3+A^4c^4}\right)+5\,(c^5x^4+2\,bc^4x^2+b^2c^3)\left(-\frac{6561\,B^4b^4-2916\,AB^3b^3c+486\,A^2B^2b^2c^2-36\,A^3Bbc^3+A^4c^4}{b^3c^{13}}\right)^{\frac{1}{4}}\log\left(5\,bc^3\left(-\frac{6561\,B^4b^4-2916\,AB^3b^3c+486\,A^2B^2b^2c^2-36\,A^3Bbc^3+A^4c^4}{b^3c^{13}}\right)^{\frac{1}{4}}-5\,(9\,Bb-Ac)\sqrt{x}\right)-5\,(c^5x^4+2\,bc^4x^2+b^2c^3)\left(-\frac{6561\,B^4b^4-2916\,AB^3b^3c+486\,A^2B^2b^2c^2-36\,A^3Bbc^3+A^4c^4}{b^3c^{13}}\right)^{\frac{1}{4}}\log\left(-5\,bc^3\left(-\frac{6561\,B^4b^4-2916\,AB^3b^3c+486\,A^2B^2b^2c^2-36\,A^3Bbc^3+A^4c^4}{b^3c^{13}}\right)^{\frac{1}{4}}-5\,(9\,Bb-Ac)\sqrt{x}\right)+4\,(32\,Bc^2x^4+45\,Bb^2-5\,Abc+9\,(9\,Bbc-Ac^2)x^2)\sqrt{x}}{64\,(c^5x^4+2\,bc^4x^2+b^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3
*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4)*arct
an((sqrt(b^2*c^6*sqrt(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c
^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13)) + (81*B^2*b^2 - 18*A*B*b*c + A^2
*c^2)*x)*b^2*c^10*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2
- 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(3/4) + (9*B*b^3*c^10 - A*b^2*c^11)
*sqrt(x)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*
B*b*c^3 + A^4*c^4)/(b^3*c^13))^(3/4))/(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 48
6*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)) + 5*(c^5*x^4 + 2*b*c^4*x^2 +
 b^2*c^3)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3
*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4)*log(5*b*c^3*(-(6561*B^4*b^4 - 2916*A*
B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/
4) - 5*(9*B*b - A*c)*sqrt(x)) - 5*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(6561
*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^
4)/(b^3*c^13))^(1/4)*log(-5*b*c^3*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*
A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4) - 5*(9*B*b -
A*c)*sqrt(x)) + 4*(32*B*c^2*x^4 + 45*B*b^2 - 5*A*b*c + 9*(9*B*b*c - A*c^2)*
x^2)*sqrt(x))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 304, normalized size = 0.94

$$\frac{2\,B\sqrt{x}}{c^3}-\frac{5\,\sqrt{2}\left(9\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^4}-\frac{5\,\sqrt{2}\left(9\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^4}-\frac{5\,\sqrt{2}\left(9\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^4}+\frac{5\,\sqrt{2}\left(9\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^4}+\frac{17\,Bbcx^{\frac{5}{2}}-9\,Ac^2x^{\frac{5}{2}}+13\,Bb^2\sqrt{x}-5\,Abc\sqrt{x}}{16\,(cx^2+b)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/c^3 - 5/64*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) - 5
/64*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) - 5/128*sqrt(2)*(9*(b
*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
sqrt(b/c))/(b*c^4) + 5/128*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c
)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^4) + 1/16*(17*B*b*
c*x^(5/2) - 9*A*c^2*x^(5/2) + 13*B*b^2*sqrt(x) - 5*A*b*c*sqrt(x))/((c*x^2 +
 b)^2*c^3)
```

Mupad [B]
time = 0.23, size = 760, normalized size = 2.36

$$\frac{\sqrt{x}\left(\frac{13Bb^2}{16}-\frac{5Abc}{16}\right)-x^{5/2}\left(\frac{9Ac^2}{16}-\frac{17Bbc}{16}\right)}{b^2c^3+2bc^4x^2+c^5x^4}+\frac{2B\sqrt{x}}{c^3}-\frac{\mathrm{atan}\left(\frac{\frac{(Ac-9Bb)\left(\frac{25\sqrt{x}\left(A^2c^2-18ABbc+81B^2b^2\right)}{64c^3}-\frac{5\left(45Bb^2-5Abc\right)(Ac-9Bb)}{64(-b)^{3/4}c^{13/4}}\right)5\mathrm{i}}{64(-b)^{3/4}c^{13/4}}+\frac{(Ac-9Bb)\left(\frac{25\sqrt{x}\left(A^2c^2-18ABbc+81B^2b^2\right)}{64c^3}+\frac{5\left(45Bb^2-5Abc\right)(Ac-9Bb)}{64(-b)^{3/4}c^{13/4}}\right)5\mathrm{i}}{64(-b)^{3/4}c^{13/4}}}{\frac{5(Ac-9Bb)\left(\frac{25\sqrt{x}\left(A^2c^2-18ABbc+81B^2b^2\right)}{64c^3}-\frac{5\left(45Bb^2-5Abc\right)(Ac-9Bb)}{64(-b)^{3/4}c^{13/4}}\right)}{64(-b)^{3/4}c^{13/4}}-\frac{5(Ac-9Bb)\left(\frac{25\sqrt{x}\left(A^2c^2-18ABbc+81B^2b^2\right)}{64c^3}+\frac{5\left(45Bb^2-5Abc\right)(Ac-9Bb)}{64(-b)^{3/4}c^{13/4}}\right)}{64(-b)^{3/4}c^{13/4}}}\right)(Ac-9Bb)\,5\mathrm{i}}{32(-b)^{3/4}c^{13/4}}-\frac{5\,\mathrm{atan}\left(\frac{\frac{5(Ac-9Bb)\left(\frac{25\sqrt{x}\left(A^2c^2-18ABbc+81B^2b^2\right)}{64c^3}-\frac{\left(45Bb^2-5Abc\right)(Ac-9Bb)5\mathrm{i}}{64(-b)^{3/4}c^{13/4}}\right)}{64(-b)^{3/4}c^{13/4}}+\frac{5(Ac-9Bb)\left(\frac{25\sqrt{x}\left(A^2c^2-18ABbc+81B^2b^2\right)}{64c^3}+\frac{\left(45Bb^2-5Abc\right)(Ac-9Bb)5\mathrm{i}}{64(-b)^{3/4}c^{13/4}}\right)}{64(-b)^{3/4}c^{13/4}}}{\frac{(Ac-9Bb)\left(\frac{25\sqrt{x}\left(A^2c^2-18ABbc+81B^2b^2\right)}{64c^3}-\frac{\left(45Bb^2-5Abc\right)(Ac-9Bb)5\mathrm{i}}{64(-b)^{3/4}c^{13/4}}\right)5\mathrm{i}}{64(-b)^{3/4}c^{13/4}}-\frac{(Ac-9Bb)\left(\frac{25\sqrt{x}\left(A^2c^2-18ABbc+81B^2b^2\right)}{64c^3}+\frac{\left(45Bb^2-5Abc\right)(Ac-9Bb)5\mathrm{i}}{64(-b)^{3/4}c^{13/4}}\right)5\mathrm{i}}{64(-b)^{3/4}c^{13/4}}}\right)(Ac-9Bb)}{32(-b)^{3/4}c^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^(1/2)*((13*B*b^2)/16 - (5*A*b*c)/16) - x^(5/2)*((9*A*c^2)/16 - (17*B*b*c
)/16))/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (2*B*x^(1/2))/c^3 - (atan((((A*c
 - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) - (5*(
45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b))/(64*(-b)^(3/4)*c^(13/4)))*5i)/(64*(-b)^(
3/4)*c^(13/4)) + ((A*c - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B
*b*c))/(64*c^3) + (5*(45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b))/(64*(-b)^(3/4)*c^(
13/4)))*5i)/(64*(-b)^(3/4)*c^(13/4)))/((5*(A*c - 9*B*b)*((25*x^(1/2)*(A^2*c
^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) - (5*(45*B*b^2 - 5*A*b*c)*(A*c - 9*
B*b))/(64*(-b)^(3/4)*c^(13/4))))/(64*(-b)^(3/4)*c^(13/4)) - (5*(A*c - 9*B*b
)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) + (5*(45*B*b^2
 - 5*A*b*c)*(A*c - 9*B*b))/(64*(-b)^(3/4)*c^(13/4))))/(64*(-b)^(3/4)*c^(13/
4))))*(A*c - 9*B*b)*5i)/(32*(-b)^(3/4)*c^(13/4)) - (5*atan(((5*(A*c - 9*B*b
)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) - ((45*B*b^2 -
 5*A*b*c)*(A*c - 9*B*b)*5i)/(64*(-b)^(3/4)*c^(13/4))))/(64*(-b)^(3/4)*c^(13
/4)) + (5*(A*c - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(
64*c^3) + ((45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b)*5i)/(64*(-b)^(3/4)*c^(13/4)))
)/(64*(-b)^(3/4)*c^(13/4)))/(((A*c - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*
b^2 - 18*A*B*b*c))/(64*c^3) - ((45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b)*5i)/(64*(
-b)^(3/4)*c^(13/4)))*5i)/(64*(-b)^(3/4)*c^(13/4)) - ((A*c - 9*B*b)*((25*x^(
1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) + ((45*B*b^2 - 5*A*b*c)*
(A*c - 9*B*b)*5i)/(64*(-b)^(3/4)*c^(13/4)))*5i)/(64*(-b)^(3/4)*c^(13/4))))*
(A*c - 9*B*b))/(32*(-b)^(3/4)*c^(13/4))
```

3.210 $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=293

$$-\frac{(bB-Ac)x^{7/2}}{4bc\,(b+cx^2)^2}-\frac{(7bB+Ac)x^{3/2}}{16bc^2\,(b+cx^2)}-\frac{3(7bB+Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{5/4}c^{11/4}}+\frac{3(7bB+Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\cdots}{\cdots}\right)}{32\sqrt{2}\,b^{5/4}c^{11/4}}$$

```
[Out] -1/4*(-A*c+B*b)*x^(7/2)/b/c/(c*x^2+b)^2-1/16*(A*c+7*B*b)*x^(3/2)/b/c^2/(c*x
^2+b)-3/64*(A*c+7*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^
(11/4)*2^(1/2)+3/64*(A*c+7*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b
^(5/4)/c^(11/4)*2^(1/2)+3/128*(A*c+7*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1
/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(11/4)*2^(1/2)-3/128*(A*c+7*B*b)*ln(b^(1/2)+
x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(11/4)*2^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3(Ac+7bB)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{5/4}c^{11/4}}+\frac{3(Ac+7bB)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{5/4}c^{11/4}}+\frac{3(Ac+7bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{5/4}c^{11/4}}-\frac{3(Ac+7bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{5/4}c^{11/4}}-\frac{x^{3/2}(Ac+7bB)}{16bc^2\,(b+cx^2)}-\frac{x^{7/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*((b*B - A*c)*x^(7/2))/(b*c*(b + c*x^2)^2) - ((7*b*B + A*c)*x^(3/2))/(1
6*b*c^2*(b + c*x^2)) - (3*(7*b*B + A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(11/4)) + (3*(7*b*B + A*c)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(11/4)) + (3*(7*b*B
+ A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt
[2]*b^(5/4)*c^(11/4)) - (3*(7*b*B + A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(5/4)*c^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \int \frac{x^{5/2}(A+Bx^2)}{(b+cx^2)^3}\,dx$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\,(b+cx^2)^2} + \frac{\left(\frac{7bB}{2}+\frac{Ac}{2}\right)\int \frac{x^{5/2}}{(b+cx^2)^2}\,dx}{4bc}$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\,(b+cx^2)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\,(b+cx^2)} + \frac{(3(7bB+Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32bc^2}$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\,(b+cx^2)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\,(b+cx^2)} + \frac{(3(7bB+Ac))\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16bc^2}$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\,(b+cx^2)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\,(b+cx^2)} - \frac{(3(7bB+Ac))\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{\right.}}{32bc^{5/2}}$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\,(b+cx^2)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\,(b+cx^2)} + \frac{(3(7bB+Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\right.}{64bc^3}$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\,(b+cx^2)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\,(b+cx^2)} + \frac{3(7bB+Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{64\sqrt{2}\,b^{5/4}c^{11/4}}$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\,(b+cx^2)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\,(b+cx^2)} - \frac{3(7bB+Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{5/4}c^{11/4}} + \cdots$$

Mathematica [A]

time = 0.59, size = 171, normalized size = 0.58

$$\frac{-\frac{4\sqrt[4]{b}\,c^{3/4}x^{3/2}(7b^2B-3Ac^2x^2+bc(A+11Bx^2))}{(b+cx^2)^2}-3\sqrt{2}\,(7bB+Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)-3\sqrt{2}\,(7bB+Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{64b^{5/4}c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(1/4)*c^(3/4)*x^(3/2)*(7*b^2*B - 3*A*c^2*x^2 + b*c*(A + 11*B*x^2)))/
(b + c*x^2)^2 - 3*Sqrt[2]*(7*b*B + A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 3*Sqrt[2]*(7*b*B + A*c)*ArcTanh[(Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(64*b^(5/4)*c^(11/4))
```

Maple [A]
time = 0.38, size = 166, normalized size = 0.57

method	result
derivativedivides	$\frac{\frac{(3Ac-11Bb)x^{\frac{7}{2}}}{16bc}-\frac{(Ac+7Bb)x^{\frac{3}{2}}}{16c^2}}{(c x^2+b)^2}+\frac{3(Ac+7Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2}{128c^3b\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{(3Ac-11Bb)x^{\frac{7}{2}}}{16bc}-\frac{(Ac+7Bb)x^{\frac{3}{2}}}{16c^2}}{(c x^2+b)^2}+\frac{3(Ac+7Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2}{128c^3b\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*(3*A*c-11*B*b)/b/c*x^(7/2)-1/32*(A*c+7*B*b)/c^2*x^(3/2))/(c*x^2+b)^
2+3/128*(A*c+7*B*b)/c^3/b/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^
(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^
(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.51, size = 251, normalized size = 0.86

$$-\frac{(11\,Bbc-3\,Ac^2)x^{\frac{7}{2}}+(7\,Bb^2+Abc)x^{\frac{3}{2}}}{16\,(bc^4x^4+2\,b^2c^3x^2+b^3c^2)}+\frac{3\,(7\,Bb+Ac)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{128\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((11*B*b*c - 3*A*c^2)*x^(7/2) + (7*B*b^2 + A*b*c)*x^(3/2))/(b*c^4*x^4
 + 2*b^2*c^3*x^2 + b^3*c^2) + 3/128*(7*B*b + A*c)*(2*sqrt(2)*arctan(1/2*sqr
t(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(
sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(
1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt
(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + s
qrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) +
sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/(b*c^2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 990 vs. $2(213) = 426$.
time = 2.17, size = 990, normalized size = 3.38

$$[illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(2401*B^4*b^4 + 1372*A*B^
3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4)
*arctan((sqrt((117649*B^6*b^6 + 100842*A*B^5*b^5*c + 36015*A^2*B^4*b^4*c^2
+ 6860*A^3*B^3*b^3*c^3 + 735*A^4*B^2*b^2*c^4 + 42*A^5*B*b*c^5 + A^6*c^6)*x
- (2401*B^4*b^7*c^5 + 1372*A*B^3*b^6*c^6 + 294*A^2*B^2*b^5*c^7 + 28*A^3*B*b
^4*c^8 + A^4*b^3*c^9)*sqrt(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*
b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11)))*b*c^3*(-(2401*B^4*b^4 + 13
72*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11)
)^(1/4) - (343*B^3*b^4*c^3 + 147*A*B^2*b^3*c^4 + 21*A^2*B*b^2*c^5 + A^3*b*c
^6)*sqrt(x)*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A
^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4))/(2401*B^4*b^4 + 1372*A*B^3*b^3*c +
 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)) - 3*(b*c^4*x^4 + 2*b^2*c^
3*x^2 + b^3*c^2)*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 +
 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4)*log(27*b^4*c^8*(-(2401*B^4*b^4
 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*
c^11))^(3/4) + 27*(343*B^3*b^3 + 147*A*B^2*b^2*c + 21*A^2*B*b*c^2 + A^3*c^3
)*sqrt(x)) + 3*(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(2401*B^4*b^4 + 1372
*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^
(1/4)*log(-27*b^4*c^8*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*
c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(3/4) + 27*(343*B^3*b^3 + 147*A
*B^2*b^2*c + 21*A^2*B*b*c^2 + A^3*c^3)*sqrt(x)) + 4*((11*B*b*c - 3*A*c^2)*x
^3 + (7*B*b^2 + A*b*c)*x)*sqrt(x))/(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 293, normalized size = 1.00

$$-\frac{11\,Bbcx^{\frac{7}{2}}-3\,Ac^2x^{\frac{7}{2}}+7\,Bb^2x^{\frac{3}{2}}+Abcx^{\frac{3}{2}}}{16\,(cx^2+b)^2bc^2}+\frac{3\,\sqrt{2}\left(7\,(bc^3)^{\frac{3}{4}}\,Bb+(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^5}+\frac{3\,\sqrt{2}\left(7\,(bc^3)^{\frac{3}{4}}\,Bb+(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^5}-\frac{3\,\sqrt{2}\left(7\,(bc^3)^{\frac{3}{4}}\,Bb+(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^5}+\frac{3\,\sqrt{2}\left(7\,(bc^3)^{\frac{3}{4}}\,Bb+(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*B*b*c*x^(7/2) - 3*A*c^2*x^(7/2) + 7*B*b^2*x^(3/2) + A*b*c*x^(3/2)
)/((c*x^2 + b)^2*b*c^2) + 3/64*sqrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)
*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^
2*c^5) + 3/64*sqrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^5) - 3/128*s
qrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^
(1/4) + x + sqrt(b/c))/(b^2*c^5) + 3/128*sqrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*
c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^5)
```

Mupad [B]
time = 0.17, size = 122, normalized size = 0.42

$$\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)\,\left(A\,c+7\,B\,b\right)}{32\,{(-b)}^{5/4}\,c^{11/4}}-\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)\,\left(A\,c+7\,B\,b\right)}{32\,{(-b)}^{5/4}\,c^{11/4}}-\frac{\frac{x^{3/2}\,\left(A\,c+7\,B\,b\right)}{16\,c^2}-\frac{x^{7/2}\,\left(3\,A\,c-11\,B\,b\right)}{16\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c + 7*B*b))/(32*(-b)^(5/4)*c^(11/
4)) - (3*atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c + 7*B*b))/(32*(-b)^(5/4)*c
^(11/4)) - ((x^(3/2)*(A*c + 7*B*b))/(16*c^2) - (x^(7/2)*(3*A*c - 11*B*b))/(
16*b*c))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

3.211 $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=298

$$-\frac{(bB-Ac)x^{5/2}}{4bc\,(b+cx^2)^2}-\frac{(5bB+3Ac)\sqrt{x}}{16bc^2\,(b+cx^2)}-\frac{(5bB+3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{7/4}c^{9/4}}+\frac{(5bB+3Ac)\tan^{-1}\left(1+\frac{\sqrt{}}{}\right.}{32\sqrt{2}\,b^{7/4}c^{9/4}}$$

```
[Out] -1/4*(-A*c+B*b)*x^(5/2)/b/c/(c*x^2+b)^2-1/64*(3*A*c+5*B*b)*arctan(1-c^(1/4)
*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(9/4)*2^(1/2)+1/64*(3*A*c+5*B*b)*arctan
(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(9/4)*2^(1/2)-1/128*(3*A*c+5*
B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(9/4)*
2^(1/2)+1/128*(3*A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^
(1/2))/b^(7/4)/c^(9/4)*2^(1/2)-1/16*(3*A*c+5*B*b)*x^(1/2)/b/c^2/(c*x^2+b)
```

Rubi [A]
time = 0.19, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 294, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{(3Ac+5bB)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{7/4}c^{9/4}}+\frac{(3Ac+5bB)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{7/4}c^{9/4}}-\frac{(3Ac+5bB)\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{7/4}c^{9/4}}+\frac{(3Ac+5bB)\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{7/4}c^{9/4}}-\frac{\sqrt{x}\,(3Ac+5bB)}{16bc^2\,(b+cx^2)}-\frac{x^{5/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*((b*B - A*c)*x^(5/2))/(b*c*(b + c*x^2)^2) - ((5*b*B + 3*A*c)*Sqrt[x])/
(16*b*c^2*(b + c*x^2)) - ((5*b*B + 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[
x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(9/4)) + ((5*b*B + 3*A*c)*ArcTan[1 + (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(9/4)) - ((5*b*B +
3*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt
[2]*b^(7/4)*c^(9/4)) + ((5*b*B + 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(7/4)*c^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \int \frac{x^{3/2}(A+Bx^2)}{(b+cx^2)^3}\,dx$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\,(b+cx^2)^2} + \frac{\left(\frac{5bB}{2}+\frac{3Ac}{2}\right)\int \frac{x^{3/2}}{(b+cx^2)^2}\,dx}{4bc}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\,(b+cx^2)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\,(b+cx^2)} + \frac{(5bB+3Ac)\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{32bc^2}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\,(b+cx^2)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\,(b+cx^2)} + \frac{(5bB+3Ac)\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16bc^2}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\,(b+cx^2)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\,(b+cx^2)} + \frac{(5bB+3Ac)\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{\right.}{32b^{3/2}c^2}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\,(b+cx^2)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\,(b+cx^2)} + \frac{(5bB+3Ac)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\right.}{64b^{3/2}c^{5/2}}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\,(b+cx^2)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\,(b+cx^2)} - \frac{(5bB+3Ac)\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}\right.}{64\sqrt{2}\,b^{7/4}c^{9/4}}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\,(b+cx^2)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\,(b+cx^2)} - \frac{(5bB+3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{7/4}c^{9/4}} +$$

Mathematica [A]

time = 0.58, size = 173, normalized size = 0.58

$$\frac{-\frac{4b^{3/4}\sqrt[4]{c}\,\sqrt{x}\,(5b^2B-Ac^2x^2+3bc(A+3Bx^2))}{(b+cx^2)^2}-\sqrt{2}\,(5bB+3Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)+\sqrt{2}\,(5bB+3Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{64b^{7/4}c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(3/4)*c^(1/4)*Sqrt[x]*(5*b^2*B - A*c^2*x^2 + 3*b*c*(A + 3*B*x^2)))/(
b + c*x^2)^2 - Sqrt[2]*(5*b*B + 3*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x])] + Sqrt[2]*(5*b*B + 3*A*c)*ArcTanh[(Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(64*b^(7/4)*c^(9/4))
```

Maple [A]
time = 0.37, size = 167, normalized size = 0.56

method	result
derivativedivides	$\frac{\frac{(Ac-9Bb)x^{\frac{5}{2}}}{16bc}-\frac{(3Ac+5Bb)\sqrt{x}}{16c^2}}{(cx^2+b)^2}+\frac{(3Ac+5Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128c^2b^2}$
default	$\frac{\frac{(Ac-9Bb)x^{\frac{5}{2}}}{16bc}-\frac{(3Ac+5Bb)\sqrt{x}}{16c^2}}{(cx^2+b)^2}+\frac{(3Ac+5Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128c^2b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*(A*c-9*B*b)/b/c*x^(5/2)-1/32*(3*A*c+5*B*b)/c^2*x^(1/2))/(c*x^2+b)^2
+1/128*(3*A*c+5*B*b)/c^2/b^2*(b/c)^(1/4)*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)
*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.52, size = 280, normalized size = 0.94

$$-\frac{(9\,Bbc-Ac^2)x^{\frac{5}{2}}+(5\,Bb^2+3\,Abc)\sqrt{x}}{16\,(bc^4x^4+2\,b^2c^3x^2+b^3c^2)}+\frac{\frac{2\sqrt{2}\,(5\,Bb+3\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,(5\,Bb+3\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,(5\,Bb+3\,Ac)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,(5\,Bb+3\,Ac)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{128\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((9*B*b*c - A*c^2)*x^(5/2) + (5*B*b^2 + 3*A*b*c)*sqrt(x))/(b*c^4*x^4
+ 2*b^2*c^3*x^2 + b^3*c^2) + 1/128*(2*sqrt(2)*(5*B*b + 3*A*c)*arctan(1/2*sq
```

```
rt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/
(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(5*B*b + 3*A*c)*arctan(-1/2*sqr
t(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(
sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(5*B*b + 3*A*c)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(5*B*b
 + 3*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3
/4)*c^(1/4)))/(b*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(218) = 436$.
time = 2.24, size = 806, normalized size = 2.70

$$\frac{4\,(bc^4x^4+2\,b^2c^3x^2+b^3c^2)\left(-\frac{625\,B^4b^4+1500\,AB^3b^3c+1350\,A^2B^2b^2c^2+540\,A^3Bbc^3+81\,A^4c^4}{b^7c^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^4c^4\sqrt{-\frac{625\,B^4b^4+1500\,AB^3b^3c+1350\,A^2B^2b^2c^2+540\,A^3Bbc^3+81\,A^4c^4}{b^7c^9}}+(25\,B^2b^2+30\,ABbc+9\,A^2c^2)x}\,b^5c^7\left(-\frac{625\,B^4b^4+\dots}{b^7c^9}\right)^{\frac{3}{4}}-(5\,Bb^6c^7+3\,Ab^5c^8)\sqrt{x}\left(-\frac{625\,B^4b^4+\dots}{b^7c^9}\right)^{\frac{3}{4}}}{625\,B^4b^4+1500\,AB^3b^3c+1350\,A^2B^2b^2c^2+540\,A^3Bbc^3+81\,A^4c^4}\right)+\dots}{64\,(bc^4x^4+2\,b^2c^3x^2+b^3c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(4*(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B^3*b
^3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4
)*arctan((sqrt(b^4*c^4*sqrt(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2
*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9)) + (25*B^2*b^2 + 30*A*B*
b*c + 9*A^2*c^2)*x)*b^5*c^7*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^
2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(3/4) - (5*B*b^6*c^7 +
 3*A*b^5*c^8)*sqrt(x)*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2*
c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(3/4))/(625*B^4*b^4 + 1500*A
*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)) + (b*c^4
*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^
2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4)*log(b^2*c^2*
(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3
+ 81*A^4*c^4)/(b^7*c^9))^(1/4) + (5*B*b + 3*A*c)*sqrt(x)) - (b*c^4*x^4 + 2*
b^2*c^3*x^2 + b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2
*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4)*log(-b^2*c^2*(-(625*B
^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4
*c^4)/(b^7*c^9))^(1/4) + (5*B*b + 3*A*c)*sqrt(x)) - 4*(5*B*b^2 + 3*A*b*c +
(9*B*b*c - A*c^2)*x^2)*sqrt(x))/(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]

time = 0.47, size = 298, normalized size = 1.00

$$\frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb+3\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^3}+\frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb+3\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^3}+\frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb+3\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^3}-\frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb+3\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^3}-\frac{9\,Bbcx^{\frac{5}{2}}-Ac^2x^{\frac{5}{2}}+5\,Bb^2\sqrt{x}+3\,Abc\sqrt{x}}{16\,(cx^2+b)^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(5*(b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1/64*sqrt(2)*(5
*(b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c
)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1/128*sqrt(2)*(5*(b*c^3)^(1/4
)*B*b + 3*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c
))/(b^2*c^3) - 1/128*sqrt(2)*(5*(b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*lo
g(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^3) - 1/16*(9*B*b*c*x
^(5/2) - A*c^2*x^(5/2) + 5*B*b^2*sqrt(x) + 3*A*b*c*sqrt(x))/((c*x^2 + b)^2*
b*c^2)
```

Mupad [B]

time = 0.38, size = 799, normalized size = 2.68

$$-\frac{\frac{\sqrt{x}\,(3\,A\,c+5\,B\,b)}{16\,c^2}-\frac{x^{5/2}\,(A\,c-9\,B\,b)}{16\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}+\frac{\mathrm{atan}\left(\frac{\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\,(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2)}{64\,b^2\,c}-\frac{(3\,A\,c^2+5\,B\,b\,c)\,(3\,A\,c+5\,B\,b)}{64\,(-b)^{7/4}\,c^{9/4}}\right)1\mathrm{i}}{64\,(-b)^{7/4}\,c^{9/4}}+\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\,(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2)}{64\,b^2\,c}+\frac{(3\,A\,c^2+5\,B\,b\,c)\,(3\,A\,c+5\,B\,b)}{64\,(-b)^{7/4}\,c^{9/4}}\right)1\mathrm{i}}{64\,(-b)^{7/4}\,c^{9/4}}}{\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\,(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2)}{64\,b^2\,c}-\frac{(3\,A\,c^2+5\,B\,b\,c)\,(3\,A\,c+5\,B\,b)}{64\,(-b)^{7/4}\,c^{9/4}}\right)}{64\,(-b)^{7/4}\,c^{9/4}}-\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\,(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2)}{64\,b^2\,c}+\frac{(3\,A\,c^2+5\,B\,b\,c)\,(3\,A\,c+5\,B\,b)}{64\,(-b)^{7/4}\,c^{9/4}}\right)}{64\,(-b)^{7/4}\,c^{9/4}}}\right)(3\,A\,c+5\,B\,b)\,1\mathrm{i}}{32\,(-b)^{7/4}\,c^{9/4}}+\frac{\mathrm{atan}\left(\frac{\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\,(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2)}{64\,b^2\,c}-\frac{(3\,A\,c^2+5\,B\,b\,c)\,(3\,A\,c+5\,B\,b)\,1\mathrm{i}}{64\,(-b)^{7/4}\,c^{9/4}}\right)}{64\,(-b)^{7/4}\,c^{9/4}}+\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\,(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2)}{64\,b^2\,c}+\frac{(3\,A\,c^2+5\,B\,b\,c)\,(3\,A\,c+5\,B\,b)\,1\mathrm{i}}{64\,(-b)^{7/4}\,c^{9/4}}\right)}{64\,(-b)^{7/4}\,c^{9/4}}}{\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\,(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2)}{64\,b^2\,c}-\frac{(3\,A\,c^2+5\,B\,b\,c)\,(3\,A\,c+5\,B\,b)\,1\mathrm{i}}{64\,(-b)^{7/4}\,c^{9/4}}\right)1\mathrm{i}}{64\,(-b)^{7/4}\,c^{9/4}}-\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\,(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2)}{64\,b^2\,c}+\frac{(3\,A\,c^2+5\,B\,b\,c)\,(3\,A\,c+5\,B\,b)\,1\mathrm{i}}{64\,(-b)^{7/4}\,c^{9/4}}\right)1\mathrm{i}}{64\,(-b)^{7/4}\,c^{9/4}}}\right)(3\,A\,c+5\,B\,b)}{32\,(-b)^{7/4}\,c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (atan((((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(6
4*b^2*c) - ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b))/(64*(-b)^(7/4)*c^(9/4)))*1
i)/(64*(-b)^(7/4)*c^(9/4)) + ((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2
*b^2 + 30*A*B*b*c))/(64*b^2*c) + ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b))/(64*
(-b)^(7/4)*c^(9/4)))*1i)/(64*(-b)^(7/4)*c^(9/4)))/(((3*A*c + 5*B*b)*((x^(1/
2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(64*b^2*c) - ((3*A*c^2 + 5*B*b*c)
*(3*A*c + 5*B*b))/(64*(-b)^(7/4)*c^(9/4))))/(64*(-b)^(7/4)*c^(9/4)) - ((3*A
*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(64*b^2*c) + (
(3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b))/(64*(-b)^(7/4)*c^(9/4))))/(64*(-b)^(7/
4)*c^(9/4))))*(3*A*c + 5*B*b)*1i)/(32*(-b)^(7/4)*c^(9/4)) - ((x^(1/2)*(3*A*
c + 5*B*b))/(16*c^2) - (x^(5/2)*(A*c - 9*B*b))/(16*b*c))/(b^2 + c^2*x^4 + 2
*b*c*x^2) + (atan((((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*
A*B*b*c))/(64*b^2*c) - ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b)*1i)/(64*(-b)^(7
/4)*c^(9/4))))/(64*(-b)^(7/4)*c^(9/4)) + ((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*
c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(64*b^2*c) + ((3*A*c^2 + 5*B*b*c)*(3*A*c +
5*B*b)*1i)/(64*(-b)^(7/4)*c^(9/4))))/(64*(-b)^(7/4)*c^(9/4)))/(((3*A*c + 5*
B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(64*b^2*c) - ((3*A*c^
2 + 5*B*b*c)*(3*A*c + 5*B*b)*1i)/(64*(-b)^(7/4)*c^(9/4)))*1i)/(64*(-b)^(7/4
```

```
)*c^(9/4)) - ((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*
c))/(64*b^2*c) + ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b)*1i)/(64*(-b)^(7/4)*c^
(9/4)))*1i)/(64*(-b)^(7/4)*c^(9/4))))*(3*A*c + 5*B*b))/(32*(-b)^(7/4)*c^(9/
4))
```

3.212 $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=298

$$-\frac{(bB-Ac)x^{3/2}}{4bc\,(b+cx^2)^2}+\frac{(3bB+5Ac)x^{3/2}}{16b^2c\,(b+cx^2)}-\frac{(3bB+5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{9/4}c^{7/4}}+\frac{(3bB+5Ac)\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{32\sqrt{2}\,b^{9/4}c^{7/4}}$$

```
[Out] -1/4*(-A*c+B*b)*x^(3/2)/b/c/(c*x^2+b)^2+1/16*(5*A*c+3*B*b)*x^(3/2)/b^2/c/(c
*x^2+b)-1/64*(5*A*c+3*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4
)/c^(7/4)*2^(1/2)+1/64*(5*A*c+3*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/
4))/b^(9/4)/c^(7/4)*2^(1/2)+1/128*(5*A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4
)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)/c^(7/4)*2^(1/2)-1/128*(5*A*c+3*B*b)*ln(b
^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)/c^(7/4)*2^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{(5Ac+3bB)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{9/4}c^{7/4}}+\frac{(5Ac+3bB)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{9/4}c^{7/4}}+\frac{(5Ac+3bB)\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{9/4}c^{7/4}}-\frac{(5Ac+3bB)\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{9/4}c^{7/4}}+\frac{x^{3/2}(5Ac+3bB)}{16b^2c\,(b+cx^2)}-\frac{x^{3/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*((b*B - A*c)*x^(3/2))/(b*c*(b + c*x^2)^2) + ((3*b*B + 5*A*c)*x^(3/2))/
(16*b^2*c*(b + c*x^2)) - ((3*b*B + 5*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[
x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(7/4)) + ((3*b*B + 5*A*c)*ArcTan[1 + (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(7/4)) + ((3*b*B +
5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt
[2]*b^(9/4)*c^(7/4)) - ((3*b*B + 5*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(9/4)*c^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \int \frac{\sqrt{x}\,(A+Bx^2)}{(b+cx^2)^3}\,dx$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\,(b+cx^2)^2} + \frac{\left(\frac{3bB}{2}+\frac{5Ac}{2}\right)\int \frac{\sqrt{x}}{(b+cx^2)^2}\,dx}{4bc}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\,(b+cx^2)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\,(b+cx^2)} + \frac{(3bB+5Ac)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^2c}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\,(b+cx^2)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\,(b+cx^2)} + \frac{(3bB+5Ac)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^2c}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\,(b+cx^2)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\,(b+cx^2)} - \frac{(3bB+5Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right.}{32b^2c^{3/2}}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\,(b+cx^2)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\,(b+cx^2)} + \frac{(3bB+5Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx\right.}{64b^2c^2}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\,(b+cx^2)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\,(b+cx^2)} + \frac{(3bB+5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{64\sqrt{2}\,b^{9/4}c^{7/4}}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\,(b+cx^2)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\,(b+cx^2)} - \frac{(3bB+5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{9/4}c^{7/4}} + \cdots$$

Mathematica [A]

time = 0.60, size = 175, normalized size = 0.59

$$\frac{\frac{4\sqrt[4]{b}\,c^{3/4}x^{3/2}\left(-b^2B+9Abc+3bBcx^2+5Ac^2x^2\right)}{\left(b+cx^2\right)^2}-\sqrt{2}\,(3bB+5Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)-\sqrt{2}\,(3bB+5Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{64b^{9/4}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*b^(1/4)*c^(3/4)*x^(3/2)*(-(b^2*B) + 9*A*b*c + 3*b*B*c*x^2 + 5*A*c^2*x^2
))/(b + c*x^2)^2 - Sqrt[2]*(3*b*B + 5*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - Sqrt[2]*(3*b*B + 5*A*c)*ArcTanh[(Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(64*b^(9/4)*c^(7/4))
```

Maple [A]
time = 0.37, size = 168, normalized size = 0.56

method	result
derivativedivides	$\frac{\frac{(5Ac+3Bb)x^{\frac{7}{2}}}{16b^2}+\frac{(9Ac-Bb)x^{\frac{3}{2}}}{16bc}}{(c\,x^2+b)^2}+\frac{(5Ac+3Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+\right.}{128b^2c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{(5Ac+3Bb)x^{\frac{7}{2}}}{16b^2}+\frac{(9Ac-Bb)x^{\frac{3}{2}}}{16bc}}{(c\,x^2+b)^2}+\frac{(5Ac+3Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+\right.}{128b^2c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*(5*A*c+3*B*b)/b^2*x^(7/2)+1/32*(9*A*c-B*b)/b/c*x^(3/2))/(c*x^2+b)^2
+1/128*(5*A*c+3*B*b)/b^2/c^2/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)
*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.55, size = 253, normalized size = 0.85

$$\frac{(3\,Bbc+5\,Ac^2)x^{\frac{7}{2}}-(Bb^2-9\,Abc)x^{\frac{3}{2}}}{16\,(b^2c^3x^4+2\,b^3c^2x^2+b^4c)}+\frac{(3\,Bb+5\,Ac)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{128\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((3*B*b*c + 5*A*c^2)*x^(7/2) - (B*b^2 - 9*A*b*c)*x^(3/2))/(b^2*c^3*x^4
 + 2*b^3*c^2*x^2 + b^4*c) + 1/128*(3*B*b + 5*A*c)*(2*sqrt(2)*arctan(1/2*sqr
t(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(
sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(
1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt
(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + s
qrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) +
sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/(b^2*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1005 vs. $2(218) = 436$.
time = 2.26, size = 1005, normalized size = 3.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-(81*B^4*b^4 + 540*A*B^3*b^
3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(1/
4)*arctan((sqrt((729*B^6*b^6 + 7290*A*B^5*b^5*c + 30375*A^2*B^4*b^4*c^2 + 6
7500*A^3*B^3*b^3*c^3 + 84375*A^4*B^2*b^2*c^4 + 56250*A^5*B*b*c^5 + 15625*A^
6*c^6)*x - (81*B^4*b^9*c^3 + 540*A*B^3*b^8*c^4 + 1350*A^2*B^2*b^7*c^5 + 150
0*A^3*B*b^6*c^6 + 625*A^4*b^5*c^7)*sqrt(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 13
50*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7)))*b^2*c^2*(-
(81*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 6
25*A^4*c^4)/(b^9*c^7))^(1/4) - (27*B^3*b^5*c^2 + 135*A*B^2*b^4*c^3 + 225*A^
2*B*b^3*c^4 + 125*A^3*b^2*c^5)*sqrt(x)*(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 13
50*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(1/4))/(81*
B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A
^4*c^4)) - (b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-(81*B^4*b^4 + 540*A*B^3*
b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(
1/4)*log(b^7*c^5*(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1
500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(3/4) + (27*B^3*b^3 + 135*A*B^2*b
^2*c + 225*A^2*B*b*c^2 + 125*A^3*c^3)*sqrt(x)) + (b^2*c^3*x^4 + 2*b^3*c^2*x
^2 + b^4*c)*(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A
^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(1/4)*log(-b^7*c^5*(-(81*B^4*b^4 + 540
*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*
c^7))^(3/4) + (27*B^3*b^3 + 135*A*B^2*b^2*c + 225*A^2*B*b*c^2 + 125*A^3*c^3
)*sqrt(x)) - 4*((3*B*b*c + 5*A*c^2)*x^3 - (B*b^2 - 9*A*b*c)*x)*sqrt(x))/(b^
2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 298, normalized size = 1.00

$$\frac{3\,Bbcx^{\frac{7}{2}}+5\,Ac^{2}x^{\frac{7}{2}}-Bb^{2}x^{\frac{3}{2}}+9\,Abcx^{\frac{3}{2}}}{16\,(cx^{2}+b)^{2}b^{2}c}+\frac{\sqrt{2}\left(3\,(bc^{3})^{\frac{3}{4}}Bb+5\,(bc^{3})^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{3}c^{4}}+\frac{\sqrt{2}\left(3\,(bc^{3})^{\frac{3}{4}}Bb+5\,(bc^{3})^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{3}c^{4}}-\frac{\sqrt{2}\left(3\,(bc^{3})^{\frac{3}{4}}Bb+5\,(bc^{3})^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{3}c^{4}}+\frac{\sqrt{2}\left(3\,(bc^{3})^{\frac{3}{4}}Bb+5\,(bc^{3})^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{3}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*B*b*c*x^(7/2) + 5*A*c^2*x^(7/2) - B*b^2*x^(3/2) + 9*A*b*c*x^(3/2))/
((c*x^2 + b)^2*b^2*c) + 1/64*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + 5*(b*c^3)^(3/4)
*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^
3*c^4) + 1/64*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + 5*(b*c^3)^(3/4)*A*c)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^4) - 1/128
*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + 5*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b
/c)^(1/4) + x + sqrt(b/c))/(b^3*c^4) + 1/128*sqrt(2)*(3*(b*c^3)^(3/4)*B*b +
 5*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^
3*c^4)
```

Mupad [B]
time = 0.16, size = 124, normalized size = 0.42

$$\frac{\frac{x^{7/2}\,\left(5\,A\,c+3\,B\,b\right)}{16\,b^2}+\frac{x^{3/2}\,\left(9\,A\,c-B\,b\right)}{16\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}+\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)\,\left(5\,A\,c+3\,B\,b\right)}{32\,{\left(-b\right)}^{9/4}\,c^{7/4}}-\frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)\,\left(5\,A\,c+3\,B\,b\right)}{32\,{\left(-b\right)}^{9/4}\,c^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^(7/2)*(5*A*c + 3*B*b))/(16*b^2) + (x^(3/2)*(9*A*c - B*b))/(16*b*c))/(b^
2 + c^2*x^4 + 2*b*c*x^2) + (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(5*A*c + 3*B
*b))/(32*(-b)^(9/4)*c^(7/4)) - (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(5*A*c
+ 3*B*b))/(32*(-b)^(9/4)*c^(7/4))
```

3.213 $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=293

$$-\frac{(bB-Ac)\sqrt{x}}{4bc\,(b+cx^2)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\,(b+cx^2)}-\frac{3(bB+7Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{11/4}c^{5/4}}+\frac{3(bB+7Ac)\tan^{-1}\left(1+\sqrt{2}\right.}{32\sqrt{2}\,b^{11/4}c^{5/4}}$$

```
[Out] -3/64*(7*A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)/c^(5/4
)*2^(1/2)+3/64*(7*A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/
4)/c^(5/4)*2^(1/2)-3/128*(7*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2
^(1/2)*x^(1/2))/b^(11/4)/c^(5/4)*2^(1/2)+3/128*(7*A*c+B*b)*ln(b^(1/2)+x*c^(
1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)/c^(5/4)*2^(1/2)-1/4*(-A*c+B*
b)*x^(1/2)/b/c/(c*x^2+b)^2+1/16*(7*A*c+B*b)*x^(1/2)/b^2/c/(c*x^2+b)
```

Rubi [A]

time = 0.33, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3(7Ac+bB)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{11/4}c^{5/4}}+\frac{3(7Ac+bB)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{11/4}c^{5/4}}-\frac{3(7Ac+bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{11/4}c^{5/4}}+\frac{3(7Ac+bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{11/4}c^{5/4}}+\frac{\sqrt{x}\,(7Ac+bB)}{16b^2c\,(b+cx^2)}-\frac{\sqrt{x}\,(bB-Ac)}{4bc\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*((b*B - A*c)*Sqrt[x])/(b*c*(b + c*x^2)^2) + ((b*B + 7*A*c)*Sqrt[x])/(1
6*b^2*c*(b + c*x^2)) - (3*(b*B + 7*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(5/4)) + (3*(b*B + 7*A*c)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(5/4)) - (3*(b*B +
7*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt
[2]*b^(11/4)*c^(5/4)) + (3*(b*B + 7*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(11/4)*c^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \int \frac{A+Bx^2}{\sqrt{x}\,(b+cx^2)^3}\,dx$$

$$= -\frac{(bB-Ac)\sqrt{x}}{4bc\,(b+cx^2)^2} + \frac{\left(\frac{bB}{2}+\frac{7Ac}{2}\right)\int \frac{1}{\sqrt{x}\,(b+cx^2)^2}\,dx}{4bc}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{4bc\,(b+cx^2)^2} + \frac{(bB+7Ac)\sqrt{x}}{16b^2c\,(b+cx^2)} + \frac{(3(bB+7Ac))\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{32b^2c}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{4bc\,(b+cx^2)^2} + \frac{(bB+7Ac)\sqrt{x}}{16b^2c\,(b+cx^2)} + \frac{(3(bB+7Ac))\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^2c}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{4bc\,(b+cx^2)^2} + \frac{(bB+7Ac)\sqrt{x}}{16b^2c\,(b+cx^2)} + \frac{(3(bB+7Ac))\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{\ldots}\right.}{32b^{5/2}c}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{4bc\,(b+cx^2)^2} + \frac{(bB+7Ac)\sqrt{x}}{16b^2c\,(b+cx^2)} + \frac{(3(bB+7Ac))\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,d\ldots\right.}{64b^{5/2}c^{3/2}}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{4bc\,(b+cx^2)^2} + \frac{(bB+7Ac)\sqrt{x}}{16b^2c\,(b+cx^2)} - \frac{3(bB+7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\ldots\right.}{64\sqrt{2}\,b^{11/4}c^{5/4}}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{4bc\,(b+cx^2)^2} + \frac{(bB+7Ac)\sqrt{x}}{16b^2c\,(b+cx^2)} - \frac{3(bB+7Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{11/4}c^{5/4}} + \ldots$$

Mathematica [A]

time = 0.59, size = 172, normalized size = 0.59

$$\frac{\frac{4b^{3/4}\sqrt[4]{c}\,\sqrt{x}\,(-3b^2B+11Abc+bBcx^2+7Ac^2x^2)}{(b+cx^2)^2}-3\sqrt{2}\,(bB+7Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)+3\sqrt{2}\,(bB+7Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{64b^{11/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*b^(3/4)*c^(1/4)*Sqrt[x]*(-3*b^2*B + 11*A*b*c + b*B*c*x^2 + 7*A*c^2*x^2)
)/(b + c*x^2)^2 - 3*Sqrt[2]*(b*B + 7*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(b*B + 7*A*c)*ArcTanh[(Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(64*b^(11/4)*c^(5/4))
```

Maple [A]
time = 0.37, size = 166, normalized size = 0.57

method	result
derivativedivides	$\frac{\frac{(7Ac+Bb)x^{\frac{5}{2}}}{16b^2}+\frac{(11Ac-3Bb)\sqrt{x}}{16bc}}{(c\,x^2+b)^2}+\frac{3(7Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128b^3c}$
default	$\frac{\frac{(7Ac+Bb)x^{\frac{5}{2}}}{16b^2}+\frac{(11Ac-3Bb)\sqrt{x}}{16bc}}{(c\,x^2+b)^2}+\frac{3(7Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128b^3c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*(7*A*c+B*b)/b^2*x^(5/2)+1/32*(11*A*c-3*B*b)/b/c*x^(1/2))/(c*x^2+b)^
2+3/128*(7*A*c+B*b)/b^3/c*(b/c)^(1/4)*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^
(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^
(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.54, size = 276, normalized size = 0.94

$$\frac{(Bbc+7\,Ac^2)x^{\frac{5}{2}}-(3\,Bb^2-11\,Abc)\sqrt{x}}{16\,(b^2c^3x^4+2\,b^3c^2x^2+b^4c)}+\frac{3\left(\frac{2\,\sqrt{2}\,(Bb+7\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,(Bb+7\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,(Bb+7\,Ac)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Bb+7\,Ac)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{128\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((B*b*c + 7*A*c^2)*x^(5/2) - (3*B*b^2 - 11*A*b*c)*sqrt(x))/(b^2*c^3*x^
4 + 2*b^3*c^2*x^2 + b^4*c) + 3/128*(2*sqrt(2)*(B*b + 7*A*c)*arctan(1/2*sqrt
```

```
(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(s
qrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(B*b + 7*A*c)*arctan(-1/2*sqrt(2)
*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt
(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(B*b + 7*A*c)*log(sqrt(2)*b^(1/4)*c^(1
/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b + 7*A*c
)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1
/4)))/(b^2*c)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(213) = 426$.
time = 3.29, size = 793, normalized size = 2.71

$$\frac{12\,(b^2c^3x^4+2\,b^3c^2x^2+b^4c)\left(-\frac{B^4b^4+28\,AB^3b^3c+294\,A^2B^2b^2c^2+1372\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^6c^2\sqrt{-\frac{B^4b^4+28\,AB^3b^3c+294\,A^2B^2b^2c^2+1372\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c^5}}+(B^2b^2+14\,ABbc+49\,A^2c^2)x}\,b^8c^4\left(-\frac{B^4b^4+28\,AB^3b^3c+294\,A^2B^2b^2c^2+1372\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c^5}\right)^{\frac{3}{4}}-(Bb^9c^4+7\,Ab^8c^5)\sqrt{x}\left(-\frac{B^4b^4+28\,AB^3b^3c+294\,A^2B^2b^2c^2+1372\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c^5}\right)^{\frac{3}{4}}}{B^4b^4+28\,AB^3b^3c+294\,A^2B^2b^2c^2+1372\,A^3Bbc^3+2401\,A^4c^4}\right)+3\,(b^2c^3x^4+2\,b^3c^2x^2+b^4c)\left(-\frac{B^4b^4+28\,AB^3b^3c+294\,A^2B^2b^2c^2+1372\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c^5}\right)^{\frac{1}{4}}\log\left(3\,b^3c\left(-\frac{B^4b^4+28\,AB^3b^3c+294\,A^2B^2b^2c^2+1372\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c^5}\right)^{\frac{1}{4}}+3\,(Bb+7\,Ac)\sqrt{x}\right)-3\,(b^2c^3x^4+2\,b^3c^2x^2+b^4c)\left(-\frac{B^4b^4+28\,AB^3b^3c+294\,A^2B^2b^2c^2+1372\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c^5}\right)^{\frac{1}{4}}\log\left(-3\,b^3c\left(-\frac{B^4b^4+28\,AB^3b^3c+294\,A^2B^2b^2c^2+1372\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c^5}\right)^{\frac{1}{4}}+3\,(Bb+7\,Ac)\sqrt{x}\right)-4\,(3\,Bb^2-11\,Abc-(Bbc+7\,Ac^2)x^2)\sqrt{x}}{64\,(b^2c^3x^4+2\,b^3c^2x^2+b^4c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(12*(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-(B^4*b^4 + 28*A*B^3*b^3*c
+ 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4)*
arctan((sqrt(b^6*c^2*sqrt(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2
+ 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5)) + (B^2*b^2 + 14*A*B*b*c + 49
*A^2*c^2)*x)*b^8*c^4*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 13
72*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(3/4) - (B*b^9*c^4 + 7*A*b^8*c^5
)*sqrt(x)*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*
c^3 + 2401*A^4*c^4)/(b^11*c^5))^(3/4))/(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*
B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)) + 3*(b^2*c^3*x^4 + 2*b^3*c^
2*x^2 + b^4*c)*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3
*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4)*log(3*b^3*c*(-(B^4*b^4 + 28*A*B^
3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5)
)^(1/4) + 3*(B*b + 7*A*c)*sqrt(x)) - 3*(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c
)*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 24
01*A^4*c^4)/(b^11*c^5))^(1/4)*log(-3*b^3*c*(-(B^4*b^4 + 28*A*B^3*b^3*c + 29
4*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4) + 3*
(B*b + 7*A*c)*sqrt(x)) - 4*(3*B*b^2 - 11*A*b*c - (B*b*c + 7*A*c^2)*x^2)*sqr
t(x))/(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 293, normalized size = 1.00

$$\frac{3\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb+7\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{3}c^{2}}+\frac{3\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb+7\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{3}c^{2}}+\frac{3\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb+7\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{3}c^{2}}-\frac{3\sqrt{2}\left(\left(bc^{3}\right)^{\frac{1}{4}}Bb+7\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{3}c^{2}}+\frac{Bbcx^{\frac{5}{2}}+7\,Ac^{2}x^{\frac{5}{2}}-3\,Bb^{2}\sqrt{x}+11\,Abc\sqrt{x}}{16\,(cx^{2}+b)^{2}b^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*((b*c^3)^(1/4)*B*b + 7*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 3/64*sqrt(2)*((b*
c^3)^(1/4)*B*b + 7*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 3/128*sqrt(2)*((b*c^3)^(1/4)*B*b
+ 7*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^
3*c^2) - 3/128*sqrt(2)*((b*c^3)^(1/4)*B*b + 7*(b*c^3)^(1/4)*A*c)*log(-sqrt(
2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) + 1/16*(B*b*c*x^(5/2) + 7
*A*c^2*x^(5/2) - 3*B*b^2*sqrt(x) + 11*A*b*c*sqrt(x))/((c*x^2 + b)^2*b^2*c)
```

Mupad [B]
time = 0.39, size = 780, normalized size = 2.66

$$\frac{\frac{x^{5/2}\,(7\,A\,c+B\,b)}{16\,b^2}+\frac{\sqrt{x}\,(11\,A\,c-3\,B\,b)}{16\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{\mathrm{atan}\left(\frac{\frac{(7\,A\,c+B\,b)\left(\frac{9\sqrt{x}\,(49\,A^2c^3+14\,A\,B\,b\,c^2+B^2b^2c)}{64\,b^4}-\frac{9\,(7\,A\,c+B\,b)\,(7\,A\,c^3+B\,b\,c^2)}{64\,(-b)^{15/4}\,c^{5/4}}\right)3i}{64\,(-b)^{11/4}\,c^{5/4}}+\frac{(7\,A\,c+B\,b)\left(\frac{9\sqrt{x}\,(49\,A^2c^3+14\,A\,B\,b\,c^2+B^2b^2c)}{64\,b^4}+\frac{9\,(7\,A\,c+B\,b)\,(7\,A\,c^3+B\,b\,c^2)}{64\,(-b)^{15/4}\,c^{5/4}}\right)3i}{64\,(-b)^{11/4}\,c^{5/4}}}{\frac{3\,(7\,A\,c+B\,b)\left(\frac{9\sqrt{x}\,(49\,A^2c^3+14\,A\,B\,b\,c^2+B^2b^2c)}{64\,b^4}-\frac{9\,(7\,A\,c+B\,b)\,(7\,A\,c^3+B\,b\,c^2)}{64\,(-b)^{15/4}\,c^{5/4}}\right)}{64\,(-b)^{11/4}\,c^{5/4}}-\frac{3\,(7\,A\,c+B\,b)\left(\frac{9\sqrt{x}\,(49\,A^2c^3+14\,A\,B\,b\,c^2+B^2b^2c)}{64\,b^4}+\frac{9\,(7\,A\,c+B\,b)\,(7\,A\,c^3+B\,b\,c^2)}{64\,(-b)^{15/4}\,c^{5/4}}\right)}{64\,(-b)^{11/4}\,c^{5/4}}}\right)(7\,A\,c+B\,b)\,3i}{32\,(-b)^{11/4}\,c^{5/4}}-\frac{3\,\mathrm{atan}\left(\frac{\frac{3\,(7\,A\,c+B\,b)\left(\frac{9\sqrt{x}\,(49\,A^2c^3+14\,A\,B\,b\,c^2+B^2b^2c)}{64\,b^4}-\frac{(7\,A\,c+B\,b)\,(7\,A\,c^3+B\,b\,c^2)\,9i}{64\,(-b)^{15/4}\,c^{5/4}}\right)}{64\,(-b)^{11/4}\,c^{5/4}}+\frac{3\,(7\,A\,c+B\,b)\left(\frac{9\sqrt{x}\,(49\,A^2c^3+14\,A\,B\,b\,c^2+B^2b^2c)}{64\,b^4}+\frac{(7\,A\,c+B\,b)\,(7\,A\,c^3+B\,b\,c^2)\,9i}{64\,(-b)^{15/4}\,c^{5/4}}\right)}{64\,(-b)^{11/4}\,c^{5/4}}}{\frac{(7\,A\,c+B\,b)\left(\frac{9\sqrt{x}\,(49\,A^2c^3+14\,A\,B\,b\,c^2+B^2b^2c)}{64\,b^4}-\frac{(7\,A\,c+B\,b)\,(7\,A\,c^3+B\,b\,c^2)\,9i}{64\,(-b)^{15/4}\,c^{5/4}}\right)3i}{64\,(-b)^{11/4}\,c^{5/4}}-\frac{(7\,A\,c+B\,b)\left(\frac{9\sqrt{x}\,(49\,A^2c^3+14\,A\,B\,b\,c^2+B^2b^2c)}{64\,b^4}+\frac{(7\,A\,c+B\,b)\,(7\,A\,c^3+B\,b\,c^2)\,9i}{64\,(-b)^{15/4}\,c^{5/4}}\right)3i}{64\,(-b)^{11/4}\,c^{5/4}}}\right)(7\,A\,c+B\,b)}{32\,(-b)^{11/4}\,c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^(5/2)*(7*A*c + B*b))/(16*b^2) + (x^(1/2)*(11*A*c - 3*B*b))/(16*b*c))/(b
^2 + c^2*x^4 + 2*b*c*x^2) - (atan((((7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 +
 B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) - (9*(7*A*c + B*b)*(7*A*c^3 + B*b*c^2)
)/(64*(-b)^(15/4)*c^(5/4)))*3i)/(64*(-b)^(11/4)*c^(5/4)) + ((7*A*c + B*b)*(
(9*x^(1/2)*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) + (9*(7*A*c +
B*b)*(7*A*c^3 + B*b*c^2))/(64*(-b)^(15/4)*c^(5/4)))*3i)/(64*(-b)^(11/4)*c^(
5/4)))/((3*(7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2
))/(64*b^4) - (9*(7*A*c + B*b)*(7*A*c^3 + B*b*c^2))/(64*(-b)^(15/4)*c^(5/4)
)))/(64*(-b)^(11/4)*c^(5/4)) - (3*(7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 + B
^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) + (9*(7*A*c + B*b)*(7*A*c^3 + B*b*c^2))/
(64*(-b)^(15/4)*c^(5/4))))/(64*(-b)^(11/4)*c^(5/4))))*(7*A*c + B*b)*3i)/(32
*(-b)^(11/4)*c^(5/4)) - (3*atan(((3*(7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 +
 B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) - ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2)*9
i)/(64*(-b)^(15/4)*c^(5/4))))/(64*(-b)^(11/4)*c^(5/4)) + (3*(7*A*c + B*b)*(
(9*x^(1/2)*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) + ((7*A*c + B*
b)*(7*A*c^3 + B*b*c^2)*9i)/(64*(-b)^(15/4)*c^(5/4))))/(64*(-b)^(11/4)*c^(5/
4)))/(((7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2))/(
64*b^4) - ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2)*9i)/(64*(-b)^(15/4)*c^(5/4)))*
3i)/(64*(-b)^(11/4)*c^(5/4)) - ((7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 + B^2
```

```
*b^2*c + 14*A*B*b*c^2))/(64*b^4) + ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2)*9i)/(
64*(-b)^(15/4)*c^(5/4)))*3i)/(64*(-b)^(11/4)*c^(5/4))))*(7*A*c + B*b))/(32*
(-b)^(11/4)*c^(5/4))
```

3.214 $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=316

$$\frac{5(bB-9Ac)}{16b^3c\sqrt{x}}-\frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2}-\frac{bB-9Ac}{16b^2c\sqrt{x}\,(b+cx^2)}-\frac{5(bB-9Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{13/4}c^{3/4}}+\frac{5(bB-}{}$$

```
[Out] -5/64*(-9*A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)/c^(3/
4)*2^(1/2)+5/64*(-9*A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1
3/4)/c^(3/4)*2^(1/2)+5/128*(-9*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4
)*2^(1/2)*x^(1/2))/b^(13/4)/c^(3/4)*2^(1/2)-5/128*(-9*A*c+B*b)*ln(b^(1/2)+x
*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)/c^(3/4)*2^(1/2)+5/16*(-9
*A*c+B*b)/b^3/c/x^(1/2)+1/4*(A*c-B*b)/b/c/(c*x^2+b)^2/x^(1/2)+1/16*(9*A*c-B
*b)/b^2/c/(c*x^2+b)/x^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1598, 468, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{5(bB-9Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{13/4}c^{3/4}}+\frac{5(bB-9Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{13/4}c^{3/4}}+\frac{5(bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{13/4}c^{3/4}}-\frac{5(bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{13/4}c^{3/4}}+\frac{5(bB-9Ac)}{16b^3c\sqrt{x}}-\frac{bB-9Ac}{16b^2c\sqrt{x}\,(b+cx^2)}-\frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (5*(b*B - 9*A*c))/(16*b^3*c*Sqrt[x]) - (b*B - A*c)/(4*b*c*Sqrt[x]*(b + c*x^
2)^2) - (b*B - 9*A*c)/(16*b^2*c*Sqrt[x]*(b + c*x^2)) - (5*(b*B - 9*A*c)*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(13/4)*c^(3/4)) +
 (5*(b*B - 9*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2
]*b^(13/4)*c^(3/4)) + (5*(b*B - 9*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4)*c^(3/4)) - (5*(b*B - 9*A*c)*Lo
g[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13
/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \int \frac{A+Bx^2}{x^{3/2}(b+cx^2)^3}\,dx$$

$$= -\frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2} + \frac{\left(-\frac{bB}{2}+\frac{9Ac}{2}\right)\int \frac{1}{x^{3/2}(b+cx^2)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\,(b+cx^2)} - \frac{(5(bB-9Ac))\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{32b^2c}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\,(b+cx^2)} + \frac{(5(bB-9Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^3}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\,(b+cx^2)} + \frac{(5(bB-9Ac))\mathrm{Subst}\left(\int\right.}{16b^3}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\,(b+cx^2)} - \frac{(5(bB-9Ac))\mathrm{Subst}\left(\int\right.}{32b^3}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\,(b+cx^2)} + \frac{(5(bB-9Ac))\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\,(b+cx^2)} + \frac{5(bB-9Ac)\log\left(\sqrt{b}-\right.}{64\sqrt{2}}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\,(b+cx^2)} - \frac{5(bB-9Ac)\tan^{-1}\left(1-\right.}{32\sqrt{2}\,b^{13/4}c}$$

Mathematica [A]
time = 0.64, size = 187, normalized size = 0.59

$$\frac{-\frac{4\sqrt[4]{b}\,(-bBx^2(9b+5cx^2)+A(32b^2+81bcx^2+45c^2x^4))}{\sqrt{x}\,(b+cx^2)^2} + \frac{5\sqrt{2}\,(-bB+9Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{c^{3/4}} + \frac{5\sqrt{2}\,(-bB+9Ac)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{c^{3/4}}}{64b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(1/4)*(-(b*B*x^2*(9*b + 5*c*x^2)) + A*(32*b^2 + 81*b*c*x^2 + 45*c^2*
x^4)))/(Sqrt[x]*(b + c*x^2)^2) + (5*Sqrt[2]*(-(b*B) + 9*A*c)*ArcTan[(Sqrt[b
```

```
] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/c^(3/4) + (5*Sqrt[2]*(-(
b*B) + 9*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*
x)])/c^(3/4))/(64*b^(13/4))
```

Maple [A]
time = 0.41, size = 173, normalized size = 0.55

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{13}{32}Ac^{2}-\frac{5}{32}bBc\right)x^{\frac{7}{2}}+\frac{b(17Ac-9Bb)x^{\frac{3}{2}}}{32}}{\left(cx^{2}+b\right)^{2}}+\frac{\left(\frac{45Ac}{32}-\frac{5Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\cdots}{\left(\frac{b}{c}\right)^{\frac{1}{4}}\cdots}\right.\cdots}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots}{b^{3}}$
default	$-\frac{2\left(\frac{\left(\frac{13}{32}Ac^{2}-\frac{5}{32}bBc\right)x^{\frac{7}{2}}+\frac{b(17Ac-9Bb)x^{\frac{3}{2}}}{32}}{\left(cx^{2}+b\right)^{2}}+\frac{\left(\frac{45Ac}{32}-\frac{5Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\cdots}{\left(\frac{b}{c}\right)^{\frac{1}{4}}\cdots}\right.\cdots}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots}{b^{3}}$
risch	$-\frac{2A}{b^{3}\sqrt{x}}-\frac{13x^{\frac{7}{2}}Ac^{2}}{16b^{3}(cx^{2}+b)^{2}}+\frac{5x^{\frac{7}{2}}Bc}{16b^{2}(cx^{2}+b)^{2}}-\frac{17Ax^{\frac{3}{2}}c}{16b^{2}(cx^{2}+b)^{2}}+\frac{9Bx^{\frac{3}{2}}}{16b(cx^{2}+b)^{2}}-\frac{45\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\cdots}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\cdots}\right.\cdots}{128b^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^3*(((13/32*A*c^2-5/32*b*B*c)*x^(7/2)+1/32*b*(17*A*c-9*B*b)*x^(3/2))/(c
*x^2+b)^2+1/8*(45/32*A*c-5/32*B*b)/c/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)
*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+
2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)-1)))-2*A/b^3/x^(1/2)
```

Maxima [A]
time = 0.52, size = 255, normalized size = 0.81

$$\frac{5\,(Bbc-9\,Ac^{2})x^{4}-32\,Ab^{2}+9\,(Bb^{2}-9\,Abc)x^{2}}{16\left(b^{3}c^{2}x^{\frac{9}{2}}+2\,b^{4}cx^{\frac{5}{2}}+b^{5}\sqrt{x}\right)}+\frac{5\,(Bb-9\,Ac)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{128\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(5*(B*b*c - 9*A*c^2)*x^4 - 32*A*b^2 + 9*(B*b^2 - 9*A*b*c)*x^2)/(b^3*c^
2*x^(9/2) + 2*b^4*c*x^(5/2) + b^5*sqrt(x)) + 5/128*(B*b - 9*A*c)*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt
(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(
2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sq
rt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
+ sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(
1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 988 vs. $2(232) = 464$.
time = 2.74, size = 988, normalized size = 3.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-(B^4*b^4 - 36*A*B^3*b^3*c +
486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1/4)*ar
ctan((sqrt((B^6*b^6 - 54*A*B^5*b^5*c + 1215*A^2*B^4*b^4*c^2 - 14580*A^3*B^3
*b^3*c^3 + 98415*A^4*B^2*b^2*c^4 - 354294*A^5*B*b*c^5 + 531441*A^6*c^6)*x -
 (B^4*b^11*c - 36*A*B^3*b^10*c^2 + 486*A^2*B^2*b^9*c^3 - 2916*A^3*B*b^8*c^4
 + 6561*A^4*b^7*c^5)*sqrt(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2
- 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3)))*b^3*c*(-(B^4*b^4 - 36*A*B^3
*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))
^(1/4) + (B^3*b^6*c - 27*A*B^2*b^5*c^2 + 243*A^2*B*b^4*c^3 - 729*A^3*b^3*c^
4)*sqrt(x)*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b
*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1/4))/(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2
*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)) - 5*(b^3*c^2*x^5 + 2*b^4*c
*x^3 + b^5*x)*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*
B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1/4)*log(125*b^10*c^2*(-(B^4*b^4 - 36*
A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*
c^3))^(3/4) - 125*(B^3*b^3 - 27*A*B^2*b^2*c + 243*A^2*B*b*c^2 - 729*A^3*c^3
)*sqrt(x)) + 5*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-(B^4*b^4 - 36*A*B^3*b^
3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1
/4)*log(-125*b^10*c^2*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2
916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(3/4) - 125*(B^3*b^3 - 27*A*B^2
*b^2*c + 243*A^2*B*b*c^2 - 729*A^3*c^3)*sqrt(x)) + 4*(5*(B*b*c - 9*A*c^2)*x
^4 - 32*A*b^2 + 9*(B*b^2 - 9*A*b*c)*x^2)*sqrt(x))/(b^3*c^2*x^5 + 2*b^4*c*x^
3 + b^5*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 300, normalized size = 0.95

$$-\frac{2\,A}{b^3\sqrt{x}}+\frac{5\,Bbcx^{\frac{7}{2}}-13\,Ac^2x^{\frac{7}{2}}+9\,Bb^2x^{\frac{3}{2}}-17\,Abcx^{\frac{3}{2}}}{16\,(cx^2+b)^2b^3}+\frac{5\,\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb-9\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c^3}+\frac{5\,\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb-9\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c^3}-\frac{5\,\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb-9\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^4c^3}+\frac{5\,\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb-9\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -2*A/(b^3*sqrt(x)) + 1/16*(5*B*b*c*x^(7/2) - 13*A*c^2*x^(7/2) + 9*B*b^2*x^(
3/2) - 17*A*b*c*x^(3/2))/((c*x^2 + b)^2*b^3) + 5/64*sqrt(2)*((b*c^3)^(3/4)*
B*b - 9*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b^4*c^3) + 5/64*sqrt(2)*((b*c^3)^(3/4)*B*b - 9*(b*c^3)^(
3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4)
)/(b^4*c^3) - 5/128*sqrt(2)*((b*c^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*log(s
qrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c^3) + 5/128*sqrt(2)*((b*c
^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/(b^4*c^3)
```

Mupad [B]
time = 0.26, size = 133, normalized size = 0.42

$$\frac{5\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)\,\left(9\,A\,c-B\,b\right)}{32\,{(-b)}^{13/4}\,c^{3/4}}-\frac{\frac{2\,A}{b}+\frac{9\,x^2\,\left(9\,A\,c-B\,b\right)}{16\,b^2}+\frac{5\,c\,x^4\,\left(9\,A\,c-B\,b\right)}{16\,b^3}}{b^2\,\sqrt{x}+c^2\,x^{9/2}+2\,b\,c\,x^{5/2}}-\frac{5\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)\,\left(9\,A\,c-B\,b\right)}{32\,{(-b)}^{13/4}\,c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (5*atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(9*A*c - B*b))/(32*(-b)^(13/4)*c^(3/4
)) - ((2*A)/b + (9*x^2*(9*A*c - B*b))/(16*b^2) + (5*c*x^4*(9*A*c - B*b))/(1
6*b^3))/(b^2*x^(1/2) + c^2*x^(9/2) + 2*b*c*x^(5/2)) - (5*atanh((c^(1/4)*x^(
1/2))/(-b)^(1/4))*(9*A*c - B*b))/(32*(-b)^(13/4)*c^(3/4))
```

3.215 $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=322

$$\frac{7(3bB-11Ac)}{48b^3cx^{3/2}}-\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2}-\frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)}-\frac{7(3bB-11Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{15/4}\sqrt[4]{c}}+\frac{7(3bB$$

```
[Out] 7/48*(-11*A*c+3*B*b)/b^3/c/x^(3/2)+1/4*(A*c-B*b)/b/c/x^(3/2)/(c*x^2+b)^2+1/
16*(11*A*c-3*B*b)/b^2/c/x^(3/2)/(c*x^2+b)-7/64*(-11*A*c+3*B*b)*arctan(1-c^(
1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)/c^(1/4)*2^(1/2)+7/64*(-11*A*c+3*B*b)
*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)/c^(1/4)*2^(1/2)-7/128*(
-11*A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/
4)/c^(1/4)*2^(1/2)+7/128*(-11*A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/
4)*2^(1/2)*x^(1/2))/b^(15/4)/c^(1/4)*2^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1598, 468, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{7(3bB-11Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{15/4}\sqrt[4]{c}}+\frac{7(3bB-11Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{15/4}\sqrt[4]{c}}-\frac{7(3bB-11Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{15/4}\sqrt[4]{c}}+\frac{7(3bB-11Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{15/4}\sqrt[4]{c}}+\frac{7(3bB-11Ac)}{48b^3cx^{3/2}}-\frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)}-\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (7*(3*b*B - 11*A*c))/(48*b^3*c*x^(3/2)) - (b*B - A*c)/(4*b*c*x^(3/2)*(b + c
*x^2)^2) - (3*b*B - 11*A*c)/(16*b^2*c*x^(3/2)*(b + c*x^2)) - (7*(3*b*B - 11
*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(15/4)*c
^(1/4)) + (7*(3*b*B - 11*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
)/(32*Sqrt[2]*b^(15/4)*c^(1/4)) - (7*(3*b*B - 11*A*c)*Log[Sqrt[b] - Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4)*c^(1/4)) + (7*(
3*b*B - 11*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
/(64*Sqrt[2]*b^(15/4)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \int \frac{A+Bx^2}{x^{5/2}\,(b+cx^2)^3}\,dx$$

$$= -\frac{bB-Ac}{4bcx^{3/2}\,(b+cx^2)^2} + \frac{\left(-\frac{3bB}{2}+\frac{11Ac}{2}\right)\int \frac{1}{x^{5/2}(b+cx^2)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bcx^{3/2}\,(b+cx^2)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\,(b+cx^2)} - \frac{(7(3bB-11Ac))\int \frac{1}{x^{5/2}(b+cx^2)}\,dx}{32b^2c}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\,(b+cx^2)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\,(b+cx^2)} + \frac{(7(3bB-11Ac))\int -}{32b^3}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\,(b+cx^2)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\,(b+cx^2)} + \frac{(7(3bB-11Ac))\mathrm{Subs}}{16}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\,(b+cx^2)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\,(b+cx^2)} + \frac{(7(3bB-11Ac))\mathrm{Subs}}{}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\,(b+cx^2)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\,(b+cx^2)} + \frac{(7(3bB-11Ac))\mathrm{Subs}}{}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\,(b+cx^2)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\,(b+cx^2)} - \frac{7(3bB-11Ac)\log\left(\right.}{64}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\,(b+cx^2)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\,(b+cx^2)} - \frac{7(3bB-11Ac)\tan^{-1}}{32\sqrt{2}\,b}$$

<u>Mathematica</u> [A]
time = 0.54, size = 187, normalized size = 0.58

$$\frac{-\frac{4b^{3/4}(-3bBx^2(11b+7cx^2)+A(32b^2+121bcx^2+77c^2x^4))}{x^{3/2}(b+cx^2)^2}+\frac{21\sqrt{2}\,(-3bB+11Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)}{\sqrt[4]{c}}+\frac{21\sqrt{2}\,(3bB-11Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt[4]{c}}}{192b^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(3/4)*(-3*b*B*x^2*(11*b + 7*c*x^2) + A*(32*b^2 + 121*b*c*x^2 + 77*c^
2*x^4)))/(x^(3/2)*(b + c*x^2)^2) + (21*Sqrt[2]*(-3*b*B + 11*A*c)*ArcTan[(Sq
rt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/c^(1/4) + (21*Sqrt[2
```

```
]*(3*b*B - 11*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqr
t[c]*x)])/c^(1/4))/(192*b^(15/4))
```

Maple [A]

time = 0.41, size = 173, normalized size = 0.54

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{15}{32}A\,c^{2}-\frac{7}{32}bBc\right)x^{\frac{5}{2}}+\frac{b(19Ac-11Bb)\sqrt{x}}{32}}{\left(c\,x^{2}+b\right)^{2}}+\frac{7(11Ac-3Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\sqrt{\cdots}\right.\right.}{256b}\cdots\right.}{b^{3}}$
default	$-\frac{2\left(\frac{\left(\frac{15}{32}A\,c^{2}-\frac{7}{32}bBc\right)x^{\frac{5}{2}}+\frac{b(19Ac-11Bb)\sqrt{x}}{32}}{\left(c\,x^{2}+b\right)^{2}}+\frac{7(11Ac-3Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\sqrt{\cdots}\right.\right.}{256b}\cdots\right.}{b^{3}}$
risch	$-\frac{2A}{3b^{3}x^{\frac{3}{2}}}-\frac{15x^{\frac{5}{2}}A\,c^{2}}{16b^{3}\left(c\,x^{2}+b\right)^{2}}+\frac{7x^{\frac{5}{2}}Bc}{16b^{2}\left(c\,x^{2}+b\right)^{2}}-\frac{19A\sqrt{x}\,c}{16b^{2}\left(c\,x^{2}+b\right)^{2}}+\frac{11B\sqrt{x}}{16b\left(c\,x^{2}+b\right)^{2}}-\frac{77\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\cdots}{\left(\frac{b}{c}\right)\cdots}\right.}{64b^{4}}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^3*(((15/32*A*c^2-7/32*b*B*c)*x^(5/2)+1/32*b*(19*A*c-11*B*b)*x^(1/2))/(
c*x^2+b)^2+7/256*(11*A*c-3*B*b)*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)^(1/4)*x^
(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*a
rctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1
)))-2/3*A/b^3/x^(3/2)
```

Maxima [A]

time = 0.50, size = 285, normalized size = 0.89

$$\frac{7\,(3\,Bbc-11\,Ac^2)x^4-32\,Ab^2+11\,(3\,Bb^2-11\,Abc)x^2}{48\left(b^3c^2x^{\frac{11}{2}}+2\,b^4cx^{\frac{7}{2}}+b^5x^{\frac{3}{2}}\right)}+\frac{7\left(\frac{2\sqrt{2}\,(3\,Bb-11\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\,(3\,Bb-11\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\,(3\,Bb-11\,Ac)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,(3\,Bb-11\,Ac)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{128\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/48*(7*(3*B*b*c - 11*A*c^2)*x^4 - 32*A*b^2 + 11*(3*B*b^2 - 11*A*b*c)*x^2)/
(b^3*c^2*x^(11/2) + 2*b^4*c*x^(7/2) + b^5*x^(3/2)) + 7/128*(2*sqrt(2)*(3*B*
b - 11*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(3*B*b
 - 11*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(3*B*b -
 11*A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4
)*c^(1/4)) - sqrt(2)*(3*B*b - 11*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
+ sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^3
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 809 vs. $2(238) = 476$.
time = 2.62, size = 809, normalized size = 2.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(84*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-(81*B^4*b^4 - 1188*A*B^3
*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c)
)^(1/4)*arctan((sqrt(b^8*sqrt(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^
2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c)) + (9*B^2*b^2 - 66*
A*B*b*c + 121*A^2*c^2)*x)*b^11*c*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^
2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(3/4) + (3*B*b
^12*c - 11*A*b^11*c^2)*sqrt(x)*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*
B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(3/4))/(81*B^4*b
^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^
4*c^4)) + 21*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-(81*B^4*b^4 - 1188*A*B
^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*
c))^(1/4)*log(7*b^4*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2
 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(1/4) - 7*(3*B*b - 11*A*c)*
sqrt(x)) - 21*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-(81*B^4*b^4 - 1188*A*
B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15
*c))^(1/4)*log(-7*b^4*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c
^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(1/4) - 7*(3*B*b - 11*A*c
)*sqrt(x)) - 4*(7*(3*B*b*c - 11*A*c^2)*x^4 - 32*A*b^2 + 11*(3*B*b^2 - 11*A*
b*c)*x^2)*sqrt(x))/(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.56, size = 304, normalized size = 0.94

$$\frac{7\sqrt{2}\left(3\left(bc^{3}\right)^{\frac{1}{4}}Bb-11\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{4}c}+\frac{7\sqrt{2}\left(3\left(bc^{3}\right)^{\frac{1}{4}}Bb-11\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{4}c}+\frac{7\sqrt{2}\left(3\left(bc^{3}\right)^{\frac{1}{4}}Bb-11\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{4}c}-\frac{7\sqrt{2}\left(3\left(bc^{3}\right)^{\frac{1}{4}}Bb-11\left(bc^{3}\right)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{4}c}-\frac{2\,A}{3\,b^{3}x^{\frac{3}{2}}}+\frac{7\,Bbcx^{\frac{5}{2}}-15\,Ac^{2}x^{\frac{5}{2}}+11\,Bb^{2}\sqrt{x}-19\,Abc\sqrt{x}}{16\,\left(cx^{2}+b\right)^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 7/64*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 7/64*sqrt(2)*(3*
(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c
)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 7/128*sqrt(2)*(3*(b*c^3)^(1/4)*
B*b - 11*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c)
)/(b^4*c) - 7/128*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*log(
-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c) - 2/3*A/(b^3*x^(3/2))
 + 1/16*(7*B*b*c*x^(5/2) - 15*A*c^2*x^(5/2) + 11*B*b^2*sqrt(x) - 19*A*b*c*s
qrt(x))/((c*x^2 + b)^2*b^3)
```

Mupad [B]
time = 0.45, size = 888, normalized size = 2.76

$$-\frac{\frac{2A}{3b}+\frac{11x^{2}\left(11Ac-3Bb\right)}{48b^{2}}+\frac{7cx^{4}\left(11Ac-3Bb\right)}{48b^{3}}}{b^{2}x^{3/2}+c^{2}x^{11/2}+2bcx^{7/2}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(11Ac-3Bb\right)\left(\sqrt{x}\left(97140736A^{2}b^{9}c^{5}-52985856ABb^{10}c^{4}+7225344B^{2}b^{11}c^{3}\right)-\frac{7\left(11Ac-3Bb\right)\left(80740352Ab^{13}c^{4}-22020096Bb^{14}c^{3}\right)}{64\left(-b\right)^{15/4}c^{1/4}}\right)7\mathrm{i}}{64\left(-b\right)^{15/4}c^{1/4}}+\frac{\left(11Ac-3Bb\right)\left(\sqrt{x}\left(97140736A^{2}b^{9}c^{5}-52985856ABb^{10}c^{4}+7225344B^{2}b^{11}c^{3}\right)+\frac{7\left(11Ac-3Bb\right)\left(80740352Ab^{13}c^{4}-22020096Bb^{14}c^{3}\right)}{64\left(-b\right)^{15/4}c^{1/4}}\right)7\mathrm{i}}{64\left(-b\right)^{15/4}c^{1/4}}}{\frac{7\left(11Ac-3Bb\right)\left(\sqrt{x}\left(97140736A^{2}b^{9}c^{5}-52985856ABb^{10}c^{4}+7225344B^{2}b^{11}c^{3}\right)-\frac{7\left(11Ac-3Bb\right)\left(80740352Ab^{13}c^{4}-22020096Bb^{14}c^{3}\right)}{64\left(-b\right)^{15/4}c^{1/4}}\right)}{64\left(-b\right)^{15/4}c^{1/4}}-\frac{7\left(11Ac-3Bb\right)\left(\sqrt{x}\left(97140736A^{2}b^{9}c^{5}-52985856ABb^{10}c^{4}+7225344B^{2}b^{11}c^{3}\right)+\frac{7\left(11Ac-3Bb\right)\left(80740352Ab^{13}c^{4}-22020096Bb^{14}c^{3}\right)}{64\left(-b\right)^{15/4}c^{1/4}}\right)}{64\left(-b\right)^{15/4}c^{1/4}}}\right)\left(11Ac-3Bb\right)7\mathrm{i}}{32\left(-b\right)^{15/4}c^{1/4}}-\frac{7\,\mathrm{atan}\left(\frac{\frac{7\left(11Ac-3Bb\right)\left(\sqrt{x}\left(97140736A^{2}b^{9}c^{5}-52985856ABb^{10}c^{4}+7225344B^{2}b^{11}c^{3}\right)-\frac{\left(11Ac-3Bb\right)\left(80740352Ab^{13}c^{4}-22020096Bb^{14}c^{3}\right)7\mathrm{i}}{64\left(-b\right)^{15/4}c^{1/4}}\right)}{64\left(-b\right)^{15/4}c^{1/4}}+\frac{7\left(11Ac-3Bb\right)\left(\sqrt{x}\left(97140736A^{2}b^{9}c^{5}-52985856ABb^{10}c^{4}+7225344B^{2}b^{11}c^{3}\right)+\frac{\left(11Ac-3Bb\right)\left(80740352Ab^{13}c^{4}-22020096Bb^{14}c^{3}\right)7\mathrm{i}}{64\left(-b\right)^{15/4}c^{1/4}}\right)}{64\left(-b\right)^{15/4}c^{1/4}}}{\frac{\left(11Ac-3Bb\right)\left(\sqrt{x}\left(97140736A^{2}b^{9}c^{5}-52985856ABb^{10}c^{4}+7225344B^{2}b^{11}c^{3}\right)-\frac{\left(11Ac-3Bb\right)\left(80740352Ab^{13}c^{4}-22020096Bb^{14}c^{3}\right)7\mathrm{i}}{64\left(-b\right)^{15/4}c^{1/4}}\right)7\mathrm{i}}{64\left(-b\right)^{15/4}c^{1/4}}-\frac{\left(11Ac-3Bb\right)\left(\sqrt{x}\left(97140736A^{2}b^{9}c^{5}-52985856ABb^{10}c^{4}+7225344B^{2}b^{11}c^{3}\right)+\frac{\left(11Ac-3Bb\right)\left(80740352Ab^{13}c^{4}-22020096Bb^{14}c^{3}\right)7\mathrm{i}}{64\left(-b\right)^{15/4}c^{1/4}}\right)7\mathrm{i}}{64\left(-b\right)^{15/4}c^{1/4}}}\right)\left(11Ac-3Bb\right)}{32\left(-b\right)^{15/4}c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((2*A)/(3*b) + (11*x^2*(11*A*c - 3*B*b))/(48*b^2) + (7*c*x^4*(11*A*c - 3*
B*b))/(48*b^3))/(b^2*x^(3/2) + c^2*x^(11/2) + 2*b*c*x^(7/2)) - (atan((((11*
A*c - 3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 529858
56*A*B*b^10*c^4) - (7*(11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*B*b^
14*c^3))/(64*(-b)^(15/4)*c^(1/4)))*7i)/(64*(-b)^(15/4)*c^(1/4)) + ((11*A*c
- 3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A
*B*b^10*c^4) + (7*(11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*B*b^14*c
^3))/(64*(-b)^(15/4)*c^(1/4)))*7i)/(64*(-b)^(15/4)*c^(1/4)))/((7*(11*A*c -
3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A*B
*b^10*c^4) - (7*(11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*B*b^14*c^3
))/(64*(-b)^(15/4)*c^(1/4))))/(64*(-b)^(15/4)*c^(1/4)) - (7*(11*A*c - 3*B*b
)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A*B*b^10
*c^4) + (7*(11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*B*b^14*c^3))/(6
4*(-b)^(15/4)*c^(1/4))))/(64*(-b)^(15/4)*c^(1/4))))*(11*A*c - 3*B*b)*7i)/(3
2*(-b)^(15/4)*c^(1/4)) - (7*atan(((7*(11*A*c - 3*B*b)*(x^(1/2)*(97140736*A^
2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A*B*b^10*c^4) - ((11*A*c - 3*B*
b)*(80740352*A*b^13*c^4 - 22020096*B*b^14*c^3)*7i)/(64*(-b)^(15/4)*c^(1/4))
```

```
))/(64*(-b)^(15/4)*c^(1/4)) + (7*(11*A*c - 3*B*b)*(x^(1/2)*(97140736*A^2*b^
9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A*B*b^10*c^4) + ((11*A*c - 3*B*b)*(
80740352*A*b^13*c^4 - 22020096*B*b^14*c^3)*7i)/(64*(-b)^(15/4)*c^(1/4))))/(
64*(-b)^(15/4)*c^(1/4)))/(((11*A*c - 3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5
+ 7225344*B^2*b^11*c^3 - 52985856*A*B*b^10*c^4) - ((11*A*c - 3*B*b)*(807403
52*A*b^13*c^4 - 22020096*B*b^14*c^3)*7i)/(64*(-b)^(15/4)*c^(1/4)))*7i)/(64*
(-b)^(15/4)*c^(1/4)) - ((11*A*c - 3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7
225344*B^2*b^11*c^3 - 52985856*A*B*b^10*c^4) + ((11*A*c - 3*B*b)*(80740352*
A*b^13*c^4 - 22020096*B*b^14*c^3)*7i)/(64*(-b)^(15/4)*c^(1/4)))*7i)/(64*(-b
)^(15/4)*c^(1/4))))*(11*A*c - 3*B*b))/(32*(-b)^(15/4)*c^(1/4))
```

3.216 $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=343

$$\frac{9(5bB-13Ac)}{80b^3cx^{5/2}}-\frac{9(5bB-13Ac)}{16b^4\sqrt{x}}-\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}-\frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)}+\frac{9\sqrt[4]{c}\,(5bB-13Ac)\tan^{-1}\left(1-\right.}{32\sqrt{2}\,b^{17/4}}$$

```
[Out] 9/80*(-13*A*c+5*B*b)/b^3/c/x^(5/2)+1/4*(A*c-B*b)/b/c/x^(5/2)/(c*x^2+b)^2+1/
16*(13*A*c-5*B*b)/b^2/c/x^(5/2)/(c*x^2+b)+9/64*c^(1/4)*(-13*A*c+5*B*b)*arct
an(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)-9/64*c^(1/4)*(-13*A*
c+5*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)-9/128*c
^(1/4)*(-13*A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2)
)/b^(17/4)*2^(1/2)+9/128*c^(1/4)*(-13*A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/
4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(17/4)*2^(1/2)-9/16*(-13*A*c+5*B*b)/b^4/x^(1/
2)
```

Rubi [A]
time = 0.19, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = $\{1598, 468, 296, 331, 335, 303, 1176, 631, 210, 1179, 642\}$

$$\frac{9\sqrt[4]{c}\,(5bB-13Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{17/4}}-\frac{9\sqrt[4]{c}\,(5bB-13Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{17/4}}-\frac{9\sqrt[4]{c}\,(5bB-13Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{17/4}}+\frac{9\sqrt[4]{c}\,(5bB-13Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{17/4}}-\frac{9(5bB-13Ac)}{16b^4\sqrt{x}}+\frac{9(5bB-13Ac)}{80b^3cx^{5/2}}-\frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)}-\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (9*(5*b*B - 13*A*c))/(80*b^3*c*x^(5/2)) - (9*(5*b*B - 13*A*c))/(16*b^4*Sqrt
[x]) - (b*B - A*c)/(4*b*c*x^(5/2)*(b + c*x^2)^2) - (5*b*B - 13*A*c)/(16*b^2
*c*x^(5/2)*(b + c*x^2)) + (9*c^(1/4)*(5*b*B - 13*A*c)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) - (9*c^(1/4)*(5*b*B - 13*A*
c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) - (
9*c^(1/4)*(5*b*B - 13*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(64*Sqrt[2]*b^(17/4)) + (9*c^(1/4)*(5*b*B - 13*A*c)*Log[Sqrt[b]
 + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= \int \frac{A+Bx^2}{x^{7/2}\left(b+cx^2\right)^3}\,dx \\
&= -\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} + \frac{\left(-\frac{5bB}{2}+\frac{13Ac}{2}\right)\int \frac{1}{x^{7/2}\left(b+cx^2\right)^2}\,dx}{4bc} \\
&= -\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} - \frac{(9(5bB-13Ac))\int \frac{1}{x^{7/2}(b+cx^2)}\,dx}{32b^2c} \\
&= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} + \frac{(9(5bB-13Ac))\int \cdots}{32b^3} \\
&= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} - \cdots \\
&= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} - \cdots \\
&= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} + \cdots \\
&= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} - \cdots \\
&= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} - \cdots \\
&= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} + \cdots
\end{aligned}
$$

Mathematica [A]
time = 0.55, size = 209, normalized size = 0.61

$$
\frac{-\frac{4\sqrt[4]{b}\left(5bBx^2\left(32b^2+81bcx^2+45c^2x^4\right)+A\left(32b^3-416b^2cx^2-1053bc^2x^4-585c^3x^6\right)\right)}{x^{5/2}\left(b+cx^2\right)^2}+45\sqrt{2}\sqrt[4]{c}\,(5bB-13Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+45\sqrt{2}\sqrt[4]{c}\,(5bB-13Ac)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{320b^{17/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
[Out] ((-4*b^(1/4)*(5*b*B*x^2*(32*b^2 + 81*b*c*x^2 + 45*c^2*x^4) + A*(32*b^3 - 41
6*b^2*c*x^2 - 1053*b*c^2*x^4 - 585*c^3*x^6)))/(x^(5/2)*(b + c*x^2)^2) + 45*
```

```
Sqrt[2]*c^(1/4)*(5*b*B - 13*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x])] + 45*Sqrt[2]*c^(1/4)*(5*b*B - 13*A*c)*ArcTanh[(Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(320*b^(17/4))
```

Maple [A]

time = 0.42, size = 190, normalized size = 0.55

method	result
derivativedivides	$\frac{2c\left(\frac{\left(\frac{21}{32}Ac^2-\frac{13}{32}bBc\right)x^{\frac{7}{2}}+\frac{b(25Ac-17Bb)x^{\frac{3}{2}}}{32}}{\left(cx^2+b\right)^2}+\frac{\left(\frac{117Ac}{32}-\frac{45Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{b^4}$
default	$\frac{2c\left(\frac{\left(\frac{21}{32}Ac^2-\frac{13}{32}bBc\right)x^{\frac{7}{2}}+\frac{b(25Ac-17Bb)x^{\frac{3}{2}}}{32}}{\left(cx^2+b\right)^2}+\frac{\left(\frac{117Ac}{32}-\frac{45Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{b^4}$
risch	$-\frac{2\left(-15Acx^2+5bBx^2+Ab\right)}{5b^4x^{\frac{5}{2}}}+\frac{21c^3x^{\frac{7}{2}}A}{16b^4(cx^2+b)^2}-\frac{13c^2x^{\frac{7}{2}}B}{16b^3(cx^2+b)^2}+\frac{25c^2Ax^{\frac{3}{2}}}{16b^3(cx^2+b)^2}-\frac{17cBx^{\frac{3}{2}}}{16b^2(cx^2+b)^2}+\frac{117c\sqrt{2}\,A}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*c*(((21/32*A*c^2-13/32*b*B*c)*x^(7/2)+1/32*b*(25*A*c-17*B*b)*x^(3/2))
/(c*x^2+b)^2+1/8*(117/32*A*c-45/32*B*b)/c/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^
(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/
2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)-1)))-2/5*A/b^3/x^(5/2)-2*(-3*A*c+B*b)/b^4/x^(1/2)
```

Maxima [A]

time = 0.53, size = 285, normalized size = 0.83

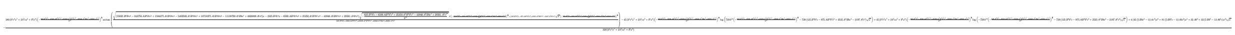
$$-\frac{45\,(5\,Bbc^2-13\,Ac^3)x^6+81\,(5\,Bb^2c-13\,Abc^2)x^4+32\,Ab^3+32\,(5\,Bb^3-13\,Ab^2c)x^2}{80\left(b^4c^2x^{\frac{13}{2}}+2\,b^5cx^{\frac{9}{2}}+b^6x^{\frac{5}{2}}\right)}-\frac{9\,(5\,Bbc-13\,Ac^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{128\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/80*(45*(5*B*b*c^2 - 13*A*c^3)*x^6 + 81*(5*B*b^2*c - 13*A*b*c^2)*x^4 + 32
*A*b^3 + 32*(5*B*b^3 - 13*A*b^2*c)*x^2)/(b^4*c^2*x^(13/2) + 2*b^5*c*x^(9/2)
 + b^6*x^(5/2)) - 9/128*(5*B*b*c - 13*A*c^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*
(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(
sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*
c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*
sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b
))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(
c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^4
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(255) = 510$.
time = 2.26, size = 1043, normalized size = 3.04



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(180*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-(625*B^4*b^4*c - 6500*A
*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b
^17)^(1/4)*arctan((sqrt((15625*B^6*b^6*c^2 - 243750*A*B^5*b^5*c^3 + 1584375
*A^2*B^4*b^4*c^4 - 5492500*A^3*B^3*b^3*c^5 + 10710375*A^4*B^2*b^2*c^6 - 111
38790*A^5*B*b*c^7 + 4826809*A^6*c^8)*x - (625*B^4*b^13*c - 6500*A*B^3*b^12*
c^2 + 25350*A^2*B^2*b^11*c^3 - 43940*A^3*B*b^10*c^4 + 28561*A^4*b^9*c^5)*sq
rt(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3
*B*b*c^4 + 28561*A^4*c^5)/b^17))*b^4*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2
+ 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4) +
(125*B^3*b^7*c - 975*A*B^2*b^6*c^2 + 2535*A^2*B*b^5*c^3 - 2197*A^3*b^4*c^4)
*sqrt(x)*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43
940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4))/(625*B^4*b^4*c - 6500*A*B^3*b
^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)) - 45*(
b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2
+ 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4)*lo
g(729*b^13*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 -
43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(3/4) - 729*(125*B^3*b^3*c - 975*A
*B^2*b^2*c^2 + 2535*A^2*B*b*c^3 - 2197*A^3*c^4)*sqrt(x)) + 45*(b^4*c^2*x^7
+ 2*b^5*c*x^5 + b^6*x^3)*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*
B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4)*log(-729*b^13*
(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B
*b*c^4 + 28561*A^4*c^5)/b^17)^(3/4) - 729*(125*B^3*b^3*c - 975*A*B^2*b^2*c^
2 + 2535*A^2*B*b*c^3 - 2197*A^3*c^4)*sqrt(x)) + 4*(45*(5*B*b*c^2 - 13*A*c^3
)*x^6 + 81*(5*B*b^2*c - 13*A*b*c^2)*x^4 + 32*A*b^3 + 32*(5*B*b^3 - 13*A*b^2
*c)*x^2)*sqrt(x))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 326, normalized size = 0.95

$$-\frac{9\sqrt{2}\left(5\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^5c^2}-\frac{9\sqrt{2}\left(5\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64b^5c^2}+\frac{9\sqrt{2}\left(5\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128b^5c^2}-\frac{9\sqrt{2}\left(5\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128b^5c^2}-\frac{13Bbc^2x^{\frac{7}{2}}-21Ac^3x^{\frac{7}{2}}+17Bb^2cx^{\frac{3}{2}}-25Abc^2x^{\frac{3}{2}}}{16\left(cx^2+b\right)^2b^4}-\frac{2\left(5Bbx^2-15Acx^2+Ab\right)}{5b^4x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -9/64*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(
2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^5*c^2) - 9/64*sqrt(2)*
(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^5*c^2) + 9/128*sqrt(2)*(5*(b*c^3)^(
3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt
(b/c))/(b^5*c^2) - 9/128*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*
c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c^2) - 1/16*(13*B
*b*c^2*x^(7/2) - 21*A*c^3*x^(7/2) + 17*B*b^2*c*x^(3/2) - 25*A*b*c^2*x^(3/2)
)/((c*x^2 + b)^2*b^4) - 2/5*(5*B*b*x^2 - 15*A*c*x^2 + A*b)/(b^4*x^(5/2))
```

Mupad [B]
time = 0.19, size = 152, normalized size = 0.44

$$\frac{\frac{2\,x^2\,(13\,A\,c-5\,B\,b)}{5\,b^2}-\frac{2\,A}{5\,b}+\frac{9\,c^2\,x^6\,(13\,A\,c-5\,B\,b)}{16\,b^4}+\frac{81\,c\,x^4\,(13\,A\,c-5\,B\,b)}{80\,b^3}}{b^2\,x^{5/2}+c^2\,x^{13/2}+2\,b\,c\,x^{9/2}}+\frac{9\,{(-c)}^{1/4}\,\mathrm{atan}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(13\,A\,c-5\,B\,b)}{32\,b^{17/4}}-\frac{9\,{(-c)}^{1/4}\,\mathrm{atanh}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(13\,A\,c-5\,B\,b)}{32\,b^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((2*x^2*(13*A*c - 5*B*b))/(5*b^2) - (2*A)/(5*b) + (9*c^2*x^6*(13*A*c - 5*B*
b))/(16*b^4) + (81*c*x^4*(13*A*c - 5*B*b))/(80*b^3))/(b^2*x^(5/2) + c^2*x^(
13/2) + 2*b*c*x^(9/2)) + (9*(-c)^(1/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(
13*A*c - 5*B*b))/(32*b^(17/4)) - (9*(-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))/b
^(1/4))*(13*A*c - 5*B*b))/(32*b^(17/4))
```

3.217 $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=343

$$\frac{11(7bB-15Ac)}{112b^3cx^{7/2}}-\frac{11(7bB-15Ac)}{48b^4x^{3/2}}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}-\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}+\frac{11c^{3/4}(7bB-15Ac)\tan^{-1}\Big(}{32\sqrt{2}\,b^{19/4}}$$

```
[Out] 11/112*(-15*A*c+7*B*b)/b^3/c/x^(7/2)-11/48*(-15*A*c+7*B*b)/b^4/x^(3/2)+1/4*
(A*c-B*b)/b/c/x^(7/2)/(c*x^2+b)^2+1/16*(15*A*c-7*B*b)/b^2/c/x^(7/2)/(c*x^2+
b)+11/64*c^(3/4)*(-15*A*c+7*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/
b^(19/4)*2^(1/2)-11/64*c^(3/4)*(-15*A*c+7*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(
1/2)/b^(1/4))/b^(19/4)*2^(1/2)+11/128*c^(3/4)*(-15*A*c+7*B*b)*ln(b^(1/2)+x*
c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(19/4)*2^(1/2)-11/128*c^(3/4)*(-
15*A*c+7*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(19/4
)*2^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1598, 468, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{11c^{3/4}(7bB-15Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{19/4}}-\frac{11c^{3/4}(7bB-15Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{19/4}}+\frac{11c^{3/4}(7bB-15Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{19/4}}-\frac{11c^{3/4}(7bB-15Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{19/4}}-\frac{11(7bB-15Ac)}{48b^4x^{3/2}}+\frac{11(7bB-15Ac)}{112b^3cx^{7/2}}-\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (11*(7*b*B - 15*A*c))/(112*b^3*c*x^(7/2)) - (11*(7*b*B - 15*A*c))/(48*b^4*x
^(3/2)) - (b*B - A*c)/(4*b*c*x^(7/2)*(b + c*x^2)^2) - (7*b*B - 15*A*c)/(16*
b^2*c*x^(7/2)*(b + c*x^2)) + (11*c^(3/4)*(7*b*B - 15*A*c)*ArcTan[1 - (Sqrt[
2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)) - (11*c^(3/4)*(7*b*B -
15*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)
) + (11*c^(3/4)*(7*b*B - 15*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19/4)) - (11*c^(3/4)*(7*b*B - 15*A*c)*Log[
Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19/4
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \int \frac{A+Bx^2}{x^{9/2}\,(b+cx^2)^3}\,dx$$

$$= -\frac{bB-Ac}{4bcx^{7/2}\,(b+cx^2)^2} + \frac{\left(-\frac{7bB}{2}+\frac{15Ac}{2}\right)\int \frac{1}{x^{9/2}(b+cx^2)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bcx^{7/2}\,(b+cx^2)^2} - \frac{7bB-15Ac}{16b^2cx^{7/2}\,(b+cx^2)} - \frac{(11(7bB-15Ac))\int \frac{1}{x^{9/2}(b+cx^2)}\,dx}{32b^2c}$$

$$= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{bB-Ac}{4bcx^{7/2}\,(b+cx^2)^2} - \frac{7bB-15Ac}{16b^2cx^{7/2}\,(b+cx^2)} + \frac{(11(7bB-15Ac))\int}{32b^3}$$

$$= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB-15Ac)}{48b^4x^{3/2}} - \frac{bB-Ac}{4bcx^{7/2}\,(b+cx^2)^2} - \frac{7bB-15Ac}{16b^2cx^{7/2}\,(b+cx^2)} - \frac{(1}{}$$

$$= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB-15Ac)}{48b^4x^{3/2}} - \frac{bB-Ac}{4bcx^{7/2}\,(b+cx^2)^2} - \frac{7bB-15Ac}{16b^2cx^{7/2}\,(b+cx^2)} - \frac{(1}{}$$

$$= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB-15Ac)}{48b^4x^{3/2}} - \frac{bB-Ac}{4bcx^{7/2}\,(b+cx^2)^2} - \frac{7bB-15Ac}{16b^2cx^{7/2}\,(b+cx^2)} - \frac{(1}{}$$

$$= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB-15Ac)}{48b^4x^{3/2}} - \frac{bB-Ac}{4bcx^{7/2}\,(b+cx^2)^2} - \frac{7bB-15Ac}{16b^2cx^{7/2}\,(b+cx^2)} - \frac{(1}{}$$

$$= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB-15Ac)}{48b^4x^{3/2}} - \frac{bB-Ac}{4bcx^{7/2}\,(b+cx^2)^2} - \frac{7bB-15Ac}{16b^2cx^{7/2}\,(b+cx^2)} + \frac{11}{}$$

$$= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB-15Ac)}{48b^4x^{3/2}} - \frac{bB-Ac}{4bcx^{7/2}\,(b+cx^2)^2} - \frac{7bB-15Ac}{16b^2cx^{7/2}\,(b+cx^2)} + \frac{11}{}$$

<u>Mathematica</u> [A]
time = 0.56, size = 210, normalized size = 0.61

$$\frac{-\frac{4b^{3/4}\left(7bBx^2\left(32b^2+121bcx^2+77c^2x^4\right)+3A\left(32b^3-160b^2cx^2-605bc^2x^4-385c^3x^6\right)\right)}{x^{7/2}(b+cx^2)^2}+231\sqrt{2}\,c^{3/4}(7bB-15Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)+231\sqrt{2}\,c^{3/4}(-7bB+15Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{1344b^{19/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(3/4)*(7*b*B*x^2*(32*b^2 + 121*b*c*x^2 + 77*c^2*x^4) + 3*A*(32*b^3 -
 160*b^2*c*x^2 - 605*b*c^2*x^4 - 385*c^3*x^6)))/(x^(7/2)*(b + c*x^2)^2) + 2
```

```
31*Sqrt[2]*c^(3/4)*(7*b*B - 15*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x])] + 231*Sqrt[2]*c^(3/4)*(-7*b*B + 15*A*c)*ArcTanh[(S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(1344*b^(19/4))
```

<u>Maple</u> [A]

time = 0.41, size = 190, normalized size = 0.55

method	result
derivativedivides	$\frac{2c\left(\frac{\left(\frac{23}{32}Ac^2-\frac{15}{32}bBc\right)x^{\frac{5}{2}}+\frac{b(27Ac-19Bb)\sqrt{x}}{32}}{\left(cx^2+b\right)^2}+\frac{11(15Ac-7Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\cdots\right)\right)}{256b}\cdots\right)}{b^4}$
default	$\frac{2c\left(\frac{\left(\frac{23}{32}Ac^2-\frac{15}{32}bBc\right)x^{\frac{5}{2}}+\frac{b(27Ac-19Bb)\sqrt{x}}{32}}{\left(cx^2+b\right)^2}+\frac{11(15Ac-7Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\cdots\right)\right)}{256b}\cdots\right)}{b^4}$
risch	$-\frac{2(-21Acx^2+7bBx^2+3Ab)}{21b^4x^{\frac{7}{2}}}+\frac{23c^3x^{\frac{5}{2}}A}{16b^4(cx^2+b)^2}-\frac{15c^2x^{\frac{5}{2}}B}{16b^3(cx^2+b)^2}+\frac{27c^2A\sqrt{x}}{16b^3(cx^2+b)^2}-\frac{19cB\sqrt{x}}{16b^2(cx^2+b)^2}+\frac{165c^2\left(\frac{b}{c}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*c*(((23/32*A*c^2-15/32*b*B*c)*x^(5/2)+1/32*b*(27*A*c-19*B*b)*x^(1/2))
/(c*x^2+b)^2+11/256*(15*A*c-7*B*b)*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)^(1/4)
*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+
2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)-1)))-2/7*A/b^3/x^(7/2)-2/3*(-3*A*c+B*b)/b^4/x^(3/2)
```

<u>Maxima</u> [A]

time = 0.54, size = 321, normalized size = 0.94

$$-\frac{77\,(7\,Bbc^2-15\,Ac^3)x^6+121\,(7\,Bb^2c-15\,Abc^2)x^4+96\,Ab^3+32\,(7\,Bb^3-15\,Ab^2c)x^2}{336\left(b^4c^2x^{\frac{15}{2}}+2\,b^5cx^{\frac{11}{2}}+b^6x^{\frac{7}{2}}\right)}-\frac{11\left(\frac{2\sqrt{2}\,(7\,Bbc-15\,Ac^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\,(7\,Bbc-15\,Ac^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\,(7\,Bbc-15\,Ac^2)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,(7\,Bbc-15\,Ac^2)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{128\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/336*(77*(7*B*b*c^2 - 15*A*c^3)*x^6 + 121*(7*B*b^2*c - 15*A*b*c^2)*x^4 +
96*A*b^3 + 32*(7*B*b^3 - 15*A*b^2*c)*x^2)/(b^4*c^2*x^(15/2) + 2*b^5*c*x^(11
/2) + b^6*x^(7/2)) - 11/128*(2*sqrt(2)*(7*B*b*c - 15*A*c^2)*arctan(1/2*sqrt
(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(s
qrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(7*B*b*c - 15*A*c^2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c))
)/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(7*B*b*c - 15*A*c^2)*log(sqrt(2
)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2
)*(7*B*b*c - 15*A*c^2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + s
qrt(b))/(b^(3/4)*c^(1/4)))/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 864 vs. $2(255) = 510$.
time = 2.76, size = 864, normalized size = 2.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/1344*(924*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-(2401*B^4*b^4*c^3 - 205
80*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^
7)/b^19)^(1/4)*arctan((sqrt(b^10*sqrt(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*
c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19) + (4
9*B^2*b^2*c^2 - 210*A*B*b*c^3 + 225*A^2*c^4)*x)*b^14*(-(2401*B^4*b^4*c^3 -
20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4
*c^7)/b^19)^(3/4) + (7*B*b^15*c - 15*A*b^14*c^2)*sqrt(x)*(-(2401*B^4*b^4*c^
3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625
*A^4*c^7)/b^19)^(3/4))/(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*
B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)) + 231*(b^4*c^2*x^8 + 2*b^
5*c*x^6 + b^6*x^4)*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^
2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1/4)*log(11*b^5*(-(24
01*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*
b*c^6 + 50625*A^4*c^7)/b^19)^(1/4) - 11*(7*B*b*c - 15*A*c^2)*sqrt(x)) - 231
*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^
3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1
/4)*log(-11*b^5*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b
^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1/4) - 11*(7*B*b*c - 15*
A*c^2)*sqrt(x)) - 4*(77*(7*B*b*c^2 - 15*A*c^3)*x^6 + 121*(7*B*b^2*c - 15*A*
b*c^2)*x^4 + 96*A*b^3 + 32*(7*B*b^3 - 15*A*b^2*c)*x^2)*sqrt(x))/(b^4*c^2*x^
8 + 2*b^5*c*x^6 + b^6*x^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [A]
time = 0.62, size = 315, normalized size = 0.92

$$-\frac{11\,\sqrt{2}\left(7\,(bc^3)^{\frac{1}{4}}\,Bb-15\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5}-\frac{11\,\sqrt{2}\left(7\,(bc^3)^{\frac{1}{4}}\,Bb-15\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5}-\frac{11\,\sqrt{2}\left(7\,(bc^3)^{\frac{1}{4}}\,Bb-15\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5}+\frac{11\,\sqrt{2}\left(7\,(bc^3)^{\frac{1}{4}}\,Bb-15\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5}-\frac{15\,Bbc^2x^{\frac{5}{2}}-23\,Ac^3x^{\frac{5}{2}}+19\,Bb^2c\sqrt{x}-27\,Abc^2\sqrt{x}}{16\,(cx^2+b)^2b^4}-\frac{2\,(7\,Bbx^2-21\,Acx^2+3\,Ab)}{21\,b^4x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -11/64*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 15*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt
(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^5 - 11/64*sqrt(2)*(7*(
b*c^3)^(1/4)*B*b - 15*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)
^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^5 - 11/128*sqrt(2)*(7*(b*c^3)^(1/4)*B*b
- 15*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^
5 + 11/128*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 15*(b*c^3)^(1/4)*A*c)*log(-sqrt(2
)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^5 - 1/16*(15*B*b*c^2*x^(5/2) - 23*
A*c^3*x^(5/2) + 19*B*b^2*c*sqrt(x) - 27*A*b*c^2*sqrt(x))/((c*x^2 + b)^2*b^4
) - 2/21*(7*B*b*x^2 - 21*A*c*x^2 + 3*A*b)/(b^4*x^(7/2))
```

<u>Mupad</u> [B]
time = 0.45, size = 626, normalized size = 1.83

$$\frac{\frac{2\,x^2\,(15\,A\,c-7\,B\,b)}{21\,b^2}-\frac{2\,A}{7\,b}+\frac{11\,c^2\,x^6\,(15\,A\,c-7\,B\,b)}{48\,b^4}+\frac{121\,c\,x^4\,(15\,A\,c-7\,B\,b)}{336\,b^3}}{b^2\,x^{7/2}+c^2\,x^{15/2}+2\,b\,c\,x^{11/2}}+\frac{11\,(-c)^{3/4}\,\mathrm{atan}\left(\frac{\frac{11\,(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\left(\sqrt{x}\,\left(446054400\,A^2\,b^{12}\,c^7-416317440\,A\,B\,b^{13}\,c^6+97140736\,B^2\,b^{14}\,c^5\right)-\frac{(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\left(173015040\,A\,b^{17}\,c^5-80740352\,B\,b^{18}\,c^4\right)\,11\mathrm{i}}{64\,b^{19/4}}\right)}{64\,b^{19/4}}+\frac{11\,(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\left(\sqrt{x}\,\left(446054400\,A^2\,b^{12}\,c^7-416317440\,A\,B\,b^{13}\,c^6+97140736\,B^2\,b^{14}\,c^5\right)+\frac{(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\left(173015040\,A\,b^{17}\,c^5-80740352\,B\,b^{18}\,c^4\right)\,11\mathrm{i}}{64\,b^{19/4}}\right)}{64\,b^{19/4}}}{\frac{(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\left(\sqrt{x}\,\left(446054400\,A^2\,b^{12}\,c^7-416317440\,A\,B\,b^{13}\,c^6+97140736\,B^2\,b^{14}\,c^5\right)-\frac{(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\left(173015040\,A\,b^{17}\,c^5-80740352\,B\,b^{18}\,c^4\right)\,11\mathrm{i}}{64\,b^{19/4}}\right)\,11\mathrm{i}}{64\,b^{19/4}}-\frac{(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\left(\sqrt{x}\,\left(446054400\,A^2\,b^{12}\,c^7-416317440\,A\,B\,b^{13}\,c^6+97140736\,B^2\,b^{14}\,c^5\right)+\frac{(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\left(173015040\,A\,b^{17}\,c^5-80740352\,B\,b^{18}\,c^4\right)\,11\mathrm{i}}{64\,b^{19/4}}\right)\,11\mathrm{i}}{64\,b^{19/4}}}\right)\,(15\,A\,c-7\,B\,b)}{32\,b^{19/4}}-\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{A^3\,c^8\,\sqrt{x}\,3375\mathrm{i}-B^3\,b^3\,c^5\,\sqrt{x}\,343\mathrm{i}-A^2\,B\,b\,c^7\,\sqrt{x}\,4725\mathrm{i}+A\,B^2\,b^2\,c^6\,\sqrt{x}\,2205\mathrm{i}}{b^{1/4}\,(-c)^{19/4}\,\left(c\,\left(c\,\left(3375\,A^3\,c-4725\,A^2\,B\,b\right)+2205\,A\,B^2\,b^2\right)-343\,B^3\,b^3\right)}\right)\,(15\,A\,c-7\,B\,b)\,11\mathrm{i}}{32\,b^{19/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((2*x^2*(15*A*c - 7*B*b))/(21*b^2) - (2*A)/(7*b) + (11*c^2*x^6*(15*A*c - 7*
B*b))/(48*b^4) + (121*c*x^4*(15*A*c - 7*B*b))/(336*b^3))/(b^2*x^(7/2) + c^2
*x^(15/2) + 2*b*c*x^(11/2)) + (11*(-c)^(3/4)*atan(((11*(-c)^(3/4)*(15*A*c -
 7*B*b)*(x^(1/2)*(446054400*A^2*b^12*c^7 + 97140736*B^2*b^14*c^5 - 41631744
0*A*B*b^13*c^6) - ((-c)^(3/4)*(15*A*c - 7*B*b)*(173015040*A*b^17*c^5 - 8074
0352*B*b^18*c^4)*11i)/(64*b^(19/4))))/(64*b^(19/4)) + (11*(-c)^(3/4)*(15*A*
c - 7*B*b)*(x^(1/2)*(446054400*A^2*b^12*c^7 + 97140736*B^2*b^14*c^5 - 41631
7440*A*B*b^13*c^6) + ((-c)^(3/4)*(15*A*c - 7*B*b)*(173015040*A*b^17*c^5 - 8
0740352*B*b^18*c^4)*11i)/(64*b^(19/4))))/(64*b^(19/4)))/(((-c)^(3/4)*(15*A*
c - 7*B*b)*(x^(1/2)*(446054400*A^2*b^12*c^7 + 97140736*B^2*b^14*c^5 - 41631
7440*A*B*b^13*c^6) - ((-c)^(3/4)*(15*A*c - 7*B*b)*(173015040*A*b^17*c^5 - 8
0740352*B*b^18*c^4)*11i)/(64*b^(19/4)))*11i)/(64*b^(19/4)) - ((-c)^(3/4)*(1
5*A*c - 7*B*b)*(x^(1/2)*(446054400*A^2*b^12*c^7 + 97140736*B^2*b^14*c^5 - 4
16317440*A*B*b^13*c^6) + ((-c)^(3/4)*(15*A*c - 7*B*b)*(173015040*A*b^17*c^5
```

```
 - 80740352*B*b^18*c^4)*11i)/(64*b^(19/4)))*11i)/(64*b^(19/4))))*(15*A*c -
7*B*b))/(32*b^(19/4)) - ((-c)^(3/4)*atan((A^3*c^8*x^(1/2)*3375i - B^3*b^3*c
^5*x^(1/2)*343i - A^2*B*b*c^7*x^(1/2)*4725i + A*B^2*b^2*c^6*x^(1/2)*2205i)/
(b^(1/4)*(-c)^(19/4)*(c*(c*(3375*A^3*c - 4725*A^2*B*b) + 2205*A*B^2*b^2) -
343*B^3*b^3)))*(15*A*c - 7*B*b)*11i)/(32*b^(19/4))
```

3.218 $\int \frac{\sqrt{x}\,(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=365

$$\frac{13(9bB-17Ac)}{144b^3cx^{9/2}}-\frac{13(9bB-17Ac)}{80b^4x^{5/2}}+\frac{13c(9bB-17Ac)}{16b^5\sqrt{x}}-\frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2}-\frac{9bB-17Ac}{16b^2cx^{9/2}\left(b+cx^2\right)}-\frac{13c^{5/4}(9b}{}$$

```
[Out] 13/144*(-17*A*c+9*B*b)/b^3/c/x^(9/2)-13/80*(-17*A*c+9*B*b)/b^4/x^(5/2)+1/4*
(A*c-B*b)/b/c/x^(9/2)/(c*x^2+b)^2+1/16*(17*A*c-9*B*b)/b^2/c/x^(9/2)/(c*x^2+
b)-13/64*c^(5/4)*(-17*A*c+9*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/
b^(21/4)*2^(1/2)+13/64*c^(5/4)*(-17*A*c+9*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(
1/2)/b^(1/4))/b^(21/4)*2^(1/2)+13/128*c^(5/4)*(-17*A*c+9*B*b)*ln(b^(1/2)+x*
c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(21/4)*2^(1/2)-13/128*c^(5/4)*(-
17*A*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(21/4
)*2^(1/2)+13/16*c*(-17*A*c+9*B*b)/b^5/x^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1598, 468, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{13c^{5/4}(9bB-17Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{21/4}}+\frac{13c^{5/4}(9bB-17Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{21/4}}+\frac{13c^{5/4}(9bB-17Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{21/4}}-\frac{13c^{5/4}(9bB-17Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{21/4}}+\frac{13c(9bB-17Ac)}{16b^5\sqrt{x}}-\frac{13(9bB-17Ac)}{80b^4x^{5/2}}+\frac{13(9bB-17Ac)}{144b^3cx^{9/2}}-\frac{9bB-17Ac}{16b^2cx^{9/2}\left(b+cx^2\right)}-\frac{bB-Ac}{4bcx^{9/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (13*(9*b*B - 17*A*c))/(144*b^3*c*x^(9/2)) - (13*(9*b*B - 17*A*c))/(80*b^4*x
^(5/2)) + (13*c*(9*b*B - 17*A*c))/(16*b^5*Sqrt[x]) - (b*B - A*c)/(4*b*c*x^(
9/2)*(b + c*x^2)^2) - (9*b*B - 17*A*c)/(16*b^2*c*x^(9/2)*(b + c*x^2)) - (13
*c^(5/4)*(9*b*B - 17*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*b^(21/4)) + (13*c^(5/4)*(9*b*B - 17*A*c)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(21/4)) + (13*c^(5/4)*(9*b*B - 17*A*c)
*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^
(21/4)) - (13*c^(5/4)*(9*b*B - 17*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}\,(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \int \frac{A+Bx^2}{x^{11/2}\,(b+cx^2)^3}\,dx$$

$$= -\frac{bB-Ac}{4bcx^{9/2}\,(b+cx^2)^2} + \frac{\left(-\frac{9bB}{2}+\frac{17Ac}{2}\right)\int \frac{1}{x^{11/2}(b+cx^2)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bcx^{9/2}\,(b+cx^2)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}\,(b+cx^2)} - \frac{(13(9bB-17Ac))\int \frac{1}{x^{11/2}(b+cx^2)}\,dx}{32b^2c}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{bB-Ac}{4bcx^{9/2}\,(b+cx^2)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}\,(b+cx^2)} + \frac{(13(9bB-17Ac))\int}{32b^3}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} - \frac{bB-Ac}{4bcx^{9/2}\,(b+cx^2)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}\,(b+cx^2)} -$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\,(b+cx^2)^2} -$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\,(b+cx^2)^2} -$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\,(b+cx^2)^2} -$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\,(b+cx^2)^2} -$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\,(b+cx^2)^2} -$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}\,(b+cx^2)^2} -$$

Mathematica [A]
time = 0.59, size = 231, normalized size = 0.63

$$\frac{-\frac{4\sqrt[4]{b}\,(-9bBx^2(-32b^3+416b^2cx^2+1053bc^2x^4+585c^3x^6)+A(160b^4-544b^3cx^2+7072b^2c^2x^4+17901bc^3x^6+9945c^4x^8))}{x^{9/2}(b+cx^2)^2} + 585\sqrt{2}\,c^{5/4}(-9bB+17Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right) + 585\sqrt{2}\,c^{5/4}(-9bB+17Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{2880b^{21/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(1/4)*(-9*b*B*x^2*(-32*b^3 + 416*b^2*c*x^2 + 1053*b*c^2*x^4 + 585*c^
3*x^6) + A*(160*b^4 - 544*b^3*c*x^2 + 7072*b^2*c^2*x^4 + 17901*b*c^3*x^6 +
9945*c^4*x^8)))/(x^(9/2)*(b + c*x^2)^2) + 585*Sqrt[2]*c^(5/4)*(-9*b*B + 17*
A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 585*
Sqrt[2]*c^(5/4)*(-9*b*B + 17*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])
/(Sqrt[b] + Sqrt[c]*x)])/(2880*b^(21/4))
```

Maple [A]
time = 0.42, size = 210, normalized size = 0.58

method	result
derivativedivides	$-\frac{2A}{9b^{3}x^{\frac{9}{2}}}-\frac{2(-3Ac+Bb)}{5b^{4}x^{\frac{5}{2}}}-\frac{6c(2Ac-Bb)}{b^{5}\sqrt{x}}-\frac{2c^{2}\left(\frac{\frac{c(29Ac-21Bb)x^{\frac{7}{2}}}{32}+\left(\frac{33}{32}Abc-\frac{25}{32}b^{2}B\right)x^{\frac{3}{2}}}{\left(cx^{2}+b\right)^{2}}+\frac{\left(\frac{221Ac}{32}-\frac{117Bb}{32}\right)\sqrt{2}\,\left(\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{2A}{9b^{3}x^{\frac{9}{2}}}-\frac{2(-3Ac+Bb)}{5b^{4}x^{\frac{5}{2}}}-\frac{6c(2Ac-Bb)}{b^{5}\sqrt{x}}-\frac{2c^{2}\left(\frac{\frac{c(29Ac-21Bb)x^{\frac{7}{2}}}{32}+\left(\frac{33}{32}Abc-\frac{25}{32}b^{2}B\right)x^{\frac{3}{2}}}{\left(cx^{2}+b\right)^{2}}+\frac{\left(\frac{221Ac}{32}-\frac{117Bb}{32}\right)\sqrt{2}\,\left(\cdots\right.}{\cdots}\right.}{\cdots}$
risch	$-\frac{2\left(270Ac^{2}x^{4}-135x^{4}bBc-27Abcx^{2}+9b^{2}Bx^{2}+5b^{2}A\right)}{45b^{5}x^{\frac{9}{2}}}-\frac{29c^{4}Ax^{\frac{7}{2}}}{16b^{5}(cx^{2}+b)^{2}}+\frac{21c^{3}Bx^{\frac{7}{2}}}{16b^{4}(cx^{2}+b)^{2}}-\frac{33c^{3}x^{\frac{3}{2}}A}{16b^{4}(cx^{2}+b)^{2}}+\frac{\cdots}{16\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*A/b^3/x^(9/2)-2/5*(-3*A*c+B*b)/b^4/x^(5/2)-6*c*(2*A*c-B*b)/b^5/x^(1/2)
-2/b^5*c^2*((1/32*c*(29*A*c-21*B*b)*x^(7/2)+(33/32*A*b*c-25/32*b^2*B)*x^(3/
2))/(c*x^2+b)^2+1/8*(221/32*A*c-117/32*B*b)/c/(b/c)^(1/4)*2^(1/2)*(ln((x-(b
/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)
^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/
4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.55, size = 311, normalized size = 0.85

$$\frac{585\,(9\,Bbc^3-17\,Ac^4)x^8+1053\,(9\,Bb^2c^2-17\,Abc^3)x^6-160\,Ab^4+416\,(9\,Bb^3c-17\,Ab^2c^2)x^4-32\,(9\,Bb^4-17\,Ab^3c)x^2}{720\left(b^5c^2x^{\frac{17}{2}}+2\,b^6cx^{\frac{13}{2}}+b^7x^{\frac{9}{2}}\right)}+\frac{13\,(9\,Bbc^2-17\,Ac^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{128\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/720*(585*(9*B*b*c^3 - 17*A*c^4)*x^8 + 1053*(9*B*b^2*c^2 - 17*A*b*c^3)*x^6
 - 160*A*b^4 + 416*(9*B*b^3*c - 17*A*b^2*c^2)*x^4 - 32*(9*B*b^4 - 17*A*b^3*
c)*x^2)/(b^5*c^2*x^(17/2) + 2*b^6*c*x^(13/2) + b^7*x^(9/2)) + 13/128*(9*B*b
*c^2 - 17*A*c^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2
*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))
/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(
2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(
2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(
3/4)))/b^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1093 vs. 2(273) = 546.
time = 2.76, size = 1093, normalized size = 2.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2880*(2340*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-(6561*B^4*b^4*c^5 - 49
572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4
*c^9)/b^21)^(1/4)*arctan((sqrt((531441*B^6*b^6*c^8 - 6022998*A*B^5*b^5*c^9
+ 28441935*A^2*B^4*b^4*c^10 - 71631540*A^3*B^3*b^3*c^11 + 101478015*A^4*B^2
*b^2*c^12 - 76672278*A^5*B*b*c^13 + 24137569*A^6*c^14)*x - (6561*B^4*b^15*c
^5 - 49572*A*B^3*b^14*c^6 + 140454*A^2*B^2*b^13*c^7 - 176868*A^3*B*b^12*c^8
 + 83521*A^4*b^11*c^9)*sqrt(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 1404
54*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21))*b^5*(-(6561
*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*
b*c^8 + 83521*A^4*c^9)/b^21)^(1/4) + (729*B^3*b^8*c^4 - 4131*A*B^2*b^7*c^5
+ 7803*A^2*B*b^6*c^6 - 4913*A^3*b^5*c^7)*sqrt(x)*(-(6561*B^4*b^4*c^5 - 4957
2*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c
^9)/b^21)^(1/4))/(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b
^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)) - 585*(b^5*c^2*x^9 + 2*b^6*c*
x^7 + b^7*x^5)*(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b
^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(1/4)*log(2197*b^16*(-(6
561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3
```

```
*B*b*c^8 + 83521*A^4*c^9)/b^21)^(3/4) - 2197*(729*B^3*b^3*c^4 - 4131*A*B^2*
b^2*c^5 + 7803*A^2*B*b*c^6 - 4913*A^3*c^7)*sqrt(x)) + 585*(b^5*c^2*x^9 + 2*
b^6*c*x^7 + b^7*x^5)*(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2
*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(1/4)*log(-2197*b^
16*(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176
868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(3/4) - 2197*(729*B^3*b^3*c^4 - 4131
*A*B^2*b^2*c^5 + 7803*A^2*B*b*c^6 - 4913*A^3*c^7)*sqrt(x)) + 4*(585*(9*B*b*
c^3 - 17*A*c^4)*x^8 + 1053*(9*B*b^2*c^2 - 17*A*b*c^3)*x^6 - 160*A*b^4 + 416
*(9*B*b^3*c - 17*A*b^2*c^2)*x^4 - 32*(9*B*b^4 - 17*A*b^3*c)*x^2)*sqrt(x))/(
b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.55, size = 351, normalized size = 0.96

$$\frac{13\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}\,Bb-17\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6c}+\frac{13\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}\,Bb-17\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6c}-\frac{13\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}\,Bb-17\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6c}+\frac{13\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}\,Bb-17\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6c}+\frac{21\,Bbc^3x^{\frac{7}{2}}-29\,Ac^4x^{\frac{7}{2}}+25\,Bb^2c^2x^{\frac{3}{2}}-33\,Abc^3x^{\frac{3}{2}}}{16\,(cx^2+b)^2b^5}+\frac{2\,(135\,Bbcx^4-270\,Ac^2x^4-9\,Bb^2x^2+27\,Abcx^2-5\,Ab^2)}{45\,b^5x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 13/64*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 17*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(
2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^6*c) + 13/64*sqrt(2)*(
9*(b*c^3)^(3/4)*B*b - 17*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b
/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^6*c) - 13/128*sqrt(2)*(9*(b*c^3)^(3/
4)*B*b - 17*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b
/c))/(b^6*c) + 13/128*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 17*(b*c^3)^(3/4)*A*c)*
log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^6*c) + 1/16*(21*B*b*c^
3*x^(7/2) - 29*A*c^4*x^(7/2) + 25*B*b^2*c^2*x^(3/2) - 33*A*b*c^3*x^(3/2))/(
(c*x^2 + b)^2*b^5) + 2/45*(135*B*b*c*x^4 - 270*A*c^2*x^4 - 9*B*b^2*x^2 + 27
*A*b*c*x^2 - 5*A*b^2)/(b^5*x^(9/2))
```

Mupad [**B**]
time = 0.29, size = 173, normalized size = 0.47

$$\frac{13\,{(-c)}^{5/4}\,\mathrm{atan}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,\left(17\,A\,c-9\,B\,b\right)}{32\,b^{21/4}}-\frac{\frac{2\,A}{9\,b}-\frac{2\,x^2\,\left(17\,A\,c-9\,B\,b\right)}{45\,b^2}+\frac{117\,c^2\,x^6\,\left(17\,A\,c-9\,B\,b\right)}{80\,b^4}+\frac{13\,c^3\,x^8\,\left(17\,A\,c-9\,B\,b\right)}{16\,b^5}+\frac{26\,c\,x^4\,\left(17\,A\,c-9\,B\,b\right)}{45\,b^3}}{b^2\,x^{9/2}+c^2\,x^{17/2}+2\,b\,c\,x^{13/2}}-\frac{13\,{(-c)}^{5/4}\,\mathrm{atanh}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,\left(17\,A\,c-9\,B\,b\right)}{32\,b^{21/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (13*(-c)^(5/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(17*A*c - 9*B*b))/(32*b^(
21/4)) - ((2*A)/(9*b) - (2*x^2*(17*A*c - 9*B*b))/(45*b^2) + (117*c^2*x^6*(1
7*A*c - 9*B*b))/(80*b^4) + (13*c^3*x^8*(17*A*c - 9*B*b))/(16*b^5) + (26*c*x
^4*(17*A*c - 9*B*b))/(45*b^3))/(b^2*x^(9/2) + c^2*x^(17/2) + 2*b*c*x^(13/2)
) - (13*(-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4))*(17*A*c - 9*B*b))/(3
2*b^(21/4))
```

3.219 $\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=365

$$\frac{15(11bB-19Ac)}{176b^3cx^{11/2}}-\frac{15(11bB-19Ac)}{112b^4x^{7/2}}+\frac{5c(11bB-19Ac)}{16b^5x^{3/2}}-\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}-\frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)}-\frac{15c^{7/}}{}$$

```
[Out] 15/176*(-19*A*c+11*B*b)/b^3/c/x^(11/2)-15/112*(-19*A*c+11*B*b)/b^4/x^(7/2)+
5/16*c*(-19*A*c+11*B*b)/b^5/x^(3/2)+1/4*(A*c-B*b)/b/c/x^(11/2)/(c*x^2+b)^2+
1/16*(19*A*c-11*B*b)/b^2/c/x^(11/2)/(c*x^2+b)-15/64*c^(7/4)*(-19*A*c+11*B*b
)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(23/4)*2^(1/2)+15/64*c^(7/4)*
(-19*A*c+11*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(23/4)*2^(1/2)
-15/128*c^(7/4)*(-19*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/
2)*x^(1/2))/b^(23/4)*2^(1/2)+15/128*c^(7/4)*(-19*A*c+11*B*b)*ln(b^(1/2)+x*c
^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(23/4)*2^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1598, 468, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{15c^{7/4}(11bB-19Ac)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{23/4}}+\frac{15c^{7/4}(11bB-19Ac)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{23/4}}-\frac{15c^{7/4}(11bB-19Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\,b^{23/4}}+\frac{15c^{7/4}(11bB-19Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\,b^{23/4}}+\frac{5c(11bB-19Ac)}{16b^5x^{3/2}}-\frac{15(11bB-19Ac)}{112b^4x^{7/2}}+\frac{15(11bB-19Ac)}{176b^3cx^{11/2}}-\frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)}-\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] (15*(11*b*B - 19*A*c))/(176*b^3*c*x^(11/2)) - (15*(11*b*B - 19*A*c))/(112*b
^4*x^(7/2)) + (5*c*(11*b*B - 19*A*c))/(16*b^5*x^(3/2)) - (b*B - A*c)/(4*b*c
*x^(11/2)*(b + c*x^2)^2) - (11*b*B - 19*A*c)/(16*b^2*c*x^(11/2)*(b + c*x^2)
) - (15*c^(7/4)*(11*b*B - 19*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(32*Sqrt[2]*b^(23/4)) + (15*c^(7/4)*(11*b*B - 19*A*c)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(23/4)) - (15*c^(7/4)*(11*b*B
 - 19*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*
Sqrt[2]*b^(23/4)) + (15*c^(7/4)*(11*b*B - 19*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\,\left(bx^2+cx^4\right)^3}\,dx = \int \frac{A+Bx^2}{x^{13/2}\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} + \frac{\left(-\frac{11bB}{2}+\frac{19Ac}{2}\right)\int \frac{1}{x^{13/2}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)} - \frac{(15(11bB-19Ac))\int \frac{1}{x^{13/2}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)} + \frac{(15(11bB-19A}{3}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right.}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}$$

Mathematica [A]
time = 0.58, size = 231, normalized size = 0.63

$$\frac{-\frac{4b^{3/4}\left(-11bBx^2\left(-32b^3+160b^2cx^2+605bc^2x^4+385c^3x^6\right)+A\left(224b^4-608b^3cx^2+3040b^2c^2x^4+11495bc^3x^6+7315c^4x^8\right)\right)}{x^{11/2}\left(b+cx^2\right)^2}+1155\sqrt{2}\,c^{7/4}(-11bB+19Ac)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)+1155\sqrt{2}\,c^{7/4}(11bB-19Ac)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{4928b^{23/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-4*b^(3/4)*(-11*b*B*x^2*(-32*b^3 + 160*b^2*c*x^2 + 605*b*c^2*x^4 + 385*c^
3*x^6) + A*(224*b^4 - 608*b^3*c*x^2 + 3040*b^2*c^2*x^4 + 11495*b*c^3*x^6 +
7315*c^4*x^8)))/(x^(11/2)*(b + c*x^2)^2) + 1155*Sqrt[2]*c^(7/4)*(-11*b*B +
19*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 1
155*Sqrt[2]*c^(7/4)*(11*b*B - 19*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x])/(Sqrt[b] + Sqrt[c]*x)])/(4928*b^(23/4))
```

Maple [A]
time = 0.41, size = 210, normalized size = 0.58

method	result
derivativedivides	$-\frac{2A}{11b^3x^{\frac{11}{2}}}-\frac{2(-3Ac+Bb)}{7b^4x^{\frac{7}{2}}}-\frac{2c(2Ac-Bb)}{b^5x^{\frac{3}{2}}}-\frac{2c^2\left(\frac{\left(\frac{31}{32}Ac^2-\frac{23}{32}bBc\right)x^{\frac{5}{2}}+\frac{b(35Ac-27Bb)\sqrt{x}}{32}}{\left(cx^2+b\right)^2}+\frac{15(19Ac-11Bb)\left(\frac{b}{c}\right)\cdots}{\cdots}\cdots\right.}{\cdots}$
default	$-\frac{2A}{11b^3x^{\frac{11}{2}}}-\frac{2(-3Ac+Bb)}{7b^4x^{\frac{7}{2}}}-\frac{2c(2Ac-Bb)}{b^5x^{\frac{3}{2}}}-\frac{2c^2\left(\frac{\left(\frac{31}{32}Ac^2-\frac{23}{32}bBc\right)x^{\frac{5}{2}}+\frac{b(35Ac-27Bb)\sqrt{x}}{32}}{\left(cx^2+b\right)^2}+\frac{15(19Ac-11Bb)\left(\frac{b}{c}\right)\cdots}{\cdots}\cdots\right.}{\cdots}$
risch	$-\frac{2\left(154Ac^2x^4-77x^4bBc-33Abcx^2+11b^2Bx^2+7b^2A\right)}{77b^5x^{\frac{11}{2}}}-\frac{31c^4x^{\frac{5}{2}}A}{16b^5(cx^2+b)^2}+\frac{23c^3x^{\frac{5}{2}}B}{16b^4(cx^2+b)^2}-\frac{35c^3A\sqrt{x}}{16b^4(cx^2+b)^2}+\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/11*A/b^3/x^(11/2)-2/7*(-3*A*c+B*b)/b^4/x^(7/2)-2*c*(2*A*c-B*b)/b^5/x^(3/
2)-2/b^5*c^2*(((31/32*A*c^2-23/32*b*B*c)*x^(5/2)+1/32*b*(35*A*c-27*B*b)*x^(
1/2))/(c*x^2+b)^2+15/256*(19*A*c-11*B*b)*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)
^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1
/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*
x^(1/2)-1)))
```

Maxima [A]
time = 0.54, size = 353, normalized size = 0.97

$$\frac{385\,(11\,Bbc^3-19\,Ac^4)x^8+605\,(11\,Bb^2c^2-19\,Abc^3)x^6-224\,Ab^4+160\,(11\,Bb^3c-19\,Ab^2c^2)x^4-32\,(11\,Bb^4-19\,Ab^3c)x^2}{1232\left(b^5c^2x^{\frac{19}{2}}+2\,b^6cx^{\frac{15}{2}}+b^7x^{\frac{11}{2}}\right)}+\frac{15\left(\frac{2\sqrt{2}\,(11\,Bbc^2-19\,Ac^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,(11\,Bbc^2-19\,Ac^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,(11\,Bbc^2-19\,Ac^3)\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,(11\,Bbc^2-19\,Ac^3)\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{128\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/1232*(385*(11*B*b*c^3 - 19*A*c^4)*x^8 + 605*(11*B*b^2*c^2 - 19*A*b*c^3)*x
^6 - 224*A*b^4 + 160*(11*B*b^3*c - 19*A*b^2*c^2)*x^4 - 32*(11*B*b^4 - 19*A*
b^3*c)*x^2)/(b^5*c^2*x^(19/2) + 2*b^6*c*x^(15/2) + b^7*x^(11/2)) + 15/128*(
2*sqrt(2)*(11*B*b*c^2 - 19*A*c^3)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/
4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c
))) + 2*sqrt(2)*(11*B*b*c^2 - 19*A*c^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4
)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)
*sqrt(c))) + sqrt(2)*(11*B*b*c^2 - 19*A*c^3)*log(sqrt(2)*b^(1/4)*c^(1/4)*sq
rt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(11*B*b*c^2 - 19*A
*c^3)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*
c^(1/4)))/b^5
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(273) = 546$.
time = 2.15, size = 894, normalized size = 2.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4928*(4620*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-(14641*B^4*b^4*c^7 -
 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 1303
21*A^4*c^11)/b^23)^(1/4)*arctan((sqrt(b^12*sqrt(-(14641*B^4*b^4*c^7 - 10115
6*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4
*c^11)/b^23) + (121*B^2*b^2*c^4 - 418*A*B*b*c^5 + 361*A^2*c^6)*x)*b^17*(-(1
4641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A
^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(3/4) + (11*B*b^18*c^2 - 19*A*b^17*c^3
)*sqrt(x)*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*
c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(3/4))/(14641*B^4*b^4*c^
7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 1
30321*A^4*c^11)) + 1155*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-(14641*B^4
*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c
^10 + 130321*A^4*c^11)/b^23)^(1/4)*log(15*b^6*(-(14641*B^4*b^4*c^7 - 101156
*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*
c^11)/b^23)^(1/4) - 15*(11*B*b*c^2 - 19*A*c^3)*sqrt(x)) - 1155*(b^5*c^2*x^1
0 + 2*b^6*c*x^8 + b^7*x^6)*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 26
2086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(1/4)*l
og(-15*b^6*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2
*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(1/4) - 15*(11*B*b*c^2
- 19*A*c^3)*sqrt(x)) - 4*(385*(11*B*b*c^3 - 19*A*c^4)*x^8 + 605*(11*B*b^2*c
^2 - 19*A*b*c^3)*x^6 - 224*A*b^4 + 160*(11*B*b^3*c - 19*A*b^2*c^2)*x^4 - 32
```

```
*(11*B*b^4 - 19*A*b^3*c)*x^2)*sqrt(x))/(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^
6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.51, size = 351, normalized size = 0.96

$$\frac{15\sqrt{2}\left(11\,(bc^3)^{\frac14}Bbc-19\,(bc^3)^{\frac14}Ac^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{64\,b^6}+\frac{15\sqrt{2}\left(11\,(bc^3)^{\frac14}Bbc-19\,(bc^3)^{\frac14}Ac^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{64\,b^6}+\frac{15\sqrt{2}\left(11\,(bc^3)^{\frac14}Bbc-19\,(bc^3)^{\frac14}Ac^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6}-\frac{15\sqrt{2}\left(11\,(bc^3)^{\frac14}Bbc-19\,(bc^3)^{\frac14}Ac^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6}+\frac{23\,Bbc^3x^{\frac52}-31\,Ac^4x^{\frac52}+27\,Bb^2c^2\sqrt{x}-35\,Abc^3\sqrt{x}}{16\,(cx^2+b)^2b^5}+\frac{2\left(77\,Bbcx^4-154\,Ac^2x^4-11\,Bb^2x^2+33\,Abcx^2-7\,Ab^2\right)}{77\,b^5x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 15/64*sqrt(2)*(11*(b*c^3)^(1/4)*B*b*c - 19*(b*c^3)^(1/4)*A*c^2)*arctan(1/2*
sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^6 + 15/64*sqrt(2)*
(11*(b*c^3)^(1/4)*B*b*c - 19*(b*c^3)^(1/4)*A*c^2)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^6 + 15/128*sqrt(2)*(11*(b*c^3)^
(1/4)*B*b*c - 19*(b*c^3)^(1/4)*A*c^2)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/b^6 - 15/128*sqrt(2)*(11*(b*c^3)^(1/4)*B*b*c - 19*(b*c^3)^(1/4)
*A*c^2)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 + 1/16*(23*B*
b*c^3*x^(5/2) - 31*A*c^4*x^(5/2) + 27*B*b^2*c^2*sqrt(x) - 35*A*b*c^3*sqrt(x
))/((c*x^2 + b)^2*b^5) + 2/77*(77*B*b*c*x^4 - 154*A*c^2*x^4 - 11*B*b^2*x^2
+ 33*A*b*c*x^2 - 7*A*b^2)/(b^5*x^(11/2))
```

Mupad [**B**]
time = 0.51, size = 639, normalized size = 1.75

$$\frac{15\,{\left(-c\right)}^{7/4}\,\mathrm{atan}\left(\frac{6859\,A^3\,c^{10}\,\sqrt{x}-1331\,B^3\,b^3\,c^7\,\sqrt{x}-11913\,A^2\,B\,b\,c^9\,\sqrt{x}+6897\,A\,B^2\,b^2\,c^8\,\sqrt{x}}{b^{1/4}\,{\left(-c\right)}^{27/4}\,\left(c\,\left(c\,\left(6859\,A^3\,c-11913\,A^2\,B\,b\right)+6897\,A\,B^2\,b^2\right)-1331\,B^3\,b^3\right)}\right)\,\left(19\,A\,c-11\,B\,b\right)}{32\,b^{23/4}}-\frac{\frac{2\,A}{11\,b}-\frac{2\,x^2\,\left(19\,A\,c-11\,B\,b\right)}{77\,b^2}+\frac{55\,c^2\,x^6\,\left(19\,A\,c-11\,B\,b\right)}{112\,b^4}+\frac{5\,c^3\,x^8\,\left(19\,A\,c-11\,B\,b\right)}{16\,b^5}+\frac{10\,c\,x^4\,\left(19\,A\,c-11\,B\,b\right)}{77\,b^3}}{b^2\,x^{11/2}+c^2\,x^{19/2}+2\,b\,c\,x^{15/2}}-\frac{{\left(-c\right)}^{7/4}\,\mathrm{atan}\left(\frac{\frac{{\left(-c\right)}^{7/4}\,\left(19\,A\,c-11\,B\,b\right)\,\left(\sqrt{x}\,\left(1330790400\,A^2\,b^{15}\,c^9-1540915200\,A\,B\,b^{16}\,c^8+446054400\,B^2\,b^{17}\,c^7\right)-\frac{15\,{\left(-c\right)}^{7/4}\,\left(19\,A\,c-11\,B\,b\right)\,\left(298844160\,A\,b^{21}\,c^6-173015040\,B\,b^{22}\,c^5\right)}{64\,b^{23/4}}\right)\,15{}\mathrm{i}}{64\,b^{23/4}}+\frac{{\left(-c\right)}^{7/4}\,\left(19\,A\,c-11\,B\,b\right)\,\left(\sqrt{x}\,\left(1330790400\,A^2\,b^{15}\,c^9-1540915200\,A\,B\,b^{16}\,c^8+446054400\,B^2\,b^{17}\,c^7\right)+\frac{15\,{\left(-c\right)}^{7/4}\,\left(19\,A\,c-11\,B\,b\right)\,\left(298844160\,A\,b^{21}\,c^6-173015040\,B\,b^{22}\,c^5\right)}{64\,b^{23/4}}\right)\,15{}\mathrm{i}}{64\,b^{23/4}}}{\frac{15\,{\left(-c\right)}^{7/4}\,\left(19\,A\,c-11\,B\,b\right)\,\left(\sqrt{x}\,\left(1330790400\,A^2\,b^{15}\,c^9-1540915200\,A\,B\,b^{16}\,c^8+446054400\,B^2\,b^{17}\,c^7\right)-\frac{15\,{\left(-c\right)}^{7/4}\,\left(19\,A\,c-11\,B\,b\right)\,\left(298844160\,A\,b^{21}\,c^6-173015040\,B\,b^{22}\,c^5\right)}{64\,b^{23/4}}\right)}{64\,b^{23/4}}-\frac{15\,{\left(-c\right)}^{7/4}\,\left(19\,A\,c-11\,B\,b\right)\,\left(\sqrt{x}\,\left(1330790400\,A^2\,b^{15}\,c^9-1540915200\,A\,B\,b^{16}\,c^8+446054400\,B^2\,b^{17}\,c^7\right)+\frac{15\,{\left(-c\right)}^{7/4}\,\left(19\,A\,c-11\,B\,b\right)\,\left(298844160\,A\,b^{21}\,c^6-173015040\,B\,b^{22}\,c^5\right)}{64\,b^{23/4}}\right)}{64\,b^{23/4}}}\right)\,\left(19\,A\,c-11\,B\,b\right)\,15{}\mathrm{i}}{32\,b^{23/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^3),x)
```

```
[Out] (15*(-c)^(7/4)*atan((6859*A^3*c^10*x^(1/2) - 1331*B^3*b^3*c^7*x^(1/2) - 119
13*A^2*B*b*c^9*x^(1/2) + 6897*A*B^2*b^2*c^8*x^(1/2))/(b^(1/4)*(-c)^(27/4)*(
c*(c*(6859*A^3*c - 11913*A^2*B*b) + 6897*A*B^2*b^2) - 1331*B^3*b^3)))*(19*A
*c - 11*B*b))/(32*b^(23/4)) - ((2*A)/(11*b) - (2*x^2*(19*A*c - 11*B*b))/(77
*b^2) + (55*c^2*x^6*(19*A*c - 11*B*b))/(112*b^4) + (5*c^3*x^8*(19*A*c - 11*
B*b))/(16*b^5) + (10*c*x^4*(19*A*c - 11*B*b))/(77*b^3))/(b^2*x^(11/2) + c^2
```

```
*x^(19/2) + 2*b*c*x^(15/2)) - ((-c)^(7/4)*atan((((-c)^(7/4)*(19*A*c - 11*B*
b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 446054400*B^2*b^17*c^7 - 1540915200*
A*B*b^16*c^8) - (15*(-c)^(7/4)*(19*A*c - 11*B*b)*(298844160*A*b^21*c^6 - 17
3015040*B*b^22*c^5))/(64*b^(23/4)))*15i)/(64*b^(23/4)) + ((-c)^(7/4)*(19*A*
c - 11*B*b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 446054400*B^2*b^17*c^7 - 15
40915200*A*B*b^16*c^8) + (15*(-c)^(7/4)*(19*A*c - 11*B*b)*(298844160*A*b^21
*c^6 - 173015040*B*b^22*c^5))/(64*b^(23/4)))*15i)/(64*b^(23/4)))/((15*(-c)^
(7/4)*(19*A*c - 11*B*b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 446054400*B^2*b
^17*c^7 - 1540915200*A*B*b^16*c^8) - (15*(-c)^(7/4)*(19*A*c - 11*B*b)*(2988
44160*A*b^21*c^6 - 173015040*B*b^22*c^5))/(64*b^(23/4))))/(64*b^(23/4)) - (
15*(-c)^(7/4)*(19*A*c - 11*B*b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 4460544
00*B^2*b^17*c^7 - 1540915200*A*B*b^16*c^8) + (15*(-c)^(7/4)*(19*A*c - 11*B*
b)*(298844160*A*b^21*c^6 - 173015040*B*b^22*c^5))/(64*b^(23/4))))/(64*b^(23
/4))))*(19*A*c - 11*B*b)*15i)/(32*b^(23/4))
```

3.220 $\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=243

$$\frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}}-\frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2}-\frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2+cx^4}}{55c}+\frac{2Bx^{3/2}(b}{1}$$

```
[Out] 2/15*B*x^(3/2)*(c*x^4+b*x^2)^(3/2)/c-4/385*b*(-5*A*c+3*B*b)*x^(3/2)*(c*x^4+
b*x^2)^(1/2)/c^2-2/55*(-5*A*c+3*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c+4/231*b^
2*(-5*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)-2/231*b^(11/4)*(-5*A*c+3*B
*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)
*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(
1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(13/4)/
(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2064, 2046, 2049, 2057, 335, 226}

$$-\frac{2b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-5Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}}+\frac{4b^2\sqrt{bx^2+cx^4}(3bB-5Ac)}{231c^3\sqrt{x}}-\frac{4bx^{3/2}\sqrt{bx^2+cx^4}(3bB-5Ac)}{385c^2}-\frac{2x^{7/2}\sqrt{bx^2+cx^4}(3bB-5Ac)}{55c}+\frac{2Bx^{3/2}(bx^2+cx^4)^{3/2}}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (4*b^2*(3*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^3*Sqrt[x]) - (4*b*(3*b*B
 - 5*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(385*c^2) - (2*(3*b*B - 5*A*c)*x^(7/
2)*Sqrt[b*x^2 + c*x^4])/(55*c) + (2*B*x^(3/2)*(b*x^2 + c*x^4)^(3/2))/(15*c)
 - (2*b^(11/4)*(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sq
rt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/
(231*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{2Bx^{3/2}(bx^2+cx^4)^{3/2}}{15c} - \frac{\left(2\left(\frac{9bB}{2}-\frac{15Ac}{2}\right)\right)\int x^{5/2}\sqrt{bx^2+cx^4}\,dx}{15c}$$

$$= -\frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2+cx^4}}{55c} + \frac{2Bx^{3/2}(bx^2+cx^4)^{3/2}}{15c} - \frac{(2b(3bB-}{}$$

$$= -\frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2} - \frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2+cx^4}}{55c} +$$

$$= \frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2} - \frac{2(3}{}$$

$$= \frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2} - \frac{2(3}{}$$

$$= \frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2} - \frac{2(3}{}$$

$$= \frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2} - \frac{2(3}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.13, size = 136, normalized size = 0.56

$$\frac{2\sqrt{x^2(b+cx^2)}\left((b+cx^2)\sqrt{1+\frac{cx^2}{b}}\,(45b^2B+7c^2x^2(15A+11Bx^2)-3bc(25A+21Bx^2))+15b^2(-3bB+5Ac)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{1155c^3\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)*Sqrt[1 + (c*x^2)/b]*(45*b^2*B + 7*c^2
*x^2*(15*A + 11*B*x^2) - 3*b*c*(25*A + 21*B*x^2)) + 15*b^2*(-3*b*B + 5*A*c)
*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(1155*c^3*Sqrt[x]*Sqrt[1
 + (c*x^2)/b])
```

Maple [A]
time = 0.42, size = 307, normalized size = 1.26

method	result
risch	$-\frac{2\left(-77B\,c^{3}x^{6}-105A\,c^{3}x^{4}-14Bb\,c^{2}x^{4}-30Ab\,c^{2}x^{2}+18B\,b^{2}c\,x^{2}+50A\,b^{2}c-30B\,b^{3}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{1155\sqrt{x}\,c^{3}}+\frac{2b^{3}(5Ac-3Bb)\sqrt{-bc}}{}$
default	$\frac{2\sqrt{x^{4}c+b\,x^{2}}\,\left(77B\,c^{5}x^{9}+105A\,c^{5}x^{7}+91Bb\,c^{4}x^{7}+25A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(77*B*c^5*x^9+105*A*c^5*x^7+91
*B*b*c^4*x^7+25*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^3*c+135*A*b*c^
4*x^5-15*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^4-4*B*b^2*c^3*x^5-20*
A*b^2*c^3*x^3+12*B*b^3*c^2*x^3-50*A*b^3*c^2*x+30*B*b^4*c*x)/c^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 123, normalized size = 0.51

$$-\frac{2\left(10\left(3\,Bb^{4}-5\,Ab^{3}c\right)\sqrt{c}\,x\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)-\left(77\,Bc^{4}x^{6}+30\,Bb^{3}c-50\,Ab^{2}c^{2}+7\left(2\,Bbc^{3}+15\,Ac^{4}\right)x^{4}-6\left(3\,Bb^{2}c^{2}-5\,Abc^{3}\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}}\,\sqrt{x}\right)}{1155\,c^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(10*(3*B*b^4 - 5*A*b^3*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0,
x) - (77*B*c^4*x^6 + 30*B*b^3*c - 50*A*b^2*c^2 + 7*(2*B*b*c^3 + 15*A*c^4)*x
^4 - 6*(3*B*b^2*c^2 - 5*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^4*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}}\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**(5/2)*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{5/2}\,\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
[Out] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2), x)
```

3.221 $\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=369

$$\frac{4b^2(7bB-13Ac)x^{3/2}(b+cx^2)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{4b(7bB-13Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{585c^2}-\frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{-}$$

```
[Out] 2/13*B*(c*x^4+b*x^2)^(3/2)*x^(1/2)/c+4/195*b^2*(-13*A*c+7*B*b)*x^(3/2)*(c*x
^2+b)/c^(5/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/117*(-13*A*c+7*B*b)
*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c-4/585*b*(-13*A*c+7*B*b)*x^(1/2)*(c*x^4+b*x^2
)^(1/2)/c^2-4/195*b^(9/4)*(-13*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*ar
ctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/
(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)+2/195*b^(9/4)*(-1
3*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))
),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/
c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$,
Rules used = {2064, 2046, 2049, 2057, 335, 311, 226, 1210}

$$\frac{2b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-13Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}-\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-13Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}+\frac{4b^2x^{3/2}(b+cx^2)(7bB-13Ac)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{4b\sqrt{x}\,\sqrt{bx^2+cx^4}\,(7bB-13Ac)}{585c^2}-\frac{2x^{5/2}\sqrt{bx^2+cx^4}\,(7bB-13Ac)}{117c}+\frac{2B\sqrt{x}\,(bx^2+cx^4)^{3/2}}{13c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (4*b^2*(7*b*B - 13*A*c)*x^(3/2)*(b + c*x^2))/(195*c^(5/2)*(Sqrt[b] + Sqrt[c
]*x)*Sqrt[b*x^2 + c*x^4]) - (4*b*(7*b*B - 13*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^
4])/(585*c^2) - (2*(7*b*B - 13*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(117*c) +
(2*B*Sqrt[x]*(b*x^2 + c*x^4)^(3/2))/(13*c) - (4*b^(9/4)*(7*b*B - 13*A*c)*x*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(11/4)*Sqrt[b*x^2 + c*x^4]
) + (2*b^(9/4)*(7*b*B - 13*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(S
qrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(195*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{2B\sqrt{x}\,(bx^2+cx^4)^{3/2}}{13c} - \frac{\left(2\left(\frac{7bB}{2}-\frac{13Ac}{2}\right)\right)\int x^{3/2}\sqrt{bx^2+cx^4}\,dx}{13c}$$

$$= -\frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}\,(bx^2+cx^4)^{3/2}}{13c} - \frac{(2b(7bB-}{}$$

$$= -\frac{4b(7bB-13Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c}$$

$$= -\frac{4b(7bB-13Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c}$$

$$= -\frac{4b(7bB-13Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c}$$

$$= -\frac{4b(7bB-13Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c}$$

$$= \frac{4b^2(7bB-13Ac)x^{3/2}(b+cx^2)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{4b(7bB-13Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{585c^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.12, size = 111, normalized size = 0.30

$$\frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left(-\left((b+cx^2)\sqrt{1+\frac{cx^2}{b}}(7bB-13Ac-9Bcx^2)\right)+b(7bB-13Ac)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{117c^2\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]

[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)*Sqrt[1 + (c*x^2)/b]*(7*b*B - 13*A*c - 9*B*c*x^2)) + b*(7*b*B - 13*A*c)*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(117*c^2*Sqrt[1 + (c*x^2)/b])

Maple [A]
time = 0.51, size = 446, normalized size = 1.21

method	result
risch	$\frac{2\sqrt{x}\,(45Bc^2x^4+65Ac^2x^2+10bBx^2c+26Abc-14b^2B)\sqrt{x^2(cx^2+b)}}{585c^2} - \frac{2b^2(13Ac-7Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}}{}$
default	$-\frac{2\sqrt{x^4c+bx^2}\left(-45Bc^4x^8-65Ac^4x^6-55Bbc^3x^6+78Ab^3c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/585*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c^3*(-45*B*c^4*x^8-65*A*c^4*x^6-55*B*b*c^3*x^6+78*A*b^3*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-39*A*b^3*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-42*B*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+21*B*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(

```
1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2),1/2*2^(1/2))-91*A*b*c^3*x^4+4*B*b^2*c^2*x^4-26*A*b^2*c^2*x^2+14*B*b^3
*c*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 103, normalized size = 0.28

$$-\frac{2\left(6\left(7\,Bb^3-13\,Ab^2c\right)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\left(45\,Bc^3x^4-14\,Bb^2c+26\,Abc^2+5\left(2\,Bbc^2+13\,Ac^3\right)x^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{585\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/585*(6*(7*B*b^3 - 13*A*b^2*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierst
rassPInverse(-4*b/c, 0, x)) - (45*B*c^3*x^4 - 14*B*b^2*c + 26*A*b*c^2 + 5*(
2*B*b*c^2 + 13*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/c^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
[Out] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2), x)
```

3.222 $\int \sqrt{x}\,(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=204

$$-\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}-\frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c}+\frac{2B(bx^2+cx^4)^{3/2}}{11c\sqrt{x}}+\frac{2b^{7/4}(5bB-11Ac)x\Big(\cdots}{\cdots}$$

```
[Out] 2/11*B*(c*x^4+b*x^2)^(3/2)/c/x^(1/2)-2/77*(-11*A*c+5*B*b)*x^(3/2)*(c*x^4+b*
x^2)^(1/2)/c-4/231*b*(-11*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+2/231*
b^(7/4)*(-11*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/
cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2)
)^2)^(1/2)/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2064, 2046, 2049, 2057, 335, 226}

$$\frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(5bB-11Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{4b\sqrt{bx^2+cx^4}(5bB-11Ac)}{231c^2\sqrt{x}}-\frac{2x^{3/2}\sqrt{bx^2+cx^4}(5bB-11Ac)}{77c}+\frac{2B(bx^2+cx^4)^{3/2}}{11c\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-4*b*(5*b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) - (2*(5*b*B -
 11*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(77*c) + (2*B*(b*x^2 + c*x^4)^(3/2))/
(11*c*Sqrt[x]) + (2*b^(7/4)*(5*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(231*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \sqrt{x}\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx = \frac{2B(bx^2+cx^4)^{3/2}}{11c\sqrt{x}} - \frac{\left(2\left(\frac{5bB}{2}-\frac{11Ac}{2}\right)\right)\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx}{11c}$$

$$= -\frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2B(bx^2+cx^4)^{3/2}}{11c\sqrt{x}} - \frac{(2b(5bB-11\ldots}{}$$

$$= -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2\ldots}{}$$

$$= -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2\ldots}{}$$

$$= -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2\ldots}{}$$

$$= -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2\ldots}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.09, size = 111, normalized size = 0.54

$$\frac{2\sqrt{x^2(b+cx^2)}\left(-\left((b+cx^2)\sqrt{1+\frac{cx^2}{b}}\,(5bB-11Ac-7Bcx^2)\right)+b(5bB-11Ac)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{77c^2\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)*Sqrt[1 + (c*x^2)/b]*(5*b*B - 11*A*c
 - 7*B*c*x^2)) + b*(5*b*B - 11*A*c)*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*
x^2)/b)]))/(77*c^2*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.43, size = 283, normalized size = 1.39

method	result

risch	$\frac{2(21B\,c^2x^4+33A\,c^2x^2+6bB\,x^2c+22Abc-10b^2B)\sqrt{x^2\,(c\,x^2+b)}}{231\sqrt{x}\,c^2}-\frac{2b^2(11Ac-5Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-}}{}$
default	$-\frac{2\sqrt{x^4c+b\,x^2}\left(-21B\,c^4x^7+11A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/231*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(-21*B*c^4*x^7+11*A*((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^2*c-33*A*c^4*x^5-5*B*((c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(
-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^
(1/2))*(-b*c)^(1/2)*b^3-27*B*b*c^3*x^5-55*A*b*c^3*x^3+4*B*b^2*c^2*x^3-22*A*
b^2*c^2*x+10*B*b^3*c*x)/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.77, size = 98, normalized size = 0.48

$$\frac{2\left(2\left(5\,Bb^3-11\,Ab^2c\right)\sqrt{c}\,x\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)+\left(21\,Bc^3x^4-10\,Bb^2c+22\,Abc^2+3\left(2\,Bbc^2+11\,Ac^3\right)x^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{231\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(2*(5*B*b^3 - 11*A*b^2*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x)
 + (21*B*c^3*x^4 - 10*B*b^2*c + 22*A*b*c^2 + 3*(2*B*b*c^2 + 11*A*c^3)*x^2)*
sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\sqrt{x^2\,(b+cx^2)}\,(A+Bx^2)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{x}\,\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2), x)
```

3.223 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$

Optimal. Leaf size=326

$$-\frac{4b(bB-3Ac)x^{3/2}(b+cx^2)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2(bB-3Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{15c}+\frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}}+\frac{4b^{5/4}(bB-3Ac)x}{\cdots}$$

```
[Out] 2/9*B*(c*x^4+b*x^2)^(3/2)/c/x^(3/2)-4/15*b*(-3*A*c+B*b)*x^(3/2)*(c*x^2+b)/c
^(3/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/15*(-3*A*c+B*b)*x^(1/2)*(c
*x^4+b*x^2)^(1/2)/c+4/15*b^(5/4)*(-3*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2
+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)-2/15*b^(5/4)*(
-3*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan
(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))
,1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c
^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2064, 2046, 2057, 335, 311, 226, 1210}

$$-\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(bB-3Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}+\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(bB-3Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}-\frac{4bx^{3/2}(b+cx^2)(bB-3Ac)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{x}\,\sqrt{bx^2+cx^4}(bB-3Ac)}{15c}+\frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/Sqrt[x],x]
```

```
[Out] (-4*b*(b*B - 3*A*c)*x^(3/2)*(b + c*x^2))/(15*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[b*x^2 + c*x^4]) - (2*(b*B - 3*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(15*c)
 + (2*B*(b*x^2 + c*x^4)^(3/2))/(9*c*x^(3/2)) + (4*b^(5/4)*(b*B - 3*A*c)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(7/4)*Sqrt[b*x^2 + c*x^4]) -
 (2*b^(5/4)*(b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
 + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c
^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
```

```
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx = \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} - \frac{\left(2\left(\frac{3bB}{2}-\frac{9Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx}{9c}$$

$$= -\frac{2(bB-3Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{15c} + \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} - \frac{(2b(bB-3Ac))\int \cdots}{15c\cdots}$$

$$= -\frac{2(bB-3Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{15c} + \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} - \frac{\left(2b(bB-3Ac)x\sqrt{\cdots}\right.}{15c\sqrt{\cdots}}$$

$$= -\frac{2(bB-3Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{15c} + \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} - \frac{\left(4b(bB-3Ac)x\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{2(bB-3Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{15c} + \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} - \frac{\left(4b^{3/2}(bB-3Ac)x\cdots\right.}{\cdots}$$

$$= -\frac{4b(bB-3Ac)x^{3/2}(b+cx^2)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2(bB-3Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{15c} + \frac{2\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.07, size = 94, normalized size = 0.29

$$\frac{2\sqrt{x}\,\sqrt{x^2(b+cx^2)}\left(B(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+(-bB+3Ac)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{9c\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(B*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (-(b*
B) + 3*A*c)*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(9*c*Sqrt[1 +
 (c*x^2)/b])
```

Maple [A]
time = 0.38, size = 422, normalized size = 1.29

method	result
risch	$\frac{2\sqrt{x}\,(5Bc\,x^2+9Ac+2Bb)\sqrt{x^2\,(c\,x^2+b)}}{45c}+\frac{2b(3Ac-Bb)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-}}{}$
default	$\frac{2\sqrt{x^4c+b\,x^2}\,\left(5B\,c^3x^6+18A\,b^2c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c^2*(5*B*c^3*x^6+18*A*b^2*c*((c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2),1/2*2^(1/2))-9*A*b^2*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2
^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*E
llipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-6*B*b^3*((c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2),1/2*2^(1/2))+3*B*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1
/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Elli
pticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+9*A*c^3*x^4+7*B*
b*c^2*x^4+9*A*b*c^2*x^2+2*B*b^2*c*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/sqrt(x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.54, size = 77, normalized size = 0.24

$$\frac{2\left(6\,(Bb^2-3\,Abc)\sqrt{c}\ \mathrm{weierstrassZeta}\left(-\frac{4b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+(5\,Bc^2x^2+2\,Bbc+9\,Ac^2)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{45\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(6*(B*b^2 - 3*A*b*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPIn
verse(-4*b/c, 0, x)) + (5*B*c^2*x^2 + 2*B*b*c + 9*A*c^2)*sqrt(c*x^4 + b*x^2
)*sqrt(x))/c^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\,(b+cx^2)}\,(A+Bx^2)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/sqrt(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(1/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(1/2), x)
```

3.224 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}}-\frac{2b^{3/4}(bB-7Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\Big(2\,\mathrm{t}}{21c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/7*B*(c*x^4+b*x^2)^(3/2)/c/x^(5/2)-2/21*(-7*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/c
/x^(1/2)-2/21*b^(3/4)*(-7*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))
)^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2
)+x*c^(1/2))^2)^(1/2)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2064, 2046, 2057, 335, 226}

$$-\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(bB-7Ac)F\left(2\mathrm{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}(bB-7Ac)}{21c\sqrt{x}}+\frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(3/2),x]
```

```
[Out] (-2*(b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(21*c*Sqrt[x]) + (2*B*(b*x^2 + c*x^4
)^(3/2))/(7*c*x^(5/2)) - (2*b^(3/4)*(b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(21*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{\left(2\left(\frac{bB}{2}-\frac{7Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx}{7c}$$

$$= -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{(2b(bB-7Ac))\int \frac{}{\sqrt{bx}}}{21c}$$

$$= -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{\left(2b(bB-7Ac)x\sqrt{b+}\right.}{21c\sqrt{b}}$$

$$= -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{\left(4b(bB-7Ac)x\sqrt{b+}\right.}{}$$

$$= -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{2b^{3/4}(bB-7Ac)x\left(\sqrt{b}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 94, normalized size = 0.57

$$\frac{2\sqrt{x^2(b+cx^2)}\left(B(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+(-bB+7Ac)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{7c\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(B*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (-(b*B) + 7*A
*c)*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(7*c*Sqrt[x]*Sqrt[1 +
 (c*x^2)/b])
```

Maple [A]
time = 0.38, size = 257, normalized size = 1.56

method	result
risch	$\frac{2(3Bc\,x^2+7Ac+2Bb)\sqrt{x^2\,(c\,x^2+b)}}{21\sqrt{x}\;c}+\frac{2b(7Ac-Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-}}}}{21c^2\sqrt{c\,x^3}}$
default	$\frac{2\sqrt{x^4c+b\,x^2}\left(7A\sqrt{-bc}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(7*A*(-b*c)^(1/2)*((-c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*2^(1/2)*b*c-B*(-b*c)^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(
-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),
1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*b^2+3*B*c^3*x^
5+7*A*c^3*x^3+5*B*b*c^2*x^3+7*A*b*c^2*x+2*B*b^2*c*x)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 74, normalized size = 0.45

$$-\frac{2\left(2\,(Bb^2-7\,Abc)\sqrt{c}\,x\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-(3\,Bc^2x^2+2\,Bbc+7\,Ac^2)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{21\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] -2/21*(2*(B*b^2 - 7*A*b*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) - (3
*B*c^2*x^2 + 2*B*b*c + 7*A*c^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\,(b+cx^2)}\,(A+Bx^2)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(3/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(3/2), x)
```

3.225 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx$

Optimal. Leaf size=323

$$\frac{4(bB+5Ac)x^{3/2}(b+cx^2)}{5\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{2(bB+5Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{5b}-\frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}}-\frac{4\sqrt[4]{b}\,(bB+5Ac)x\Big(\sqrt{}}{}$$

```
[Out] -2*A*(c*x^4+b*x^2)^(3/2)/b/x^(7/2)+4/5*(5*A*c+B*b)*x^(3/2)*(c*x^2+b)/c^(1/2
)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+2/5*(5*A*c+B*b)*x^(1/2)*(c*x^4+b*
x^2)^(1/2)/b-4/5*b^(1/4)*(5*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1
/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2)^(1/2)+2/5*b^(1/4)*(5*A*c+B*b)
*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2
))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x
^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2063, 2046, 2057, 335, 311, 226, 1210}

$$\frac{2\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(5Ac+bB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}-\frac{4\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(5Ac+bB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{x}\,\sqrt{bx^2+cx^4}\,(5Ac+bB)}{5b}+\frac{4x^{3/2}(b+cx^2)\,(5Ac+bB)}{5\sqrt{c}\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(5/2),x]
```

```
[Out] (4*(b*B + 5*A*c)*x^(3/2)*(b + c*x^2))/(5*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[b*x^2 + c*x^4]) + (2*(b*B + 5*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*b) - (2
*A*(b*x^2 + c*x^4)^(3/2))/(b*x^(7/2)) - (4*b^(1/4)*(b*B + 5*A*c)*x*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[
(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[b*x^2 + c*x^4]) + (2*b^(1
/4)*(b*B + 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sq
rt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
```

```
n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = -\frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{\left(2\left(-\frac{bB}{2}-\frac{5Ac}{2}\right)\right)\int\frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx}{b}$$

$$= \frac{2(bB+5Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{5b} - \frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}} + \frac{1}{5}(2(bB+5Ac))\int \frac{}{\sqrt{b}}$$

$$= \frac{2(bB+5Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{5b} - \frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}} + \frac{\left(2(bB+5Ac)x\sqrt{b+c}\right.}{5\sqrt{bx^2}}$$

$$= \frac{2(bB+5Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{5b} - \frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}} + \frac{\left(4(bB+5Ac)x\sqrt{b+c}\right.}{}$$

$$= \frac{2(bB+5Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{5b} - \frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}} + \frac{\left(4\sqrt{b}\,(bB+5Ac)x\sqrt{b}\right.}{}$$

$$= \frac{4(bB+5Ac)x^{3/2}(b+cx^2)}{5\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{2(bB+5Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{5b} - \frac{2A(b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.14, size = 97, normalized size = 0.30

$$\frac{2\sqrt{x^2(b+cx^2)}\left(-3A(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+(bB+5Ac)x^2\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{3bx^{3/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(5/2),x]
```

[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-3*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (b*B + 5*A
*c)*x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(3*b*x^(3/2)*Sqrt
[1 + (c*x^2)/b])

Maple [A]
time = 0.39, size = 399, normalized size = 1.24

method	result
risch	$-\frac{2(-Bx^2+5A)\sqrt{x^2(cx^2+b)}}{5x^{\frac{3}{2}}}+\frac{\left(2Ac+\frac{2Bb}{5}\right)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}}{}$
default	$\frac{2\sqrt{x^4c+bx^2}\left(10A\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}bc-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/5*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(10*A*EllipticE(((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
2^(1/2)((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)
*b*c-5*A*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b*c+2*B*EllipticE(((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^
(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b^
2-B*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b^2+B*c^2*x^4-5*A*c^2*x^2+b*B*x^2*c-5*A*b*c
)/c

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(5/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 71, normalized size = 0.22

$$-\frac{2\left(2\,(Bb+5\,Ac)\sqrt{c}\,x^2\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\sqrt{cx^4+bx^2}\,(Bcx^2-5\,Ac)\sqrt{x}\right)}{5\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="fricas")

[Out] -2/5*(2*(B*b + 5*A*c)*sqrt(c)*x^2*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) - sqrt(c*x^4 + b*x^2)*(B*c*x^2 - 5*A*c)*sqrt(x))/(c*x^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\,(b+cx^2)}\,(A+Bx^2)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(5/2),x)

[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(5/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="giac")

[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(5/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(5/2),x)

[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(5/2), x)

3.226 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx$

Optimal. Leaf size=163

$$\frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}}-\frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}}+\frac{2(bB+Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\right.\right.}{3\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}$$

```
[Out] -2/3*A*(c*x^4+b*x^2)^(3/2)/b/x^(9/2)+2/3*(A*c+B*b)*(c*x^4+b*x^2)^(1/2)/b/x^
(1/2)+2/3*(A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)
^(1/2)/b^(1/4)/c^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2063, 2046, 2057, 335, 226}

$$\frac{2x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(Ac+bB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}\,(Ac+bB)}{3b\sqrt{x}}-\frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(7/2),x]
```

```
[Out] (2*(b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(3*b*Sqrt[x]) - (2*A*(b*x^2 + c*x^4)^(3
/2))/(3*b*x^(9/2)) + (2*(b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1
/2])/(3*b^(1/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = -\frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}} + -\frac{\left(2\left(-\frac{3bB}{2}-\frac{3Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx}{3b}$$

$$= \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}} + \frac{1}{3}(2(bB+Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+}}$$

$$= \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}} + \frac{\left(2(bB+Ac)x\sqrt{b+cx^2}\right)}{3\sqrt{bx^2+c}}$$

$$= \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}} + \frac{\left(4(bB+Ac)x\sqrt{b+cx^2}\right)}{3\sqrt{}}$$

$$= \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}} + \frac{2(bB+Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.37, size = 97, normalized size = 0.60

$$\frac{2\sqrt{x^2(b+cx^2)}\left(-A(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+3(bB+Ac)x^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{3bx^{5/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(7/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-(A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b]) + 3*(b*B + A
*c)*x^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(3*b*x^(5/2)*Sqrt
[1 + (c*x^2)/b])
```

Maple [A]
time = 0.38, size = 239, normalized size = 1.47

method	result

risch	$-\frac{2\left(-B\,x^{2}+A\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{3x^{\frac{5}{2}}}+\frac{\left(\frac{2Ac}{3}+\frac{2Bb}{3}\right)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}}{c\sqrt{c\,x^{3}+bx}}$
default	$\frac{2\sqrt{x^{4}c+b\,x^{2}}\left(A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(c*x^4+b*x^2)^(1/2)/x^(5/2)/(c*x^2+b)*(A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x
*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/
2*2^(1/2))*c*x+B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/
2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellip
ticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*x+B*c^2*x^4-A*c
^2*x^2+b*B*x^2*c-A*b*c)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 61, normalized size = 0.37

$$\frac{2\left(2\left(Bb+Ac\right)\sqrt{c}\,x^{3}\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^{4}+bx^{2}}\left(Bcx^{2}-Ac\right)\sqrt{x}\right)}{3\,cx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*(B*b + A*c)*sqrt(c)*x^3*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x
^4 + b*x^2)*(B*c*x^2 - A*c)*sqrt(x))/(c*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\,(b+cx^2)}\,(A+Bx^2)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(7/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(7/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(7/2), x)
```

3.227 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx$

Optimal. Leaf size=328

$$\frac{4\sqrt{c}\,(5bB+Ac)x^{3/2}(b+cx^2)}{5b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}}-\frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}}-\frac{4\sqrt[4]{c}\,(5bB+Ac)x\left(\sqrt{b}+\sqrt{\right.}}{}$$

```
[Out] -2/5*A*(c*x^4+b*x^2)^(3/2)/b/x^(11/2)+4/5*(A*c+5*B*b)*x^(3/2)*(c*x^2+b)*c^(
1/2)/b/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/5*(A*c+5*B*b)*(c*x^4+b*x^2
)^(1/2)/b/x^(3/2)-4/5*c^(1/4)*(A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*ar
ctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/
(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)+2/5*c^(1/4)*(A*c+5
*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2
^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)
/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2063, 2045, 2057, 335, 311, 226, 1210}

$$\frac{2\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(Ac+5bB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}-\frac{4\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(Ac+5bB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}+\frac{4\sqrt{c}\,x^{3/2}(b+cx^2)\,(Ac+5bB)}{5b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}\,(Ac+5bB)}{5bx^{3/2}}-\frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(9/2),x]
```

```
[Out] (4*Sqrt[c]*(5*b*B + A*c)*x^(3/2)*(b + c*x^2))/(5*b*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) - (2*(5*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(5*b*x^(3/2)) -
(2*A*(b*x^2 + c*x^4)^(3/2))/(5*b*x^(11/2)) - (4*c^(1/4)*(5*b*B + A*c)*x*(Sq
rt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^2 + c*x^4]) + (2
*c^(1/4)*(5*b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(3/
4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
```

```
n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = -\frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} - \frac{\left(2\left(-\frac{5bB}{2}-\frac{Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx}{5b}$$

$$= -\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} + \frac{(2c(5bB+Ac))\int \frac{x^3}{\sqrt{bx^2+}}}{5b}$$

$$= -\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} + \frac{\left(2c(5bB+Ac)x\sqrt{b+cx}\right.}{5b\sqrt{bx^2+}}$$

$$= -\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} + \frac{\left(4c(5bB+Ac)x\sqrt{b+cx}\right.}{5}$$

$$= -\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} + \frac{\left(4\sqrt{c}\,(5bB+Ac)x\sqrt{b+}\right.}{5}$$

$$= \frac{4\sqrt{c}\,(5bB+Ac)x^{3/2}(b+cx^2)}{5b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx}{5bx^{11/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 96, normalized size = 0.29

$$-\frac{2\sqrt{x^2(b+cx^2)}\left(A(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+(5bB+Ac)x^2\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{cx^2}{b}\right)\right)}{5bx^{7/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(9/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (5*b*B + A*c
)*x^2*Hypergeometric2F1[-1/2, -1/4, 3/4, -((c*x^2)/b)]))/(5*b*x^(7/2)*Sqrt[
1 + (c*x^2)/b])
```

Maple [A]
time = 0.39, size = 422, normalized size = 1.29

method	result
risch	$-\frac{2(2Ac\,x^2+5bB\,x^2+Ab)\sqrt{x^2\,(c\,x^2+b)}}{5x^{\frac{7}{2}}b}+\frac{2(Ac+5Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\cdots}}{\cdots}$
default	$\frac{2\sqrt{x^4c+b\,x^2}\left(2A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc\,x^2\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(1/2)/x^(7/2)/(c*x^2+b)*(2*A*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(
1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*
b*c*x^2-A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2+10*B*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*
c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))*b^2*x^2-5*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*x^2-2*A*c^2*x^4-5*x^4*b*B*c
-3*A*b*c*x^2-5*b^2*B*x^2-b^2*A)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(9/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 76, normalized size = 0.23

$$-\frac{2\left(2\,(5\,Bb+Ac)\sqrt{c}\,x^4\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}\,\left((5\,Bb+2\,Ac)x^2+Ab\right)\sqrt{x}\right)}{5\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="fricas")

[Out] -2/5*(2*(5*B*b + A*c)*sqrt(c)*x^4*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*((5*B*b + 2*A*c)*x^2 + A*b)*sqrt(x))/(b*x^4)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(9/2),x)

[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(9/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="giac")

[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(9/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}}{x^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(9/2),x)

[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(9/2), x)

3.228 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}}+\frac{2c^{3/4}(7bB-Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\right.}{21b^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/7*A*(c*x^4+b*x^2)^(3/2)/b/x^(13/2)-2/21*(-A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)
/b/x^(5/2)+2/21*c^(3/4)*(-A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1
/2)+x*c^(1/2))^2)^(1/2)/b^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2063, 2045, 2057, 335, 226}

$$\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(7bB-Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}\,(7bB-Ac)}{21bx^{5/2}}-\frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(11/2),x]
```

```
[Out] (-2*(7*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(21*b*x^(5/2)) - (2*A*(b*x^2 + c*x^4
)^(3/2))/(7*b*x^(13/2)) + (2*c^(3/4)*(7*b*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[
x])/b^(1/4)], 1/2])/(21*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx = -\frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}} - \frac{\left(2\left(-\frac{7bB}{2}+\frac{Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx}{7b}$$

$$= -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}} + \frac{(2c(7bB-Ac))\int \frac{\ldots}{\sqrt{bx^{\ldots}}}}{21b}$$

$$= -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}} + \frac{\left(2c(7bB-Ac)x\sqrt{b+\ldots}\right.}{21b\sqrt{b\ldots}}$$

$$= -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}} + \frac{\left(4c(7bB-Ac)x\sqrt{b+\ldots}\right.}{\ldots}$$

$$= -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}} + \frac{2c^{3/4}(7bB-Ac)x\left(\sqrt{b}\ldots\right.}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 98, normalized size = 0.59

$$-\frac{2\sqrt{x^2(b+cx^2)}\left(3A(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+(7bB-Ac)x^2\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{cx^2}{b}\right)\right)}{21bx^{9/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(11/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(3*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (7*b*B - A
*c)*x^2*Hypergeometric2F1[-3/4, -1/2, 1/4, -((c*x^2)/b)]))/(21*b*x^(9/2)*Sq
rt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.39, size = 255, normalized size = 1.53

method	result

risch	$-\frac{2(2Acx^2+7bBx^2+3Ab)\sqrt{x^2(cx^2+b)}}{21x^{\frac{9}{2}}b}-\frac{2(Ac-7Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\ldots}}{21b\sqrt{c\ldots}}$
default	$-\frac{2\sqrt{x^4c+b\,x^2}\left(A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\ldots\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(c*x^4+b*x^2)^(1/2)/x^(9/2)/(c*x^2+b)*(A*(-b*c)^(1/2)*((c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(
-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),
1/2*2^(1/2))*c*x^3-7*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)
*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*x^3+2*A*c
^2*x^4+7*x^4*b*B*c+5*A*b*c*x^2+7*b^2*B*x^2+3*b^2*A)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 71, normalized size = 0.43

$$\frac{2\left(2\,(7\,Bb-Ac)\sqrt{c}\,x^5\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^4+bx^2}\,((7\,Bb+2\,Ac)x^2+3\,Ab)\sqrt{x}\right)}{21\,bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] 2/21*(2*(7*B*b - A*c)*sqrt(c)*x^5*weierstrassPInverse(-4*b/c, 0, x) - sqrt(
c*x^4 + b*x^2)*((7*B*b + 2*A*c)*x^2 + 3*A*b)*sqrt(x))/(b*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 (b + cx^2)} (A + Bx^2)}{x^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(11/2),x)
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(11/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(11/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B x^2 + A) \sqrt{c x^4 + b x^2}}{x^{11/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(11/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(11/2), x)
```

3.229 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx$

Optimal. Leaf size=369

$$\frac{4c^{3/2}(3bB-Ac)x^{3/2}(b+cx^2)}{15b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}}-\frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}}-\frac{2A(bx^2+cx^4)^{3/2}}{9bx^{15/2}}$$

```
[Out] -2/9*A*(c*x^4+b*x^2)^(3/2)/b/x^(15/2)+4/15*c^(3/2)*(-A*c+3*B*b)*x^(3/2)*(c*
x^2+b)/b^2/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/15*(-A*c+3*B*b)*(c*x^4
+b*x^2)^(1/2)/b/x^(7/2)-4/15*c*(-A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^(3/2)
-4/15*c^(5/4)*(-A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/
2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1
/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)+2/15*c^(5/4)*(-A*c+3*B*b)*x*(cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1
/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2
)^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2063, 2045, 2050, 2057, 335, 311, 226, 1210}

$$\frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}-\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}+\frac{4c^{3/2}x^{3/2}(b+cx^2)(3bB-Ac)}{15b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}(3bB-Ac)}{15b^2x^{3/2}}-\frac{2\sqrt{bx^2+cx^4}(3bB-Ac)}{15bx^{7/2}}-\frac{2A(bx^2+cx^4)^{3/2}}{9bx^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(13/2),x]
```

```
[Out] (4*c^(3/2)*(3*b*B - A*c)*x^(3/2)*(b + c*x^2))/(15*b^2*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[b*x^2 + c*x^4]) - (2*(3*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(15*b*x^(7/2)
) - (4*c*(3*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(15*b^2*x^(3/2)) - (2*A*(b*x^2
+ c*x^4)^(3/2))/(9*b*x^(15/2)) - (4*c^(5/4)*(3*b*B - A*c)*x*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[b*x^2 + c*x^4]) + (2*c^(5/4)*(3
*b*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^
2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[b*
x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = -\frac{2A(bx^2+cx^4)^{3/2}}{9bx^{15/2}} - \frac{\left(2\left(-\frac{9bB}{2}+\frac{3Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx}{9b}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{9bx^{15/2}} + \frac{(2c(3bB-Ac))\int \frac{}{\sqrt{x}\,\sqrt{}}}{15b}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A(bx^2+cx^4)^{3}}{9bx^{15/2}}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A(bx^2+cx^4)^{3}}{9bx^{15/2}}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A(bx^2+cx^4)^{3}}{9bx^{15/2}}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A(bx^2+cx^4)^{3}}{9bx^{15/2}}$$

$$= \frac{4c^{3/2}(3bB-Ac)x^{3/2}(b+cx^2)}{15b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-}{15}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 99, normalized size = 0.27

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(5A(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+3(3bB-Ac)x^2\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)\right)}{45bx^{11/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(13/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(5*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + 3*(3*b*B -
 A*c)*x^2*Hypergeometric2F1[-5/4, -1/2, -1/4, -((c*x^2)/b)]))/(45*b*x^(11/2
)*Sqrt[1 + (c*x^2)/b])
```

Maple [**A**]
time = 0.39, size = 452, normalized size = 1.22

method	result
risch	$-\frac{2(-6Ac^2x^4+18x^4bBc+2Abcx^2+9b^2Bx^2+5b^2A)\sqrt{x^2\left(cx^2+b\right)}}{45x^{\frac{11}{2}}b^2}-\frac{2c(Ac-3Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\cdots}}{\cdots}$
default	$-\frac{2\sqrt{x^4c+bx^2}\left(6A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b\,c\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(c*x^4+b*x^2)^(1/2)/x^(11/2)/(c*x^2+b)*(6*A*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*b*c^2*x^4-3*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x^4-18*B*((c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(
-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),
1/2*2^(1/2))*b^2*c*x^4+9*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*
```

```
((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Elliptic
F(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c*x^4-6*A*c^3*x^
6+18*x^6*B*b*c^2-4*A*b*c^2*x^4+27*x^4*B*b^2*c+7*A*b^2*c*x^2+9*x^2*B*b^3+5*A
*b^3)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(13/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 103, normalized size = 0.28

$$-\frac{2\left(6\,(3\,Bbc-Ac^2)\sqrt{c}\,x^6\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+(6\,(3\,Bbc-Ac^2)x^4+5\,Ab^2+(9\,Bb^2+2\,Abc)x^2)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{45\,b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] -2/45*(6*(3*B*b*c - A*c^2)*sqrt(c)*x^6*weierstrassZeta(-4*b/c, 0, weierstra
ssPInverse(-4*b/c, 0, x)) + (6*(3*B*b*c - A*c^2)*x^4 + 5*A*b^2 + (9*B*b^2 +
 2*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^2*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(13/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(13/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(13/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(13/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(13/2), x)
```

3.230 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx$

Optimal. Leaf size=204

$$-\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}}-\frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}}-\frac{2c^{7/4}(11bB-5Ac)x\left(\sqrt{b}\right.}{}$$

```
[Out] -2/11*A*(c*x^4+b*x^2)^(3/2)/b/x^(17/2)-2/77*(-5*A*c+11*B*b)*(c*x^4+b*x^2)^(
1/2)/b/x^(9/2)-4/231*c*(-5*A*c+11*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^(5/2)-2/23
1*c^(7/4)*(-5*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/
2))^2)^(1/2)/b^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2063, 2045, 2050, 2057, 335, 226}

$$-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-5Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}(11bB-5Ac)}{231b^2x^{5/2}}-\frac{2\sqrt{bx^2+cx^4}(11bB-5Ac)}{77bx^{9/2}}-\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(15/2),x]
```

```
[Out] (-2*(11*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(77*b*x^(9/2)) - (4*c*(11*b*B - 5
*A*c)*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5/2)) - (2*A*(b*x^2 + c*x^4)^(3/2))/
(11*b*x^(17/2)) - (2*c^(7/4)*(11*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b
^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = -\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}} - \frac{\left(2\left(-\frac{11bB}{2}+\frac{5Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx}{11b}$$

$$= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}} + \frac{(2c(11bB-5Ac))\int -}{77b}$$

$$= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2A(bx^2+}{11bx^{17}}$$

$$= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2A(bx^2+}{11bx^{17}}$$

$$= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2A(bx^2+}{11bx^{17}}$$

$$= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2A(bx^2+}{11bx^{17}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 98, normalized size = 0.48

$$-\frac{2\sqrt{x^2(b+cx^2)}\left(7A(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+(11bB-5Ac)x^2\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)\right)}{77bx^{13/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(15/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(7*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (11*b*B -
5*A*c)*x^2*Hypergeometric2F1[-7/4, -1/2, -3/4, -((c*x^2)/b)]))/(77*b*x^(13/
2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.39, size = 283, normalized size = 1.39

method	result

risch	$-\frac{2(-10A\,c^2x^4+22x^4bBc+6Abc\,x^2+33b^2B\,x^2+21b^2A)\sqrt{x^2\,(c\,x^2+b)}}{231x^{\frac{13}{2}}b^2}+\frac{2c(5Ac-11Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\ \cdots}{\cdots}$
default	$\frac{2\sqrt{x^4c+b\,x^2}\left(5A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{\cdots}}{2}\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/231*(c*x^4+b*x^2)^(1/2)/x^(13/2)/(c*x^2+b)*(5*A*(-b*c)^(1/2)*((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*c^2*x^5-11*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))
^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x
^5+10*A*c^3*x^6-22*x^6*B*b*c^2+4*A*b*c^2*x^4-55*x^4*B*b^2*c-27*A*b^2*c*x^2-
33*x^2*B*b^3-21*A*b^3)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(15/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.74, size = 96, normalized size = 0.47

$$-\frac{2\left(2\,(11\,Bbc-5\,Ac^2)\sqrt{c}\,x^7\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)+\left(2\,(11\,Bbc-5\,Ac^2)x^4+21\,Ab^2+3\,(11\,Bb^2+2\,Abc)x^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{231\,b^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] -2/231*(2*(11*B*b*c - 5*A*c^2)*sqrt(c)*x^7*weierstrassPInverse(-4*b/c, 0, x
) + (2*(11*B*b*c - 5*A*c^2)*x^4 + 21*A*b^2 + 3*(11*B*b^2 + 2*A*b*c)*x^2)*sq
rt(c*x^4 + b*x^2)*sqrt(x))/(b^2*x^7)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(15/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(15/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}}{x^{15/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(15/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(15/2), x)
```

3.231 $\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=486

$$\frac{88b^5(3bB-5Ac)x^{3/2}(b+cx^2)}{16575c^{9/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{88b^4(3bB-5Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{49725c^4}+\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+c}}{69615c^3}$$

```
[Out] -2/105*(-5*A*c+3*B*b)*x^(9/2)*(c*x^4+b*x^2)^(3/2)/c+2/25*B*x^(5/2)*(c*x^4+b
*x^2)^(5/2)/c+88/16575*b^5*(-5*A*c+3*B*b)*x^(3/2)*(c*x^2+b)/c^(9/2)/(b^(1/2
)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+88/69615*b^3*(-5*A*c+3*B*b)*x^(5/2)*(c*x^4
+b*x^2)^(1/2)/c^3-8/7735*b^2*(-5*A*c+3*B*b)*x^(9/2)*(c*x^4+b*x^2)^(1/2)/c^2
-4/595*b*(-5*A*c+3*B*b)*x^(13/2)*(c*x^4+b*x^2)^(1/2)/c-88/49725*b^4*(-5*A*c
+3*B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^4-88/16575*b^(21/4)*(-5*A*c+3*B*b)*x*
(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*
(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(19/4)/(c*x^4
+b*x^2)^(1/2)+44/16575*b^(21/4)*(-5*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^
2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(19/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 486, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2064, 2046, 2049, 2057, 335, 311, 226, 1210}

$$\frac{44b^{21/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-5Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{16575c^{19/4}\sqrt{bx^2+cx^4}}-\frac{88b^{21/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-5Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{16575c^{19/4}\sqrt{bx^2+cx^4}}+\frac{88b^5x^{3/2}(b+cx^2)(3bB-5Ac)}{16575c^{9/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{88b^4\sqrt{x}\sqrt{bx^2+cx^4}(3bB-5Ac)}{49725c^4}+\frac{88b^3x^{5/2}\sqrt{bx^2+cx^4}(3bB-5Ac)}{69615c^3}-\frac{8b^2x^{9/2}\sqrt{bx^2+cx^4}(3bB-5Ac)}{7735c^2}-\frac{4bx^{13/2}\sqrt{bx^2+cx^4}(3bB-5Ac)}{595c}-\frac{2x^{9/2}\left(bx^2+cx^4\right)^{3/2}(3bB-5Ac)}{105c}+\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (88*b^5*(3*b*B - 5*A*c)*x^(3/2)*(b + c*x^2))/(16575*c^(9/2)*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[b*x^2 + c*x^4]) - (88*b^4*(3*b*B - 5*A*c)*Sqrt[x]*Sqrt[b*x^2 +
c*x^4])/(49725*c^4) + (88*b^3*(3*b*B - 5*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/
(69615*c^3) - (8*b^2*(3*b*B - 5*A*c)*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(7735*c^2
) - (4*b*(3*b*B - 5*A*c)*x^(13/2)*Sqrt[b*x^2 + c*x^4])/(595*c) - (2*(3*b*B
- 5*A*c)*x^(9/2)*(b*x^2 + c*x^4)^(3/2))/(105*c) + (2*B*x^(5/2)*(b*x^2 + c*x
^4)^(5/2))/(25*c) - (88*b^(21/4)*(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x]
)/b^(1/4)], 1/2])/(16575*c^(19/4)*Sqrt[b*x^2 + c*x^4]) + (44*b^(21/4)*(3*b*
B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(16575*c^(19/4)*Sqrt
[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{2Bx^{5/2}(bx^2+cx^4)^{5/2}}{25c}-\frac{\left(2\left(\frac{15bB}{2}-\frac{25Ac}{2}\right)\right)\int x^{7/2}(bx^2+cx^4)^{3/2}\,dx}{25c}$$

$$=-\frac{2(3bB-5Ac)x^{9/2}(bx^2+cx^4)^{3/2}}{105c}+\frac{2Bx^{5/2}(bx^2+cx^4)^{5/2}}{25c}-\ldots$$

$$=-\frac{4b(3bB-5Ac)x^{13/2}\sqrt{bx^2+cx^4}}{595c}-\frac{2(3bB-5Ac)x^{9/2}(bx^2+cx^4)^{3/2}}{105c}\ldots$$

$$=-\frac{8b^2(3bB-5Ac)x^{9/2}\sqrt{bx^2+cx^4}}{7735c^2}-\frac{4b(3bB-5Ac)x^{13/2}\sqrt{bx^2+cx^4}}{595c}\ldots$$

$$=\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3}-\frac{8b^2(3bB-5Ac)x^{9/2}\sqrt{bx^2+cx^4}}{7735c^2}\ldots$$

$$=-\frac{88b^4(3bB-5Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{49725c^4}+\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+\ldots}}{69615c^3}\ldots$$

$$=-\frac{88b^4(3bB-5Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{49725c^4}+\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+\ldots}}{69615c^3}\ldots$$

$$=-\frac{88b^4(3bB-5Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{49725c^4}+\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+\ldots}}{69615c^3}\ldots$$

$$=-\frac{88b^4(3bB-5Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{49725c^4}+\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+\ldots}}{69615c^3}\ldots$$

$$=\frac{88b^5(3bB-5Ac)x^{3/2}(b+cx^2)}{16575c^{9/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{88b^4(3bB-5Ac)\sqrt{x}\,\sqrt{bx^2\ldots}}{49725c^4}\ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.16, size = 160, normalized size = 0.33

$$\frac{2\sqrt{x}\,\sqrt{x^2(b+cx^2)}\left(-(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}\left(1155b^3B-221c^3x^4(25A+21Bx^2)-55b^2c(35A+39Bx^2)+65bc^2x^2(55A+51Bx^2)\right)+385b^4(3bB-5Ac)\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{116025c^4\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(1155 *b^3*B - 221*c^3*x^4*(25*A + 21*B*x^2) - 55*b^2*c*(35*A + 39*B*x^2) + 65*b* c^2*x^2*(55*A + 51*B*x^2))) + 385*b^4*(3*b*B - 5*A*c)*Hypergeometric2F1[-3/ 2, 3/4, 7/4, -((c*x^2)/b)]))/(116025*c^4*Sqrt[1 + (c*x^2)/b])

Maple [A]
time = 0.40, size = 518, normalized size = 1.07

method	result
risch	$\frac{2\sqrt{x}\,(13923B\,c^5x^{10}+16575A\,c^5x^8+17901Bb\,c^4x^8+22425Ab\,c^4x^6+468B\,b^2c^3x^6+900A\,b^2c^3x^4-540B\,b^3c^2x^4-1100A\,b^3c^2x^2+660}{348075c^4}$
default	$\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(13923B\,c^7x^{14}+16575A\,c^7x^{12}+31824Bb\,c^6x^{12}+39000Ab\,c^6x^{10}+18369B\,b^2c^5x^{10}+23325A\,b^2c^5x^8-72B\,b^3c^4x^8-200\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/348075*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^5*(13923*B*c^7*x^14+1657 5*A*c^7*x^12+31824*B*b*c^6*x^12+39000*A*b*c^6*x^10+18369*B*b^2*c^5*x^10+233 25*A*b^2*c^5*x^8-72*B*b^3*c^4*x^8-200*A*b^3*c^4*x^6+120*B*b^4*c^3*x^6+2310* A*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b* c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1 /2)*(-x*c/(-b*c)^(1/2))^(1/2)*b^6*c-4620*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^6* c-1386*B*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c *x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1 /2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b^7+2772*B*((c*x+(-b*c)^(1/2))/(-b*c)^ (1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^ (1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)) *b^7+440*A*b^4*c^3*x^4-264*B*b^5*c^2*x^4+1540*A*b^5*c^2*x^2-924*B*b^6*c*x^2)

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.79, size = 175, normalized size = 0.36

$$-\frac{2\left(924\,(3\,Bb^6-5\,Ab^5c)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-(13923\,Bc^6x^{10}+663\,(27\,Bbc^5+25\,Ac^6)x^8-924\,Bb^5c+1540\,Ab^4c^2+39\,(12\,Bb^2c^4+575\,Abc^5)x^6-180\,(3\,Bb^3c^3-5\,Ab^2c^4)x^4+220\,(3\,Bb^4c^2-5\,Ab^3c^3)x^2)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{348075\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/348075*(924*(3*B*b^6 - 5*A*b^5*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, wei
erstrassPInverse(-4*b/c, 0, x)) - (13923*B*c^6*x^10 + 663*(27*B*b*c^5 + 25*
A*c^6)*x^8 - 924*B*b^5*c + 1540*A*b^4*c^2 + 39*(12*B*b^2*c^4 + 575*A*b*c^5)
*x^6 - 180*(3*B*b^3*c^3 - 5*A*b^2*c^4)*x^4 + 220*(3*B*b^4*c^2 - 5*A*b^3*c^3
)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/c^5
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5984 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{7/2}\,\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.232 $\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=321

$$-\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}}+\frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3}-\frac{8b^2(13bB-23Ac)x^{7/2}\sqrt{bx^2+cx^4}}{24035c^2}$$

```
[Out] -2/437*(-23*A*c+13*B*b)*x^(7/2)*(c*x^4+b*x^2)^(3/2)/c+2/23*B*x^(3/2)*(c*x^4
+b*x^2)^(5/2)/c+72/168245*b^3*(-23*A*c+13*B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/
c^3-8/24035*b^2*(-23*A*c+13*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c^2-4/2185*b*(
-23*A*c+13*B*b)*x^(11/2)*(c*x^4+b*x^2)^(1/2)/c-24/33649*b^4*(-23*A*c+13*B*b
)*(c*x^4+b*x^2)^(1/2)/c^4/x^(1/2)+12/33649*b^(19/4)*(-23*A*c+13*B*b)*x*(cos
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(
1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(17/4)/(c*x^4+b*x
^2)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2064, 2046, 2049, 2057, 335, 226}

$$\frac{12b^{19/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(13bB-23Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{33649c^{17/4}\sqrt{bx^2+cx^4}}-\frac{24b^4\sqrt{bx^2+cx^4}(13bB-23Ac)}{33649c^4\sqrt{x}}+\frac{72b^3x^{3/2}\sqrt{bx^2+cx^4}(13bB-23Ac)}{168245c^3}-\frac{8b^2x^{7/2}\sqrt{bx^2+cx^4}(13bB-23Ac)}{24035c^2}-\frac{4bx^{11/2}\sqrt{bx^2+cx^4}(13bB-23Ac)}{2185c}-\frac{2x^{7/2}\left(bx^2+cx^4\right)^{3/2}(13bB-23Ac)}{437c}+\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-24*b^4*(13*b*B - 23*A*c)*Sqrt[b*x^2 + c*x^4])/(33649*c^4*Sqrt[x]) + (72*b
^3*(13*b*B - 23*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(168245*c^3) - (8*b^2*(13
*b*B - 23*A*c)*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(24035*c^2) - (4*b*(13*b*B - 23
*A*c)*x^(11/2)*Sqrt[b*x^2 + c*x^4])/(2185*c) - (2*(13*b*B - 23*A*c)*x^(7/2)
*(b*x^2 + c*x^4)^(3/2))/(437*c) + (2*B*x^(3/2)*(b*x^2 + c*x^4)^(5/2))/(23*c
) + (12*b^(19/4)*(13*b*B - 23*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(33649*c^(17/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{2Bx^{3/2}(bx^2+cx^4)^{5/2}}{23c} - \frac{\left(2\left(\frac{13bB}{2}-\frac{23Ac}{2}\right)\right)\int x^{5/2}(bx^2+cx^4)^{3/2}\,dx}{23c}$$

$$= -\frac{2(13bB-23Ac)x^{7/2}(bx^2+cx^4)^{3/2}}{437c} + \frac{2Bx^{3/2}(bx^2+cx^4)^{5/2}}{23c} - \frac{(6b(\cdots}{\cdots}$$

$$= -\frac{4b(13bB-23Ac)x^{11/2}\sqrt{bx^2+cx^4}}{2185c} - \frac{2(13bB-23Ac)x^{7/2}(bx^2+\cdots}{437c}$$

$$= -\frac{8b^2(13bB-23Ac)x^{7/2}\sqrt{bx^2+cx^4}}{24035c^2} - \frac{4b(13bB-23Ac)x^{11/2}\sqrt{bx^2\cdots}}{2185c}$$

$$= \frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3} - \frac{8b^2(13bB-23Ac)x^{7/2}\sqrt{bx^2\cdots}}{24035c^2}$$

$$= -\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}} + \frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+\cdots}}{168245c^3}$$

$$= -\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}} + \frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+\cdots}}{168245c^3}$$

$$= -\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}} + \frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+\cdots}}{168245c^3}$$

$$= -\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}} + \frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+\cdots}}{168245c^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.15, size = 160, normalized size = 0.50

$$\frac{2\sqrt{x^2(b+cx^2)}\left(-(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}\,(195b^3B-55c^3x^4(23A+19Bx^2)+11bc^2x^2(69A+65Bx^2)-3b^2c(115A+143Bx^2))+15b^4(13bB-23Ac)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{24035c^4\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(195*b^3*B -
55*c^3*x^4*(23*A + 19*B*x^2) + 11*b*c^2*x^2*(69*A + 65*B*x^2) - 3*b^2*c*(11
5*A + 143*B*x^2))) + 15*b^4*(13*b*B - 23*A*c)*Hypergeometric2F1[-3/2, 1/4,
5/4, -((c*x^2)/b)]))/(24035*c^4*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.39, size = 355, normalized size = 1.11

method	result
risch	$\frac{2\left(7315 B\,c^{5} x^{10}+8855 A\,c^{5} x^{8}+9625 B b\,c^{4} x^{8}+12397 A b\,c^{4} x^{6}+308 B\,b^{2} c^{3} x^{6}+644 A\,b^{2} c^{3} x^{4}-364 B\,b^{3} c^{2} x^{4}-828 A\,b^{3} c^{2} x^{2}+468 B\,b^{4} c\,x^{2}+13\right.}{168245 c^{4} \sqrt{x}}$
default	$-\frac{2\left(x^{4} c+b\,x^{2}\right)^{\frac{3}{2}} \left(-7315 B\,c^{7} x^{13}-8855 A\,c^{7} x^{11}-16940 B b\,c^{6} x^{11}-21252 A b\,c^{6} x^{9}-9933 B\,b^{2} c^{5} x^{9}-13041 A\,b^{2} c^{5} x^{7}+56 B\,b^{3} c^{4} x^{7}+690 A\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/168245*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(-7315*B*c^7*x^13-8855*A*
c^7*x^11-16940*B*b*c^6*x^11-21252*A*b*c^6*x^9-9933*B*b^2*c^5*x^9-13041*A*b^
2*c^5*x^7+56*B*b^3*c^4*x^7+690*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^
(1/2)*(-b*c)^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1
/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b
^5*c+184*A*b^3*c^4*x^5-390*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2
)*(-b*c)^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))
^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^6-1
04*B*b^4*c^3*x^5-552*A*b^4*c^3*x^3+312*B*b^5*c^2*x^3-1380*A*b^5*c^2*x+780*B
*b^6*c*x)/c^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.99, size = 170, normalized size = 0.53

$$\frac{2\left(60\,(13\,Bb^{6}-23\,Ab^{5}c)\sqrt{c}\,x\,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+(7315\,Bc^{6}x^{10}+385\,(25\,Bbc^{5}+23\,Ac^{6})x^{8}-780\,Bb^{5}c+1380\,Ab^{4}c^{2}+77\,(4\,Bb^{2}c^{4}+161\,Abc^{5})x^{6}-28\,(13\,Bb^{3}c^{3}-23\,Ab^{2}c^{4})x^{4}+36\,(13\,Bb^{4}c^{2}-23\,Ab^{3}c^{3})x^{2})\sqrt{cx^{4}+bx^{2}}\,\sqrt{x}\right)}{168245\,c^{5}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/168245*(60*(13*B*b^6 - 23*A*b^5*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c,
0, x) + (7315*B*c^6*x^10 + 385*(25*B*b*c^5 + 23*A*c^6)*x^8 - 780*B*b^5*c +
1380*A*b^4*c^2 + 77*(4*B*b^2*c^4 + 161*A*b*c^5)*x^6 - 28*(13*B*b^3*c^3 - 23
*A*b^2*c^4)*x^4 + 36*(13*B*b^4*c^2 - 23*A*b^3*c^3)*x^2)*sqrt(c*x^4 + b*x^2)
*sqrt(x))/(c^5*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4060 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{5/2}\,\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.233 $\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=447

$$-\frac{8b^4(11bB-21Ac)x^{3/2}(b+cx^2)}{3315c^{7/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{8b^3(11bB-21Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{9945c^3}-\frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+}}{13923c^2}$$

```
[Out] -2/357*(-21*A*c+11*B*b)*x^(5/2)*(c*x^4+b*x^2)^(3/2)/c+2/21*B*(c*x^4+b*x^2)^
(5/2)*x^(1/2)/c-8/3315*b^4*(-21*A*c+11*B*b)*x^(3/2)*(c*x^2+b)/c^(7/2)/(b^(1
/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-8/13923*b^2*(-21*A*c+11*B*b)*x^(5/2)*(c*
x^4+b*x^2)^(1/2)/c^2-4/1547*b*(-21*A*c+11*B*b)*x^(9/2)*(c*x^4+b*x^2)^(1/2)/
c+8/9945*b^3*(-21*A*c+11*B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^3+8/3315*b^(17/
4)*(-21*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)
^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)-4/3315*b^(17/4)*(-21*A*c+11*B*b)*x*(cos
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(
1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(15/4)/(c*x^4+b*x
^2)^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {2064, 2046, 2049, 2057, 335, 311, 226, 1210}

$$-\frac{4b^{17/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-21Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3315c^{15/4}\sqrt{bx^2+cx^4}}+\frac{8b^{17/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-21Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3315c^{15/4}\sqrt{bx^2+cx^4}}-\frac{8b^4x^{3/2}(b+cx^2)(11bB-21Ac)}{3315c^{7/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{8b^3\sqrt{x}\,\sqrt{bx^2+cx^4}(11bB-21Ac)}{9945c^3}-\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}(11bB-21Ac)}{13923c^2}-\frac{4bx^{9/2}\sqrt{bx^2+cx^4}(11bB-21Ac)}{1547c}-\frac{2x^{5/2}\left(bx^2+cx^4\right)^{3/2}(11bB-21Ac)}{357c}+\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-8*b^4*(11*b*B - 21*A*c)*x^(3/2)*(b + c*x^2))/(3315*c^(7/2)*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[b*x^2 + c*x^4]) + (8*b^3*(11*b*B - 21*A*c)*Sqrt[x]*Sqrt[b*x^2
+ c*x^4])/(9945*c^3) - (8*b^2*(11*b*B - 21*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4]
)/(13923*c^2) - (4*b*(11*b*B - 21*A*c)*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(1547*c
) - (2*(11*b*B - 21*A*c)*x^(5/2)*(b*x^2 + c*x^4)^(3/2))/(357*c) + (2*B*Sqrt
[x]*(b*x^2 + c*x^4)^(5/2))/(21*c) + (8*b^(17/4)*(11*b*B - 21*A*c)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3315*c^(15/4)*Sqrt[b*x^2 + c*x^4]) - (4
*b^(17/4)*(11*b*B - 21*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(33
15*c^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c} - \frac{\left(2\left(\frac{11bB}{2}-\frac{21Ac}{2}\right)\right)\int x^{3/2}(bx^2+cx^4)^{3/2}\,dx}{21c}$$

$$= -\frac{2(11bB-21Ac)x^{5/2}(bx^2+cx^4)^{3/2}}{357c} + \frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c} - \frac{(2b$$

$$= -\frac{4b(11bB-21Ac)x^{9/2}\sqrt{bx^2+cx^4}}{1547c} - \frac{2(11bB-21Ac)x^{5/2}(bx^2+c$$

$$= -\frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2} - \frac{4b(11bB-21Ac)x^{9/2}\sqrt{bx^2}$$

$$= \frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{9945c^3} - \frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+}$$

$$= \frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{9945c^3} - \frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+}$$

$$= \frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{9945c^3} - \frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+}$$

$$= \frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{9945c^3} - \frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+}$$

$$= -\frac{8b^4(11bB-21Ac)x^{3/2}(b+cx^2)}{3315c^{7/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.13, size = 138, normalized size = 0.31

$$\frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left((b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}\,(77b^2B+13c^2x^2(21A+17Bx^2)-bc(147A+143Bx^2))+7b^3(-11bB+21Ac)\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{4641c^3\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(77*b^2
*B + 13*c^2*x^2*(21*A + 17*B*x^2) - b*c*(147*A + 143*B*x^2)) + 7*b^3*(-11*b
```

```
*B + 21*A*c)*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(4641*c^3*Sq
rt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.38, size = 494, normalized size = 1.11

method	result
risch	$-\frac{2\sqrt{x}\,\left(-3315 B\,c^{4} x^{8}-4095 A\,c^{4} x^{6}-4485 B b\,c^{3} x^{6}-5985 A b\,c^{3} x^{4}-180 B\,b^{2} c^{2} x^{4}-420 A\,b^{2} c^{2} x^{2}+220 B\,b^{3} c\,x^{2}+588 A\,b^{3} c-308 B\,b^{4}\right) \sqrt{}}{69615 c^{3}}$
default	$\frac{2\left(x^{4} c+b\,x^{2}\right)^{\frac{3}{2}} \left(3315 B\,c^{6} x^{12}+4095 A\,c^{6} x^{10}+7800 B b\,c^{5} x^{10}+10080 A b\,c^{5} x^{8}+4665 B\,b^{2} c^{4} x^{8}+6405 A\,b^{2} c^{4} x^{6}-40 B\,b^{3} c^{3} x^{6}+1764 A \sqrt{\frac{cx}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/69615*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^4*(3315*B*c^6*x^12+4095*A
*c^6*x^10+7800*B*b*c^5*x^10+10080*A*b*c^5*x^8+4665*B*b^2*c^4*x^8+6405*A*b^2
*c^4*x^6-40*B*b^3*c^3*x^6+1764*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^
(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*El
lipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^5*c-882*A*((
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2),1/2*2^(1/2))*b^5*c-924*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/
2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^6+462*B
*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2),1/2*2^(1/2))*b^6-168*A*b^3*c^3*x^4+88*B*b^4*c^2*x^4-588*A*b
^4*c^2*x^2+308*B*b^5*c*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 150, normalized size = 0.34

$$\frac{2\left(84\,(11\,Bb^5-21\,Ab^4c)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(3315\,Bc^5x^8+195\,(23\,Bbc^4+21\,Ac^5)x^6+308\,Bb^4c-588\,Ab^3c^2+45\,(4\,Bb^2c^3+133\,Abc^4)x^4-20\,(11\,Bb^3c^2-21\,Ab^2c^3)x^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{69615\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/69615*(84*(11*B*b^5 - 21*A*b^4*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weie
rstrassPInverse(-4*b/c, 0, x)) + (3315*B*c^5*x^8 + 195*(23*B*b*c^4 + 21*A*c
^5)*x^6 + 308*B*b^4*c - 588*A*b^3*c^2 + 45*(4*B*b^2*c^3 + 133*A*b*c^4)*x^4
- 20*(11*B*b^3*c^2 - 21*A*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/c^4
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5984 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\,\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.234 $\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=282

$$\frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}}-\frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2}-\frac{4b(9bB-19Ac)x^{7/2}\sqrt{bx^2+cx^4}}{1045c}-\frac{2(9bE}{}$$

```
[Out] -2/285*(-19*A*c+9*B*b)*x^(3/2)*(c*x^4+b*x^2)^(3/2)/c+2/19*B*(c*x^4+b*x^2)^(
5/2)/c/x^(1/2)-8/7315*b^2*(-19*A*c+9*B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^2-4
/1045*b*(-19*A*c+9*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c+8/4389*b^3*(-19*A*c+9
*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)-4/4389*b^(15/4)*(-19*A*c+9*B*b)*x*(co
s(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/
b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^
(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(13/4)/(c*x^4+b*
x^2)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2064, 2046, 2049, 2057, 335, 226}

$$-\frac{4b^{15/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(9bB-19Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4389c^{13/4}\sqrt{bx^2+cx^4}}+\frac{8b^3\sqrt{bx^2+cx^4}\,(9bB-19Ac)}{4389c^3\sqrt{x}}-\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}\,(9bB-19Ac)}{7315c^2}-\frac{4bx^{7/2}\sqrt{bx^2+cx^4}\,(9bB-19Ac)}{1045c}-\frac{2x^{3/2}\left(bx^2+cx^4\right)^{3/2}(9bB-19Ac)}{285c}+\frac{2B\left(bx^2+cx^4\right)^{5/2}}{19c\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (8*b^3*(9*b*B - 19*A*c)*Sqrt[b*x^2 + c*x^4])/(4389*c^3*Sqrt[x]) - (8*b^2*(9
*b*B - 19*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7315*c^2) - (4*b*(9*b*B - 19*A
*c)*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(1045*c) - (2*(9*b*B - 19*A*c)*x^(3/2)*(b*
x^2 + c*x^4)^(3/2))/(285*c) + (2*B*(b*x^2 + c*x^4)^(5/2))/(19*c*Sqrt[x]) -
(4*b^(15/4)*(9*b*B - 19*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt
[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(4
389*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{2B(bx^2+cx^4)^{5/2}}{19c\sqrt{x}} - \frac{\left(2\left(\frac{9bB}{2}-\frac{19Ac}{2}\right)\right)\int \sqrt{x}\left(bx^2+cx^4\right)^{3/2}\,dx}{19c}$$

$$= -\frac{2(9bB-19Ac)x^{3/2}(bx^2+cx^4)^{3/2}}{285c} + \frac{2B(bx^2+cx^4)^{5/2}}{19c\sqrt{x}} - \frac{(2b(9bB-$$

$$= -\frac{4b(9bB-19Ac)x^{7/2}\sqrt{bx^2+cx^4}}{1045c} - \frac{2(9bB-19Ac)x^{3/2}(bx^2+cx^4)^{3}}{285c}$$

$$= -\frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2} - \frac{4b(9bB-19Ac)x^{7/2}\sqrt{bx^2+c}}{1045c}$$

$$= \frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}} - \frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2} -$$

$$= \frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}} - \frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2} -$$

$$= \frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}} - \frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2} -$$

$$= \frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}} - \frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.13, size = 138, normalized size = 0.49

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(\left(b+cx^2\right)^2\sqrt{1+\frac{cx^2}{b}}\left(45b^2B+11c^2x^2(19A+15Bx^2)-bc(95A+99Bx^2)\right)+5b^3(-9bB+19Ac)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{3135c^3\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(45*b^2*B + 11*
c^2*x^2*(19*A + 15*B*x^2) - b*c*(95*A + 99*B*x^2)) + 5*b^3*(-9*b*B + 19*A*c
)*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(3135*c^3*Sqrt[x]*Sqrt[
1 + (c*x^2)/b])
```

Maple [A]

time = 0.53, size = 331, normalized size = 1.17

method	result
risch	$-\frac{2(-1155B\,c^4x^8-1463A\,c^4x^6-1617Bb\,c^3x^6-2261Ab\,c^3x^4-84B\,b^2c^2x^4-228A\,b^2c^2x^2+108B\,b^3c\,x^2+380A\,b^3c-180B\,b^4)\sqrt{x^2(}}{21945c^3\sqrt{x}}$
default	$\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(1155B\,c^6x^{11}+1463A\,c^6x^9+2772Bb\,c^5x^9+3724Ab\,c^5x^7+1701B\,b^2c^4x^7+190A\sqrt{-\frac{xc}{\sqrt{-bc}}}\ \text{EllipticF}\left(\sqrt{\frac{cx+}{\sqrt{}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21945*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(1155*B*c^6*x^11+1463*A*c^6
*x^9+2772*B*b*c^5*x^9+3724*A*b*c^5*x^7+1701*B*b^2*c^4*x^7+190*A*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*(-b*c)^(1/2)*b^4*c+2489*A*b^2*c^4*x^5-90*B*(-x*c/(-b*c)^(1
/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*(-b*c)^(1/2)*b^5-24*B*b^3*c^3*x^5-152*A*b^3*c^3*x^3+72*B*b^4*c
^2*x^3-380*A*b^4*c^2*x+180*B*b^5*c*x)/c^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*sqrt(x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.96, size = 147, normalized size = 0.52

$$-\frac{2\left(20\,(9\,Bb^5-19\,Ab^4c)\sqrt{c}\,x\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-(1155\,Bc^5x^8+77\,(21\,Bbc^4+19\,Ac^5)x^6+180\,Bb^4c-380\,Ab^3c^2+7\,(12\,Bb^2c^3+323\,Abc^4)x^4-12\,(9\,Bb^3c^2-19\,Ab^2c^3)x^2)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{21945\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] -2/21945*(20*(9*B*b^5 - 19*A*b^4*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0
, x) - (1155*B*c^5*x^8 + 77*(21*B*b*c^4 + 19*A*c^5)*x^6 + 180*B*b^4*c - 380
```

```
*A*b^3*c^2 + 7*(12*B*b^2*c^3 + 323*A*b*c^4)*x^4 - 12*(9*B*b^3*c^2 - 19*A*b^
2*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}} \left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{x}\,\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.235 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=408

$$\frac{8b^3(7bB-17Ac)x^{3/2}(b+cx^2)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{8b^2(7bB-17Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{3315c^2}-\frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c}$$

```
[Out] 2/17*B*(c*x^4+b*x^2)^(5/2)/c/x^(3/2)-2/221*(-17*A*c+7*B*b)*(c*x^4+b*x^2)^(3
/2)*x^(1/2)/c+8/1105*b^3*(-17*A*c+7*B*b)*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)
+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-4/663*b*(-17*A*c+7*B*b)*x^(5/2)*(c*x^4+b*x^
2)^(1/2)/c-8/3315*b^2*(-17*A*c+7*B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^2-8/110
5*b^(13/4)*(-17*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/
2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1
/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)+4/1105*b^(13/4)*(-17*A*c+7*B*b)*
x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2)
)*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x
^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2064, 2046, 2049, 2057, 335, 311, 226, 1210}

$$\frac{4b^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-17Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}-\frac{8b^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-17Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}+\frac{8b^3x^{3/2}(b+cx^2)(7bB-17Ac)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}(7bB-17Ac)}{3315c^2}-\frac{2\sqrt{x}(bx^2+cx^4)^{3/2}(7bB-17Ac)}{221c}-\frac{4bx^{5/2}\sqrt{bx^2+cx^4}(7bB-17Ac)}{663c}+\frac{2B(bx^2+cx^4)^{5/2}}{17cx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/Sqrt[x],x]
```

```
[Out] (8*b^3*(7*b*B - 17*A*c)*x^(3/2)*(b + c*x^2))/(1105*c^(5/2)*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[b*x^2 + c*x^4]) - (8*b^2*(7*b*B - 17*A*c)*Sqrt[x]*Sqrt[b*x^2 + c
*x^4])/(3315*c^2) - (4*b*(7*b*B - 17*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(663
*c) - (2*(7*b*B - 17*A*c)*Sqrt[x]*(b*x^2 + c*x^4)^(3/2))/(221*c) + (2*B*(b*
x^2 + c*x^4)^(5/2))/(17*c*x^(3/2)) - (8*b^(13/4)*(7*b*B - 17*A*c)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(1105*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (4
*b^(13/4)*(7*b*B - 17*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b
] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(110
5*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \frac{2B(bx^2+cx^4)^{5/2}}{17cx^{3/2}} - \frac{\left(2\left(\frac{7bB}{2}-\frac{17Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx}{17c}$$

$$= -\frac{2(7bB-17Ac)\sqrt{x}\,(bx^2+cx^4)^{3/2}}{221c} + \frac{2B(bx^2+cx^4)^{5/2}}{17cx^{3/2}} - \frac{(6b(7bB-17A}{}$$

$$= -\frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB-17Ac)\sqrt{x}\,(bx^2+cx^4)^{3/2}}{221c} +$$

$$= -\frac{8b^2(7bB-17Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} -$$

$$= -\frac{8b^2(7bB-17Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} -$$

$$= -\frac{8b^2(7bB-17Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} -$$

$$= -\frac{8b^2(7bB-17Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} -$$

$$= \frac{8b^3(7bB-17Ac)x^{3/2}(b+cx^2)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{8b^2(7bB-17Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{3315c^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.12, size = 115, normalized size = 0.28

$$\frac{2\sqrt{x}\,\sqrt{x^2(b+cx^2)}\left(-(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}\,(7bB-17Ac-13Bcx^2)+b^2(7bB-17Ac)\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{221c^2\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(7*b*
B - 17*A*c - 13*B*c*x^2)) + b^2*(7*b*B - 17*A*c)*Hypergeometric2F1[-3/2, 3/
4, 7/4, -((c*x^2)/b)]))/(221*c^2*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.42, size = 470, normalized size = 1.15

method	result
risch	$\frac{2\sqrt{x}\,(195 B c^{3} x^{6}+255 A c^{3} x^{4}+285 B b c^{2} x^{4}+425 A b c^{2} x^{2}+20 B b^{2} c x^{2}+68 A b^{2} c-28 B b^{3}) \sqrt{x^{2} (c x^{2}+b)}}{3315 c^{2}} - \frac{4 b^{3} (17 A c-7 B b)}{}$
default	$-\frac{2 (x^{4} c+b x^{2})^{\frac{3}{2}} \left(-195 B c^{5} x^{10}-255 A c^{5} x^{8}-480 B b c^{4} x^{8}-680 A b c^{4} x^{6}-305 B b^{2} c^{3} x^{6}+204 A \sqrt{-\frac{x c}{\sqrt{-b c}}}\, \mathrm{EllipticE}\left(\sqrt{\frac{c x+\sqrt{}}{\sqrt{-}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3315*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^3*(-195*B*c^5*x^10-255*A*
c^5*x^8-480*B*b*c^4*x^8-680*A*b*c^4*x^6-305*B*b^2*c^3*x^6+204*A*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*b^4*c-102*A*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*b^4*c-84*B*(-x*c/(
-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^
(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*b^5+42*B*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*b^5-493*A*b^2*c^3
*x^4+8*B*b^3*c^2*x^4-68*A*b^3*c^2*x^2+28*B*b^4*c*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/sqrt(x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.74, size = 127, normalized size = 0.31

$$-\frac{2\left(12\left(7 Bb^{4}-17 Ab^{3}c\right)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)-\left(195 Bc^{4}x^{6}-28 Bb^{3}c+68 Ab^{2}c^{2}+15\left(19 Bbc^{3}+17 Ac^{4}\right)x^{4}+5\left(4 Bb^{2}c^{2}+85 Abc^{3}\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}}\,\sqrt{x}\right)}{3315\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3315*(12*(7*B*b^4 - 17*A*b^3*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weier
strassPInverse(-4*b/c, 0, x)) - (195*B*c^4*x^6 - 28*B*b^3*c + 68*A*b^2*c^2
+ 15*(19*B*b*c^3 + 17*A*c^4)*x^4 + 5*(4*B*b^2*c^2 + 85*A*b*c^3)*x^2)*sqrt(c
*x^4 + b*x^2)*sqrt(x))/c^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}(b+cx^{2})\right)^{\frac{3}{2}}\left(A+Bx^{2}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/sqrt(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^{2}+A\right)\,{\left(c\,x^{4}+b\,x^{2}\right)}^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(1/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(1/2), x)
```

3.236 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}-\frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c}-\frac{2(bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{33c\sqrt{x}}+\frac{2B(bx^2+cx^4}{15cx^{5/2}}$$

[Out] 2/15*B*(c*x^4+b*x^2)^(5/2)/c/x^(5/2)-2/33*(-3*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/c/x^(1/2)-4/77*b*(-3*A*c+B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c-8/231*b^2*(-3*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+4/231*b^(11/4)*(-3*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(9/4)/(c*x^4+b*x^2)^(1/2)

Rubi [A]
time = 0.26, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2064, 2046, 2049, 2057, 335, 226}

$$\frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(bB-3Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{8b^2\sqrt{bx^2+cx^4}(bB-3Ac)}{231c^2\sqrt{x}}-\frac{2(bx^2+cx^4)^{3/2}(bB-3Ac)}{33c\sqrt{x}}-\frac{4bx^{3/2}\sqrt{bx^2+cx^4}(bB-3Ac)}{77c}+\frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2),x]

[Out] (-8*b^2*(b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) - (4*b*(b*B - 3*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(77*c) - (2*(b*B - 3*A*c)*(b*x^2 + c*x^4)^(3/2))/(33*c*Sqrt[x]) + (2*B*(b*x^2 + c*x^4)^(5/2))/(15*c*x^(5/2)) + (4*b^(11/4)*(b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*c^(9/4)*Sqrt[b*x^2 + c*x^4])

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}} - \frac{\left(2\left(\frac{5bB}{2}-\frac{15Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx}{15c}$$

$$= -\frac{2(bB-3Ac)(bx^2+cx^4)^{3/2}}{33c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}} - \frac{(2b(bB-3Ac))\int \sqrt{}}{11}$$

$$= -\frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)(bx^2+cx^4)^{3/2}}{33c\sqrt{x}} + \frac{2B(bx}{1}$$

$$= -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB}{}$$

$$= -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB}{}$$

$$= -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB}{}$$

$$= -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.12, size = 115, normalized size = 0.48

$$\frac{2\sqrt{x^2(b+cx^2)}\left(-(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}(5bB-15Ac-11Bcx^2)+5b^2(bB-3Ac)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{165c^2\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(5*b*B - 15*A
*c - 11*B*c*x^2)) + 5*b^2*(b*B - 3*A*c)*Hypergeometric2F1[-3/2, 1/4, 5/4, -
((c*x^2)/b)]))/(165*c^2*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.38, size = 307, normalized size = 1.28

method	result
risch	$\frac{2(77B\,c^3x^6+105A\,c^3x^4+119Bb\,c^2x^4+195Ab\,c^2x^2+12B\,b^2c\,x^2+60A\,b^2c-20B\,b^3)\sqrt{x^2\,(c\,x^2+b)}}{1155c^2\sqrt{x}} - \frac{4b^3(3Ac-Bb)\sqrt{-bc}\,\sqrt{}}{}$
default	$-\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(-77B\,c^5x^9-105A\,c^5x^7-196Bb\,c^4x^7+30A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/1155*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(-77*B*c^5*x^9-105*A*c^5*x^
7-196*B*b*c^4*x^7+30*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^3*c-300*A
*b*c^4*x^5-10*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^4-131*B*b^2*c^3*
x^5-255*A*b^2*c^3*x^3+8*B*b^3*c^2*x^3-60*A*b^3*c^2*x+20*B*b^4*c*x)/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.54, size = 121, normalized size = 0.51

$$\frac{2\left(20\,(Bb^4-3\,Ab^3c)\sqrt{c}\;x\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)+\left(77\,Bc^4x^6-20\,Bb^3c+60\,Ab^2c^2+7\,(17\,Bbc^3+15\,Ac^4)x^4+3\,(4\,Bb^2c^2+65\,Abc^3)x^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{1155\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(20*(B*b^4 - 3*A*b^3*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x)
+ (77*B*c^4*x^6 - 20*B*b^3*c + 60*A*b^2*c^2 + 7*(17*B*b*c^3 + 15*A*c^4)*x^4
 + 3*(4*B*b^2*c^2 + 65*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(b + cx^{2}\right)\right)^{\frac{3}{2}}\left(A + Bx^{2}\right)}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(3/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2), x)
```

3.237 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=369

$$-\frac{8b^2(3bB-13Ac)x^{3/2}(b+cx^2)}{195c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{4b(3bB-13Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{195c}-\frac{2(3bB-13Ac)\left(bx^2+cx^4\right)^{3/2}}{117cx^{3/2}}+2$$

```
[Out] -2/117*(-13*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/c/x^(3/2)+2/13*B*(c*x^4+b*x^2)^(
5/2)/c/x^(7/2)-8/195*b^2*(-13*A*c+3*B*b)*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)
+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-4/195*b*(-13*A*c+3*B*b)*x^(1/2)*(c*x^4+b*x^
2)^(1/2)/c+8/195*b^(9/4)*(-13*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)-4/195*b^(9/4)*(-13*
A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),
1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^
(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2064, 2046, 2057, 335, 311, 226, 1210}

$$-\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-13Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{7/4}\sqrt{bx^2+cx^4}}+\frac{8b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3bB-13Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{7/4}\sqrt{bx^2+cx^4}}-\frac{8b^2x^{3/2}(b+cx^2)(3bB-13Ac)}{195c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{4b\sqrt{x}\sqrt{bx^2+cx^4}(3bB-13Ac)}{195c}-\frac{2(bx^2+cx^4)^{3/2}(3bB-13Ac)}{117cx^{3/2}}+\frac{2B(bx^2+cx^4)^{5/2}}{13cx^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2),x]
```

```
[Out] (-8*b^2*(3*b*B - 13*A*c)*x^(3/2)*(b + c*x^2))/(195*c^(3/2)*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[b*x^2 + c*x^4]) - (4*b*(3*b*B - 13*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x
^4])/(195*c) - (2*(3*b*B - 13*A*c)*(b*x^2 + c*x^4)^(3/2))/(117*c*x^(3/2)) +
 (2*B*(b*x^2 + c*x^4)^(5/2))/(13*c*x^(7/2)) + (8*b^(9/4)*(3*b*B - 13*A*c)*x
*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[
2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(7/4)*Sqrt[b*x^2 + c*x^4]
) - (4*b^(9/4)*(3*b*B - 13*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(S
qrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(195*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
```

```
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = \frac{2B(bx^2+cx^4)^{5/2}}{13cx^{7/2}} - \frac{\left(2\left(\frac{3bB}{2}-\frac{13Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx}{13c}$$

$$= -\frac{2(3bB-13Ac)(bx^2+cx^4)^{3/2}}{117cx^{3/2}} + \frac{2B(bx^2+cx^4)^{5/2}}{13cx^{7/2}} - \frac{(2b(3bB-13Ac))}{39}$$

$$= -\frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)(bx^2+cx^4)^{3/2}}{117cx^{3/2}} + 2B$$

$$= -\frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)(bx^2+cx^4)^{3/2}}{117cx^{3/2}} + 2B$$

$$= -\frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)(bx^2+cx^4)^{3/2}}{117cx^{3/2}} + 2B$$

$$= -\frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)(bx^2+cx^4)^{3/2}}{117cx^{3/2}} + 2B$$

$$= -\frac{8b^2(3bB-13Ac)x^{3/2}(b+cx^2)}{195c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 98, normalized size = 0.27

$$\frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left(3B(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}+b(-3bB+13Ac)\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{39c\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2),x]
```

[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(3*B*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b
*(-3*b*B + 13*A*c)*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(39*c*
Sqrt[1 + (c*x^2)/b])

Maple [A]
time = 0.39, size = 446, normalized size = 1.21

method	result
risch	$\frac{2\sqrt{x}\,(45Bc^2x^4+65Ac^2x^2+75bBx^2c+143Abc+12b^2B)\sqrt{x^2(cx^2+b)}}{585c}+\frac{4b^2(13Ac-3Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\cdots}{\cdots}$
default	$\frac{2(x^4c+bx^2)^{\frac{3}{2}}\left(45Bc^4x^8+65Ac^4x^6+120Bbc^3x^6+156Ab^3c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{Elliptic}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/585*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^2*(45*B*c^4*x^8+65*A*c^4*x^
6+120*B*b*c^3*x^6+156*A*b^3*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/
2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellip
ticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-78*A*b^3*c*((c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2),1/2*2^(1/2))-36*B*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1
/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Elli
pticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+18*B*b^4*((c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2),1/2*2^(1/2))+208*A*b*c^3*x^4+87*B*b^2*c^2*x^4+143*A*b^2*c^2*x^2+12*
B*b^3*c*x^2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.71, size = 102, normalized size = 0.28

$$\frac{2\left(12\,(3\,Bb^3-13\,Ab^2c)\sqrt{c}\ \text{weierstrassZeta}\left(-\frac{4b}{c},0,\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+\left(45\,Bc^3x^4+12\,Bb^2c+143\,Abc^2+5\,(15\,Bbc^2+13\,Ac^3)x^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{585\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(12*(3*B*b^3 - 13*A*b^2*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierst
rassPInverse(-4*b/c, 0, x)) + (45*B*c^3*x^4 + 12*B*b^2*c + 143*A*b*c^2 + 5*
(15*B*b*c^2 + 13*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/c^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}\left(A+Bx^2\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(5/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2), x)
```

3.238 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx$

Optimal. Leaf size=201

$$-\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}}-\frac{2(bB-11Ac)(bx^2+cx^4)^{3/2}}{77cx^{5/2}}+\frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}}-\frac{4b^{7/4}(bB-11Ac)x\left(\sqrt{b}\right.}{}$$

```
[Out] -2/77*(-11*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/c/x^(5/2)+2/11*B*(c*x^4+b*x^2)^(5/2
)/c/x^(9/2)-4/77*b*(-11*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)-4/77*b^(7/4)
*(-11*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2
)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2064, 2046, 2057, 335, 226}

$$-\frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(bB-11Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}}-\frac{4b\sqrt{bx^2+cx^4}(bB-11Ac)}{77c\sqrt{x}}-\frac{2(bx^2+cx^4)^{3/2}(bB-11Ac)}{77cx^{5/2}}+\frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2),x]
```

```
[Out] (-4*b*(b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4])/(77*c*Sqrt[x]) - (2*(b*B - 11*A*c
)*(b*x^2 + c*x^4)^(3/2))/(77*c*x^(5/2)) + (2*B*(b*x^2 + c*x^4)^(5/2))/(11*c
*x^(9/2)) - (4*b^(7/4)*(b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(77*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\,dx = \frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}} - \frac{\left(2\left(\frac{bB}{2}-\frac{11Ac}{2}\right)\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\,dx}{11c}$$

$$= -\frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}} - \frac{(6b(bB-11Ac))\int}{77}$$

$$= -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B(bx^2}{11c}$$

$$= -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B(bx^2}{11c}$$

$$= -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B(bx^2}{11c}$$

$$= -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B(bx^2}{11c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 97, normalized size = 0.48

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(B(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}+b(-bB+11Ac)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{11c\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(B*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(-(b*B) +
 11*A*c)*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(11*c*Sqrt[x]*Sq
rt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.38, size = 283, normalized size = 1.41

method	result

risch	$\frac{2(7B c^2x^4+11A c^2x^2+13bB x^2c+33Abc+4b^2B)\sqrt{x^2(cx^2+b)}}{77c\sqrt{x}}+\frac{4b^2(11Ac-Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x\right.}{}}}{}$
default	$\frac{2(x^4c+b x^2)^{\frac{3}{2}}\left(7B c^4x^7+22A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/77*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(7*B*c^4*x^7+22*A*((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*(-b*c)^(1/2)*b^2*c+11*A*c^4*x^5-2*B*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*(-b*c)^(1/2)*b^3+20*B*b*c^3*x^5+44*A*b*c^3*x^3+17*B*b^2*c^2*x^3+33*A*b
^2*c^2*x+4*B*b^3*c*x)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.81, size = 97, normalized size = 0.48

$$-\frac{2\left(4\left(Bb^3-11\,Ab^2c\right)\sqrt{c}\,x\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)-\left(7\,Bc^3x^4+4\,Bb^2c+33\,Abc^2+\left(13\,Bbc^2+11\,Ac^3\right)x^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{77\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] -2/77*(4*(B*b^3 - 11*A*b^2*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) -
 (7*B*c^3*x^4 + 4*B*b^2*c + 33*A*b*c^2 + (13*B*b*c^2 + 11*A*c^3)*x^2)*sqrt(
c*x^4 + b*x^2)*sqrt(x))/(c^2*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(7/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2), x)
```

3.239 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx$

Optimal. Leaf size=356

$$\frac{8b(bB+9Ac)x^{3/2}(b+cx^2)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{4}{15}(bB+9Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}+\frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}}-\frac{2A(bx^2+}{bx^{11}}$$

```
[Out] 2/9*(9*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(3/2)-2*A*(c*x^4+b*x^2)^(5/2)/b/x^(
11/2)+8/15*b*(9*A*c+B*b)*x^(3/2)*(c*x^2+b)/c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x
^4+b*x^2)^(1/2)+4/15*(9*A*c+B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)-8/15*b^(5/4)*(
9*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),
1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^
(3/4)/(c*x^4+b*x^2)^(1/2)+4/15*b^(5/4)*(9*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF
(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((
c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2063, 2046, 2057, 335, 311, 226, 1210}

$$\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9Ac+bB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}-\frac{8b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9Ac+bB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}+\frac{4}{15}\sqrt{x}\sqrt{bx^2+cx^4}(9Ac+bB)+\frac{2\left(bx^2+cx^4\right)^{3/2}(9Ac+bB)}{9bx^{3/2}}+\frac{8bx^{3/2}\left(b+cx^2\right)(9Ac+bB)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{bx^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2),x]
```

```
[Out] (8*b*(b*B + 9*A*c)*x^(3/2)*(b + c*x^2))/(15*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) + (4*(b*B + 9*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/15 + (2
*(b*B + 9*A*c)*(b*x^2 + c*x^4)^(3/2))/(9*b*x^(3/2)) - (2*A*(b*x^2 + c*x^4)^
(5/2))/(b*x^(11/2)) - (8*b^(5/4)*(b*B + 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4]) + (4*b^(5/4)*(b*B + 9*A*c)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4
])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
```

```
n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx = -\frac{2A(bx^2+cx^4)^{5/2}}{bx^{11/2}} - \frac{\left(2\left(-\frac{bB}{2}-\frac{9Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx}{b}$$

$$= \frac{2(bB+9Ac)(bx^2+cx^4)^{3/2}}{9bx^{3/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{bx^{11/2}} + \frac{1}{3}(2(bB+9Ac))\int \frac{\sqrt{b\ldots}}{\ldots}$$

$$= \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bB+9Ac)(bx^2+cx^4)^{3/2}}{9bx^{3/2}} - \frac{2A(bx^2+\ldots}{bx^{11\ldots}}$$

$$= \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bB+9Ac)(bx^2+cx^4)^{3/2}}{9bx^{3/2}} - \frac{2A(bx^2+\ldots}{bx^{11\ldots}}$$

$$= \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bB+9Ac)(bx^2+cx^4)^{3/2}}{9bx^{3/2}} - \frac{2A(bx^2+\ldots}{bx^{11\ldots}}$$

$$= \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bB+9Ac)(bx^2+cx^4)^{3/2}}{9bx^{3/2}} - \frac{2A(bx^2+\ldots}{bx^{11\ldots}}$$

$$= \frac{8b(bB+9Ac)x^{3/2}(b+cx^2)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 85, normalized size = 0.24

$$\frac{2\sqrt{x^2(b+cx^2)}\left(-\frac{3A(b+cx^2)^2}{b} + \frac{(bB+9Ac)x^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)}{\sqrt{1+\frac{cx^2}{b}}}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((-3*A*(b + c*x^2)^2)/b + ((b*B + 9*A*c)*x^2*Hyper
geometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)])/Sqrt[1 + (c*x^2)/b]))/(3*x^(3/2
))
```

Maple [A]
time = 0.38, size = 429, normalized size = 1.21

method	result
risch	$-\frac{2(-5Bcx^4-9Acx^2-11bBx^2+45Ab)\sqrt{x^2(cx^2+b)}}{45x^{\frac{3}{2}}}+\frac{4b(9Ac+Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-b\ldots}}{c}\ldots\right)}{\sqrt{-bc}}}\ldots}{\ldots}$
default	$\frac{2(x^4c+bx^2)^{\frac{3}{2}}\left(5Bc^3x^6+108Ab^2c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\ldots\right)\ldots\right)}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(5*B*c^3*x^6+108*A*b^2*c*((c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2),1/2*2^(1/2))-54*A*b^2*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2
^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*E
llipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+12*B*b^3*((c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2),1/2*2^(1/2))-6*B*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(
1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ell
ipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+9*A*c^3*x^4+16*
B*b*c^2*x^4-36*A*b*c^2*x^2+11*B*b^2*c*x^2-45*A*b^2*c)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(9/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 94, normalized size = 0.26

$$-\frac{2\left(12\,(Bb^2+9\,Abc)\sqrt{c}\,x^2\mathrm{weierstrassZeta}\left(-\frac{4b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)-(5\,Bc^2x^4-45\,Abc+(11\,Bbc+9\,Ac^2)x^2)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{45\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="fricas")

[Out] -2/45*(12*(B*b^2 + 9*A*b*c)*sqrt(c)*x^2*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) - (5*B*c^2*x^4 - 45*A*b*c + (11*B*b*c + 9*A*c^2)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c*x^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(9/2),x)

[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(9/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(9/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(B\,x^2+A)\,{(c\,x^4+b\,x^2)}^{3/2}}{x^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2),x)

[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2), x)

3.240 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx$

Optimal. Leaf size=200

$$\frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}}+\frac{2(3bB+7Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{5/2}}-\frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}}+\frac{4b^{3/4}(3bB+7Ac)x\left(\sqrt{b}+\right.}{}$$

```
[Out] 2/21*(7*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(5/2)-2/3*A*(c*x^4+b*x^2)^(5/2)/
b/x^(13/2)+4/21*(7*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/x^(1/2)+4/21*b^(3/4)*(7*A
*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1
/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(
1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2063, 2046, 2057, 335, 226}

$$\frac{4b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7Ac+3bB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{4\sqrt{bx^2+cx^4}\,(7Ac+3bB)}{21\sqrt{x}}+\frac{2(bx^2+cx^4)^{3/2}(7Ac+3bB)}{21bx^{5/2}}-\frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2),x]
```

```
[Out] (4*(3*b*B + 7*A*c)*Sqrt[b*x^2 + c*x^4])/(21*Sqrt[x]) + (2*(3*b*B + 7*A*c)*(
b*x^2 + c*x^4)^(3/2))/(21*b*x^(5/2)) - (2*A*(b*x^2 + c*x^4)^(5/2))/(3*b*x^(
13/2)) + (4*b^(3/4)*(3*b*B + 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1
/2])/(21*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = -\frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}} - \frac{\left(2\left(-\frac{3bB}{2}-\frac{7Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx}{3b}$$

$$= \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}} + \frac{1}{7}(2(3bB+7Ac))\int$$

$$= \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+c}{3bx^{13/}}$$

$$= \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+c}{3bx^{13/}}$$

$$= \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+c}{3bx^{13/}}$$

$$= \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+c}{3bx^{13/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 101, normalized size = 0.50

$$-\frac{2\sqrt{x^2(b+cx^2)}\left(A(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}} - b(3bB+7Ac)x^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{3bx^{5/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] - b*(3*b*B +
 7*A*c)*x^2*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(3*b*x^(5/2)*
Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.39, size = 260, normalized size = 1.30

method	result

risch	$-\frac{2(-3Bc\,x^4-7Ac\,x^2-9bB\,x^2+7Ab)\sqrt{x^2\,(c\,x^2+b)}}{21x^{\frac{5}{2}}}+\frac{4b(7Ac+3Bb)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right.}{\sqrt{-bc}}}}{}$
default	$\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(14A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}\,b\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*(c*x^4+b*x^2)^(3/2)/x^(9/2)/(c*x^2+b)^2*(14*A*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*
c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))*(-b*c)^(1/2)*b*c*x+6*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*
((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Elliptic
F(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^2*x+3
*B*c^3*x^6+7*A*c^3*x^4+12*B*b*c^2*x^4+9*B*b^2*c*x^2-7*A*b^2*c)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.63, size = 86, normalized size = 0.43

$$\frac{2\left(4\,(3\,Bb^2+7\,Abc)\sqrt{c}\,x^3\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+(3\,Bc^2x^4-7\,Abc+(9\,Bbc+7\,Ac^2)x^2)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{21\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] 2/21*(4*(3*B*b^2 + 7*A*b*c)*sqrt(c)*x^3*weierstrassPInverse(-4*b/c, 0, x) +
 (3*B*c^2*x^4 - 7*A*b*c + (9*B*b*c + 7*A*c^2)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt
(x))/(c*x^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(11/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(11/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(11/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2), x)
```

3.241 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx$

Optimal. Leaf size=354

$$\frac{24\sqrt{c}\,(bB+Ac)x^{3/2}(b+cx^2)}{5\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{12c(bB+Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{5b}-\frac{2(bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}}-\frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}}$$

```
[Out] -2*(A*c+B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(7/2)-2/5*A*(c*x^4+b*x^2)^(5/2)/b/x^(1
5/2)+24/5*(A*c+B*b)*x^(3/2)*(c*x^2+b)*c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*
x^2)^(1/2)+12/5*c*(A*c+B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)/b-24/5*b^(1/4)*c^(1
/4)*(A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
)),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)
/(c*x^4+b*x^2)^(1/2)+12/5*b^(1/4)*c^(1/4)*(A*c+B*b)*x*(cos(2*arctan(c^(1/4)
*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Elliptic
F(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*(
(c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2063, 2045, 2046, 2057, 335, 311, 226, 1210}

$$\frac{12\sqrt[4]{b}\,\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(Ac+bB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}-\frac{24\sqrt[4]{b}\,\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(Ac+bB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}+\frac{12c\sqrt{x}\,\sqrt{bx^2+cx^4}\,(Ac+bB)}{5b}-\frac{2(bx^2+cx^4)^{3/2}(Ac+bB)}{bx^{7/2}}+\frac{24\sqrt{c}\,x^{3/2}(b+cx^2)(Ac+bB)}{5\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2),x]
```

```
[Out] (24*Sqrt[c]*(b*B + A*c)*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
b*x^2 + c*x^4]) + (12*c*(b*B + A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*b) - (2
*(b*B + A*c)*(b*x^2 + c*x^4)^(3/2))/(b*x^(7/2)) - (2*A*(b*x^2 + c*x^4)^(5/2
))/(5*b*x^(15/2)) - (24*b^(1/4)*c^(1/4)*(b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4]) + (12*b^(1/4)*c^(1/4)*(b*B + A
*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{13/2}}\,dx = -\frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}} + -\frac{\left(2\left(-\frac{5bB}{2}-\frac{5Ac}{2}\right)\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{9/2}}\,dx}{5b}$$

$$= -\frac{2(bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{5bx^{15/2}} + \frac{(6c(bB+Ac))\int \frac{\sqrt{bx^2}}{}}{b}$$

$$= \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+}{5bx^{15}}$$

$$= \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+}{5bx^{15}}$$

$$= \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+}{5bx^{15}}$$

$$= \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+}{5bx^{15}}$$

$$= \frac{24\sqrt{c}\,(bB+Ac)x^{3/2}(b+cx^2)}{5\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 99, normalized size = 0.28

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(A\left(b+cx^2\right)^2\sqrt{1+\frac{cx^2}{b}}+5b(bB+Ac)x^2\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{cx^2}{b}\right)\right)}{5bx^{7/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + 5*b*(b*B +
 A*c)*x^2*Hypergeometric2F1[-3/2, -1/4, 3/4, -((c*x^2)/b)]))/(5*b*x^(7/2)*S
qrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.40, size = 427, normalized size = 1.21

method	result
risch	$-\frac{2\left(-Bc\,x^4+7Ac\,x^2+5bB\,x^2+Ab\right)\sqrt{x^2\left(c\,x^2+b\right)}}{5x^{\frac{7}{2}}}+\frac{12(Ac+Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}}{}$
default	$\frac{2\left(x^4c+b\,x^2\right)^{\frac{3}{2}}\left(12A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc\,x^2-6A\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(3/2)/x^(11/2)/(c*x^2+b)^2*(12*A*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*
c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))*b*c*x^2-6*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2+12*B*((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*
c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2
*2^(1/2))*b^2*x^2-6*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*
```

```
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*x^2+B*c^2*x^6-7*A*c^2
*x^4-4*x^4*b*B*c-8*A*b*c*x^2-5*b^2*B*x^2-b^2*A)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(13/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.66, size = 81, normalized size = 0.23

$$-\frac{2\left(12\,(Bb+Ac)\sqrt{c}\,x^4\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-(Bcx^4-(5\,Bb+7\,Ac)x^2-Ab)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="fricas")
[Out] -2/5*(12*(B*b + A*c)*sqrt(c)*x^4*weierstrassZeta(-4*b/c, 0, weierstrassPInv
erse(-4*b/c, 0, x)) - (B*c*x^4 - (5*B*b + 7*A*c)*x^2 - A*b)*sqrt(c*x^4 + b*
x^2)*sqrt(x))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(13/2),x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(13/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(13/2), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2), x)
```

3.242 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx$

Optimal. Leaf size=204

$$\frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}}-\frac{2(7bB+3Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{9/2}}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{7bx^{17/2}}+\frac{4c^{3/4}(7bB+3Ac)x\left(\sqrt{b}+\sqrt{\vphantom{b}}\right.}{}$$

```
[Out] -2/21*(3*A*c+7*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(9/2)-2/7*A*(c*x^4+b*x^2)^(5/2)
/b/x^(17/2)+4/21*c*(3*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^(1/2)+4/21*c^(3/4)
*(3*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2
)/b^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2063, 2045, 2046, 2057, 335, 226}

$$\frac{4c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(3Ac+7bB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{b}\,\sqrt{bx^2+cx^4}}+\frac{4c\sqrt{bx^2+cx^4}\,(3Ac+7bB)}{21b\sqrt{x}}-\frac{2\left(bx^2+cx^4\right)^{3/2}(3Ac+7bB)}{21bx^{9/2}}-\frac{2A\left(bx^2+cx^4\right)^{5/2}}{7bx^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2),x]
```

```
[Out] (4*c*(7*b*B + 3*A*c)*Sqrt[b*x^2 + c*x^4])/(21*b*Sqrt[x]) - (2*(7*b*B + 3*A*
c)*(b*x^2 + c*x^4)^(3/2))/(21*b*x^(9/2)) - (2*A*(b*x^2 + c*x^4)^(5/2))/(7*b
*x^(17/2)) + (4*c^(3/4)*(7*b*B + 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(21*b^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx = -\frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}} - \frac{\left(2\left(-\frac{7bB}{2}-\frac{3Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx}{7b}$$

$$= -\frac{2(7bB+3Ac)(bx^2+cx^4)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}} + \frac{(2c(7bB+3Ac))\int \text{-}}{7b}$$

$$= \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)(bx^2+cx^4)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+c}{7bx^{17/2}}$$

$$= \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)(bx^2+cx^4)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+c}{7bx^{17/2}}$$

$$= \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)(bx^2+cx^4)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+c}{7bx^{17/2}}$$

$$= \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)(bx^2+cx^4)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+c}{7bx^{17/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 101, normalized size = 0.50

$$-\frac{2\sqrt{x^2(b+cx^2)}\left(3A(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}+b(7bB+3Ac)x^2\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{cx^2}{b}\right)\right)}{21bx^{9/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(3*A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(7*b*B
 + 3*A*c)*x^2*Hypergeometric2F1[-3/2, -3/4, 1/4, -((c*x^2)/b)]))/(21*b*x^(9
/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.39, size = 254, normalized size = 1.25

method	result

risch	$-\frac{2(-7Bc\,x^4+9Ac\,x^2+7bB\,x^2+3Ab)\sqrt{x^2\,(c\,x^2+b)}}{21x^{\frac{9}{2}}}+\frac{4(3Ac+7Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right.}{\sqrt{-bc}}}$
default	$\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(6A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*(c*x^4+b*x^2)^(3/2)/x^(13/2)/(c*x^2+b)^2*(6*A*(-b*c)^(1/2)*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*c*x^3+14*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^
(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*x^3+
7*B*c^2*x^6-9*A*c^2*x^4-12*A*b*c*x^2-7*b^2*B*x^2-3*b^2*A)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(15/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 75, normalized size = 0.37

$$\frac{2\left(4\,(7\,Bb+3\,Ac)\sqrt{c}\,x^5\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+(7\,Bcx^4-(7\,Bb+9\,Ac)x^2-3\,Ab)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{21\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] 2/21*(4*(7*B*b + 3*A*c)*sqrt(c)*x^5*weierstrassPInverse(-4*b/c, 0, x) + (7*
B*c*x^4 - (7*B*b + 9*A*c)*x^2 - 3*A*b)*sqrt(c*x^4 + b*x^2)*sqrt(x))/x^5
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(15/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(15/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{15/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2), x)
```

3.243 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx$

<u>**Optimal**</u>. Leaf size=364

$$\frac{8c^{3/2}(9bB+Ac)x^{3/2}(b+cx^2)}{15b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}}-\frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}}-\frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}}$$

```
[Out] -2/45*(A*c+9*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(11/2)-2/9*A*(c*x^4+b*x^2)^(5/2)/
b/x^(19/2)+8/15*c^(3/2)*(A*c+9*B*b)*x^(3/2)*(c*x^2+b)/b/(b^(1/2)+x*c^(1/2))
/(c*x^4+b*x^2)^(1/2)-4/15*c*(A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^(3/2)-8/15*
c^(5/4)*(A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)
^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)+4/15*c^(5/4)*(A*c+9*B*b)*x*(cos(2*arctan
(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*
EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^
(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)
```

<u>**Rubi**</u> [A]
time = 0.31, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2063, 2045, 2057, 335, 311, 226, 1210}

$$\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(Ac+9bB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}-\frac{8c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(Ac+9bB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}+\frac{8c^{3/2}x^{3/2}(b+cx^2)(Ac+9bB)}{15b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2(bx^2+cx^4)^{3/2}(Ac+9bB)}{45bx^{11/2}}-\frac{4c\sqrt{bx^2+cx^4}(Ac+9bB)}{15bx^{3/2}}-\frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2),x]
```

```
[Out] (8*c^(3/2)*(9*b*B + A*c)*x^(3/2)*(b + c*x^2))/(15*b*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) - (4*c*(9*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(15*b*x^(3/2)
) - (2*(9*b*B + A*c)*(b*x^2 + c*x^4)^(3/2))/(45*b*x^(11/2)) - (2*A*(b*x^2 +
 c*x^4)^(5/2))/(9*b*x^(19/2)) - (8*c^(5/4)*(9*b*B + A*c)*x*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)
*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x^2 + c*x^4]) + (4*c^(5/4)*(9*
b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x
^2 + c*x^4])
```

<u>Rule 226</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
```

```
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{17/2}}\,dx = -\frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}} - \frac{\left(2\left(-\frac{9bB}{2}-\frac{Ac}{2}\right)\right)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{13/2}}\,dx}{9b}$$

$$= -\frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}} + \frac{(2c(9bB+Ac))\int \sqrt{}}{15b}$$

$$= -\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}} - \frac{2A(bx^2+}{9bx^{19}}$$

$$= -\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}} - \frac{2A(bx^2+}{9bx^{19}}$$

$$= -\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}} - \frac{2A(bx^2+}{9bx^{19}}$$

$$= -\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}} - \frac{2A(bx^2+}{9bx^{19}}$$

$$= \frac{8c^{3/2}(9bB+Ac)x^{3/2}(b+cx^2)}{15b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - 2(9bB-$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 100, normalized size = 0.27

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(5A(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}+b(9bB+Ac)x^2\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{cx^2}{b}\right)\right)}{45bx^{11/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(5*A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(9*b*B
 + A*c)*x^2*Hypergeometric2F1[-3/2, -5/4, -1/4, -((c*x^2)/b)]))/(45*b*x^(11
/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.40, size = 452, normalized size = 1.24

method	result
risch	$-\frac{2(12Ac^2x^4+63x^4bBc+11Abcx^2+9b^2Bx^2+5b^2A)\sqrt{x^2(cx^2+b)}}{45x^{\frac{11}{2}}b}+\frac{4c(Ac+9Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2(\cdots}{\cdots}}\cdots}{\cdots}$
default	$\frac{2(x^4c+bx^2)^{\frac{3}{2}}\left(12A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc^2x^4-6A\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*(c*x^4+b*x^2)^(3/2)/x^(15/2)/(c*x^2+b)^2*(12*A*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*b*c^2*x^4-6*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x^4+108*B*((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*b^2*c*x^4-54*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1
/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Elli
pticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c*x^4-12*A*c
^3*x^6-63*x^6*B*b*c^2-23*A*b*c^2*x^4-72*x^4*B*b^2*c-16*A*b^2*c*x^2-9*x^2*B*
b^3-5*A*b^3)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(17/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.59, size = 102, normalized size = 0.28

$$-\frac{2\left(12\,(9\,Bbc+Ac^2)\sqrt{c}\,x^6\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(3\,(21\,Bbc+4\,Ac^2)x^4+5\,Ab^2+(9\,Bb^2+11\,Abc)x^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{45\,bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="fricas")
```

```
[Out] -2/45*(12*(9*B*b*c + A*c^2)*sqrt(c)*x^6*weierstrassZeta(-4*b/c, 0, weierstr
assPInverse(-4*b/c, 0, x)) + (3*(21*B*b*c + 4*A*c^2)*x^4 + 5*A*b^2 + (9*B*b
^2 + 11*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b*x^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(17/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5985 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(17/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{17/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2),x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2), x)
```

3.244 $\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=243

$$-\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}}+\frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3}-\frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+cx^4}}{165c^2}+\frac{2B}{}$$

```
[Out] 6/385*b*(-15*A*c+13*B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^3-2/165*(-15*A*c+13*
B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c^2+2/15*B*x^(11/2)*(c*x^4+b*x^2)^(1/2)/c-
2/77*b^2*(-15*A*c+13*B*b)*(c*x^4+b*x^2)^(1/2)/c^4/x^(1/2)+1/77*b^(11/4)*(-1
5*A*c+13*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
)),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)
/c^(17/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2064, 2049, 2057, 335, 226}

$$\frac{b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(13bB-15Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{17/4}\sqrt{bx^2+cx^4}}-\frac{2b^2\sqrt{bx^2+cx^4}(13bB-15Ac)}{77c^4\sqrt{x}}+\frac{6bx^{3/2}\sqrt{bx^2+cx^4}(13bB-15Ac)}{385c^3}-\frac{2x^{7/2}\sqrt{bx^2+cx^4}(13bB-15Ac)}{165c^2}+\frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(13/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*b^2*(13*b*B - 15*A*c)*Sqrt[b*x^2 + c*x^4])/(77*c^4*Sqrt[x]) + (6*b*(13*
b*B - 15*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(385*c^3) - (2*(13*b*B - 15*A*c)
*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(165*c^2) + (2*B*x^(11/2)*Sqrt[b*x^2 + c*x^4]
)/(15*c) + (b^(11/4)*(13*b*B - 15*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*
x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)]
, 1/2])/(77*c^(17/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c} - \frac{\left(2\left(\frac{13bB}{2}-\frac{15Ac}{2}\right)\right)\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx}{15c}$$

$$= -\frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+cx^4}}{165c^2} + \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c} + \frac{(3b(13bB-15Ac))\int}{55c^2}$$

$$= \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+cx^4}}{165c^2} + \frac{2Bx^{11/2}\sqrt{}}{1}$$

$$= -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-}{}$$

$$= -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-}{}$$

$$= -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-}{}$$

$$= -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.13, size = 143, normalized size = 0.59

$$\frac{2x^{3/2}\left(-\left((b+cx^2)\left(195b^3B-7c^3x^4(15A+11Bx^2)-9b^2c(25A+13Bx^2)+bc^2x^2(135A+91Bx^2)\right)\right)+15b^3(13bB-15Ac)\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{1155c^4\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(-((b + c*x^2)*(195*b^3*B - 7*c^3*x^4*(15*A + 11*B*x^2) - 9*b^2*
c*(25*A + 13*B*x^2) + b*c^2*x^2*(135*A + 91*B*x^2))) + 15*b^3*(13*b*B - 15*
A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(
1155*c^4*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.40, size = 298, normalized size = 1.23

method	result
risch	$\frac{2(77Bc^3x^6+105Ac^3x^4-91Bbc^2x^4-135Abc^2x^2+117Bb^2cx^2+225Ab^2c-195Bb^3)x^{\frac{3}{2}}(cx^2+b)}{1155c^4\sqrt{x^2(cx^2+b)}} - \frac{b^3(15Ac-13Bb)\sqrt{-bc}\sqrt{\cdots}}{\cdots}$
default	$-\frac{\sqrt{x}\left(-154Bc^5x^9-210Ac^5x^7+28Bbc^4x^7+225A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\operatorname{EllipticF}\left(\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1155/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(-154*B*c^5*x^9-210*A*c^5*x^7+28*B*b*c^
4*x^7+225*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^3*c+60*A*b*c^4*x^5-1
95*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^4-52*B*b^2*c^3*x^5-180*A*b^
2*c^3*x^3+156*B*b^3*c^2*x^3-450*A*b^3*c^2*x+390*B*b^4*c*x)/c^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 122, normalized size = 0.50

$$\frac{2\left(15\left(13Bb^4-15Ab^3c\right)\sqrt{c}\,x\,\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)+\left(77Bc^4x^6-195Bb^3c+225Ab^2c^2-7\left(13Bbc^3-15Ac^4\right)x^4+9\left(13Bb^2c^2-15Abc^3\right)x^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{1155\,c^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(15*(13*B*b^4 - 15*A*b^3*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0,
 x) + (77*B*c^4*x^6 - 195*B*b^3*c + 225*A*b^2*c^2 - 7*(13*B*b*c^3 - 15*A*c^
```

```
4)*x^4 + 9*(13*B*b^2*c^2 - 15*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c
^5*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5457 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{13/2}\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.245 $\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=369

$$-\frac{14b^2(11bB-13Ac)x^{3/2}(b+cx^2)}{195c^{7/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{14b(11bB-13Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{585c^3}-\frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+c}}{117c^2}$$

```
[Out] -14/195*b^2*(-13*A*c+11*B*b)*x^(3/2)*(c*x^2+b)/c^(7/2)/(b^(1/2)+x*c^(1/2))/
(c*x^4+b*x^2)^(1/2)-2/117*(-13*A*c+11*B*b)*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c^2+
2/13*B*x^(9/2)*(c*x^4+b*x^2)^(1/2)/c+14/585*b*(-13*A*c+11*B*b)*x^(1/2)*(c*x
^4+b*x^2)^(1/2)/c^3+14/195*b^(9/4)*(-13*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)
*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Elliptic
E(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*(
(c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)-7/195*b
^(9/4)*(-13*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/
cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2)
)^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2064, 2049, 2057, 335, 311, 226, 1210}

$$-\frac{7b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-13Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{15/4}\sqrt{bx^2+cx^4}}+\frac{14b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-13Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{15/4}\sqrt{bx^2+cx^4}}-\frac{14b^2x^{3/2}(b+cx^2)(11bB-13Ac)}{195c^{7/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{14b\sqrt{x}\,\sqrt{bx^2+cx^4}(11bB-13Ac)}{585c^3}-\frac{2x^{5/2}\sqrt{bx^2+cx^4}(11bB-13Ac)}{117c^2}+\frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(11/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-14*b^2*(11*b*B - 13*A*c)*x^(3/2)*(b + c*x^2))/(195*c^(7/2)*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[b*x^2 + c*x^4]) + (14*b*(11*b*B - 13*A*c)*Sqrt[x]*Sqrt[b*x^2 +
 c*x^4])/(585*c^3) - (2*(11*b*B - 13*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(117
*c^2) + (2*B*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(13*c) + (14*b^(9/4)*(11*b*B - 13
*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ell
ipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(15/4)*Sqrt[b*x^2
+ c*x^4]) - (7*b^(9/4)*(11*b*B - 13*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(195*c^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
```

```
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c} - \frac{\left(2\left(\frac{11bB}{2}-\frac{13Ac}{2}\right)\right)\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx}{13c}$$

$$= -\frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c} + \frac{(7b(11bB-13Ac))\int}{117c^2}$$

$$= \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}}{}$$

$$= \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}}{}$$

$$= \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}}{}$$

$$= \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}}{}$$

$$= -\frac{14b^2(11bB-13Ac)x^{3/2}(b+cx^2)}{195c^{7/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - 2($$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.11, size = 122, normalized size = 0.33

$$\frac{2x^{5/2}\left((b+cx^2)\left(77b^2B+5c^2x^2(13A+9Bx^2)-bc(91A+55Bx^2)\right)+7b^2(-11bB+13Ac)\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{585c^3\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

[Out] (2*x^(5/2)*((b + c*x^2)*(77*b^2*B + 5*c^2*x^2*(13*A + 9*B*x^2) - b*c*(91*A + 55*B*x^2)) + 7*b^2*(-11*b*B + 13*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(585*c^3*Sqrt[x^2*(b + c*x^2)])

Maple [A]
time = 0.39, size = 437, normalized size = 1.18

method	result
risch	$-\frac{2x^{\frac{5}{2}}\left(-45B\,c^2x^4-65A\,c^2x^2+55bB\,x^2c+91Abc-77b^2B\right)\left(c\,x^2+b\right)}{585c^3\sqrt{x^2\left(c\,x^2+b\right)}}+\frac{7b^2(13Ac-11Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x\right.}{}}}{}$
default	$\frac{\sqrt{x}\left(90B\,c^4x^8+130A\,c^4x^6-20Bb\,c^3x^6+546A\,b^3c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/585/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^4*(90*B*c^4*x^8+130*A*c^4*x^6-20*B*b*c^3*x^6+546*A*b^3*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-273*A*b^3*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-462*B*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+231*B*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-52*A*b*c^3*x^4+44*B*b^2*c^2*x^4-182*A*b^2*c^2*x^2+154*B*b^3*c*x^2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.65, size = 102, normalized size = 0.28

$$\frac{2\left(21\,(11\,Bb^3-13\,Ab^2c)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+(45\,Bc^3x^4+77\,Bb^2c-91\,Abc^2-5\,(11\,Bbc^2-13\,Ac^3)x^2)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{585\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/585*(21*(11*B*b^3 - 13*A*b^2*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weiers
trassPInverse(-4*b/c, 0, x)) + (45*B*c^3*x^4 + 77*B*b^2*c - 91*A*b*c^2 - 5*
(11*B*b*c^2 - 13*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/c^4
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3655 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11/2}\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.246 $\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=204

$$\frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}}-\frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2}+\frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{5b^{7/4}(9bB-11Ac)x}{}$$

```
[Out] -2/77*(-11*A*c+9*B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^2+2/11*B*x^(7/2)*(c*x^4
+b*x^2)^(1/2)/c+10/231*b*(-11*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)-5/
231*b^(7/4)*(-11*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(
1/2))^2)^(1/2)/c^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2064, 2049, 2057, 335, 226}

$$-\frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9bB-11Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}}+\frac{10b\sqrt{bx^2+cx^4}\,(9bB-11Ac)}{231c^3\sqrt{x}}-\frac{2x^{3/2}\sqrt{bx^2+cx^4}\,(9bB-11Ac)}{77c^2}+\frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (10*b*(9*b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^3*Sqrt[x]) - (2*(9*b*B -
 11*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(77*c^2) + (2*B*x^(7/2)*Sqrt[b*x^2 +
c*x^4])/(11*c) - (5*b^(7/4)*(9*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(231*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(2\left(\frac{9bB}{2}-\frac{11Ac}{2}\right)\right)\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{11c}$$

$$= -\frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} + \frac{(5b(9bB-11Ac))\int \frac{}{\sqrt{bx}}}{77c^2}$$

$$= \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+c}}{11c}$$

$$= \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+c}}{11c}$$

$$= \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+c}}{11c}$$

$$= \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+c}}{11c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.11, size = 122, normalized size = 0.60

$$\frac{2x^{3/2}\left((b+cx^2)\left(45b^2B+3c^2x^2(11A+7Bx^2)-bc(55A+27Bx^2)\right)+5b^2(-9bB+11Ac)\sqrt{1+\frac{cx^2}{b}}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{231c^3\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*((b + c*x^2)*(45*b^2*B + 3*c^2*x^2*(11*A + 7*B*x^2) - b*c*(55*A
+ 27*B*x^2)) + 5*b^2*(-9*b*B + 11*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F
1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(231*c^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.37, size = 274, normalized size = 1.34

method	result

risch	$-\frac{2(-21Bc^2x^4-33Ac^2x^2+27bBx^2c+55Abc-45b^2B)x^{\frac{3}{2}}(cx^2+b)}{231c^3\sqrt{x^2(cx^2+b)}}+\frac{5b^2(11Ac-9Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2(x}{}}}{}$
default	$\frac{\sqrt{x}\left(42Bc^4x^7+55A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/231/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(42*B*c^4*x^7+55*A*((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-
b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(
1/2))*(-b*c)^(1/2)*b^2*c+66*A*c^4*x^5-45*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2)
)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*
c)^(1/2)*b^3-12*B*b*c^3*x^5-44*A*b*c^3*x^3+36*B*b^2*c^2*x^3-110*A*b^2*c^2*x
+90*B*b^3*c*x)/c^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 99, normalized size = 0.49

$$-\frac{2\left(5\left(9Bb^3-11Ab^2c\right)\sqrt{c}\,x\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)-\left(21Bc^3x^4+45Bb^2c-55Abc^2-3\left(9Bbc^2-11Ac^3\right)x^2\right)\sqrt{cx^4+bx^2}\sqrt{x}\right)}{231c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/231*(5*(9*B*b^3 - 11*A*b^2*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x
) - (21*B*c^3*x^4 + 45*B*b^2*c - 55*A*b*c^2 - 3*(9*B*b*c^2 - 11*A*c^3)*x^2)
*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^4*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3655 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{9/2}\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.247 $\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=330

$$\frac{2b(7bB-9Ac)x^{3/2}(b+cx^2)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2(7bB-9Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{45c^2}+\frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{2b^{5/4}(7bB-9A}{}$$

```
[Out] 2/15*b*(-9*A*c+7*B*b)*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+
b*x^2)^(1/2)+2/9*B*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c-2/45*(-9*A*c+7*B*b)*x^(1/2
)*(c*x^4+b*x^2)^(1/2)/c^2-2/15*b^(5/4)*(-9*A*c+7*B*b)*x*(cos(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ellipt
icE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))
*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)+1/15*
b^(5/4)*(-9*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/c
os(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))
^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2064, 2049, 2057, 335, 311, 226, 1210}

$$\frac{b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-9Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}-\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-9Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}+\frac{2bx^{3/2}(b+cx^2)(7bB-9Ac)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{x}\,\sqrt{bx^2+cx^4}\,(7bB-9Ac)}{45c^2}+\frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*b*(7*b*B - 9*A*c)*x^(3/2)*(b + c*x^2))/(15*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[b*x^2 + c*x^4]) - (2*(7*b*B - 9*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45
*c^2) + (2*B*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*c) - (2*b^(5/4)*(7*b*B - 9*A*c
)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(11/4)*Sqrt[b*x^2 + c*x
^4]) + (b^(5/4)*(7*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(S
qrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(15*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
```

```
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} - \frac{\left(2\left(\frac{7bB}{2}-\frac{9Ac}{2}\right)\right)\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{9c}$$

$$= -\frac{2(7bB-9Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{(b(7bB-9Ac))\int \frac{x}{\sqrt{bx^2}}}{15c^2}$$

$$= -\frac{2(7bB-9Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(b(7bB-9Ac)x\sqrt{b+c}\right.}{15c^2\sqrt{bx^2}}$$

$$= -\frac{2(7bB-9Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(2b(7bB-9Ac)x\sqrt{b+}\right.}{1}$$

$$= -\frac{2(7bB-9Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(2b^{3/2}(7bB-9Ac)x\sqrt{b}\right.}{1}$$

$$= \frac{2b(7bB-9Ac)x^{3/2}(b+cx^2)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2(7bB-9Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx}}{9c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.10, size = 97, normalized size = 0.29

$$\frac{2x^{5/2}\left(-\left((b+cx^2)\left(7bB-9Ac-5Bcx^2\right)\right)+b(7bB-9Ac)\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{45c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*(-((b + c*x^2)*(7*b*B - 9*A*c - 5*B*c*x^2)) + b*(7*b*B - 9*A*c)*
Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(45*c^
2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.50, size = 413, normalized size = 1.25

method	result
risch	$\frac{2x^{\frac{5}{2}}\left(5Bc\,x^{2}+9Ac-7Bb\right)\left(c\,x^{2}+b\right)}{45c^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{b(9Ac-7Bb)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\right.}{1}$
default	$-\frac{\sqrt{x}\left(-10B\,c^{3}x^{6}+54A\,b^{2}c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/45/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^3*(-10*B*c^3*x^6+54*A*b^2*c*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))-27*A*b^2*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)
*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellipti
cF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-42*B*b^3*((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))+21*B*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*
((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Elliptic
F(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-18*A*c^3*x^4+4*B*b*c
^2*x^4-18*A*b*c^2*x^2+14*B*b^2*c*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 79, normalized size = 0.24

$$-\frac{2\left(3\left(7Bb^2-9Abc\right)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)-\left(5Bc^2x^2-7Bbc+9Ac^2\right)\sqrt{cx^4+bx^2}\sqrt{x}\right)}{45c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(3*(7*B*b^2 - 9*A*b*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrass
PInverse(-4*b/c, 0, x)) - (5*B*c^2*x^2 - 7*B*b*c + 9*A*c^2)*sqrt(c*x^4 + b*
x^2)*sqrt(x))/c^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{7}{2}}\left(A+Bx^2\right)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(7/2)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^{7/2}\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.248 $\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=167

$$-\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}}+\frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c}+\frac{b^{3/4}(5bB-7Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(\ldots\right.}{21c^{9/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/7*B*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c-2/21*(-7*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)
/c^2/x^(1/2)+1/21*b^(3/4)*(-7*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2064, 2049, 2057, 335, 226}

$$\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(5bB-7Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}(5bB-7Ac)}{21c^2\sqrt{x}}+\frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*(5*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(21*c^2*Sqrt[x]) + (2*B*x^(3/2)*Sq
rt[b*x^2 + c*x^4])/(7*c) + (b^(3/4)*(5*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(21*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} - \frac{\left(2\left(\frac{5bB}{2}-\frac{7Ac}{2}\right)\right)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{7c}$$

$$= -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{(b(5bB-7Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}}{21c^2}$$

$$= -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{\left(b(5bB-7Ac)x\sqrt{b+cx^2}\right)\int}{21c^2\sqrt{bx^2+c}}$$

$$= -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{\left(2b(5bB-7Ac)x\sqrt{b+cx^2}\right)}{21c^2\sqrt{}}$$

$$= -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{b^{3/4}(5bB-7Ac)x\left(\sqrt{b}+\sqrt{c}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.09, size = 97, normalized size = 0.58

$$\frac{2x^{3/2}\left(-\left(\left(b+cx^2\right)\left(5bB-7Ac-3Bcx^2\right)\right)+b(5bB-7Ac)\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{21c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(-((b + c*x^2)*(5*b*B - 7*A*c - 3*B*c*x^2)) + b*(5*b*B - 7*A*c)*
Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(21*c^
2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.44, size = 248, normalized size = 1.49

method	result

risch	$\frac{2(3Bc\,x^2+7Ac-5Bb)x^{\frac{3}{2}}(c\,x^2+b)}{21c^2\sqrt{x^2(c\,x^2+b)}}-\frac{b(7Ac-5Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{E}}{21c^3\sqrt{c\,x^3+bx}\sqrt{x^2(c\,x^{}}}$
default	$-\frac{\sqrt{x}\left(7A\sqrt{-bc}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\,bc-5}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(7*A*(-b*c)^(1/2)*((-c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(
1/2)*b*c-5*B*(-b*c)^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-
b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(
1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*b^2-6*B*c^3*x^5-14*A*
c^3*x^3+4*B*b*c^2*x^3-14*A*b*c^2*x+10*B*b^2*c*x)/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.67, size = 73, normalized size = 0.44

$$\frac{2\left((5\,Bb^2-7\,Abc)\sqrt{c}\,x\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)+(3\,Bc^2x^2-5\,Bbc+7\,Ac^2)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{21\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*((5*B*b^2 - 7*A*b*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) + (3*
B*c^2*x^2 - 5*B*b*c + 7*A*c^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}(A+Bx^2)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.249 $\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=293

$$-\frac{2(3bB-5Ac)x^{3/2}(b+cx^2)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{2B\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c}+\frac{2\sqrt[4]{b}\,(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c}{\left(\sqrt{b}+\sqrt{}\right.}}}{5c^{7/4}\sqrt{bx^2+c}}$$

```
[Out] -2/5*(-5*A*c+3*B*b)*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*
x^2)^(1/2)+2/5*B*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c+2/5*b^(1/4)*(-5*A*c+3*B*b)*x
*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))
*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4
+b*x^2)^(1/2)-1/5*b^(1/4)*(-5*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2064, 2057, 335, 311, 226, 1210}

$$-\frac{\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(3bB-5Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}+\frac{2\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(3bB-5Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}-\frac{2x^{3/2}(b+cx^2)(3bB-5Ac)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{2B\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*(3*b*B - 5*A*c)*x^(3/2)*(b + c*x^2))/(5*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) + (2*B*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) + (2*b^(1/4)*
(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]
*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7/4)*Sqrt
[b*x^2 + c*x^4]) - (b^(1/4)*(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
 + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(
1/4)], 1/2])/(5*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2B\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c} - \frac{\left(2\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5c}$$

$$= \frac{2B\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c} - \frac{\left(2\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5c\sqrt{bx^2+cx^4}}$$

$$= \frac{2B\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c} - \frac{\left(4\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right.}{5c\sqrt{bx^2+cx^4}}$$

$$= \frac{2B\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c} - \frac{\left(4\sqrt{b}\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x,\right.}{5c^{3/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(3bB-5Ac)x^{3/2}(b+cx^2)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{2B\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c} + \frac{2\sqrt[4]{b}\,(3bB-5Ac)x\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 81, normalized size = 0.28

$$\frac{2x^{5/2}\left(3B(b+cx^2)+(-3bB+5Ac)\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{15c\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*(3*B*(b + c*x^2) + (-3*b*B + 5*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(15*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.38, size = 378, normalized size = 1.29

method	result

risch	$\frac{2Bx^{\frac{5}{2}}(cx^2+b)}{5c\sqrt{x^2(cx^2+b)}}+\frac{(5Ac-3Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}}{}\right.}{5c^2\sqrt{cx^3}}$
default	$\frac{\sqrt{x}\left(10A\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}bc-5A\,\mathrm{EllipticF}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^2*(10*A*EllipticE(((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/
2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b*c-5
*A*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b*c-6*B*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*
((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b^2+3*B*
EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-x*c/(-b*c)^(1/2))^(1/2)*b^2+2*B*c^2*x^4+2*b*B*x^2*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 55, normalized size = 0.19

$$\frac{2\left(\sqrt{cx^4+bx^2}\,Bc\sqrt{x}+(3\,Bb-5\,Ac)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)\right)}{5\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(sqrt(c*x^4 + b*x^2)*B*c*sqrt(x) + (3*B*b - 5*A*c)*sqrt(c)*weierstrassZ
eta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)))/c^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}(A+Bx^2)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3/2}\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.250 $\int \frac{\sqrt{x}\,(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=130

$$\frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/3*B*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)-1/3*(-3*A*c+B*b)*x*(cos(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ellipti
cF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*
((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(1/4)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2064, 2057, 335, 226}

$$\frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(bB-3Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,c^{5/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*B*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - ((b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)
*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(1/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}\,(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx &= \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{\left(2\left(\frac{bB}{2}-\frac{3Ac}{2}\right)\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{3c}\\ &= \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{\left(2\left(\frac{bB}{2}-\frac{3Ac}{2}\right)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3c\sqrt{bx^2+cx^4}}\\ &= \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{\left(4\left(\frac{bB}{2}-\frac{3Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3c\sqrt{bx^2+cx^4}}\\ &= \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\cdots}\right.\right.}{3\sqrt[4]{b}\,c^{5/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 80, normalized size = 0.62

$$\frac{2x^{3/2}\left(B(b+cx^2)+(-bB+3Ac)\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{3c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(B*(b + c*x^2) + (-(b*B) + 3*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(3*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.38, size = 216, normalized size = 1.66

method	result
risch	$\frac{2 B x^{\frac{3}{2}} \left(c x^{2}+b\right)}{3 c \sqrt{x^{2} \left(c x^{2}+b\right)}}+\frac{\left(3 A c - B b\right) \sqrt{-b c}\, \sqrt{\frac{\left(x+\frac{\sqrt{-b c}}{c}\right) c}{\sqrt{-b c}}}\, \sqrt{-\frac{2 \left(x-\frac{\sqrt{-b c}}{c}\right) c}{\sqrt{-b c}}}\, \sqrt{-\frac{x c}{\sqrt{-b c}}}\, \mathrm{EllipticF}\left(\sqrt{\ldots}\right.}{3 c^{2} \sqrt{c x^{3}+b x}\, \sqrt{x^{2} \left(c x^{2}+b\right)}}$
default	$\frac{\sqrt{x}\, \left(3 A \sqrt{\frac{c x+\sqrt{-b c}}{\sqrt{-b c}}}\, \sqrt{2}\, \sqrt{\frac{-c x+\sqrt{-b c}}{\sqrt{-b c}}}\, \sqrt{-\frac{x c}{\sqrt{-b c}}}\, \mathrm{EllipticF}\left(\sqrt{\frac{c x+\sqrt{-b c}}{\sqrt{-b c}}}, \frac{\sqrt{2}}{2}\right) \sqrt{-b c}\, c - B \sqrt{\ldots}\right.}{3 \sqrt{x^{4} c+\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(3*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2
)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/
2)*c-B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b+2*B*c^2*x^3+2*B*b*c*x)/c^
2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 51, normalized size = 0.39

$$-\frac{2\left((Bb-3\,Ac)\sqrt{c}\,x\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^4+bx^2}\,Bc\sqrt{x}\right)}{3\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*((B*b - 3*A*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x^
4 + b*x^2)*B*c*sqrt(x))/(c^2*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}\,(A+Bx^2)}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.251 $\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=281

$$\frac{2(bB+Ac)x^{3/2}(b+cx^2)}{b\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}}-\frac{2(bB+Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\Big(2\mathrm{t}}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2*(A*c+B*b)*x^(3/2)*(c*x^2+b)/b/c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(
1/2)-2*A*(c*x^4+b*x^2)^(1/2)/b/x^(3/2)-2*(A*c+B*b)*x*(cos(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE
(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((
c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)+(
A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/
2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3
/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2063, 2057, 335, 311, 226, 1210}

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(Ac+bB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}-\frac{2x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(Ac+bB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{2x^{3/2}(b+cx^2)(Ac+bB)}{b\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (2*(b*B + A*c)*x^(3/2)*(b + c*x^2))/(b*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b
*x^2 + c*x^4]) - (2*A*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) - (2*(b*B + A*c)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(3/4)*Sqrt[b*x^2 + c*x^
4]) + ((b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt
[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(
3/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{(bB+Ac)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left((bB+Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left(2(bB+Ac)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left(2(bB+Ac)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\sqrt{c}\sqrt{bx^2+cx^4}} -$$

$$= \frac{2(bB+Ac)x^{3/2}(b+cx^2)}{b\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} - \frac{2(bB+Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 82, normalized size = 0.29

$$\frac{2\sqrt{x}\left(-3A(b+cx^2)+(bB+Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{3b\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[x]*(-3*A*(b + c*x^2) + (b*B + A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(3*b*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.39, size = 377, normalized size = 1.34

method	result

risch	$-\frac{2A(cx^2+b)\sqrt{x}}{b\sqrt{x^2(cx^2+b)}}+\frac{(Ac+Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}}{\cdots}\right.}{bc\sqrt{cx\cdots}}$
default	$\frac{\sqrt{x}\left(2A\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}bc-A\,\mathrm{EllipticF}\left(\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(2*A*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b*c-A*Ellipt
icF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*
c/(-b*c)^(1/2))^(1/2)*b*c+2*B*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b^2-B*EllipticF((
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*b^2-2*A*c^2*x^2-2*A*b*c)/c/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 62, normalized size = 0.22

$$-\frac{2\left((Bb+Ac)\sqrt{c}\,x^2\mathrm{weierstrassZeta}\left(-\frac{4b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}\,Ac\sqrt{x}\right)}{bcx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*((B*b + A*c)*sqrt(c)*x^2*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(
-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*A*c*sqrt(x))/(b*c*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{\sqrt{x}\,\sqrt{x^2\,(b + cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(sqrt(x)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2 + A}{\sqrt{x}\,\sqrt{c\,x^4 + b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.252 $\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=131

$$-\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}+\frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}$$

```
[Out] -2/3*A*(c*x^4+b*x^2)^(1/2)/b/x^(5/2)+1/3*(-A*c+3*B*b)*x*(cos(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ellipt
icF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))
*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(5/4)/c^(1/4)/(c*x^4+b*x^2)^(1/2
)
```

Rubi [A]
time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2063, 2057, 335, 226}

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(3bB-Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*A*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) + ((3*b*B - A*c)*x*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}} - \frac{\left(2\left(-\frac{3bB}{2}+\frac{Ac}{2}\right)\right)\int\frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{3b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}} - \frac{\left(2\left(-\frac{3bB}{2}+\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\int\frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{3b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}} - \frac{\left(4\left(-\frac{3bB}{2}+\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{3b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}} + \frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\right.\right.}{3b^{5/4}\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 82, normalized size = 0.63

$$-\frac{2\left(A(b+cx^2)+(-3bB+Ac)x^2\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{3b\sqrt{x}\,\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*(A*(b + c*x^2) + (-3*b*B + A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2
F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(3*b*Sqrt[x]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.39, size = 219, normalized size = 1.67

method	result
risch	$-\frac{2A(cx^2+b)}{3b\sqrt{x}\,\sqrt{x^2(cx^2+b)}} - \frac{(Ac-3Bb)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{Ellipt}}{3bc\sqrt{cx^3+bx}\,\sqrt{x^2(cx^2+}}$
default	$-\frac{A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)cx-3B\sqrt{-bc}}{3\sqrt{x^4c+bx}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(c*x^4+b*x^2)^(1/2)/x^(1/2)*(A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)
^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)
)*c*x-3*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*x+2*A*c^2*x^2+2*A*b*c)
/c/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 57, normalized size = 0.44

$$\frac{2\left((3\,Bb-Ac)\sqrt{c}\,x^3\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^4+bx^2}\,Ac\sqrt{x}\right)}{3\,bcx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*((3*B*b - A*c)*sqrt(c)*x^3*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x
^4 + b*x^2)*A*c*sqrt(x))/(b*c*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^{\frac{3}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**(3/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{x^{3/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.253 $\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=332

$$\frac{2\sqrt{c}\,(5bB-3Ac)x^{3/2}(b+cx^2)}{5b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}}-\frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}}-\frac{2\sqrt[4]{c}\,(5bB-3Ac)x\left(\sqrt{b}+\right.}{}$$

```
[Out] 2/5*(-3*A*c+5*B*b)*x^(3/2)*(c*x^2+b)*c^(1/2)/b^2/(b^(1/2)+x*c^(1/2))/(c*x^4
+b*x^2)^(1/2)-2/5*A*(c*x^4+b*x^2)^(1/2)/b/x^(7/2)-2/5*(-3*A*c+5*B*b)*(c*x^4
+b*x^2)^(1/2)/b^2/x^(3/2)-2/5*c^(1/4)*(-3*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ellipti
cE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*
((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)+1/5*c^(
1/4)*(-3*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)
^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2063, 2050, 2057, 335, 311, 226, 1210}

$$\frac{\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(5bB-3Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(5bB-3Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{c}\,x^{3/2}(b+cx^2)(5bB-3Ac)}{5b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}(5bB-3Ac)}{5b^2x^{3/2}}-\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[c]*(5*b*B - 3*A*c)*x^(3/2)*(b + c*x^2))/(5*b^2*(Sqrt[b] + Sqrt[c]*x
)*Sqrt[b*x^2 + c*x^4]) - (2*A*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7/2)) - (2*(5*b*
B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(5*b^2*x^(3/2)) - (2*c^(1/4)*(5*b*B - 3*A*c
)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[b*x^2 + c*x^4
]) + (c^(1/4)*(5*b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqr
t[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(
5*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
```

```
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{\left(2\left(-\frac{5bB}{2}+\frac{3Ac}{2}\right)\right)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{5b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{(c(5bB-3Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}}{5b^2}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{\left(c(5bB-3Ac)x\sqrt{b+cx^2}\right)\int}{5b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{\left(2c(5bB-3Ac)x\sqrt{b+cx^2}\right)}{5b^2\sqrt{bx}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{\left(2\sqrt{c}\,(5bB-3Ac)x\sqrt{b+cx^2}\right.}{5b^{3/2}}$$

$$= \frac{2\sqrt{c}\,(5bB-3Ac)x^{3/2}(b+cx^2)}{5b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 83, normalized size = 0.25

$$-\frac{2\left(A(b+cx^2)+(5bB-3Ac)x^2\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{cx^2}{b}\right)\right)}{5bx^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*(A*(b + c*x^2) + (5*b*B - 3*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric
2F1[-1/4, 1/2, 3/4, -((c*x^2)/b)]))/(5*b*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.40, size = 413, normalized size = 1.24

method	result
risch	$-\frac{2(cx^2+b)(-3Acx^2+5bBx^2+Ab)}{5b^2x^{\frac{3}{2}}\sqrt{x^2(cx^2+b)}}-\frac{(3Ac-5Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\Bigg(}{}$
default	$-\frac{6A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bcx^2-3A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/(c*x^4+b*x^2)^(1/2)/x^(3/2)*(6*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/
2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2-3
*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2-10*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2
))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2
*x^2+5*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*x^2-6*A*c^2*x^4+10*x^4*b*B*c-4*A*b
*c*x^2+10*b^2*B*x^2+2*b^2*A)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 76, normalized size = 0.23

$$-\frac{2\left((5\,Bb-3\,Ac)\sqrt{c}\,x^4\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}\,\left((5\,Bb-3\,Ac)x^2+Ab\right)\sqrt{x}\right)}{5\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] -2/5*((5*B*b - 3*A*c)*sqrt(c)*x^4*weierstrassZeta(-4*b/c, 0, weierstrassPIn
verse(-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*((5*B*b - 3*A*c)*x^2 + A*b)*sqrt
(x))/(b^2*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+Bx^2}{x^{\frac{5}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral((A + B*x**2)/(x**(5/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{B\,x^2+A}{x^{5/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.254 $\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=167

$$-\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}}-\frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}-\frac{c^{3/4}(7bB-5Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\Big(2\,\mathrm{ta}}{21b^{9/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/7*A*(c*x^4+b*x^2)^(1/2)/b/x^(9/2)-2/21*(-5*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2
)/b^2/x^(5/2)-1/21*c^(3/4)*(-5*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*ar
ctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/
(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2063, 2050, 2057, 335, 226}

$$-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-5Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}(7bB-5Ac)}{21b^2x^{5/2}}-\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*A*Sqrt[b*x^2 + c*x^4])/(7*b*x^(9/2)) - (2*(7*b*B - 5*A*c)*Sqrt[b*x^2 +
c*x^4])/(21*b^2*x^(5/2)) - (c^(3/4)*(7*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(21*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{\left(2\left(-\frac{7bB}{2}+\frac{5Ac}{2}\right)\right)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{7b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{(c(7bB-5Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,d}{21b^2}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{\left(c(7bB-5Ac)x\sqrt{b+cx^2}\right)\int -}{21b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{\left(2c(7bB-5Ac)x\sqrt{b+cx^2}\right)\text{Su}}{21b^2\sqrt{bx}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{c^{3/4}(7bB-5Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right.}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 85, normalized size = 0.51

$$\frac{-6A(b+cx^2)+2(-7bB+5Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{cx^2}{b}\right)}{21bx^{5/2}\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-6*A*(b + c*x^2) + 2*(-7*b*B + 5*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometr
ic2F1[-3/4, 1/2, 1/4, -((c*x^2)/b)])/(21*b*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.41, size = 247, normalized size = 1.48

method	result
risch	$-\frac{2(c\,x^2+b)(-5Ac\,x^2+7bB\,x^2+3Ab)}{21b^2x^{\frac{5}{2}}\sqrt{x^2(c\,x^2+b)}}+\frac{(5Ac-7Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\ \text{F}}{21b^2\sqrt{c\,x^3+bx}\sqrt{x^2(c\,x}}$

default	$\frac{5A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c\,x^{3}-7B\sqrt{-bc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21/(c*x^4+b*x^2)^(1/2)/x^(5/2)*(5*A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*
c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))*c*x^3-7*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*
((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Elliptic
F(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*x^3+10*A*c^2*x^4-1
4*x^4*b*B*c+4*A*b*c*x^2-14*b^2*B*x^2-6*b^2*A)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 69, normalized size = 0.41

$$-\frac{2\left((7\,Bb-5\,Ac)\sqrt{c}\,x^{5}\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^{4}+bx^{2}}\,((7\,Bb-5\,Ac)x^{2}+3\,Ab)\sqrt{x}\right)}{21\,b^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/21*((7*B*b - 5*A*c)*sqrt(c)*x^5*weierstrassPInverse(-4*b/c, 0, x) + sqrt
(c*x^4 + b*x^2)*((7*B*b - 5*A*c)*x^2 + 3*A*b)*sqrt(x))/(b^2*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+Bx^{2}}{x^{\frac{7}{2}}\sqrt{x^{2}\,(b+cx^{2})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**(7/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{x^{7/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.255 $\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=369

$$-\frac{2c^{3/2}(9bB-7Ac)x^{3/2}(b+cx^2)}{15b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}}-\frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}}+\frac{2c(9bB-7Ac)\sqrt{bx^2+}}{15b^3x^{3/2}}$$

```
[Out] -2/15*c^(3/2)*(-7*A*c+9*B*b)*x^(3/2)*(c*x^2+b)/b^3/(b^(1/2)+x*c^(1/2))/(c*x
^4+b*x^2)^(1/2)-2/9*A*(c*x^4+b*x^2)^(1/2)/b/x^(11/2)-2/45*(-7*A*c+9*B*b)*(c
*x^4+b*x^2)^(1/2)/b^2/x^(7/2)+2/15*c*(-7*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^3
/x^(3/2)+2/15*c^(5/4)*(-7*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1
/2)+x*c^(1/2))^2)^(1/2)/b^(11/4)/(c*x^4+b*x^2)^(1/2)-1/15*c^(5/4)*(-7*A*c+9
*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2
^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(11/4
)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2063, 2050, 2057, 335, 311, 226, 1210}

$$-\frac{c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9bB-7Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}+\frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9bB-7Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}-\frac{2c^{3/2}x^{3/2}(b+cx^2)(9bB-7Ac)}{15b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{2c\sqrt{bx^2+cx^4}(9bB-7Ac)}{15b^3x^{3/2}}-\frac{2\sqrt{bx^2+cx^4}(9bB-7Ac)}{45b^2x^{7/2}}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*c^(3/2)*(9*b*B - 7*A*c)*x^(3/2)*(b + c*x^2))/(15*b^3*(Sqrt[b] + Sqrt[c]
*x)*Sqrt[b*x^2 + c*x^4]) - (2*A*Sqrt[b*x^2 + c*x^4])/(9*b*x^(11/2)) - (2*(9
*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(45*b^2*x^(7/2)) + (2*c*(9*b*B - 7*A*c)*
Sqrt[b*x^2 + c*x^4])/(15*b^3*x^(3/2)) + (2*c^(5/4)*(9*b*B - 7*A*c)*x*(Sqrt[
b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(11/4)*Sqrt[b*x^2 + c*x^4]) - (c^
(5/4)*(9*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(11
/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
```

```
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{\left(2\left(-\frac{9bB}{2}+\frac{7Ac}{2}\right)\right)\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx}{9b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{(c(9bB-7Ac))\int \frac{1}{\sqrt{x}\sqrt{bx^2+}}}{15b^2}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} -$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} -$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} -$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} -$$

$$= -\frac{2c^{3/2}(9bB-7Ac)x^{3/2}(b+cx^2)}{15b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+ca}}{45b^2x^{7/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 84, normalized size = 0.23

$$-\frac{2\left(5A(b+cx^2)+(9bB-7Ac)x^2\sqrt{1+\frac{cx^2}{b}}\ {}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)\right)}{45bx^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]

[Out] (-2*(5*A*(b + c*x^2) + (9*b*B - 7*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((c*x^2)/b)]))/(45*b*x^(7/2)*Sqrt[x^2*(b + c*x^2)])

Maple [A]
time = 0.42, size = 443, normalized size = 1.20

method	result
risch	$-\frac{2(cx^2+b)(21Ac^2x^4-27x^4bBc-7Abcx^2+9b^2Bx^2+5b^2A)}{45b^3x^{\frac{7}{2}}\sqrt{x^2(cx^2+b)}}+\frac{c(7Ac-9Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right.}{\sqrt{-bc}}}}{}$
default	$\frac{42A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc^2x^4-21A\sqrt{\frac{cx+\sqrt{-b}}{\sqrt{-bc}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/45/(c*x^4+b*x^2)^(1/2)/x^(7/2)*(42*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x^4-21*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x^4-54*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c*x^4+27*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c*x^4-42*A*c^3*x^6+54*x^6*B*b*c^2-28*A*b*c^2*x^4+36*x^4*B*b^2*c+4*A*b^2*c*x^2-18*x^2*B*b^3-10*A*b^3)/b^3

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 104, normalized size = 0.28

$$\frac{2\left(3\,(9\,Bbc-7\,Ac^2)\sqrt{c}\,x^6\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+(3\,(9\,Bbc-7\,Ac^2)x^4-5\,Ab^2-(9\,Bb^2-7\,Abc)x^2)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{45\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*(9*B*b*c - 7*A*c^2)*sqrt(c)*x^6*weierstrassZeta(-4*b/c, 0, weierstr
assPInverse(-4*b/c, 0, x)) + (3*(9*B*b*c - 7*A*c^2)*x^4 - 5*A*b^2 - (9*B*b^
2 - 7*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^3*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^{\frac{9}{2}}\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(9/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**(9/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{x^{9/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.256 $\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=204

$$-\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}}-\frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}+\frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}}+\frac{5c^{7/4}(11bB-9Ac)x\left(\sqrt{b}\right.}{}$$

```
[Out] -2/11*A*(c*x^4+b*x^2)^(1/2)/b/x^(13/2)-2/77*(-9*A*c+11*B*b)*(c*x^4+b*x^2)^(
1/2)/b^2/x^(9/2)+10/231*c*(-9*A*c+11*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^(5/2)+5
/231*c^(7/4)*(-9*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(
1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^
(1/2))^2)^(1/2)/b^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2063, 2050, 2057, 335, 226}

$$\frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-9Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{13/4}\sqrt{bx^2+cx^4}}+\frac{10c\sqrt{bx^2+cx^4}(11bB-9Ac)}{231b^3x^{5/2}}-\frac{2\sqrt{bx^2+cx^4}(11bB-9Ac)}{77b^2x^{9/2}}-\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*A*Sqrt[b*x^2 + c*x^4])/(11*b*x^(13/2)) - (2*(11*b*B - 9*A*c)*Sqrt[b*x^2
 + c*x^4])/(77*b^2*x^(9/2)) + (10*c*(11*b*B - 9*A*c)*Sqrt[b*x^2 + c*x^4])/(
231*b^3*x^(5/2)) + (5*c^(7/4)*(11*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(231*b^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{\left(2\left(-\frac{11bB}{2}+\frac{9Ac}{2}\right)\right)\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx}{11b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} - \frac{(5c(11bB-9Ac))\int \frac{1}{x^{3/2}\sqrt{bx^2}}}{77b^2}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} + \frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} + \frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} + \frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} + \frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 84, normalized size = 0.41

$$-\frac{2\left(7A(b+cx^2)+(11bB-9Ac)x^2\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)\right)}{77bx^{9/2}\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*(7*A*(b + c*x^2) + (11*b*B - 9*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeomet
ric2F1[-7/4, 1/2, -3/4, -((c*x^2)/b)]))/(77*b*x^(9/2)*Sqrt[x^2*(b + c*x^2)]
)
```

Maple [A]
time = 0.41, size = 274, normalized size = 1.34

method	result

risch	$-\frac{2(c\,x^2+b)(45A\,c^2x^4-55x^4bBc-27Abc\,x^2+33b^2B\,x^2+21b^2A)}{231b^3x^{\frac{9}{2}}\sqrt{x^2\,(c\,x^2+b)}}-\frac{5c(9Ac-11Bb)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\right.}{\sqrt{}}}}{231}$
default	$-\frac{45A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c^2x^5-55B\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/231/(c*x^4+b*x^2)^(1/2)/x^(9/2)*(45*A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(
-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^
(1/2))*c^2*x^5-55*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^
(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*El
lipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^5+90*A*c
^3*x^6-110*x^6*B*b*c^2+36*A*b*c^2*x^4-44*x^4*B*b^2*c-12*A*b^2*c*x^2+66*x^2*
B*b^3+42*A*b^3)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.56, size = 96, normalized size = 0.47

$$\frac{2\left(5\,(11\,Bbc-9\,Ac^2)\sqrt{c}\,x^7\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)+\left(5\,(11\,Bbc-9\,Ac^2)x^4-21\,Ab^2-3\,(11\,Bb^2-9\,Abc)x^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{231\,b^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(5*(11*B*b*c - 9*A*c^2)*sqrt(c)*x^7*weierstrassPInverse(-4*b/c, 0, x)
 + (5*(11*B*b*c - 9*A*c^2)*x^4 - 21*A*b^2 - 3*(11*B*b^2 - 9*A*b*c)*x^2)*sqr
t(c*x^4 + b*x^2)*sqrt(x))/(b^3*x^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^{\frac{11}{2}} \sqrt{x^2 (b + cx^2)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(11/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**(11/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2 + A}{x^{11/2}\,\sqrt{c\,x^4 + b\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(11/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^(11/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.257 $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}}+\frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}}-\frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3}+\frac{(13bB-11Ac)x^{7/2}}{11bc^2}$$

```
[Out] -(-A*c+B*b)*x^(15/2)/b/c/(c*x^4+b*x^2)^(1/2)-9/77*(-11*A*c+13*B*b)*x^(3/2)*
(c*x^4+b*x^2)^(1/2)/c^3+1/11*(-11*A*c+13*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/b
/c^2+15/77*b*(-11*A*c+13*B*b)*(c*x^4+b*x^2)^(1/2)/c^4/x^(1/2)-15/154*b^(7/4
)*(-11*A*c+13*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^
(1/2)/c^(17/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2062, 2049, 2057, 335, 226}

$$-\frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(13bB-11Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{154c^{17/4}\sqrt{bx^2+cx^4}}+\frac{15b\sqrt{bx^2+cx^4}(13bB-11Ac)}{77c^4\sqrt{x}}-\frac{9x^{3/2}\sqrt{bx^2+cx^4}(13bB-11Ac)}{77c^3}+\frac{x^{7/2}\sqrt{bx^2+cx^4}(13bB-11Ac)}{11bc^2}-\frac{x^{15/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(15/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + (15*b*(13*b*B - 11*A*
c)*Sqrt[b*x^2 + c*x^4])/(77*c^4*Sqrt[x]) - (9*(13*b*B - 11*A*c)*x^(3/2)*Sqr
t[b*x^2 + c*x^4])/(77*c^3) + ((13*b*B - 11*A*c)*x^(7/2)*Sqrt[b*x^2 + c*x^4]
)/(11*b*c^2) - (15*b^(7/4)*(13*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(154*c^(17/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(\frac{13bB}{2}-\frac{11Ac}{2}\right)\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(13bB-11Ac)x^{7/2}\sqrt{bx^2+cx^4}}{11bc^2} - \frac{(9(13bB-11Ac))\int \frac{x}{\sqrt{bx^2}}}{22c^2}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}} - \frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3} + \frac{(13bB-11Ac)x^{7/2}\sqrt{bx^2}}{11bc^2}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}} + \frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} - \frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2}}{77c^3}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}} + \frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} - \frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2}}{77c^3}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}} + \frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} - \frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2}}{77c^3}$$

$$= -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}} + \frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} - \frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2}}{77c^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.13, size = 134, normalized size = 0.53

$$\frac{x^{3/2}\left(195b^3B+2c^3x^4(11A+7Bx^2)-2bc^2x^2(33A+13Bx^2)+b^2(-165Ac+78Bcx^2)+15b^2(-13bB+11Ac)\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{77c^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(195*b^3*B + 2*c^3*x^4*(11*A + 7*B*x^2) - 2*b*c^2*x^2*(33*A + 13*B
*x^2) + b^2*(-165*A*c + 78*B*c*x^2) + 15*b^2*(-13*b*B + 11*A*c)*Sqrt[1 + (c
*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(77*c^4*Sqrt[x^2*
(b + c*x^2)])
```

Maple [A]
time = 0.44, size = 281, normalized size = 1.12

method	result
default	$\frac{x^{\frac{5}{2}}(c\,x^2+b)\left(28B\,c^4x^7+165A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right.}{}$
risch	$-\frac{2(-7B\,c^2x^4-11A\,c^2x^2+20bB\,x^2c+44Abc-59b^2B)(c\,x^2+b)x^{\frac{3}{2}}}{77c^4\sqrt{x^2\,(c\,x^2+b)}}+\frac{b^2\left(\frac{121A\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right.}{\sqrt{-bc}}}}{\sqrt{c}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/154/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(28*B*c^4*x^7+165*A*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*(-b*c)^(1/2)*b^2*c+44*A*c^4*x^5-195*B*((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/
(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2
^(1/2))*(-b*c)^(1/2)*b^3-52*B*b*c^3*x^5-132*A*b*c^3*x^3+156*B*b^2*c^2*x^3-3
30*A*b^2*c^2*x+390*B*b^3*c*x)/c^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.69, size = 156, normalized size = 0.62

$$-\frac{15\left((13\,Bb^3c-11\,Ab^2c^2)x^3+(13\,Bb^4-11\,Ab^3c)x\right)\sqrt{c}\,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)-\left(14\,Bc^4x^6+195\,Bb^3c-165\,Ab^2c^2-2\,(13\,Bbc^3-11\,Ac^4)x^4+6\,(13\,Bb^2c^2-11\,Abc^3)x^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}}{77\,(c^6x^3+bc^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/77*(15*((13*B*b^3*c - 11*A*b^2*c^2)*x^3 + (13*B*b^4 - 11*A*b^3*c)*x)*sqr
t(c)*weierstrassPInverse(-4*b/c, 0, x) - (14*B*c^4*x^6 + 195*B*b^3*c - 165*
A*b^2*c^2 - 2*(13*B*b*c^3 - 11*A*c^4)*x^4 + 6*(13*B*b^2*c^2 - 11*A*b*c^3)*x
^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^6*x^3 + b*c^5*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 10661 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{17/2}\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.258 $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=377

$$-\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}}+\frac{7b(11bB-9Ac)x^{3/2}(b+cx^2)}{15c^{7/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{7(11bB-9Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{45c^3}+\frac{(11bB-9Ac)x^{5/2}}{9bc^2}$$

```
[Out] -(-A*c+B*b)*x^(13/2)/b/c/(c*x^4+b*x^2)^(1/2)+7/15*b*(-9*A*c+11*B*b)*x^(3/2)
*(c*x^2+b)/c^(7/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+1/9*(-9*A*c+11*B
*b)*x^(5/2)*(c*x^4+b*x^2)^(1/2)/b/c^2-7/45*(-9*A*c+11*B*b)*x^(1/2)*(c*x^4+b
*x^2)^(1/2)/c^3-7/15*b^(5/4)*(-9*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+
b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)+7/30*b^(5/4)*(
-9*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2
)/c^(15/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2062, 2049, 2057, 335, 311, 226, 1210}

$$\frac{7b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-9Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{30c^{15/4}\sqrt{bx^2+cx^4}}-\frac{7b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(11bB-9Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{15/4}\sqrt{bx^2+cx^4}}+\frac{7bx^{3/2}(b+cx^2)(11bB-9Ac)}{15c^{7/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{7\sqrt{x}\sqrt{bx^2+cx^4}(11bB-9Ac)}{45c^3}+\frac{x^{5/2}\sqrt{bx^2+cx^4}(11bB-9Ac)}{9bc^2}-\frac{x^{13/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(13/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + (7*b*(11*b*B - 9*A*c)
*x^(3/2)*(b + c*x^2))/(15*c^(7/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]
) - (7*(11*b*B - 9*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c^3) + ((11*b*B -
9*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*b*c^2) - (7*b^(5/4)*(11*b*B - 9*A*c)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
E[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(15/4)*Sqrt[b*x^2 + c*x^
4]) + (7*b^(5/4)*(11*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2
])/(30*c^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
```

```
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(\frac{11bB}{2}-\frac{9Ac}{2}\right)\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2} - \frac{(7(11bB-9Ac))\int \frac{x^{7/2}}{\sqrt{bx^2+c}}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+c}}{9bc^2}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+c}}{9bc^2}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+c}}{9bc^2}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+c}}{9bc^2}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} + \frac{7b(11bB-9Ac)x^{3/2}(b+cx^2)}{15c^{7/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2}}{45c^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.11, size = 110, normalized size = 0.29

$$\frac{2x^{5/2}\left(77b^2B+c^2x^2(9A+5Bx^2)-bc(63A+11Bx^2)+7b(-11bB+9Ac)\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{45c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

[Out] (2*x^(5/2)*(77*b^2*B + c^2*x^2*(9*A + 5*B*x^2) - b*c*(63*A + 11*B*x^2) + 7*b*(-11*b*B + 9*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^2)/b)]))/(45*c^3*Sqrt[x^2*(b + c*x^2)])

Maple [A]
time = 0.43, size = 420, normalized size = 1.11

method	result
default	$-\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(-20B\,c^{3}x^{6}+378A\,b^{2}c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.\right.}{}$
risch	$\frac{2x^{\frac{5}{2}}\left(5Bc\,x^{2}+9Ac-16Bb\right)\left(c\,x^{2}+b\right)}{45c^{3}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{b\left(\frac{\left(24Ac-31Bb\right)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/90/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(-20*B*c^3*x^6+378*A*b^2*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-189*A*b^2*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-462*B*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+231*B*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-36*A*c^3*x^4+44*B*b*c^2*x^4-126*A*b*c^2*x^2+154*B*b^2*c*x^2)/c^4

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 134, normalized size = 0.36

$$-\frac{21\,(11\,Bb^3-9\,Ab^2c+(11\,Bb^2c-9\,Abc^2)x^2)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-(10\,Bc^3x^4-77\,Bb^2c+63\,Abc^2-2\,(11\,Bbc^2-9\,Ac^3)x^2)\sqrt{cx^4+bx^2}\,\sqrt{x}}{45\,(c^5x^2+bc^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/45*(21*(11*B*b^3 - 9*A*b^2*c + (11*B*b^2*c - 9*A*b*c^2)*x^2)*sqrt(c)*wei
erstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) - (10*B*c^3*x^4
- 77*B*b^2*c + 63*A*b*c^2 - 2*(11*B*b*c^2 - 9*A*c^3)*x^2)*sqrt(c*x^4 + b*x^
2)*sqrt(x))/(c^5*x^2 + b*c^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7771 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{15/2}\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.259 $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}}-\frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}}+\frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2}+\frac{5b^{3/4}(9bB-7Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\cdots}{\cdots}$$

```
[Out] -(-A*c+B*b)*x^(11/2)/b/c/(c*x^4+b*x^2)^(1/2)+1/7*(-7*A*c+9*B*b)*x^(3/2)*(c*
x^4+b*x^2)^(1/2)/b/c^2-5/21*(-7*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)+
5/42*b^(3/4)*(-7*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(
1/2))^2)^(1/2)/c^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2062, 2049, 2057, 335, 226}

$$\frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9bB-7Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{42c^{13/4}\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}(9bB-7Ac)}{21c^3\sqrt{x}}+\frac{x^{3/2}\sqrt{bx^2+cx^4}(9bB-7Ac)}{7bc^2}-\frac{x^{11/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(11/2))/(b*c*Sqrt[b*x^2 + c*x^4])) - (5*(9*b*B - 7*A*c)*Sq
rt[b*x^2 + c*x^4])/(21*c^3*Sqrt[x]) + ((9*b*B - 7*A*c)*x^(3/2)*Sqrt[b*x^2 +
 c*x^4])/(7*b*c^2) + (5*b^(3/4)*(9*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(42*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(\frac{9bB}{2}-\frac{7Ac}{2}\right)\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2} - \frac{(5(9bB-7Ac))\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}}{14c^2}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} - \frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}} + \frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2} +$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} - \frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}} + \frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2} +$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} - \frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}} + \frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2} +$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} - \frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}} + \frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.12, size = 110, normalized size = 0.51

$$\frac{x^{3/2}\left(-45b^2B+bc(35A-18Bx^2)+2c^2x^2(7A+3Bx^2)+5b(9bB-7Ac)\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{21c^3\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(-45*b^2*B + b*c*(35*A - 18*B*x^2) + 2*c^2*x^2*(7*A + 3*B*x^2) + 5
*b*(9*b*B - 7*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((
c*x^2)/b)]))/(21*c^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.41, size = 255, normalized size = 1.19

method	result

default	$-\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(35A\sqrt{-bc}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\cdots\right.}{\cdots}$
risch	$\frac{2\left(3Bc\,x^{2}+7Ac-12Bb\right)x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}{21c^{3}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{b\left(\frac{28A\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{Elliptic}\cdots}{\sqrt{c\,x^{3}+bx}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/42/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(35*A*(-b*c)^(1/2)*((-c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*b*c-45*B*(-b*c)^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*b^2-12*B*
c^3*x^5-28*A*c^3*x^3+36*B*b*c^2*x^3-70*A*b*c^2*x+90*B*b^2*c*x)/c^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.71, size = 129, normalized size = 0.60

$$\frac{5\left(\left(9\,Bb^{2}c-7\,Abc^{2}\right)x^{3}+\left(9\,Bb^{3}-7\,Ab^{2}c\right)x\right)\sqrt{c}\ \text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)+\left(6\,Bc^{3}x^{4}-45\,Bb^{2}c+35\,Abc^{2}-2\left(9\,Bbc^{2}-7\,Ac^{3}\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}}\,\sqrt{x}}{21\left(c^{5}x^{3}+bc^{4}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*(5*((9*B*b^2*c - 7*A*b*c^2)*x^3 + (9*B*b^3 - 7*A*b^2*c)*x)*sqrt(c)*wei
erstrassPInverse(-4*b/c, 0, x) + (6*B*c^3*x^4 - 45*B*b^2*c + 35*A*b*c^2 - 2
```

```
*(9*B*b*c^2 - 7*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^5*x^3 + b*c^4*x
)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5457 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{13/2}\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.260 $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=340

$$-\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}}-\frac{3(7bB-5Ac)x^{3/2}(b+cx^2)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{(7bB-5Ac)\sqrt{x}\,\sqrt{bx^2+cx^4}}{5bc^2}+\frac{3\sqrt[4]{b}\,(7bB-5Ac)x\left(\right.}{}$$

```
[Out] -(-A*c+B*b)*x^(9/2)/b/c/(c*x^4+b*x^2)^(1/2)-3/5*(-5*A*c+7*B*b)*x^(3/2)*(c*x
^2+b)/c^(5/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+1/5*(-5*A*c+7*B*b)*x^
(1/2)*(c*x^4+b*x^2)^(1/2)/b/c^2+3/5*b^(1/4)*(-5*A*c+7*B*b)*x*(cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*E
llipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(
1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)-
3/10*b^(1/4)*(-5*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(
1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2062, 2049, 2057, 335, 311, 226, 1210}

$$-\frac{3\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(7bB-5Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10c^{11/4}\sqrt{bx^2+cx^4}}+\frac{3\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(7bB-5Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{11/4}\sqrt{bx^2+cx^4}}-\frac{3x^{3/2}(b+cx^2)(7bB-5Ac)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{\sqrt{x}\,\sqrt{bx^2+cx^4}\,(7bB-5Ac)}{5bc^2}-\frac{x^{9/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(9/2))/(b*c*Sqrt[b*x^2 + c*x^4])) - (3*(7*b*B - 5*A*c)*x^(
3/2)*(b + c*x^2))/(5*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (
(7*b*B - 5*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*b*c^2) + (3*b^(1/4)*(7*b*B
- 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*
EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(11/4)*Sqrt[b*x^2
 + c*x^4]) - (3*b^(1/4)*(7*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(10*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
```

```
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(\frac{7bB}{2}-\frac{5Ac}{2}\right)\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2} - \frac{(3(7bB-5Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx}}}{10c^2}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2} - \frac{\left(3(7bB-5Ac)x\sqrt{b+cx^2}\right)}{10c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2} - \frac{\left(3(7bB-5Ac)x\sqrt{b+cx^2}\right)}{5c^2\sqrt{b}}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2} - \frac{\left(3\sqrt{b}(7bB-5Ac)x\sqrt{b+cx}\right.}{5c^{5/2}}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} - \frac{3(7bB-5Ac)x^{3/2}(b+cx^2)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+}}{5bc^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.10, size = 85, normalized size = 0.25

$$\frac{2x^{5/2}\left(-7bB+5Ac+Bcx^2+(7bB-5Ac)\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{5c^2\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^(5/2)*(-7*b*B + 5*A*c + B*c*x^2 + (7*b*B - 5*A*c)*Sqrt[1 + (c*x^2)/b]*
Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^2)/b)]))/(5*c^2*Sqrt[x^2*(b + c*x^2
)])
```

Maple [A]
time = 0.52, size = 394, normalized size = 1.16

method	result
default	$\frac{x^{\frac{5}{2}}(cx^2+b)\left(30A\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,bc-15A\,\text{Ellip}\ldots}{\ldots}$
risch	$\frac{2Bx^{\frac{5}{2}}(cx^2+b)}{5c^2\sqrt{x^2(cx^2+b)}}+\frac{\left(\frac{(5Ac-8Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\ldots}{\ldots}\right.}{c\sqrt{cx^3-\ldots}}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(30*A*EllipticE(((c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2
)*b*c-15*A*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b*c-42*B*EllipticE(((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)
*b^2+21*B*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b^2+4*B*c^2*x^4-10*A*c^2*x^2+14*b*B*x
^2*c)/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 106, normalized size = 0.31

$$\frac{3\,(7\,Bb^2-5\,Abc+(7\,Bbc-5\,Ac^2)x^2)\sqrt{c}\ \text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+(2\,Bc^2x^2+7\,Bbc-5\,Ac^2)\sqrt{cx^4+bx^2}\ \sqrt{x}}{5\,(c^4x^2+bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(3*(7*B*b^2 - 5*A*b*c + (7*B*b*c - 5*A*c^2)*x^2)*sqrt(c)*weierstrassZet
a(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) + (2*B*c^2*x^2 + 7*B*b*c -
5*A*c^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^4*x^2 + b*c^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3655 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11/2}\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.261 $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=178

$$-\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}}-\frac{(5bB-3Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\right.\right.}{6\sqrt[4]{b}\,c^{9/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -(-A*c+B*b)*x^(7/2)/b/c/(c*x^4+b*x^2)^(1/2)+1/3*(-3*A*c+5*B*b)*(c*x^4+b*x^2
)^(1/2)/b/c^2/x^(1/2)-1/6*(-3*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(1/4)/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2062, 2049, 2057, 335, 226}

$$-\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(5bB-3Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{b}\,c^{9/4}\sqrt{bx^2+cx^4}}+\frac{\sqrt{bx^2+cx^4}\,(5bB-3Ac)}{3bc^2\sqrt{x}}-\frac{x^{7/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(7/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((5*b*B - 3*A*c)*Sqrt[
b*x^2 + c*x^4])/(3*b*c^2*Sqrt[x]) - ((5*b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x
)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqr
t[x])/b^(1/4)], 1/2])/(6*b^(1/4)*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(\frac{5bB}{2}-\frac{3Ac}{2}\right)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}} - \frac{(5bB-3Ac)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{6c^2}$$

$$= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}} - \frac{\left((5bB-3Ac)x\sqrt{b+cx^2}\right)\int \frac{}{\sqrt{x}\sqrt{}}}{6c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}} - \frac{\left((5bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\Big(}{3c^2\sqrt{bx^2+cx}}$$

$$= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}} - \frac{(5bB-3Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{}{(\sqrt{}}}}{6\sqrt[4]{b}\,c^{9/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.10, size = 86, normalized size = 0.48

$$\frac{x^{3/2}\left(5bB-3Ac+2Bcx^2+(-5bB+3Ac)\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{3c^2\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(5*b*B - 3*A*c + 2*B*c*x^2 + (-5*b*B + 3*A*c)*Sqrt[1 + (c*x^2)/b]*
Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(3*c^2*Sqrt[x^2*(b + c*x^2
)])
```

Maple [A]
time = 0.48, size = 230, normalized size = 1.29

method	result
default	$\frac{x^{\frac{5}{2}}(c\,x^2+b)\left(3A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\;\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}\,c-5\right.}{6(}$

risch	$\frac{2Bx^{\frac{3}{2}}(cx^2+b)}{3c^2\sqrt{x^2(cx^2+b)}} + \frac{\left(\frac{3A\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\right.}{}}\right.}{\sqrt{cx^3+bx}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(3*A*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(
1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*
(-b*c)^(1/2)*c-5*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b+4*B*c^2*x^3-6
*A*c^2*x+10*B*b*c*x)/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 102, normalized size = 0.57

$$-\frac{((5Bbc-3Ac^2)x^3+(5Bb^2-3Abc)x)\sqrt{c}\,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)-(2Bc^2x^2+5Bbc-3Ac^2)\sqrt{cx^4+bx^2}\sqrt{x}}{3(c^4x^3+bc^3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(((5*B*b*c - 3*A*c^2)*x^3 + (5*B*b^2 - 3*A*b*c)*x)*sqrt(c)*weierstrass
PInverse(-4*b/c, 0, x) - (2*B*c^2*x^2 + 5*B*b*c - 3*A*c^2)*sqrt(c*x^4 + b*x
^2)*sqrt(x))/(c^4*x^3 + b*c^3*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{9/2}\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.262 $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=299

$$-\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(3bB-Ac)x^{3/2}(b+cx^2)}{bc^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\Big(2}{b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -(-A*c+B*b)*x^(5/2)/b/c/(c*x^4+b*x^2)^(1/2)+(-A*c+3*B*b)*x^(3/2)*(c*x^2+b)/
b/c^(3/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-(-A*c+3*B*b)*x*(cos(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x
*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/c^(7/4)/(c*x^4+b*
x^2)^(1/2)+1/2*(-A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(
1/2))^2)^(1/2)/b^(3/4)/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2062, 2057, 335, 311, 226, 1210}

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(3bB-Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}-\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(3bB-Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}+\frac{x^{3/2}(b+cx^2)(3bB-Ac)}{bc^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{x^{5/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(5/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((3*b*B - A*c)*x^(3/2)
*(b + c*x^2))/(b*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - ((3*b
*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]
*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(7/4)*Sqrt
[b*x^2 + c*x^4]) + ((3*b*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2
])/(2*b^(3/4)*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(\frac{3bB}{2}-\frac{Ac}{2}\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(\left(\frac{3bB}{2}-\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{bc\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(2\left(\frac{3bB}{2}-\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{bc\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(2\left(\frac{3bB}{2}-\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\,c^{3/2}\sqrt{bx^2+cx^4}} -$$

$$= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(3bB-Ac)x^{3/2}(b+cx^2)}{bc^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 78, normalized size = 0.26

$$-\frac{2x^{5/2}\left(-3bB+(3bB-Ac)\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{3bc\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*x^(5/2)*(-3*b*B + (3*b*B - A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3
/4, 3/2, 7/4, -((c*x^2)/b)]))/(3*b*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.38, size = 388, normalized size = 1.30

method	result

default	$-\frac{x^{\frac{5}{2}}\left(c\,x^2+b\right)\left(2A\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,bc-A\,\mathrm{Ellipt}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(2*A*EllipticE(((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)
*b*c-A*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b*c-6*B*EllipticE(((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1
/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b^2+
3*B*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b^2-2*A*c^2*x^2+2*b*B*x^2*c)/c^2/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.56, size = 98, normalized size = 0.33

$$-\frac{(3\,Bb^2-Abc+(3\,Bbc-Ac^2)x^2)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}\,(Bbc-Ac^2)\sqrt{x}}{bc^3x^2+b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -((3*B*b^2 - A*b*c + (3*B*b*c - A*c^2)*x^2)*sqrt(c)*weierstrassZeta(-4*b/c,
 0, weierstrassPInverse(-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*(B*b*c - A*c^2
)*sqrt(x))/(b*c^3*x^2 + b^2*c^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{7/2}\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.263 $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(bB+Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -(-A*c+B*b)*x^(3/2)/b/c/(c*x^4+b*x^2)^(1/2)+1/2*(A*c+B*b)*x*(cos(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*El
lipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1
/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(5/4)/c^(5/4)/(c*x^4+b*x^2)^
(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2062, 2057, 335, 226}

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(Ac+bB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}c^{5/4}\sqrt{bx^2+cx^4}}-\frac{x^{3/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(3/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((b*B + A*c)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*b^(5/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx &= -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(bB+Ac)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{2bc}\\ &= -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left((bB+Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{2bc\sqrt{bx^2+cx^4}}\\ &= -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left((bB+Ac)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{bc\sqrt{bx^2+cx^4}}\\ &= -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(bB+Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\ldots\right.\right.}{2b^{5/4}c^{5/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 76, normalized size = 0.55

$$\frac{x^{3/2}\left(-bB+Ac+(bB+Ac)\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{bc\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(-(b*B) + A*c + (b*B + A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[
1/4, 1/2, 5/4, -((c*x^2)/b)]))/(b*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.37, size = 222, normalized size = 1.62

method	result
default	$\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}\,c+B\right.}{2(x^{4}c+b\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/
2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-
b*c)^(1/2)*c+B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*b+2*A*c^2*x-2*B*b*c
*x)/b/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 90, normalized size = 0.66

$$\frac{((Bbc+Ac^2)x^3+(Bb^2+Abc)x)\sqrt{c}\,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)-\sqrt{cx^4+bx^2}\,(Bbc-Ac^2)\sqrt{x}}{bc^3x^3+b^2c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

[Out] (((B*b*c + A*c^2)*x^3 + (B*b^2 + A*b*c)*x)*sqrt(c)*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x^4 + b*x^2)*(B*b*c - A*c^2)*sqrt(x))/(b*c^3*x^3 + b^2*c^2*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}(A+Bx^2)}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)

[Out] Integral(x**(5/2)*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")

[Out] integrate((B*x^2 + A)*x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)

[Out] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)

3.264 $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=318

$$-\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}}+\frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}}-\frac{(bB-3Ac)x^{3/2}(b+cx^2)}{b^2\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{\cdots}{\left(\sqrt{b}\cdots\right.}}\cdots}{b^{7/4}c^{3/4}\cdots}$$

```
[Out] (-3*A*c+B*b)*x^(5/2)/b^2/(c*x^4+b*x^2)^(1/2)-(-3*A*c+B*b)*x^(3/2)*(c*x^2+b)
/b^2/c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2*A*x^(1/2)/b/(c*x^4+b
*x^2)^(1/2)+(-3*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)
/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2
))^2)^(1/2)/b^(7/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)-1/2*(-3*A*c+B*b)*x*(cos(2*a
rctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)
+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/c^(3/4)/(c*x^4+
b*x^2)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2063, 2048, 2057, 335, 311, 226, 1210}

$$-\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(bB-3Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(bB-3Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x^{5/2}(bB-3Ac)}{b^2\sqrt{bx^2+cx^4}}-\frac{x^{3/2}(b+cx^2)(bB-3Ac)}{b^2\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*A*Sqrt[x])/(b*Sqrt[b*x^2 + c*x^4]) + ((b*B - 3*A*c)*x^(5/2))/(b^2*Sqrt[
b*x^2 + c*x^4]) - ((b*B - 3*A*c)*x^(3/2)*(b + c*x^2))/(b^2*Sqrt[c]*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + ((b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(b^(7/4)*c^(3/4)*Sqrt[b*x^2 + c*x^4]) - ((b*B - 3*A*c)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*b^(7/4)*c^(3/4)*Sqrt[b*x^2
+ c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
```

```
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{bB}{2}+\frac{3Ac}{2}\right)\right)\int \frac{x^{7/2}}{(bx^2+cx^4)^{3/2}}\,dx}{b}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{(bB-3Ac)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{2b^2}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{\left((bB-3Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{2b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{\left((bB-3Ac)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\right.}{b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{\left((bB-3Ac)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\right.}{b^{3/2}\sqrt{c}\,\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{(bB-3Ac)x^{3/2}(b+cx^2)}{b^2\sqrt{c}\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{(bB-3A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 77, normalized size = 0.24

$$\frac{2\sqrt{x}\left(-3Ab+(bB-3Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{3b^2\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

[Out] (2*Sqrt[x]*(-3*A*b + (b*B - 3*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F
1[3/4, 3/2, 7/4, -((c*x^2)/b)]))/(3*b^2*Sqrt[x^2*(b + c*x^2)])

Maple [A]
time = 0.41, size = 392, normalized size = 1.23

method	result
default	$\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(6A\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,bc-3A\,\mathrm{Ellip}\cdots\right.}{\cdots}$
risch	$-\frac{2A\left(c\,x^{2}+b\right)\sqrt{x}}{b^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(\frac{A\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\mathrm{Ellip}\cdots}{\cdots}\right.}{\sqrt{c\,x^{3}+\cdots}}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(6*A*EllipticE(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*
b*c-3*A*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b*c-2*B*EllipticE(((c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(
1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b^2
+B*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*b^2-6*A*c^2*x^2+2*b*B*x^2*c-4*A*b*c)/c/b^2

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.68, size = 115, normalized size = 0.36

$$\frac{((Bbc-3\,Ac^2)x^4+(Bb^2-3\,Abc)x^2)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\sqrt{cx^4+bx^2}\,(2\,Abc-(Bbc-3\,Ac^2)x^2)\sqrt{x}}{b^2c^2x^4+b^3cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] (((B*b*c - 3*A*c^2)*x^4 + (B*b^2 - 3*A*b*c)*x^2)*sqrt(c)*weierstrassZeta(-4
*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) - sqrt(c*x^4 + b*x^2)*(2*A*b*c
- (B*b*c - 3*A*c^2)*x^2)*sqrt(x))/(b^2*c^2*x^4 + b^3*c*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**(3/2)*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3/2}\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.265 $\int \frac{\sqrt{x}\,(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{2A}{3b\sqrt{x}\,\sqrt{bx^2+cx^4}}+\frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}}+\frac{(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\ldots}\right.\right.}{6b^{9/4}\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}$$

```
[Out] 1/3*(-5*A*c+3*B*b)*x^(3/2)/b^2/(c*x^4+b*x^2)^(1/2)-2/3*A/b/x^(1/2)/(c*x^4+b
*x^2)^(1/2)+1/6*(-5*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*
c^(1/2))^2)^(1/2)/b^(9/4)/c^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2063, 2048, 2057, 335, 226}

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(3bB-5Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}+\frac{x^{3/2}(3bB-5Ac)}{3b^2\sqrt{bx^2+cx^4}}-\frac{2A}{3b\sqrt{x}\,\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*A)/(3*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4]) + ((3*b*B - 5*A*c)*x^(3/2))/(3*b^2
*Sqrt[b*x^2 + c*x^4]) + ((3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(6*b^(9/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{\sqrt{x}\,(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{2A}{3b\sqrt{x}\,\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{3bB}{2}+\frac{5Ac}{2}\right)\right)\int \frac{x^{5/2}}{(bx^2+cx^4)^{3/2}}\,dx}{3b}$$

$$= -\frac{2A}{3b\sqrt{x}\,\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{6b^2}$$

$$= -\frac{2A}{3b\sqrt{x}\,\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{\left((3bB-5Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b}}}{6b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A}{3b\sqrt{x}\,\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{\left((3bB-5Ac)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int\right.}{3b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A}{3b\sqrt{x}\,\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b}{\left(\sqrt{b}\right.}}}{6b^{9/4}\sqrt[4]{c}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 92, normalized size = 0.55

$$\frac{-2Ab+3bBx^2-5Acx^2+(3bB-5Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)}{3b^2\sqrt{x}\,\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*A*b + 3*b*B*x^2 - 5*A*c*x^2 + (3*b*B - 5*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*H
ypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)])/(3*b^2*Sqrt[x]*Sqrt[x^2*(b +
 c*x^2)])
```

Maple [A]
time = 0.40, size = 235, normalized size = 1.41

method	result
default	$-\frac{x^{\frac{3}{2}}(c\,x^2+b)\left(5A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right.}{}$

risch	$-\frac{2A(c\,x^2+b)}{3b^2\sqrt{x}\,\sqrt{x^2\,(c\,x^2+b)}}-\frac{\left(\frac{A\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\right.}{\sqrt{c\,x^3+bx}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(c*x^4+b*x^2)^(3/2)*x^(3/2)*(c*x^2+b)*(5*A*(-b*c)^(1/2)*((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
,1/2*2^(1/2))*c*x-3*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*
EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*x+10*A*c^2
*x^2-6*b*B*x^2*c+4*A*b*c)/c/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 111, normalized size = 0.66

$$\frac{((3\,Bbc-5\,Ac^2)x^5+(3\,Bb^2-5\,Abc)x^3)\sqrt{c}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^4+bx^2}\,(2\,Abc-(3\,Bbc-5\,Ac^2)x^2)\sqrt{x}}{3\,(b^2c^2x^5+b^3cx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(((3*B*b*c - 5*A*c^2)*x^5 + (3*B*b^2 - 5*A*b*c)*x^3)*sqrt(c)*weierstras
sPInverse(-4*b/c, 0, x) - sqrt(c*x^4 + b*x^2)*(2*A*b*c - (3*B*b*c - 5*A*c^2
)*x^2)*sqrt(x))/(b^2*c^2*x^5 + b^3*c*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}\,(A + Bx^2)}{(x^2\,(b + cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(sqrt(x)*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}\,\left(B\,x^2 + A\right)}{{\left(c\,x^4 + b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.266 $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=368

$$-\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}}+\frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}}+\frac{3\sqrt{c}\,(5bB-7Ac)x^{3/2}(b+cx^2)}{5b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}-\frac{3\sqrt[4]{\ }}{}$$

```
[Out] -2/5*A/b/x^(3/2)/(c*x^4+b*x^2)^(1/2)+3/5*(-7*A*c+5*B*b)*x^(3/2)*(c*x^2+b)*c
^(1/2)/b^3/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+1/5*(-7*A*c+5*B*b)*x^(1/
2)/b^2/(c*x^4+b*x^2)^(1/2)-3/5*(-7*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^(3/
2)-3/5*c^(1/4)*(-7*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^
(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)
*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c
^(1/2))^2)^(1/2)/b^(11/4)/(c*x^4+b*x^2)^(1/2)+3/10*c^(1/4)*(-7*A*c+5*B*b)*x
*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))
*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(11/4)/(c*x^
4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2063, 2048, 2050, 2057, 335, 311, 226, 1210}

$$\frac{3\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(5bB-7Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{bx^2+cx^4}}-\frac{3\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,(5bB-7Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{bx^2+cx^4}}+\frac{3\sqrt{c}\,x^{3/2}(b+cx^2)\,(5bB-7Ac)}{5b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}\,(5bB-7Ac)}{5b^3x^{3/2}}+\frac{\sqrt{x}\,(5bB-7Ac)}{5b^2\sqrt{bx^2+cx^4}}-\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*A)/(5*b*x^(3/2)*Sqrt[b*x^2 + c*x^4]) + ((5*b*B - 7*A*c)*Sqrt[x])/(5*b^2
*Sqrt[b*x^2 + c*x^4]) + (3*Sqrt[c]*(5*b*B - 7*A*c)*x^(3/2)*(b + c*x^2))/(5*
b^3*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*(5*b*B - 7*A*c)*Sqrt[b*
x^2 + c*x^4])/(5*b^3*x^(3/2)) - (3*c^(1/4)*(5*b*B - 7*A*c)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/
4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(11/4)*Sqrt[b*x^2 + c*x^4]) + (3*c^(1/4)*(
5*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*
x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(10*b^(11/4)*Sqr
t[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
```

```
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx = -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{5bB}{2}+\frac{7Ac}{2}\right)\right)\int \frac{x^{3/2}}{(bx^2+cx^4)^{3/2}}\,dx}{5b}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} + \frac{(3(5bB-7Ac))\int \frac{1}{\sqrt{x}\,\sqrt{bx^2+cx^4}}}{10b^2}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} + \frac{(3c(5b}{}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} + \frac{\left(3c(5\right.}{}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} + \frac{\left(3c(5\right.}{}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} + \frac{\left(3\sqrt{c}\right.}{}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} + \frac{3\sqrt{c}\,(5bB-7Ac)x^{3/2}(b+cx^2)}{5b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{3}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 79, normalized size = 0.21

$$\frac{-2Ab+2(-5bB+7Ac)x^2\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{cx^2}{b}\right)}{5b^2x^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*A*b + 2*(-5*b*B + 7*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-1/4
, 3/2, 3/4, -((c*x^2)/b)])/(5*b^2*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.41, size = 420, normalized size = 1.14

method	result
default	$-\frac{\sqrt{x}\,\left(c\,x^{2}+b\right)\left(42A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc\,x^{2}\right.}{}$

risch	$-\frac{2(c\,x^2+b)(-8Ac\,x^2+5bB\,x^2+Ab)}{5b^3x^{\frac{3}{2}}\sqrt{x^2\,(c\,x^2+b)}}-\frac{c^2\left(\frac{(8Ac-5Bb)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/(c*x^4+b*x^2)^(3/2)*x^(1/2)*(c*x^2+b)*(42*A*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*b*c*x^2-21*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2-30*B*((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*
```

```
c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2
*2^(1/2))*b^2*x^2+15*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*x^2-42*A*c^2*x^4+30*
x^4*b*B*c-28*A*b*c*x^2+20*b^2*B*x^2+4*b^2*A)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 134, normalized size = 0.36

$$-\frac{3\left(\left(5\,Bbc-7\,Ac^2\right)x^6+\left(5\,Bb^2-7\,Abc\right)x^4\right)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(3\left(5\,Bbc-7\,Ac^2\right)x^4+2\,Ab^2+2\left(5\,Bb^2-7\,Abc\right)x^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}}{5\left(b^3cx^6+b^4x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*((5*B*b*c - 7*A*c^2)*x^6 + (5*B*b^2 - 7*A*b*c)*x^4)*sqrt(c)*weierst
rassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) + (3*(5*B*b*c - 7*A*
c^2)*x^4 + 2*A*b^2 + 2*(5*B*b^2 - 7*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x)
)/(b^3*c*x^6 + b^4*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+Bx^2}{\sqrt{x}\,\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(sqrt(x)*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{\sqrt{x}\,\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^(3/2)),x)
[Out] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

3.267 $\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}}+\frac{7bB-9Ac}{7b^2\sqrt{x}\,\sqrt{bx^2+cx^4}}-\frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}}-\frac{5c^{3/4}(7bB-9Ac)x\left(\sqrt{b}+\sqrt{c}\,x\right.}{4\ldots}$$

[Out] -2/7*A/b/x^(5/2)/(c*x^4+b*x^2)^(1/2)+1/7*(-9*A*c+7*B*b)/b^2/x^(1/2)/(c*x^4+b*x^2)^(1/2)-5/21*(-9*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^(5/2)-5/42*c^(3/4)*(-9*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(13/4)/(c*x^4+b*x^2)^(1/2)

Rubi [A]

time = 0.22, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2063, 2048, 2050, 2057, 335, 226}

$$-\frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(7bB-9Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{42b^{13/4}\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}\,(7bB-9Ac)}{21b^3x^{5/2}}+\frac{7bB-9Ac}{7b^2\sqrt{x}\,\sqrt{bx^2+cx^4}}-\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]

[Out] (-2*A)/(7*b*x^(5/2)*Sqrt[b*x^2 + c*x^4]) + (7*b*B - 9*A*c)/(7*b^2*Sqrt[x]*Sqrt[b*x^2 + c*x^4]) - (5*(7*b*B - 9*A*c)*Sqrt[b*x^2 + c*x^4])/(21*b^3*x^(5/2)) - (5*c^(3/4)*(7*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(42*b^(13/4)*Sqrt[b*x^2 + c*x^4])

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{7bB}{2}+\frac{9Ac}{2}\right)\right)\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^{3/2}}\,dx}{7b}$$

$$= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{(5(7bB-9Ac))\int \frac{1}{x^{3/2}\sqrt{bx^2+cx}}$$

$$= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}} - \frac{(5$$

$$= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}} - \frac{(5$$

$$= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}} - \frac{(5$$

$$5c$$

$$= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}} - $$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 79, normalized size = 0.39

$$\frac{-6Ab+2(-7bB+9Ac)x^2\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{cx^2}{b}\right)}{21b^2x^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-6*A*b + 2*(-7*b*B + 9*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-3/4
, 3/2, 1/4, -((c*x^2)/b)])/(21*b^2*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [**A**]
time = 0.40, size = 254, normalized size = 1.25

method	result

default	$\frac{(c\,x^2+b)\left(45A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c\,x^3-3\cdots\right.}{\cdots}$
risch	$-\frac{2(c\,x^2+b)(-12Ac\,x^2+7bB\,x^2+3Ab)}{21b^3x^{\frac{5}{2}}\sqrt{x^2\,(c\,x^2+b)}}+\frac{c\left(\frac{12A\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{Ellipt}\cdots}{\sqrt{c\,x^3+bx}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/42/(c*x^4+b*x^2)^(3/2)/x^(1/2)*(c*x^2+b)*(45*A*(-b*c)^(1/2)*((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
),1/2*2^(1/2))*c*x^3-35*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1
/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*x^3+90
*A*c^2*x^4-70*x^4*b*B*c+36*A*b*c*x^2-28*b^2*B*x^2-12*b^2*A)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 126, normalized size = 0.62

$$-\frac{5\left((7\,Bbc-9\,Ac^2)x^7+(7\,Bb^2-9\,Abc)x^5\right)\sqrt{c}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(5\,(7\,Bbc-9\,Ac^2)x^4+6\,Ab^2+2\,(7\,Bb^2-9\,Abc)x^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}}{21\,(b^3cx^7+b^4x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/21*(5*((7*B*b*c - 9*A*c^2)*x^7 + (7*B*b^2 - 9*A*b*c)*x^5)*sqrt(c)*weiers
trassPInverse(-4*b/c, 0, x) + (5*(7*B*b*c - 9*A*c^2)*x^4 + 6*A*b^2 + 2*(7*B
*b^2 - 9*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^3*c*x^7 + b^4*x^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{x^{3/2}\,{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

3.268 $\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=405

$$-\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}}+\frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}}-\frac{7c^{3/2}(9bB-11Ac)x^{3/2}(b+cx^2)}{15b^4\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}}-$$

```
[Out] -2/9*A/b/x^(7/2)/(c*x^4+b*x^2)^(1/2)+1/9*(-11*A*c+9*B*b)/b^2/x^(3/2)/(c*x^4
+b*x^2)^(1/2)-7/15*c^(3/2)*(-11*A*c+9*B*b)*x^(3/2)*(c*x^2+b)/b^4/(b^(1/2)+x
*c^(1/2))/(c*x^4+b*x^2)^(1/2)-7/45*(-11*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/
x^(7/2)+7/15*c*(-11*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^4/x^(3/2)+7/15*c^(5/4)
*(-11*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*a
rctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1
/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1
/2)/b^(15/4)/(c*x^4+b*x^2)^(1/2)-7/30*c^(5/4)*(-11*A*c+9*B*b)*x*(cos(2*arct
an(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))
)*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*
c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(15/4)/(c*x^4+b*x^2)^(1/
2)
```

Rubi [A]
time = 0.36, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2063, 2048, 2050, 2057, 335, 311, 226, 1210}

$$-\frac{7c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9bB-11Ac)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}}+\frac{7c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}(9bB-11Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{15/4}\sqrt{bx^2+cx^4}}-\frac{7c^{3/2}x^{3/2}(b+cx^2)(9bB-11Ac)}{15b^4\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{7c\sqrt{bx^2+cx^4}(9bB-11Ac)}{15b^4x^{3/2}}-\frac{7\sqrt{bx^2+cx^4}(9bB-11Ac)}{45b^3x^{7/2}}+\frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}}-\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*A)/(9*b*x^(7/2)*Sqrt[b*x^2 + c*x^4]) + (9*b*B - 11*A*c)/(9*b^2*x^(3/2)*
Sqrt[b*x^2 + c*x^4]) - (7*c^(3/2)*(9*b*B - 11*A*c)*x^(3/2)*(b + c*x^2))/(15
*b^4*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (7*(9*b*B - 11*A*c)*Sqrt[
b*x^2 + c*x^4])/(45*b^3*x^(7/2)) + (7*c*(9*b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4
])/(15*b^4*x^(3/2)) + (7*c^(5/4)*(9*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(15*b^(15/4)*Sqrt[b*x^2 + c*x^4]) - (7*c^(5/4)*(9*b*B -
11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*E
llipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(30*b^(15/4)*Sqrt[b*x^2
 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
```

```
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{9bB}{2}+\frac{11Ac}{2}\right)\right)\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx}{9b}$$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} + \frac{(7(9bB-11Ac))\int \frac{1}{x^{5/2}\sqrt{bx^2+c}}$$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - $$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + $$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + $$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + $$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + $$

$$= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7c^{3/2}(9bB-11Ac)x^{3/2}(b+cx^2)}{15b^4\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 79, normalized size = 0.20

$$\frac{-10Ab+2(-9bB+11Ac)x^2\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)}{45b^2x^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-10*A*b + 2*(-9*b*B + 11*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-5
/4, 3/2, -1/4, -((c*x^2)/b)])/(45*b^2*x^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.42, size = 450, normalized size = 1.11

method	result
default	$\frac{(c x^2+b)\left(462A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right) b c^2 x^4-231A\right.}{}$
risch	$-\frac{2(c x^2+b)(93A c^2x^4-72x^4bBc-16Abc x^2+9b^2B x^2+5b^2A)}{45b^4x^{\frac{7}{2}}\sqrt{x^2(c x^2+b)}}+\frac{c^2\left(\frac{(31Ac-24Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-}}{}\right.}{\sqrt{-}}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/90/(c*x^4+b*x^2)^(3/2)/x^(3/2)*(c*x^2+b)*(462*A*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*b*c^2*x^4-231*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x^4-378*B*((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*b^2*c*x^4+189*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(
1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ell
ipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c*x^4-462*A
*c^3*x^6+378*x^6*B*b*c^2-308*A*b*c^2*x^4+252*x^4*B*b^2*c+44*A*b^2*c*x^2-36*
x^2*B*b^3-20*A*b^3)/b^4
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 163, normalized size = 0.40

$$\frac{21\left((9\,Bbc^2-11\,Ac^3)x^8+(9\,Bb^2c-11\,Abc^2)x^6\right)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(21\,(9\,Bbc^2-11\,Ac^3)x^6+14\,(9\,Bb^2c-11\,Abc^2)x^4-10\,Ab^3-2\,(9\,Bb^3-11\,Ab^2c)x^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}}{45\,(b^4cx^8+b^5x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/45*(21*((9*B*b*c^2 - 11*A*c^3)*x^8 + (9*B*b^2*c - 11*A*b*c^2)*x^6)*sqrt(c
)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) + (21*(9*B*
b*c^2 - 11*A*c^3)*x^6 + 14*(9*B*b^2*c - 11*A*b*c^2)*x^4 - 10*A*b^3 - 2*(9*B
*b^3 - 11*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^4*c*x^8 + b^5*x^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{x^{5/2}\,{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

3.269 $\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=96

$$\frac{Ab^3x^{7+m}}{7+m}+\frac{b^2(bB+3Ac)x^{9+m}}{9+m}+\frac{3bc(bB+Ac)x^{11+m}}{11+m}+\frac{c^2(3bB+Ac)x^{13+m}}{13+m}+\frac{Bc^3x^{15+m}}{15+m}$$

```
[Out] A*b^3*x^(7+m)/(7+m)+b^2*(3*A*c+B*b)*x^(9+m)/(9+m)+3*b*c*(A*c+B*b)*x^(11+m)/
(11+m)+c^2*(A*c+3*B*b)*x^(13+m)/(13+m)+B*c^3*x^(15+m)/(15+m)
```

Rubi [A]
time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {1598, 459}

$$\frac{Ab^3x^{m+7}}{m+7}+\frac{b^2x^{m+9}(3Ac+bB)}{m+9}+\frac{c^2x^{m+13}(Ac+3bB)}{m+13}+\frac{3bcx^{m+11}(Ac+bB)}{m+11}+\frac{Bc^3x^{m+15}}{m+15}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (A*b^3*x^(7 + m))/(7 + m) + (b^2*(b*B + 3*A*c)*x^(9 + m))/(9 + m) + (3*b*c*
(b*B + A*c)*x^(11 + m))/(11 + m) + (c^2*(3*b*B + A*c)*x^(13 + m))/(13 + m)
+ (B*c^3*x^(15 + m))/(15 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx&=\int x^{6+m}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx\\&=\int\left(Ab^3x^{6+m}+b^2(bB+3Ac)x^{8+m}+3bc(bB+Ac)x^{10+m}+c^2(3bB+Ac\right.\\&=\frac{Ab^3x^{7+m}}{7+m}+\frac{b^2(bB+3Ac)x^{9+m}}{9+m}+\frac{3bc(bB+Ac)x^{11+m}}{11+m}+\frac{c^2(3bB+Ac)x}{13+m}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 89, normalized size = 0.93

$$x^{7+m}\left(\frac{Ab^3}{7+m}+\frac{b^2(bB+3Ac)x^2}{9+m}+\frac{3bc(bB+Ac)x^4}{11+m}+\frac{c^2(3bB+Ac)x^6}{13+m}+\frac{Bc^3x^8}{15+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^(7 + m)*((A*b^3)/(7 + m) + (b^2*(b*B + 3*A*c)*x^2)/(9 + m) + (3*b*c*(b*B
+ A*c)*x^4)/(11 + m) + (c^2*(3*b*B + A*c)*x^6)/(13 + m) + (B*c^3*x^8)/(15 +
 m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(96) = 192.
time = 0.38, size = 474, normalized size = 4.94

method	result
gosper	$x^{7+m}(B c^3m^4x^8+40B c^3m^3x^8+A c^3m^4x^6+3Bb c^2m^4x^6+590B c^3m^2x^8+42A c^3m^3x^6+126Bb c^2m^3x^6+3800m x^8B c^3+3Ab c^2m^4$
risch	$x^m(B c^3m^4x^8+40B c^3m^3x^8+A c^3m^4x^6+3Bb c^2m^4x^6+590B c^3m^2x^8+42A c^3m^3x^6+126Bb c^2m^3x^6+3800m x^8B c^3+3Ab c^2m^4$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x^(7+m)*(B*c^3*m^4*x^8+40*B*c^3*m^3*x^8+A*c^3*m^4*x^6+3*B*b*c^2*m^4*x^6+590
*B*c^3*m^2*x^8+42*A*c^3*m^3*x^6+126*B*b*c^2*m^3*x^6+3800*B*c^3*m*x^8+3*A*b*
c^2*m^4*x^4+644*A*c^3*m^2*x^6+3*B*b^2*c*m^4*x^4+1932*B*b*c^2*m^2*x^6+9009*B
*c^3*x^8+132*A*b*c^2*m^3*x^4+4278*A*c^3*m*x^6+132*B*b^2*c*m^3*x^4+12834*B*b
*c^2*m*x^6+3*A*b^2*c*m^4*x^2+2118*A*b*c^2*m^2*x^4+10395*A*c^3*x^6+B*b^3*m^4
*x^2+2118*B*b^2*c*m^2*x^4+31185*B*b*c^2*x^6+138*A*b^2*c*m^3*x^2+14652*A*b*c
^2*m*x^4+46*B*b^3*m^3*x^2+14652*B*b^2*c*m*x^4+A*b^3*m^4+2328*A*b^2*c*m^2*x^
2+36855*A*b*c^2*x^4+776*B*b^3*m^2*x^2+36855*B*b^2*c*x^4+48*A*b^3*m^3+16998*
A*b^2*c*m*x^2+5666*B*b^3*m*x^2+854*A*b^3*m^2+45045*A*b^2*c*x^2+15015*B*b^3*
x^2+6672*A*b^3*m+19305*A*b^3)/(15+m)/(13+m)/(11+m)/(9+m)/(7+m)
```

Maxima [A]
time = 0.31, size = 129, normalized size = 1.34

$$\frac{Bc^3x^{m+15}}{m+15}+\frac{3\,Bbc^2x^{m+13}}{m+13}+\frac{Ac^3x^{m+13}}{m+13}+\frac{3\,Bb^2cx^{m+11}}{m+11}+\frac{3\,Abc^2x^{m+11}}{m+11}+\frac{Bb^3x^{m+9}}{m+9}+\frac{3\,Ab^2cx^{m+9}}{m+9}+\frac{Ab^3x^{m+7}}{m+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] B*c^3*x^(m + 15)/(m + 15) + 3*B*b*c^2*x^(m + 13)/(m + 13) + A*c^3*x^(m + 13
)/(m + 13) + 3*B*b^2*c*x^(m + 11)/(m + 11) + 3*A*b*c^2*x^(m + 11)/(m + 11)
+ B*b^3*x^(m + 9)/(m + 9) + 3*A*b^2*c*x^(m + 9)/(m + 9) + A*b^3*x^(m + 7)/(
m + 7)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(96) = 192$.
time = 2.42, size = 381, normalized size = 3.97

$$\frac{\left(\left(Bc^3m^4+40\,Bc^3m^3+590\,Bc^3m^2+3800\,Bc^3m+9009\,Bc^3\right)x^{15}+\left(\left(3\,Bbc^2+Ac^3\right)m^4+31185\,Bbc^2+10395\,Ac^3+42\,\left(3\,Bbc^2+Ac^3\right)m^3+644\,\left(3\,Bbc^2+Ac^3\right)m^2+4278\,\left(3\,Bbc^2+Ac^3\right)m\right)x^{13}+3\left(\left(Bb^2c+Abc^2\right)m^4+12285\,Bb^2c+12285\,Abc^2+44\,\left(Bb^2c+Abc^2\right)m^3+706\,\left(Bb^2c+Abc^2\right)m^2+4884\,\left(Bb^2c+Abc^2\right)m\right)x^{11}+\left(\left(Bb^3+3\,Ab^2c\right)m^4+15015\,Bb^3+45045\,Ab^2c+46\,\left(Bb^3+3\,Ab^2c\right)m^3+776\,\left(Bb^3+3\,Ab^2c\right)m^2+5666\,\left(Bb^3+3\,Ab^2c\right)m\right)x^9+\left(Ab^3m^4+48\,Ab^3m^3+854\,Ab^3m^2+6672\,Ab^3m+19305\,Ab^3\right)x^7\right)x^m}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] ((B*c^3*m^4 + 40*B*c^3*m^3 + 590*B*c^3*m^2 + 3800*B*c^3*m + 9009*B*c^3)*x^1
5 + ((3*B*b*c^2 + A*c^3)*m^4 + 31185*B*b*c^2 + 10395*A*c^3 + 42*(3*B*b*c^2
+ A*c^3)*m^3 + 644*(3*B*b*c^2 + A*c^3)*m^2 + 4278*(3*B*b*c^2 + A*c^3)*m)*x^
13 + 3*((B*b^2*c + A*b*c^2)*m^4 + 12285*B*b^2*c + 12285*A*b*c^2 + 44*(B*b^2
*c + A*b*c^2)*m^3 + 706*(B*b^2*c + A*b*c^2)*m^2 + 4884*(B*b^2*c + A*b*c^2)*
m)*x^11 + ((B*b^3 + 3*A*b^2*c)*m^4 + 15015*B*b^3 + 45045*A*b^2*c + 46*(B*b^
3 + 3*A*b^2*c)*m^3 + 776*(B*b^3 + 3*A*b^2*c)*m^2 + 5666*(B*b^3 + 3*A*b^2*c)
*m)*x^9 + (A*b^3*m^4 + 48*A*b^3*m^3 + 854*A*b^3*m^2 + 6672*A*b^3*m + 19305*
A*b^3)*x^7)*x^m/(m^5 + 55*m^4 + 1190*m^3 + 12650*m^2 + 66009*m + 135135)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2077 vs. $2(87) = 174$.
time = 1.07, size = 2077, normalized size = 21.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

```
[Out] Piecewise((-A*b**3/(8*x**8) - A*b**2*c/(2*x**6) - 3*A*b*c**2/(4*x**4) - A*c
**3/(2*x**2) - B*b**3/(6*x**6) - 3*B*b**2*c/(4*x**4) - 3*B*b*c**2/(2*x**2)
+ B*c**3*log(x), Eq(m, -15)), (-A*b**3/(6*x**6) - 3*A*b**2*c/(4*x**4) - 3*A
*b*c**2/(2*x**2) + A*c**3*log(x) - B*b**3/(4*x**4) - 3*B*b**2*c/(2*x**2) +
3*B*b*c**2*log(x) + B*c**3*x**2/2, Eq(m, -13)), (-A*b**3/(4*x**4) - 3*A*b**
2*c/(2*x**2) + 3*A*b*c**2*log(x) + A*c**3*x**2/2 - B*b**3/(2*x**2) + 3*B*b*
*2*c*log(x) + 3*B*b*c**2*x**2/2 + B*c**3*x**4/4, Eq(m, -11)), (-A*b**3/(2*x
**2) + 3*A*b**2*c*log(x) + 3*A*b*c**2*x**2/2 + A*c**3*x**4/4 + B*b**3*log(x
) + 3*B*b**2*c*x**2/2 + 3*B*b*c**2*x**4/4 + B*c**3*x**6/6, Eq(m, -9)), (A*b
**3*log(x) + 3*A*b**2*c*x**2/2 + 3*A*b*c**2*x**4/4 + A*c**3*x**6/6 + B*b**3
*x**2/2 + 3*B*b**2*c*x**4/4 + B*b*c**2*x**6/2 + B*c**3*x**8/8, Eq(m, -7)),
(A*b**3*m**4*x**7*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m +
 135135) + 48*A*b**3*m**3*x**7*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**
```

2 + 66009*m + 135135) + 854*A*b**3*m**2*x**7*x**m/(m**5 + 55*m**4 + 1190*m*
*3 + 12650*m**2 + 66009*m + 135135) + 6672*A*b**3*m*x**7*x**m/(m**5 + 55*m*
*4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 19305*A*b**3*x**7*x**m/(m
5 + 55*m4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 3*A*b**2*c*m**
4*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) +
138*A*b**2*c*m**3*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 2328*A*b**2*c*m**2*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 +
12650*m**2 + 66009*m + 135135) + 16998*A*b**2*c*m*x**9*x**m/(m**5 + 55*m**4
 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 45045*A*b**2*c*x**9*x**m/(m
5 + 55*m4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 3*A*b*c**2*m**
4*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) +
 132*A*b*c**2*m**3*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66
009*m + 135135) + 2118*A*b*c**2*m**2*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3
 + 12650*m**2 + 66009*m + 135135) + 14652*A*b*c**2*m*x**11*x**m/(m**5 + 55*
m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 36855*A*b*c**2*x**11*x*
*m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + A*c**3*m*
*4*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135)
 + 42*A*c**3*m**3*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 644*A*c**3*m**2*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12
650*m**2 + 66009*m + 135135) + 4278*A*c**3*m*x**13*x**m/(m**5 + 55*m**4 + 1
190*m**3 + 12650*m**2 + 66009*m + 135135) + 10395*A*c**3*x**13*x**m/(m**5 +
 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + B*b**3*m**4*x**9*x*
*m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 46*B*b**3
*m**3*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135
) + 776*B*b**3*m**2*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66
009*m + 135135) + 5666*B*b**3*m*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 126
50*m**2 + 66009*m + 135135) + 15015*B*b**3*x**9*x**m/(m**5 + 55*m**4 + 1190
*m**3 + 12650*m**2 + 66009*m + 135135) + 3*B*b**2*c*m**4*x**11*x**m/(m**5 +
 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 132*B*b**2*c*m**3*x
11*xm/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 21
18*B*b**2*c*m**2*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 14652*B*b**2*c*m*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 1
2650*m**2 + 66009*m + 135135) + 36855*B*b**2*c*x**11*x**m/(m**5 + 55*m**4 +
 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 3*B*b*c**2*m**4*x**13*x**m/(m
5 + 55*m4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 126*B*b*c**2*m
3*x13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135)
 + 1932*B*b*c**2*m**2*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 +
 66009*m + 135135) + 12834*B*b*c**2*m*x**13*x**m/(m**5 + 55*m**4 + 1190*m**
3 + 12650*m**2 + 66009*m + 135135) + 31185*B*b*c**2*x**13*x**m/(m**5 + 55*m
4 + 1190*m3 + 12650*m**2 + 66009*m + 135135) + B*c**3*m**4*x**15*x**m/(
m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 40*B*c**3*m**
3*x**15*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) +
 590*B*c**3*m**2*x**15*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 3800*B*c**3*m*x**15*x**m/(m**5 + 55*m**4 + 1190*m**3 + 1265
0*m**2 + 66009*m + 135135) + 9009*B*c**3*x**15*x**m/(m**5 + 55*m**4 + 1190*

```
m**3 + 12650*m**2 + 66009*m + 135135), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(96) = 192.
time = 1.41, size = 603, normalized size = 6.28

$$\frac{Bc^3m^4x^{15}x^m + 40\,Bc^3m^3x^{15}x^m + 3\,Bbc^2m^4x^{13}x^m + Ac^3m^4x^{13}x^m + 590\,Bc^3m^2x^{15}x^m + 126\,Bbc^2m^3x^{13}x^m + 42\,Ac^3m^3x^{13}x^m + 3800\,Bc^3mx^{15}x^m + 3\,Bb^2cm^4x^{11}x^m + 3\,Abc^2m^4x^{11}x^m + 1932\,Bbc^2m^2x^{13}x^m + 644\,Ac^3m^2x^{13}x^m + 9009\,Bc^3x^{15}x^m + 132\,Bb^2cm^3x^{11}x^m + 132\,Abc^2m^3x^{11}x^m + 12834\,Bbc^2mx^{13}x^m + 4278\,Ac^3mx^{13}x^m + Bb^3m^4x^9x^m + 3\,Ab^2cm^4x^9x^m + 2118\,Bb^2cm^2x^{11}x^m + 2118\,Abc^2m^2x^{11}x^m + 31185\,Bbc^2x^{13}x^m + 10395\,Ac^3x^{13}x^m + 46\,Bb^3m^3x^9x^m + 138\,Ab^2cm^3x^9x^m + 14652\,Bb^2cmx^{11}x^m + 14652\,Abc^2mx^{11}x^m + Ab^3m^4x^7x^m + 776\,Bb^3m^2x^9x^m + 2328\,Ab^2cm^2x^9x^m + 36855\,Bb^2cx^{11}x^m + 36855\,Abc^2x^{11}x^m + 48\,Ab^3m^3x^7x^m + 5666\,Bb^3mx^9x^m + 16998\,Ab^2cmx^9x^m + 854\,Ab^3m^2x^7x^m + 15015\,Bb^3x^9x^m + 45045\,Ab^2cx^9x^m + 6672\,Ab^3mx^7x^m + 19305\,Ab^3x^7x^m}{m^5 + 55\,m^4 + 1190\,m^3 + 12650\,m^2 + 66009\,m + 135135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] (B*c^3*m^4*x^15*x^m + 40*B*c^3*m^3*x^15*x^m + 3*B*b*c^2*m^4*x^13*x^m + A*c^
3*m^4*x^13*x^m + 590*B*c^3*m^2*x^15*x^m + 126*B*b*c^2*m^3*x^13*x^m + 42*A*c
^3*m^3*x^13*x^m + 3800*B*c^3*m*x^15*x^m + 3*B*b^2*c*m^4*x^11*x^m + 3*A*b*c^
2*m^4*x^11*x^m + 1932*B*b*c^2*m^2*x^13*x^m + 644*A*c^3*m^2*x^13*x^m + 9009*
B*c^3*x^15*x^m + 132*B*b^2*c*m^3*x^11*x^m + 132*A*b*c^2*m^3*x^11*x^m + 1283
4*B*b*c^2*m*x^13*x^m + 4278*A*c^3*m*x^13*x^m + B*b^3*m^4*x^9*x^m + 3*A*b^2*
c*m^4*x^9*x^m + 2118*B*b^2*c*m^2*x^11*x^m + 2118*A*b*c^2*m^2*x^11*x^m + 311
85*B*b*c^2*x^13*x^m + 10395*A*c^3*x^13*x^m + 46*B*b^3*m^3*x^9*x^m + 138*A*b
^2*c*m^3*x^9*x^m + 14652*B*b^2*c*m*x^11*x^m + 14652*A*b*c^2*m*x^11*x^m + A*
b^3*m^4*x^7*x^m + 776*B*b^3*m^2*x^9*x^m + 2328*A*b^2*c*m^2*x^9*x^m + 36855*
B*b^2*c*x^11*x^m + 36855*A*b*c^2*x^11*x^m + 48*A*b^3*m^3*x^7*x^m + 5666*B*b
^3*m*x^9*x^m + 16998*A*b^2*c*m*x^9*x^m + 854*A*b^3*m^2*x^7*x^m + 15015*B*b^
3*x^9*x^m + 45045*A*b^2*c*x^9*x^m + 6672*A*b^3*m*x^7*x^m + 19305*A*b^3*x^7*
x^m)/(m^5 + 55*m^4 + 1190*m^3 + 12650*m^2 + 66009*m + 135135)
```

Mupad [B]
time = 0.48, size = 291, normalized size = 3.03

$$\frac{A\,b^3\,x^m\,x^7\,\left(m^4+48\,m^3+854\,m^2+6672\,m+19305\right)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}+\frac{B\,c^3\,x^m\,x^{15}\,\left(m^4+40\,m^3+590\,m^2+3800\,m+9009\right)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}+\frac{b^2\,x^m\,x^9\,\left(3\,A\,c+B\,b\right)\,\left(m^4+46\,m^3+776\,m^2+5666\,m+15015\right)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}+\frac{c^2\,x^m\,x^{13}\,\left(A\,c+3\,B\,b\right)\,\left(m^4+42\,m^3+644\,m^2+4278\,m+10395\right)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}+\frac{3\,b\,c\,x^m\,x^{11}\,\left(A\,c+B\,b\right)\,\left(m^4+44\,m^3+706\,m^2+4884\,m+12285\right)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (A*b^3*x^m*x^7*(6672*m + 854*m^2 + 48*m^3 + m^4 + 19305))/(66009*m + 12650*
m^2 + 1190*m^3 + 55*m^4 + m^5 + 135135) + (B*c^3*x^m*x^15*(3800*m + 590*m^2
 + 40*m^3 + m^4 + 9009))/(66009*m + 12650*m^2 + 1190*m^3 + 55*m^4 + m^5 + 1
35135) + (b^2*x^m*x^9*(3*A*c + B*b)*(5666*m + 776*m^2 + 46*m^3 + m^4 + 1501
5))/(66009*m + 12650*m^2 + 1190*m^3 + 55*m^4 + m^5 + 135135) + (c^2*x^m*x^1
3*(A*c + 3*B*b)*(4278*m + 644*m^2 + 42*m^3 + m^4 + 10395))/(66009*m + 12650
*m^2 + 1190*m^3 + 55*m^4 + m^5 + 135135) + (3*b*c*x^m*x^11*(A*c + B*b)*(488
4*m + 706*m^2 + 44*m^3 + m^4 + 12285))/(66009*m + 12650*m^2 + 1190*m^3 + 55
*m^4 + m^5 + 135135)
```

3.270 $\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=71

$$\frac{Ab^2x^{5+m}}{5+m}+\frac{b(bB+2Ac)x^{7+m}}{7+m}+\frac{c(2bB+Ac)x^{9+m}}{9+m}+\frac{Bc^2x^{11+m}}{11+m}$$

```
[Out] A*b^2*x^(5+m)/(5+m)+b*(2*A*c+B*b)*x^(7+m)/(7+m)+c*(A*c+2*B*b)*x^(9+m)/(9+m)
+B*c^2*x^(11+m)/(11+m)
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {1598, 459}

$$\frac{Ab^2x^{m+5}}{m+5}+\frac{bx^{m+7}(2Ac+bB)}{m+7}+\frac{cx^{m+9}(Ac+2bB)}{m+9}+\frac{Bc^2x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (A*b^2*x^(5 + m))/(5 + m) + (b*(b*B + 2*A*c)*x^(7 + m))/(7 + m) + (c*(2*b*B
 + A*c)*x^(9 + m))/(9 + m) + (B*c^2*x^(11 + m))/(11 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx&=\int x^{4+m}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\&=\int\left(Ab^2x^{4+m}+b(bB+2Ac)x^{6+m}+c(2bB+Ac)x^{8+m}+Bc^2x^{10+m}\right)\,dx\\&=\frac{Ab^2x^{5+m}}{5+m}+\frac{b(bB+2Ac)x^{7+m}}{7+m}+\frac{c(2bB+Ac)x^{9+m}}{9+m}+\frac{Bc^2x^{11+m}}{11+m}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 66, normalized size = 0.93

$$x^{5+m}\left(\frac{Ab^2}{5+m}+\frac{b(bB+2Ac)x^2}{7+m}+\frac{c(2bB+Ac)x^4}{9+m}+\frac{Bc^2x^6}{11+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] x^(5 + m)*((A*b^2)/(5 + m) + (b*(b*B + 2*A*c)*x^2)/(7 + m) + (c*(2*b*B + A*
c)*x^4)/(9 + m) + (B*c^2*x^6)/(11 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(71) = 142$.
time = 0.36, size = 262, normalized size = 3.69

method	result
gosper	$\frac{x^{5+m}(Bc^2m^3x^6+21Bc^2m^2x^6+Ac^2m^3x^4+2Bbcm^3x^4+143mx^6Bc^2+23Ac^2m^2x^4+46Bbcm^2x^4+315Bc^2x^6+2Abcm^3x^2+167Ac^2}{}$
risch	$\frac{x^m(Bc^2m^3x^6+21Bc^2m^2x^6+Ac^2m^3x^4+2Bbcm^3x^4+143mx^6Bc^2+23Ac^2m^2x^4+46Bbcm^2x^4+315Bc^2x^6+2Abcm^3x^2+167Ac^2m}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^(5+m)*(B*c^2*m^3*x^6+21*B*c^2*m^2*x^6+A*c^2*m^3*x^4+2*B*b*c*m^3*x^4+143*B
*c^2*m*x^6+23*A*c^2*m^2*x^4+46*B*b*c*m^2*x^4+315*B*c^2*x^6+2*A*b*c*m^3*x^2+
167*A*c^2*m*x^4+B*b^2*m^3*x^2+334*B*b*c*m*x^4+50*A*b*c*m^2*x^2+385*A*c^2*x^
4+25*B*b^2*m^2*x^2+770*B*b*c*x^4+A*b^2*m^3+398*A*b*c*m*x^2+199*B*b^2*m*x^2+
27*A*b^2*m^2+990*A*b*c*x^2+495*B*b^2*x^2+239*A*b^2*m+693*A*b^2)/(11+m)/(9+m
)/(7+m)/(5+m)
```

Maxima [A]
time = 0.28, size = 91, normalized size = 1.28

$$\frac{Bc^2x^{m+11}}{m+11}+\frac{2\,Bbcx^{m+9}}{m+9}+\frac{Ac^2x^{m+9}}{m+9}+\frac{Bb^2x^{m+7}}{m+7}+\frac{2\,Abcx^{m+7}}{m+7}+\frac{Ab^2x^{m+5}}{m+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] B*c^2*x^(m + 11)/(m + 11) + 2*B*b*c*x^(m + 9)/(m + 9) + A*c^2*x^(m + 9)/(m
+ 9) + B*b^2*x^(m + 7)/(m + 7) + 2*A*b*c*x^(m + 7)/(m + 7) + A*b^2*x^(m + 5
)/(m + 5)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(71) = 142$.
time = 1.90, size = 217, normalized size = 3.06

$$\frac{\left(\left(Bc^2m^3 + 21\,Bc^2m^2 + 143\,Bc^2m + 315\,Bc^2\right)x^{11} + \left((2\,Bbc + Ac^2)m^3 + 770\,Bbc + 385\,Ac^2 + 23\,(2\,Bbc + Ac^2)m^2 + 167\,(2\,Bbc + Ac^2)m\right)x^9 + \left((Bb^2 + 2\,Abc)m^3 + 495\,Bb^2 + 990\,Abc + 25\,(Bb^2 + 2\,Abc)m^2 + 199\,(Bb^2 + 2\,Abc)m\right)x^7 + \left(Ab^2m^3 + 27\,Ab^2m^2 + 239\,Ab^2m + 693\,Ab^2\right)x^5\right)x^m}{m^4 + 32\,m^3 + 374\,m^2 + 1888\,m + 3465}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] ((B*c^2*m^3 + 21*B*c^2*m^2 + 143*B*c^2*m + 315*B*c^2)*x^11 + ((2*B*b*c + A*
c^2)*m^3 + 770*B*b*c + 385*A*c^2 + 23*(2*B*b*c + A*c^2)*m^2 + 167*(2*B*b*c
+ A*c^2)*m)*x^9 + ((B*b^2 + 2*A*b*c)*m^3 + 495*B*b^2 + 990*A*b*c + 25*(B*b^
2 + 2*A*b*c)*m^2 + 199*(B*b^2 + 2*A*b*c)*m)*x^7 + (A*b^2*m^3 + 27*A*b^2*m^2
 + 239*A*b^2*m + 693*A*b^2)*x^5)*x^m/(m^4 + 32*m^3 + 374*m^2 + 1888*m + 346
5)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1051 vs. $2(63) = 126$.
time = 0.62, size = 1051, normalized size = 14.80

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] Piecewise((-A*b**2/(6*x**6) - A*b*c/(2*x**4) - A*c**2/(2*x**2) - B*b**2/(4*
x**4) - B*b*c/x**2 + B*c**2*log(x), Eq(m, -11)), (-A*b**2/(4*x**4) - A*b*c/
x**2 + A*c**2*log(x) - B*b**2/(2*x**2) + 2*B*b*c*log(x) + B*c**2*x**2/2, Eq
(m, -9)), (-A*b**2/(2*x**2) + 2*A*b*c*log(x) + A*c**2*x**2/2 + B*b**2*log(x
) + B*b*c*x**2 + B*c**2*x**4/4, Eq(m, -7)), (A*b**2*log(x) + A*b*c*x**2 + A
*c**2*x**4/4 + B*b**2*x**2/2 + B*b*c*x**4/2 + B*c**2*x**6/6, Eq(m, -5)), (A
*b**2*m**3*x**5*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 27*A*b**
2*m**2*x**5*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 239*A*b**2*m
*x**5*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 693*A*b**2*x**5*x*
*m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 2*A*b*c*m**3*x**7*x**m/(m*
*4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 50*A*b*c*m**2*x**7*x**m/(m**4 +
32*m**3 + 374*m**2 + 1888*m + 3465) + 398*A*b*c*m*x**7*x**m/(m**4 + 32*m**3
 + 374*m**2 + 1888*m + 3465) + 990*A*b*c*x**7*x**m/(m**4 + 32*m**3 + 374*m*
*2 + 1888*m + 3465) + A*c**2*m**3*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 18
88*m + 3465) + 23*A*c**2*m**2*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m
 + 3465) + 167*A*c**2*m*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 346
5) + 385*A*c**2*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + B*b
**2*m**3*x**7*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 25*B*b**2*
m**2*x**7*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 199*B*b**2*m*x
**7*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 495*B*b**2*x**7*x**m
```

```
/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 2*B*b*c*m**3*x**9*x**m/(m**4
 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 46*B*b*c*m**2*x**9*x**m/(m**4 + 32
*m**3 + 374*m**2 + 1888*m + 3465) + 334*B*b*c*m*x**9*x**m/(m**4 + 32*m**3 +
 374*m**2 + 1888*m + 3465) + 770*B*b*c*x**9*x**m/(m**4 + 32*m**3 + 374*m**2
 + 1888*m + 3465) + B*c**2*m**3*x**11*x**m/(m**4 + 32*m**3 + 374*m**2 + 188
8*m + 3465) + 21*B*c**2*m**2*x**11*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m
 + 3465) + 143*B*c**2*m*x**11*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 34
65) + 315*B*c**2*x**11*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465), Tr
ue))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(71) = 142$.
time = 1.34, size = 340, normalized size = 4.79

$$\frac{Bc^2m^3x^{11}x^m + 21\,Bc^2m^2x^{11}x^m + 2\,Bbcm^3x^9x^m + Ac^2m^3x^9x^m + 143\,Bc^2mx^{11}x^m + 46\,Bbcm^2x^9x^m + 23\,Ac^2m^2x^9x^m + 315\,Bc^2x^{11}x^m + Bb^2m^3x^7x^m + 2\,Abcm^3x^7x^m + 334\,Bbcmx^9x^m + 167\,Ac^2mx^9x^m + 25\,Bb^2m^2x^7x^m + 50\,Abcm^2x^7x^m + 770\,Bbcx^9x^m + 385\,Ac^2x^9x^m + Ab^2m^3x^5x^m + 199\,Bb^2mx^7x^m + 398\,Abcmx^7x^m + 27\,Ab^2m^2x^5x^m + 495\,Bb^2x^7x^m + 990\,Abcx^7x^m + 239\,Ab^2mx^5x^m + 693\,Ab^2x^5x^m}{m^4 + 32\,m^3 + 374\,m^2 + 1888\,m + 3465}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] (B*c^2*m^3*x^11*x^m + 21*B*c^2*m^2*x^11*x^m + 2*B*b*c*m^3*x^9*x^m + A*c^2*m
^3*x^9*x^m + 143*B*c^2*m*x^11*x^m + 46*B*b*c*m^2*x^9*x^m + 23*A*c^2*m^2*x^9
*x^m + 315*B*c^2*x^11*x^m + B*b^2*m^3*x^7*x^m + 2*A*b*c*m^3*x^7*x^m + 334*B
*b*c*m*x^9*x^m + 167*A*c^2*m*x^9*x^m + 25*B*b^2*m^2*x^7*x^m + 50*A*b*c*m^2*
x^7*x^m + 770*B*b*c*x^9*x^m + 385*A*c^2*x^9*x^m + A*b^2*m^3*x^5*x^m + 199*B
*b^2*m*x^7*x^m + 398*A*b*c*m*x^7*x^m + 27*A*b^2*m^2*x^5*x^m + 495*B*b^2*x^7
*x^m + 990*A*b*c*x^7*x^m + 239*A*b^2*m*x^5*x^m + 693*A*b^2*x^5*x^m)/(m^4 +
32*m^3 + 374*m^2 + 1888*m + 3465)
```

Mupad [**B**]
time = 0.34, size = 179, normalized size = 2.52

$$x^m\left(\frac{A\,b^2\,x^5\,\left(m^3+27\,m^2+239\,m+693\right)}{m^4+32\,m^3+374\,m^2+1888\,m+3465}+\frac{B\,c^2\,x^{11}\,\left(m^3+21\,m^2+143\,m+315\right)}{m^4+32\,m^3+374\,m^2+1888\,m+3465}+\frac{b\,x^7\,\left(2\,A\,c+B\,b\right)\,\left(m^3+25\,m^2+199\,m+495\right)}{m^4+32\,m^3+374\,m^2+1888\,m+3465}+\frac{c\,x^9\,\left(A\,c+2\,B\,b\right)\,\left(m^3+23\,m^2+167\,m+385\right)}{m^4+32\,m^3+374\,m^2+1888\,m+3465}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^m*((A*b^2*x^5*(239*m + 27*m^2 + m^3 + 693))/(1888*m + 374*m^2 + 32*m^3 +
m^4 + 3465) + (B*c^2*x^11*(143*m + 21*m^2 + m^3 + 315))/(1888*m + 374*m^2 +
 32*m^3 + m^4 + 3465) + (b*x^7*(2*A*c + B*b)*(199*m + 25*m^2 + m^3 + 495))/
(1888*m + 374*m^2 + 32*m^3 + m^4 + 3465) + (c*x^9*(A*c + 2*B*b)*(167*m + 23
*m^2 + m^3 + 385))/(1888*m + 374*m^2 + 32*m^3 + m^4 + 3465))
```

3.271 $\int x^m(A + Bx^2)\left(bx^2 + cx^4\right) dx$

Optimal. Leaf size=45

$$\frac{Abx^{3+m}}{3+m} + \frac{(bB + Ac)x^{5+m}}{5+m} + \frac{Bcx^{7+m}}{7+m}$$

```
[Out] A*b*x^(3+m)/(3+m)+(A*c+B*b)*x^(5+m)/(5+m)+B*c*x^(7+m)/(7+m)
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1598, 459}

$$\frac{x^{m+5}(Ac + bB)}{m+5} + \frac{Abx^{m+3}}{m+3} + \frac{Bcx^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^(3 + m))/(3 + m) + ((b*B + A*c)*x^(5 + m))/(5 + m) + (B*c*x^(7 + m))
/(7 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^m\left(A + Bx^2\right)\left(bx^2 + cx^4\right) dx &= \int x^{2+m}\left(A + Bx^2\right)\left(b + cx^2\right) dx \\
&= \int \left(Abx^{2+m} + (bB + Ac)x^{4+m} + Bcx^{6+m}\right) dx \\
&= \frac{Abx^{3+m}}{3+m} + \frac{(bB + Ac)x^{5+m}}{5+m} + \frac{Bcx^{7+m}}{7+m}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 42, normalized size = 0.93

$$x^{3+m}\left(\frac{Ab}{3+m}+\frac{(bB+Ac)x^2}{5+m}+\frac{Bcx^4}{7+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4),x]
[Out] x^(3 + m)*((A*b)/(3 + m) + ((b*B + A*c)*x^2)/(5 + m) + (B*c*x^4)/(7 + m))
```

Maple [A]
time = 0.03, size = 55, normalized size = 1.22

method	result	si
norman	$\frac{(Ac+Bb)x^5 e^{m\ln(x)}}{5+m}+\frac{Ab\,x^3 e^{m\ln(x)}}{3+m}+\frac{Bc\,x^7 e^{m\ln(x)}}{7+m}$	55
gosper	$\frac{x^{3+m}(Bc\,m^2x^4+8Bcm\,x^4+Ac\,m^2x^2+Bb\,m^2x^2+15Bc\,x^4+10Acm\,x^2+10Bbm\,x^2+Ab\,m^2+21Ac\,x^2+21bB\,x^2+12Abm+35Ab)}{(7+m)(5+m)(3+m)}$	11
risch	$\frac{x^m(Bc\,m^2x^4+8Bcm\,x^4+Ac\,m^2x^2+Bb\,m^2x^2+15Bc\,x^4+10Acm\,x^2+10Bbm\,x^2+Ab\,m^2+21Ac\,x^2+21bB\,x^2+12Abm+35Ab)x^3}{(7+m)(5+m)(3+m)}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] (A*c+B*b)/(5+m)*x^5*exp(m*ln(x))+A*b/(3+m)*x^3*exp(m*ln(x))+B*c/(7+m)*x^7*e
xp(m*ln(x))
```

Maxima [A]
time = 0.27, size = 53, normalized size = 1.18

$$\frac{Bcx^{m+7}}{m+7}+\frac{Bbx^{m+5}}{m+5}+\frac{Acx^{m+5}}{m+5}+\frac{Abx^{m+3}}{m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] B*c*x^(m + 7)/(m + 7) + B*b*x^(m + 5)/(m + 5) + A*c*x^(m + 5)/(m + 5) + A*b
*x^(m + 3)/(m + 3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(45) = 90.
time = 2.46, size = 94, normalized size = 2.09

$$\frac{((Bcm^2+8\,Bcm+15\,Bc)x^7+((Bb+Ac)m^2+21\,Bb+21\,Ac+10\,(Bb+Ac)m)x^5+(Abm^2+12\,Abm+35\,Ab)x^3)x^m}{m^3+15\,m^2+71\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] ((B*c*m^2 + 8*B*c*m + 15*B*c)*x^7 + ((B*b + A*c)*m^2 + 21*B*b + 21*A*c + 10
*(B*b + A*c)*m)*x^5 + (A*b*m^2 + 12*A*b*m + 35*A*b)*x^3)*x^m/(m^3 + 15*m^2
+ 71*m + 105)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(37) = 74$.
time = 0.33, size = 415, normalized size = 9.22

$$\begin{cases} -\frac{Ab}{4x^4} - \frac{Ac}{2x^2} - \frac{Bb}{2x^2} + Bc\log(x) & \text{for } m = -7 \\ -\frac{Ab}{2x^2} + Ac\log(x) + Bb\log(x) + \frac{Bcx^2}{2} & \text{for } m = -5 \\ Ab\log(x) + \frac{Acx^2}{2} + \frac{Bbx^2}{2} + \frac{Bcx^4}{4} & \text{for } m = -3 \\ \frac{Abm^2x^3x^m}{m^3+15m^2+71m+105} + \frac{12Abmx^3x^m}{m^3+15m^2+71m+105} + \frac{35Abx^3x^m}{m^3+15m^2+71m+105} + \frac{Acm^2x^5x^m}{m^3+15m^2+71m+105} + \frac{10Acmx^5x^m}{m^3+15m^2+71m+105} + \frac{21Acx^5x^m}{m^3+15m^2+71m+105} + \frac{Bbm^2x^5x^m}{m^3+15m^2+71m+105} + \frac{10Bbmx^5x^m}{m^3+15m^2+71m+105} + \frac{21Bbx^5x^m}{m^3+15m^2+71m+105} + \frac{Bcm^2x^7x^m}{m^3+15m^2+71m+105} + \frac{8Bcmx^7x^m}{m^3+15m^2+71m+105} + \frac{15Bcx^7x^m}{m^3+15m^2+71m+105} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((-A*b/(4*x**4) - A*c/(2*x**2) - B*b/(2*x**2) + B*c*log(x), Eq(m,
-7)), (-A*b/(2*x**2) + A*c*log(x) + B*b*log(x) + B*c*x**2/2, Eq(m, -5)), (A
*b*log(x) + A*c*x**2/2 + B*b*x**2/2 + B*c*x**4/4, Eq(m, -3)), (A*b*m**2*x**
3*x**m/(m**3 + 15*m**2 + 71*m + 105) + 12*A*b*m*x**3*x**m/(m**3 + 15*m**2 +
 71*m + 105) + 35*A*b*x**3*x**m/(m**3 + 15*m**2 + 71*m + 105) + A*c*m**2*x*
*5*x**m/(m**3 + 15*m**2 + 71*m + 105) + 10*A*c*m*x**5*x**m/(m**3 + 15*m**2
+ 71*m + 105) + 21*A*c*x**5*x**m/(m**3 + 15*m**2 + 71*m + 105) + B*b*m**2*x
**5*x**m/(m**3 + 15*m**2 + 71*m + 105) + 10*B*b*m*x**5*x**m/(m**3 + 15*m**2
 + 71*m + 105) + 21*B*b*x**5*x**m/(m**3 + 15*m**2 + 71*m + 105) + B*c*m**2*
x**7*x**m/(m**3 + 15*m**2 + 71*m + 105) + 8*B*c*m*x**7*x**m/(m**3 + 15*m**2
 + 71*m + 105) + 15*B*c*x**7*x**m/(m**3 + 15*m**2 + 71*m + 105), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(45) = 90$.
time = 1.03, size = 149, normalized size = 3.31

$$\frac{Bcm^2x^7x^m + 8\,Bcmx^7x^m + Bbm^2x^5x^m + Acm^2x^5x^m + 15\,Bcx^7x^m + 10\,Bbmx^5x^m + 10\,Acmx^5x^m + Abm^2x^3x^m + 21\,Bbx^5x^m + 21\,Acx^5x^m + 12\,Abmx^3x^m + 35\,Abx^3x^m}{m^3 + 15\,m^2 + 71\,m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*c*m^2*x^7*x^m + 8*B*c*m*x^7*x^m + B*b*m^2*x^5*x^m + A*c*m^2*x^5*x^m + 15
*B*c*x^7*x^m + 10*B*b*m*x^5*x^m + 10*A*c*m*x^5*x^m + A*b*m^2*x^3*x^m + 21*B
*b*x^5*x^m + 21*A*c*x^5*x^m + 12*A*b*m*x^3*x^m + 35*A*b*x^3*x^m)/(m^3 + 15*
m^2 + 71*m + 105)
```

Mupad [B]
time = 0.25, size = 97, normalized size = 2.16

$$x^m\left(\frac{x^5\,\left(A\,c+B\,b\right)\,\left(m^2+10\,m+21\right)}{m^3+15\,m^2+71\,m+105}+\frac{A\,b\,x^3\,\left(m^2+12\,m+35\right)}{m^3+15\,m^2+71\,m+105}+\frac{B\,c\,x^7\,\left(m^2+8\,m+15\right)}{m^3+15\,m^2+71\,m+105}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

```
[Out] x^m*((x^5*(A*c + B*b)*(10*m + m^2 + 21))/(71*m + 15*m^2 + m^3 + 105) + (A*b
*x^3*(12*m + m^2 + 35))/(71*m + 15*m^2 + m^3 + 105) + (B*c*x^7*(8*m + m^2 +
 15))/(71*m + 15*m^2 + m^3 + 105))
```

3.272 $\int \frac{x^m(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=71

$$-\frac{Bx^{-1+m}}{c(1-m)}+\frac{(bB-Ac)x^{-1+m}\,{}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{cx^2}{b}\right)}{bc(1-m)}$$

```
[Out] -B*x^(-1+m)/c/(1-m)+(-A*c+B*b)*x^(-1+m)*hypergeom([1, -1/2+1/2*m],[1/2+1/2*
m],-c*x^2/b)/b/c/(1-m)
```

Rubi [A]

time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 470, 371}

$$\frac{x^{m-1}(bB-Ac)\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{cx^2}{b}\right)}{bc(1-m)}-\frac{Bx^{m-1}}{c(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -((B*x^(-1 + m))/(c*(1 - m))) + ((b*B - A*c)*x^(-1 + m)*Hypergeometric2F1[1
, (-1 + m)/2, (1 + m)/2, -((c*x^2)/b)])/(b*c*(1 - m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m(A+Bx^2)}{bx^2+cx^4}\,dx &= \int \frac{x^{-2+m}(A+Bx^2)}{b+cx^2}\,dx\\ &= -\frac{Bx^{-1+m}}{c(1-m)} - \frac{(bB(-1+m)-Ac(-1+m))\int \frac{x^{-2+m}}{b+cx^2}\,dx}{c(-1+m)}\\ &= -\frac{Bx^{-1+m}}{c(1-m)} + \frac{(bB-Ac)x^{-1+m}\,{}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{cx^2}{b}\right)}{bc(1-m)}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 55, normalized size = 0.77

$$\frac{x^{-1+m}\left(bB+(-bB+Ac)\,{}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{cx^2}{b}\right)\right)}{bc(-1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] (x^(-1 + m)*(b*B + (-(b*B) + A*c)*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/
2, -((c*x^2)/b)]))/(b*c*(-1 + m))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^m(B\,x^2+A)}{x^4c+b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)/(c*x^4+b*x^2),x)
[Out] int(x^m*(B*x^2+A)/(c*x^4+b*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] integral((B*x^2 + A)*x^m/(c*x^4 + b*x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m(A + Bx^2)}{x^2\,(b + cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)/(c*x**4+b*x**2),x)
[Out] Integral(x**m*(A + B*x**2)/(x**2*(b + c*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(B\,x^2+A\right)}{c\,x^4+b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(A + B*x^2))/(b*x^2 + c*x^4),x)
[Out] int((x^m*(A + B*x^2))/(b*x^2 + c*x^4), x)
```

3.273 $\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=98

$$-\frac{(bB-Ac)x^{-3+m}}{2bc\,(b+cx^2)}+\frac{(bB(3-m)-Ac(5-m))x^{-3+m}\,{}_2F_1\left(1,\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);-\frac{cx^2}{b}\right)}{2b^2c(3-m)}$$

```
[Out] -1/2*(-A*c+B*b)*x^(-3+m)/b/c/(c*x^2+b)+1/2*(b*B*(3-m)-A*c*(5-m))*x^(-3+m)*h
ypergeom([1, -3/2+1/2*m],[-1/2+1/2*m],-c*x^2/b)/b^2/c/(3-m)
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 468, 371}

$$\frac{x^{m-3}\left(\frac{bB}{c}-\frac{A(5-m)}{3-m}\right)\,{}_2F_1\left(1,\frac{m-3}{2};\frac{m-1}{2};-\frac{cx^2}{b}\right)}{2b^2}-\frac{x^{m-3}(bB-Ac)}{2bc\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((b*B - A*c)*x^(-3 + m))/(b*c*(b + c*x^2)) + (((b*B)/c - (A*(5 - m))/(
3 - m))*x^(-3 + m)*Hypergeometric2F1[1, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b
)])/(2*b^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \int \frac{x^{-4+m}(A+Bx^2)}{(b+cx^2)^2}\,dx\\ &= -\frac{(bB-Ac)x^{-3+m}}{2bc\,(b+cx^2)}+\frac{(-Ac(-5+m)+bB(-3+m))\int \frac{x^{-4+m}}{b+cx^2}\,dx}{2bc}\\ &= -\frac{(bB-Ac)x^{-3+m}}{2bc\,(b+cx^2)}+\frac{\left(\frac{bB}{c}-\frac{A(5-m)}{3-m}\right)x^{-3+m}\,{}_2F_1\left(1,\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);-\frac{cx^2}{b}\right)}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 131, normalized size = 1.34

$$\frac{x^{-3+m}\left(b\left(\frac{Ab}{-3+m}+\frac{bBx^2}{-1+m}-\frac{2Acx^2}{-1+m}\right)+\frac{c(-bB+2Ac)x^4\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{cx^2}{b}\right)}{1+m}+\frac{c(-bB+Ac)x^4\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{cx^2}{b}\right)}{1+m}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^(-3 + m)*(b*((A*b)/(-3 + m) + (b*B*x^2)/(-1 + m) - (2*A*c*x^2)/(-1 + m))
 + (c*(-(b*B) + 2*A*c)*x^4*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((c*
x^2)/b)])/(1 + m) + (c*(-(b*B) + A*c)*x^4*Hypergeometric2F1[2, (1 + m)/2, (
3 + m)/2, -((c*x^2)/b)])/(1 + m)))/b^4
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^m(B\,x^2+A)}{(x^4c+b\,x^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] int(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] integral((B*x^2 + A)*x^m/(c^2*x^8 + 2*b*c*x^6 + b^2*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m(A + Bx^2)}{x^4\,(b + cx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
[Out] Integral(x**m*(A + B*x**2)/(x**4*(b + c*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,(B\,x^2 + A)}{(c\,x^4 + b\,x^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
[Out] int((x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2, x)
```

3.274 $\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=140

$$\frac{Bx^{-1+m}(bx^2+cx^4)^{1+p}}{c(3+m+4p)}-\frac{(bB(1+m+2p)-Ac(3+m+4p))x^{1+m}\left(1+\frac{cx^2}{b}\right)^{-p}(bx^2+cx^4)^p\,{}_2F_1\left(-p,\frac{1}{2}(1+\right.}{c(1+m+2p)(3+m+4p)}$$

[Out] B*x^(-1+m)*(c*x^4+b*x^2)^(1+p)/c/(3+m+4*p)-(b*B*(1+m+2*p)-A*c*(3+m+4*p))*x^(1+m)*(c*x^4+b*x^2)^p*hypergeom([-p, 1/2+1/2*m+p],[3/2+1/2*m+p],-c*x^2/b)/c/(1+m+2*p)/(3+m+4*p)/((1+c*x^2/b)^p)

Rubi [A]
time = 0.09, antiderivative size = 126, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2064, 2057, 372, 371}

$$x^{m+1}\left(\frac{cx^2}{b}+1\right)^{-p}(bx^2+cx^4)^p\left(\frac{A}{m+2p+1}-\frac{bB}{c(m+4p+3)}\right){}_2F_1\left(-p,\frac{1}{2}(m+2p+1);\frac{1}{2}(m+2p+3);-\frac{cx^2}{b}\right)+\frac{Bx^{m-1}(bx^2+cx^4)^{p+1}}{c(m+4p+3)}$$

Antiderivative was successfully verified.

[In] Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p,x]

[Out] (B*x^(-1 + m)*(b*x^2 + c*x^4)^(1 + p))/(c*(3 + m + 4*p)) + ((A/(1 + m + 2*p) - (b*B)/(c*(3 + m + 4*p)))*x^(1 + m)*(b*x^2 + c*x^4)^p*Hypergeometric2F1[-p, (1 + m + 2*p)/2, (3 + m + 2*p)/2, -((c*x^2)/b)])/(1 + (c*x^2)/b)^p

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^p\,dx &= \frac{Bx^{-1+m}\left(bx^2+cx^4\right)^{1+p}}{c(3+m+4p)}-\left(-A+\frac{bB(1+m+2p)}{c(3+m+4p)}\right)\int x^m\left(bx^2+cx^4\right)^p \\ &= \frac{Bx^{-1+m}\left(bx^2+cx^4\right)^{1+p}}{c(3+m+4p)}-\left(\left(-A+\frac{bB(1+m+2p)}{c(3+m+4p)}\right)x^{-2p}\left(b+cx^2\right)^{-p}\right. \\ &= \frac{Bx^{-1+m}\left(bx^2+cx^4\right)^{1+p}}{c(3+m+4p)}-\left(\left(-A+\frac{bB(1+m+2p)}{c(3+m+4p)}\right)x^{-2p}\left(1+\frac{cx^2}{b}\right)^{-}\right. \\ &= \frac{Bx^{-1+m}\left(bx^2+cx^4\right)^{1+p}}{c(3+m+4p)}+\left(\frac{A}{1+m+2p}-\frac{bB}{c(3+m+4p)}\right)x^{1+m}\left(1+\frac{c}{}\right.\end{aligned}$$

Mathematica [A]
time = 0.12, size = 135, normalized size = 0.96

$$\frac{x^{1+m}\left(x^2\left(b+cx^2\right)\right)^p\left(1+\frac{cx^2}{b}\right)^{-p}\left(A(3+m+2p)\,{}_2F_1\left(-p,\frac{1}{2}+\frac{m}{2}+p;\frac{3}{2}+\frac{m}{2}+p;-\frac{cx^2}{b}\right)+B(1+m+2p)x^2\,{}_2F_1\left(-p,\frac{3}{2}+\frac{m}{2}+p;\frac{5}{2}+\frac{m}{2}+p;-\frac{cx^2}{b}\right)\right)}{(1+m+2p)(3+m+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^(1 + m)*(x^2*(b + c*x^2))^p*(A*(3 + m + 2*p)*Hypergeometric2F1[-p, 1/2 +
 m/2 + p, 3/2 + m/2 + p, -((c*x^2)/b)] + B*(1 + m + 2*p)*x^2*Hypergeometric
2F1[-p, 3/2 + m/2 + p, 5/2 + m/2 + p, -((c*x^2)/b)]))/((1 + m + 2*p)*(3 + m
 + 2*p)*(1 + (c*x^2)/b)^p)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int x^m\left(B\,x^2+A\right)\left(x^4c+b\,x^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x)
[Out] int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*(c*x^4 + b*x^2)^p*x^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x, algorithm="fricas")
[Out] integral((B*x^2 + A)*(c*x^4 + b*x^2)^p*x^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\left(x^2\left(b+cx^2\right)\right)^p\left(A+Bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2)**p,x)
[Out] Integral(x**m*(x**2*(b + c*x**2))**p*(A + B*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(c*x^4 + b*x^2)^p*x^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \left(B x^2 + A\right) \left(c x^4 + b x^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p,x)
```

```
[Out] int(x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p, x)
```

3.275 $\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx$

Optimal. Leaf size=95

$$-\frac{(ad-bc(2+p))x^{-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{b^2n(1+p)(2+p)}+\frac{dx^{n-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{bn(2+p)}$$

```
[Out] -(a*d-b*c*(2+p))*(a*x^j+b*x^(j+n))^(1+p)/b^2/n/(p^2+3*p+2)/(x^(j*(1+p)))+d*
x^(n-j*(1+p))*(a*x^j+b*x^(j+n))^(1+p)/b/n/(2+p)
```

Rubi [A]
time = 0.11, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2064, 2039}

$$\frac{dx^{n-j(p+1)}\left(ax^j+bx^{j+n}\right)^{p+1}}{bn(p+2)}-\frac{x^{-j(p+1)}(ad-bc(p+2))\left(ax^j+bx^{j+n}\right)^{p+1}}{b^2n(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n - j*p)*(c + d*x^n)*(a*x^j + b*x^(j + n))^p,x]
```

```
[Out] -(((a*d - b*c*(2 + p))*(a*x^j + b*x^(j + n))^(1 + p))/(b^2*n*(1 + p)*(2 + p
)*x^(j*(1 + p)))) + (d*x^(n - j*(1 + p))*(a*x^j + b*x^(j + n))^(1 + p))/(b*
n*(2 + p))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx = \frac{dx^{n-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{bn(2+p)} - \left(-c+\frac{ad}{b(2+p)}\right)\int x^{-1+n-jp}(ax^j$$

$$= \frac{\left(c-\frac{ad}{b(2+p)}\right)x^{-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{bn(1+p)} + \frac{dx^{n-j(1+p)}\left(ax^j+bx^{j+n}\right)}{bn(2+p)}$$

Mathematica [A]
time = 0.09, size = 63, normalized size = 0.66

$$\frac{x^{-jp}(a+bx^n)\left(x^j(a+bx^n)\right)^p\left(-ad+bc(2+p)+bd(1+p)x^n\right)}{b^2n(1+p)(2+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n - j*p)*(c + d*x^n)*(a*x^j + b*x^(j + n))^p,x]
```

```
[Out] ((a + b*x^n)*(x^j*(a + b*x^n))^p*(-(a*d) + b*c*(2 + p) + b*d*(1 + p)*x^n))/
(b^2*n*(1 + p)*(2 + p)*x^(j*p))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int x^{-jp+n-1}(c+d\,x^n)\left(a\,x^j+b\,x^{j+n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x)
```

```
[Out] int(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x)
```

Maxima [A]
time = 0.38, size = 112, normalized size = 1.18

$$\frac{(bx^n+a)ce^{\left(-jp\log(x)+p\log(bx^n+a)+p\log(x^j)\right)}}{bn(p+1)} + \frac{\left(b^2(p+1)x^{2n}+abpx^n-a^2\right)de^{\left(-jp\log(x)+p\log(bx^n+a)+p\log(x^j)\right)}}{(p^2+3p+2)b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x, algorithm="maxima")
```

```
[Out] (b*x^n + a)*c*e^(-j*p*log(x) + p*log(b*x^n + a) + p*log(x^j))/(b*n*(p + 1))
 + (b^2*(p + 1)*x^(2*n) + a*b*p*x^n - a^2)*d*e^(-j*p*log(x) + p*log(b*x^n +
 a) + p*log(x^j))/((p^2 + 3*p + 2)*b^2*n)
```

Fricas [A]
time = 2.01, size = 140, normalized size = 1.47

$$\frac{\left(\left(b^{2} d p+b^{2} d\right) x x^{-j p+n-1} x^{2 n}+\left(2 b^{2} c+\left(b^{2} c+a b d\right) p\right) x x^{-j p+n-1} x^{n}+\left(a b c p+2 a b c-a^{2} d\right) x x^{-j p+n-1}\right)\left(\frac{\left(b x^{n}+a\right) x^{j+n}}{x^{n}}\right)^{p}}{\left(b^{2} n p^{2}+3 b^{2} n p+2 b^{2} n\right) x^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x, algorithm="fricas")
```

```
[Out] ((b^2*d*p + b^2*d)*x*x^(-j*p + n - 1)*x^(2*n) + (2*b^2*c + (b^2*c + a*b*d)*
p)*x*x^(-j*p + n - 1)*x^n + (a*b*c*p + 2*a*b*c - a^2*d)*x*x^(-j*p + n - 1))
*((b*x^n + a)*x^(j + n)/x^n)^p/((b^2*n*p^2 + 3*b^2*n*p + 2*b^2*n)*x^n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-j*p+n-1)*(c+d*x**n)*(a*x**j+b*x**(j+n))**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)*(b*x^(j + n) + a*x^j)^p*x^(-j*p + n - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{n-j\,p-1}\,{\left(a\,x^j+b\,x^{j+n}\right)}^p\,\left(c+d\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - j*p - 1)*(a*x^j + b*x^(j + n))^p*(c + d*x^n),x)
```

```
[Out] int(x^(n - j*p - 1)*(a*x^j + b*x^(j + n))^p*(c + d*x^n), x)
```

3.276 $\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p dx$

Optimal. Leaf size=113

$$\frac{x(ex)^m \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \left(ax^j+bx^{j+n}\right)^p F_1\left(\frac{1+m+jp}{n};-p,-q;\frac{1+m+n+jp}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{1+m+jp}$$

```
[Out] x*(e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p*AppellF1((j*p+m+1)/n,-p,-q,(j*p+m
+n+1)/n,-b*x^n/a,-d*x^n/c)/(j*p+m+1)/((1+b*x^n/a)^p)/((1+d*x^n/c)^q)
```

Rubi [A]
time = 0.15, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2067, 525, 524}

$$\frac{x(ex)^m \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \left(ax^j+bx^{j+n}\right)^p F_1\left(\frac{m+jp+1}{n};-p,-q;\frac{m+n+jp+1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{jp+m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^p,x]
```

```
[Out] (x*(e*x)^m*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^p*AppellF1[(1 + m + j*p)/n,
-p, -q, (1 + m + n + j*p)/n, -((b*x^n)/a), -((d*x^n)/c)])/((1 + m + j*p)*(1
 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2067

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
```

```
^n)^FracPart[p])), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p \, dx = \left(x^{-m-jp}(ex)^m (a+bx^n)^{-p} \left(ax^j+bx^{j+n}\right)^p\right) \int x^{m+jp}(a+bx^n)^p (c-$$

$$= \left(x^{-m-jp}(ex)^m \left(1+\frac{bx^n}{a}\right)^{-p} \left(ax^j+bx^{j+n}\right)^p\right) \int x^{m+jp}\left(1+\frac{bx^n}{a}\right.$$

$$= \left(x^{-m-jp}(ex)^m \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \left(ax^j+bx^{j+}\right.\right.$$

$$= \frac{x(ex)^m \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \left(ax^j+bx^{j+n}\right)^p F_1\left(\frac{1+m-}{n}\right.}{1+m+jp}$$

Mathematica [A]
time = 0.15, size = 111, normalized size = 0.98

$$\frac{x(ex)^m \left(x^j(a+bx^n)\right)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} F_1\left(\frac{1+m+jp}{n};-p,-q;\frac{1+m+n+jp}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{1+m+jp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^p,x]
[Out] (x*(e*x)^m*(x^j*(a + b*x^n))^p*(c + d*x^n)^q*AppellF1[(1 + m + j*p)/n, -p,
-q, (1 + m + n + j*p)/n, -((b*x^n)/a), -((d*x^n)/c)])/((1 + m + j*p)*(1 + (
b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Maple [F]
time = 1.55, size = 0, normalized size = 0.00

$$\int (ex)^m (c+d\,x^n)^q \left(a\,x^j+b\,x^{j+n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x)
[Out] int((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x, algorithm="maxima")
[Out] integrate((b*x^(j + n) + a*x^j)^p*(d*x^n + c)^q*(x*e)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x, algorithm="fricas")
[Out] integral((b*x^(j + n) + a*x^j)^p*(d*x^n + c)^q*(x*e)^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(c+d*x**n)**q*(a*x**j+b*x**(j+n))**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x, algorithm="giac")
[Out] integrate((b*x^(j + n) + a*x^j)^p*(d*x^n + c)^q*(x*e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\,x^j+b\,x^{j+n}\right)^p\,{\left(e\,x\right)}^m\,{\left(c+d\,x^n\right)}^q \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^(j + n))^p*(e*x)^m*(c + d*x^n)^q,x)
[Out] int((a*x^j + b*x^(j + n))^p*(e*x)^m*(c + d*x^n)^q, x)
```

3.277 $\int (ex)^{7/4} (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^{5/3} dx$

Optimal. Leaf size=129

$$\frac{12aex^{2+j}(ex)^{3/4}(c+dx^n)^q\left(1+\frac{dx^n}{c}\right)^{-q}\left(ax^j+bx^{j+n}\right)^{2/3}F_1\left(\frac{33+20j}{12n};-\frac{5}{3},-q;\frac{33+20j+12n}{12n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(33+20j)\left(1+\frac{bx^n}{a}\right)^{2/3}}$$

```
[Out] 12*a*e*x^(2+j)*(e*x)^(3/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(2/3)*AppellF1(1/1
2*(33+20*j)/n,-5/3,-q,1+(11/4+5/3*j)/n,-b*x^n/a,-d*x^n/c)/(33+20*j)/(1+b*x^
n/a)^(2/3)/((1+d*x^n/c)^q)
```

Rubi [A]
time = 0.24, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2067, 525, 524}

$$\frac{12ae(ex)^{3/4}x^{j+2}\left(ax^j+bx^{j+n}\right)^{2/3}(c+dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}F_1\left(\frac{20j+33}{12n};-\frac{5}{3},-q;\frac{20j+12n+33}{12n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(20j+33)\left(\frac{bx^n}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/4)*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^(5/3),x]
```

```
[Out] (12*a*e*x^(2 + j)*(e*x)^(3/4)*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^(2/3)*App
ellF1[(33 + 20*j)/(12*n), -5/3, -q, (33 + 20*j + 12*n)/(12*n), -((b*x^n)/a)
, -((d*x^n)/c)])/((33 + 20*j)*(1 + (b*x^n)/a)^(2/3)*(1 + (d*x^n)/c)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2067

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[e^IntPart[m]*(e*x)^FracPart[m]*(
```

```
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p])), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\begin{aligned}\int (ex)^{7/4}(c+dx^n)^q\left(ax^j+bx^{j+n}\right)^{5/3}\,dx &= \frac{\left(ex^{-\frac{3}{4}-\frac{2j}{3}}(ex)^{3/4}\left(ax^j+bx^{j+n}\right)^{2/3}\right)\int x^{\frac{7}{4}+\frac{5j}{3}}(a+bx^n)^{5/3}(c+dx^n)}{(a+bx^n)^{2/3}}\\ &= \frac{\left(aex^{-\frac{3}{4}-\frac{2j}{3}}(ex)^{3/4}\left(ax^j+bx^{j+n}\right)^{2/3}\right)\int x^{\frac{7}{4}+\frac{5j}{3}}\left(1+\frac{bx^n}{a}\right)^{5/3}(c+dx}{\left(1+\frac{bx^n}{a}\right)^{2/3}}\\ &= \frac{\left(aex^{-\frac{3}{4}-\frac{2j}{3}}(ex)^{3/4}(c+dx^n)^q\left(1+\frac{dx^n}{c}\right)^{-q}\left(ax^j+bx^{j+n}\right)^{2/3}\right)\int x^{\frac{7}{4}}}{\left(1+\frac{bx^n}{a}\right)^{2/3}}\\ &= \frac{12aex^{2+j}(ex)^{3/4}(c+dx^n)^q\left(1+\frac{dx^n}{c}\right)^{-q}\left(ax^j+bx^{j+n}\right)^{2/3}F_1\left(\frac{33+20}{12n}\right.}{(33+20j)\left(1+\frac{bx^n}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 210, normalized size = 1.63

$$\frac{12x^{1+j}(ex)^{7/4}\left(x^j(a+bx^n)\right)^{2/3}(c+dx^n)^q\left(1+\frac{dx^n}{c}\right)^{-q}\left(a(33+20j+12n)F_1\left(\frac{33+20j}{12n};-\frac{2}{3},-q;\frac{\frac{11}{4}+\frac{5j}{3}+n}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+b(33+20j)x^nF_1\left(\frac{33+20j+12n}{12n};-\frac{2}{3},-q;\frac{33+20j+24n}{12n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)\right)}{(33+20j)(33+20j+12n)\left(1+\frac{bx^n}{a}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/4)*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^(5/3),x]
```

```
[Out] (12*x^(1 + j)*(e*x)^(7/4)*(x^j*(a + b*x^n))^(2/3)*(c + d*x^n)^q*(a*(33 + 20
*j + 12*n)*AppellF1[(33 + 20*j)/(12*n), -2/3, -q, (11/4 + (5*j)/3 + n)/n, -
((b*x^n)/a), -((d*x^n)/c)] + b*(33 + 20*j)*x^n*AppellF1[(33 + 20*j + 12*n)/
(12*n), -2/3, -q, (33 + 20*j + 24*n)/(12*n), -((b*x^n)/a), -((d*x^n)/c)]))/
((33 + 20*j)*(33 + 20*j + 12*n)*(1 + (b*x^n)/a)^(2/3)*(1 + (d*x^n)/c)^q)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (ex)^{\frac{7}{4}}(c+d\,x^n)^q\left(a\,x^j+b\,x^{j+n}\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x)
```

```
[Out] int((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x, algorithm="max
ima")
[Out] e^(7/4)*integrate((b*x^(j + n) + a*x^j)^(5/3)*(d*x^n + c)^q*x^(7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x, algorithm="fri
cas")
[Out] integral((b*x*x^(j + n)*e^(7/4) + a*x*x^j*e^(7/4))*(b*x^(j + n) + a*x^j)^(2
/3)*(d*x^n + c)^q*x^(3/4), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/4)*(c+d*x**n)**q*(a*x**j+b*x**(j+n))**(5/3),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x, algorithm="gia
c")
[Out] integrate((b*x^(j + n) + a*x^j)^(5/3)*(d*x^n + c)^q*x^(7/4)*e^(7/4), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\,x^j+b\,x^{j+n}\right)^{5/3}\,{\left(e\,x\right)}^{7/4}\,{\left(c+d\,x^n\right)}^q \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^(j + n))^(5/3)*(e*x)^(7/4)*(c + d*x^n)^q,x)
[Out] int((a*x^j + b*x^(j + n))^(5/3)*(e*x)^(7/4)*(c + d*x^n)^q, x)
```

3.278 $\int \frac{4+3x^4}{5x+2x^5}\,dx$

Optimal. Leaf size=19

$$\frac{4\log(x)}{5}+\frac{7}{40}\log\left(5+2x^4\right)$$

```
[Out] 4/5*ln(x)+7/40*ln(2*x^4+5)
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1607, 457, 78}

$$\frac{7}{40}\log\left(2x^4+5\right)+\frac{4\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^4)/(5*x + 2*x^5),x]
```

```
[Out] (4*Log[x])/5 + (7*Log[5 + 2*x^4])/40
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{4+3x^4}{5x+2x^5}\,dx &= \int \frac{4+3x^4}{x\,(5+2x^4)}\,dx\\ &= \frac{1}{4}\mathrm{Subst}\left(\int \frac{4+3x}{x(5+2x)}\,dx, x, x^4\right)\\ &= \frac{1}{4}\mathrm{Subst}\left(\int \left(\frac{4}{5x}+\frac{7}{5(5+2x)}\right)\,dx, x, x^4\right)\\ &= \frac{4\log(x)}{5}+\frac{7}{40}\log\left(5+2x^4\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$\frac{4\log(x)}{5}+\frac{7}{40}\log\left(5+2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^4)/(5*x + 2*x^5),x]
```

```
[Out] (4*Log[x])/5 + (7*Log[5 + 2*x^4])/40
```

Maple [A]
time = 0.39, size = 16, normalized size = 0.84

method	result	size
default	$\frac{4\ln(x)}{5}+\frac{7\ln(2x^4+5)}{40}$	16
norman	$\frac{4\ln(x)}{5}+\frac{7\ln(2x^4+5)}{40}$	16
risch	$\frac{4\ln(x)}{5}+\frac{7\ln(2x^4+5)}{40}$	16
meijerg	$\frac{7\ln\left(1+\frac{2x^4}{5}\right)}{40}+\frac{4\ln(x)}{5}+\frac{\ln(2)}{5}-\frac{\ln(5)}{5}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4+4)/(2*x^5+5*x),x,method=_RETURNVERBOSE)
```

```
[Out] 4/5*ln(x)+7/40*ln(2*x^4+5)
```

Maxima [A]
time = 0.49, size = 15, normalized size = 0.79

$$\frac{7}{40}\,\log\left(2\,x^4+5\right)+\frac{4}{5}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/(2*x^5+5*x),x, algorithm="maxima")
[Out] 7/40*log(2*x^4 + 5) + 4/5*log(x)
```

Fricas [A]
time = 1.51, size = 15, normalized size = 0.79

$$\frac{7}{40}\,\log\left(2\,x^4+5\right)+\frac{4}{5}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/(2*x^5+5*x),x, algorithm="fricas")
[Out] 7/40*log(2*x^4 + 5) + 4/5*log(x)
```

Sympy [A]
time = 0.03, size = 17, normalized size = 0.89

$$\frac{4\log\left(x\right)}{5}+\frac{7\log\left(2x^4+5\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4+4)/(2*x**5+5*x),x)
[Out] 4*log(x)/5 + 7*log(2*x**4 + 5)/40
```

Giac [A]
time = 0.53, size = 17, normalized size = 0.89

$$\frac{7}{40}\,\log\left(2\,x^4+5\right)+\frac{1}{5}\,\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/(2*x^5+5*x),x, algorithm="giac")
[Out] 7/40*log(2*x^4 + 5) + 1/5*log(x^4)
```

Mupad [B]
time = 0.17, size = 13, normalized size = 0.68

$$\frac{7\,\ln\left(x^4+\frac{5}{2}\right)}{40}+\frac{4\,\ln\left(x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4 + 4)/(5*x + 2*x^5),x)
[Out] (7*log(x^4 + 5/2))/40 + (4*log(x))/5
```

3.279 $\int \frac{1+x^6}{x-x^7} \, dx$

Optimal. Leaf size=15

$$\log(x) - \frac{1}{3}\log\left(1 - x^6\right)$$

[Out] ln(x)-1/3*ln(-x^6+1)

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1607, 457, 78}

$$\log(x) - \frac{1}{3}\log\left(1 - x^6\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^6)/(x - x^7),x]

[Out] Log[x] - Log[1 - x^6]/3

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^6}{x-x^7}\,dx &= \int \frac{1+x^6}{x\left(1-x^6\right)}\,dx \\
&= \frac{1}{6}\mathrm{Subst}\left(\int \frac{1+x}{(1-x)x}\,dx, x, x^6\right) \\
&= \frac{1}{6}\mathrm{Subst}\left(\int \left(-\frac{2}{-1+x}+\frac{1}{x}\right)dx, x, x^6\right) \\
&= \log(x)-\frac{1}{3}\log\left(1-x^6\right)
\end{aligned}
$$

Mathematica [**A**]
time = 0.00, size = 15, normalized size = 1.00

$$\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(x - x^7),x]
```

```
[Out] Log[x] - Log[1 - x^6]/3
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(13) = 26$.
time = 0.41, size = 36, normalized size = 2.40

method	result	size
risch	$\ln(x)-\frac{\ln(x^6-1)}{3}$	12
meijerg	$-\frac{\ln(-x^6+1)}{3}+\ln(x)+\frac{i\pi}{6}$	18
default	$-\frac{\ln(x+1)}{3}-\frac{\ln(x^2+x+1)}{3}-\frac{\ln(x-1)}{3}+\ln(x)-\frac{\ln(x^2-x+1)}{3}$	36
norman	$-\frac{\ln(x+1)}{3}-\frac{\ln(x^2+x+1)}{3}-\frac{\ln(x-1)}{3}+\ln(x)-\frac{\ln(x^2-x+1)}{3}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/(-x^7+x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*ln(x+1)-1/3*ln(x^2+x+1)-1/3*ln(x-1)+ln(x)-1/3*ln(x^2-x+1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(13) = 26$.
time = 0.52, size = 35, normalized size = 2.33

$$-\frac{1}{3}\log\left(x^2+x+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)-\frac{1}{3}\log\left(x-1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(-x^7+x),x, algorithm="maxima")
```

```
[Out] -1/3*log(x^2 + x + 1) - 1/3*log(x^2 - x + 1) - 1/3*log(x + 1) - 1/3*log(x -
 1) + log(x)
```

Fricas [A]
time = 1.74, size = 11, normalized size = 0.73

$$-\frac{1}{3}\log\left(x^6-1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(-x^7+x),x, algorithm="fricas")
```

```
[Out] -1/3*log(x^6 - 1) + log(x)
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.67

$$\log\left(x\right)-\frac{\log\left(x^6-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/(-x**7+x),x)
```

```
[Out] log(x) - log(x**6 - 1)/3
```

Giac [A]
time = 0.61, size = 16, normalized size = 1.07

$$\frac{1}{6}\log\left(x^6\right)-\frac{1}{3}\log\left(\left|x^6-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(-x^7+x),x, algorithm="giac")
```

```
[Out] 1/6*log(x^6) - 1/3*log(abs(x^6 - 1))
```

Mupad [B]
time = 0.15, size = 11, normalized size = 0.73

$$\ln\left(x\right)-\frac{\ln\left(x^6-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)/(x - x^7),x)
```

```
[Out] log(x) - log(x^6 - 1)/3
```

3.280 $\int \frac{8+5x^{10}}{2x-x^{11}} \, dx$

Optimal. Leaf size=17

$$4\log(x)-\frac{9}{10}\log\left(2-x^{10}\right)$$

[Out] 4*ln(x)-9/10*ln(-x^10+2)

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1607, 457, 78}

$$4\log(x)-\frac{9}{10}\log\left(2-x^{10}\right)$$

Antiderivative was successfully verified.

[In] Int[(8 + 5*x^10)/(2*x - x^11),x]

[Out] 4*Log[x] - (9*Log[2 - x^10])/10

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{8+5x^{10}}{2x-x^{11}}\,dx &= \int \frac{8+5x^{10}}{x\left(2-x^{10}\right)}\,dx \\
&= \frac{1}{10}\text{Subst}\left(\int \frac{8+5x}{(2-x)x}\,dx, x, x^{10}\right) \\
&= \frac{1}{10}\text{Subst}\left(\int \left(-\frac{9}{-2+x}+\frac{4}{x}\right)\,dx, x, x^{10}\right) \\
&= 4\log(x)-\frac{9}{10}\log\left(2-x^{10}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$4\log(x)-\frac{9}{10}\log\left(2-x^{10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 5*x^10)/(2*x - x^11),x]
```

```
[Out] 4*Log[x] - (9*Log[2 - x^10])/10
```

Maple [A]
time = 0.40, size = 14, normalized size = 0.82

method	result	size
default	$4\ln\left(x\right)-\frac{9\ln\left(x^{10}-2\right)}{10}$	14
norman	$4\ln\left(x\right)-\frac{9\ln\left(x^{10}-2\right)}{10}$	14
risch	$4\ln\left(x\right)-\frac{9\ln\left(x^{10}-2\right)}{10}$	14
meijerg	$-\frac{9\ln\left(1-\frac{x^{10}}{2}\right)}{10}+4\ln\left(x\right)-\frac{2\ln\left(2\right)}{5}+\frac{2i\pi}{5}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^10+8)/(-x^11+2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 4*ln(x)-9/10*ln(x^10-2)
```

Maxima [A]
time = 0.49, size = 13, normalized size = 0.76

$$-\frac{9}{10}\log\left(x^{10}-2\right)+4\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^10+8)/(-x^11+2*x),x, algorithm="maxima")
[Out] -9/10*log(x^10 - 2) + 4*log(x)
```

Fricas [A]
time = 1.64, size = 13, normalized size = 0.76

$$-\frac{9}{10}\log\left(x^{10}-2\right)+4\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^10+8)/(-x^11+2*x),x, algorithm="fricas")
[Out] -9/10*log(x^10 - 2) + 4*log(x)
```

Sympy [A]
time = 0.05, size = 14, normalized size = 0.82

$$4\log\left(x\right)-\frac{9\log\left(x^{10}-2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**10+8)/(-x**11+2*x),x)
[Out] 4*log(x) - 9*log(x**10 - 2)/10
```

Giac [A]
time = 0.50, size = 16, normalized size = 0.94

$$\frac{2}{5}\log\left(x^{10}\right)-\frac{9}{10}\log\left(\left|x^{10}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^10+8)/(-x^11+2*x),x, algorithm="giac")
[Out] 2/5*log(x^10) - 9/10*log(abs(x^10 - 2))
```

Mupad [B]
time = 0.14, size = 13, normalized size = 0.76

$$4\ln\left(x\right)-\frac{9\ln\left(x^{10}-2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^10 + 8)/(2*x - x^11),x)
[Out] 4*log(x) - (9*log(x^10 - 2))/10
```

3.281 $\int \frac{-3+2x}{-x^2+x^3} dx$

Optimal. Leaf size=16

$$-\frac{3}{x} - \log(1-x) + \log(x)$$

```
[Out] -3/x-ln(1-x)+ln(x)
```

Rubi [A]

time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1607, 78}

$$-\frac{3}{x} - \log(1-x) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*x)/(-x^2 + x^3),x]
```

```
[Out] -3/x - Log[1 - x] + Log[x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3+2x}{-x^2+x^3} dx &= \int \frac{-3+2x}{(-1+x)x^2} dx \\
&= \int \left(\frac{1}{1-x} + \frac{3}{x^2} + \frac{1}{x}\right) dx \\
&= -\frac{3}{x} - \log(1-x) + \log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-\frac{3}{x}-\log(1-x)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 2*x)/(-x^2 + x^3),x]
[Out] -3/x - Log[1 - x] + Log[x]
```

Maple [A]
time = 0.42, size = 15, normalized size = 0.94

method	result	size
default	$-\ln\left(x-1\right)+\ln\left(x\right)-\frac{3}{x}$	15
norman	$-\ln\left(x-1\right)+\ln\left(x\right)-\frac{3}{x}$	15
risch	$-\ln\left(x-1\right)+\ln\left(x\right)-\frac{3}{x}$	15
meijerg	$-\ln\left(1-x\right)+\ln\left(x\right)+i\pi-\frac{3}{x}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*x)/(x^3-x^2),x,method=_RETURNVERBOSE)
[Out] -ln(x-1)+ln(x)-3/x
```

Maxima [A]
time = 0.30, size = 14, normalized size = 0.88

$$-\frac{3}{x}-\log\left(x-1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(x^3-x^2),x, algorithm="maxima")
[Out] -3/x - log(x - 1) + log(x)
```

Fricas [A]
time = 2.22, size = 18, normalized size = 1.12

$$-\frac{x\log\left(x-1\right)-x\log\left(x\right)+3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(x^3-x^2),x, algorithm="fricas")
```

```
[Out] -(x*log(x - 1) - x*log(x) + 3)/x
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.62

$$\log(x) - \log(x-1) - \frac{3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(x**3-x**2),x)
[Out] log(x) - log(x - 1) - 3/x
```

Giac [A]
time = 0.50, size = 16, normalized size = 1.00

$$-\frac{3}{x} - \log(|x-1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(x^3-x^2),x, algorithm="giac")
[Out] -3/x - log(abs(x - 1)) + log(abs(x))
```

Mupad [B]
time = 0.13, size = 14, normalized size = 0.88

$$2\,\mathrm{atanh}(2\,x-1) - \frac{3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 3)/(x^2 - x^3),x)
[Out] 2*atanh(2*x - 1) - 3/x
```

3.282 $\int \frac{ax^m+bx^n}{cx^m+dx^n}\,dx$

Optimal. Leaf size=54

$$\frac{ax}{c}+\frac{(bc-ad)x\,{}_2F_1\left(1,\frac{1}{m-n};1+\frac{1}{m-n};-\frac{cx^{m-n}}{d}\right)}{cd}$$

```
[Out] a*x/c+(-a*d+b*c)*x*hypergeom([1, 1/(m-n)],[1+1/(m-n)],-c*x^(m-n)/d)/c/d
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1607, 1598, 396, 251}

$$\frac{x(bc-ad)\,{}_2F_1\left(1,\frac{1}{m-n};1+\frac{1}{m-n};-\frac{cx^{m-n}}{d}\right)}{cd}+\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m + b*x^n)/(c*x^m + d*x^n),x]
```

```
[Out] (a*x)/c + ((b*c - a*d)*x*Hypergeometric2F1[1, (m - n)^(-1), 1 + (m - n)^(-1
), -((c*x^(m - n))/d)])/(c*d)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{ax^m+bx^n}{cx^m+dx^n}\,dx &= \int \frac{x^n(b+ax^{m-n})}{cx^m+dx^n}\,dx\\ &= \int \frac{b+ax^{m-n}}{d+cx^{m-n}}\,dx\\ &= \frac{ax}{c}-\frac{(-bc+ad)\int \frac{1}{d+cx^{m-n}}\,dx}{c}\\ &= \frac{ax}{c}+\frac{(bc-ad)x\,{}_2F_1\left(1,\frac{1}{m-n};1+\frac{1}{m-n};-\frac{cx^{m-n}}{d}\right)}{cd}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 52, normalized size = 0.96

$$\frac{x\left(ad+(bc-ad)\,{}_2F_1\left(1,\frac{1}{m-n};1+\frac{1}{m-n};-\frac{cx^{m-n}}{d}\right)\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m + b*x^n)/(c*x^m + d*x^n),x]
[Out] (x*(a*d + (b*c - a*d)*Hypergeometric2F1[1, (m - n)^(-1), 1 + (m - n)^(-1),
-((c*x^(m - n))/d)]))/(c*d)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a\,x^m+b\,x^n}{c\,x^m+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m+b*x^n)/(c*x^m+d*x^n),x)
[Out] int((a*x^m+b*x^n)/(c*x^m+d*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^n)/(c*x^m+d*x^n),x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)*integrate(x^m/(c*d*x^m + d^2*x^n), x) + b*x/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^n)/(c*x^m+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((a*x^m + b*x^n)/(c*x^m + d*x^n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^m + bx^n}{cx^m + dx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m+b*x**n)/(c*x**m+d*x**n),x)
```

```
[Out] Integral((a*x**m + b*x**n)/(c*x**m + d*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^n)/(c*x^m+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*x^n)/(c*x^m + d*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a\,x^m + b\,x^n}{c\,x^m + d\,x^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m + b*x^n)/(c*x^m + d*x^n),x)
```

```
[Out] int((a*x^m + b*x^n)/(c*x^m + d*x^n), x)
```

3.283 $\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p$

Optimal. Leaf size=18

$$x^{1+m+q}(a+bx^n)^{1+p}$$

```
[Out] x^(1+m+q)*(a+b*x^n)^(1+p)
```

Rubi [A]
time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1598, 460}

$$x^{m+q+1}(a+bx^n)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^n)^p*(a*(1 + m + q)*x^q + b*(1 + m + n*(1 + p) + q)*x^(n +
 q)),x]
```

```
[Out] x^(1 + m + q)*(a + b*x^n)^(1 + p)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}\right)\,dx=\int x^{m+q}(a+bx^n)^p\left(a(1+m+q)+\right.$$
$$=x^{1+m+q}(a+bx^n)^{1+p}$$

Mathematica [A]
time = 0.28, size = 18, normalized size = 1.00

$$x^{1+m+q}(a+bx^n)^{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^n)^p*(a*(1 + m + q)*x^q + b*(1 + m + n*(1 + p) + q)*
x^(n + q)),x]
```

```
[Out] x^(1 + m + q)*(a + b*x^n)^(1 + p)
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int x^{m}(a+b\,x^{n})^{p}\left(a(1+m+q)\,x^{q}+b(1+m+n(1+p)+q)\,x^{n+q}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x)
```

```
[Out] int(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.
time = 0.33, size = 37, normalized size = 2.06

$$\left(axx^{m}+bxe^{(m\log(x)+n\log(x))}\right)e^{(p\log(bx^{n}+a)+q\log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x, algo
rithm="maxima")
```

```
[Out] (a*x*x^m + b*x*e^(m*log(x) + n*log(x)))*e^(p*log(b*x^n + a) + q*log(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(18) = 36$.
time = 1.72, size = 43, normalized size = 2.39

$$\left(bxx^{m}x^{n+q}+axx^{m}x^{q}\right)\left(\frac{bx^{n+q}+ax^{q}}{x^{q}}\right)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x, algo
rithm="fricas")
```

```
[Out] (b*x*x^m*x^(n + q) + a*x*x^m*x^q)*((b*x^(n + q) + a*x^q)/x^q)^p
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**n)**p*(a*(1+m+q)*x**q+b*(1+m+n*(1+p)+q)*x**(n+q)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.
time = 0.66, size = 48, normalized size = 2.67

$$(bx^n + a)^p bxx^n e^{(m\log(x)+q\log(x))} + (bx^n + a)^p axe^{(m\log(x)+q\log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x, algo
rithm="giac")
```

```
[Out] (b*x^n + a)^p*b*x*x^n*e^(m*log(x) + q*log(x)) + (b*x^n + a)^p*a*x*e^(m*log(
x) + q*log(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int x^m \left(a\,x^q\,(m+q+1) + b\,x^{n+q}\,(m+q+n\,(p+1)+1)\right)\,(a+b\,x^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*x^q*(m + q + 1) + b*x^(n + q)*(m + q + n*(p + 1) + 1))*(a + b*x^
n)^p,x)
```

```
[Out] int(x^m*(a*x^q*(m + q + 1) + b*x^(n + q)*(m + q + n*(p + 1) + 1))*(a + b*x^
n)^p, x)
```

3.284 $\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx} \, dx$

Optimal. Leaf size=64

$$\frac{\left(a+\frac{b}{x}\right)^n \left(1+\frac{b}{ax}\right)^{-n} x^m F_1\left(-m;-n,1;1-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{dm}$$

```
[Out] (a+b/x)^n*x^m*AppellF1(-m,-n,1,1-m,-b/a/x,-c/d/x)/d/m/((1+b/a/x)^n)
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {528, 511, 140, 138}

$$\frac{x^m \left(a+\frac{b}{x}\right)^n \left(\frac{b}{ax}+1\right)^{-n} F_1\left(-m;-n,1;1-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x^m)/(c + d*x),x]
```

```
[Out] ((a + b/x)^n*x^m*AppellF1[-m, -n, 1, 1 - m, -(b/(a*x)), -(c/(d*x))])/(d*m*(
1 + b/(a*x))^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Dist[(-(e*x)^m)*(x^(-1))^m, Subst[Int[(a + b/x^n)^p*((
c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q},
 x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n x^{-1+m}}{d+\frac{c}{x}}\,dx\\ &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\operatorname{Subst}\left(\int \frac{x^{-1-m}(a+bx)^n}{d+cx}\,dx,x,\frac{1}{x}\right)\right)\\ &= -\left(\left(\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{ax}\right)^{-n}\left(\frac{1}{x}\right)^m x^m\right)\operatorname{Subst}\left(\int \frac{x^{-1-m}\left(1+\frac{bx}{a}\right)^n}{d+cx}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{ax}\right)^{-n} x^m F_1\left(-m;-n,1;1-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{dm}\end{aligned}$$

Mathematica [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b/x)^n*x^m)/(c + d*x),x]
[Out] Integrate[((a + b/x)^n*x^m)/(c + d*x), x]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^n*x^m/(d*x+c),x)
[Out] int((a+1/x*b)^n*x^m/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n*x^m/(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(x^m*((a*x + b)/x)^n/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x**m/(d*x+c),x)
```

```
[Out] Integral(x**m*(a + b/x)**n/(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n*x^m/(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,{\left(a+\frac{b}{x}\right)}^n}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b/x)^n)/(c + d*x),x)
```

```
[Out] int((x^m*(a + b/x)^n)/(c + d*x), x)
```

3.285 $\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx}\,dx$

Optimal. Leaf size=195

$$-\frac{\left(2ac+bd(1-n)\right)\left(a+\frac{b}{x}\right)^{1+n}x}{2a^2d^2}+\frac{\left(a+\frac{b}{x}\right)^{1+n}x^2}{2ad}-\frac{c^3\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^3(ac-bd)(1+n)}+\frac{(2a^2c^2-2a}{}$$

```
[Out] -1/2*(2*a*c+b*d*(1-n))*(a+b/x)^(1+n)*x/a^2/d^2+1/2*(a+b/x)^(1+n)*x^2/a/d-c^
3*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/d^3/(a*c-b*d)
/(1+n)+1/2*(2*a^2*c^2-2*a*b*c*d*n-b^2*d^2*(1-n)*n)*(a+b/x)^(1+n)*hypergeom(
[1, 1+n],[2+n],1+b/a/x)/a^3/d^3/(1+n)
```

Rubi [A]
time = 0.15, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {528, 457, 105, 156, 162, 67, 70}

$$-\frac{x\left(a+\frac{b}{x}\right)^{n+1}(2ac+bd(1-n))}{2a^2d^2}+\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(2a^2c^2-2abcdn-b^2d^2(1-n)n\right)\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{2a^3d^3(n+1)}-\frac{c^3\left(a+\frac{b}{x}\right)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^3(n+1)(ac-bd)}+\frac{x^2\left(a+\frac{b}{x}\right)^{n+1}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x^2)/(c + d*x),x]
```

```
[Out] -1/2*((2*a*c + b*d*(1 - n))*(a + b/x)^(1 + n)*x)/(a^2*d^2) + ((a + b/x)^(1
+ n)*x^2)/(2*a*d) - (c^3*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 +
n, (c*(a + b/x))/(a*c - b*d)])/(d^3*(a*c - b*d)*(1 + n)) + ((2*a^2*c^2 - 2*
a*b*c*d*n - b^2*d^2*(1 - n)*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n
, 2 + n, 1 + b/(a*x)])/(2*a^3*d^3*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x}{d+\frac{c}{x}}\,dx$$

$$= -\text{Subst}\left(\int \frac{(a+bx)^n}{x^3(d+cx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2ad} + \frac{\text{Subst}\left(\int \frac{(a+bx)^n(2ac+bd(1-n)+bc(1-n)x)}{x^2(d+cx)}\,dx, x, \frac{1}{x}\right)}{2ad}$$

$$= -\frac{(2ac+bd(1-n))\left(a+\frac{b}{x}\right)^{1+n} x}{2a^2d^2} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2ad} - \frac{\text{Subst}\left(\int \frac{(a+bx)^n(2a^2c^2-2abcdn-b^2d^2(1-}{x(d+cx}\right.}{2a^2d^2}$$

$$= -\frac{(2ac+bd(1-n))\left(a+\frac{b}{x}\right)^{1+n} x}{2a^2d^2} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2ad} + \frac{c^3\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{d^3} - (2a$$

$$= -\frac{(2ac+bd(1-n))\left(a+\frac{b}{x}\right)^{1+n} x}{2a^2d^2} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2ad} - \frac{c^3\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\right.}{d^3(ac-bd)(1+n)}$$

Mathematica [A]
time = 0.21, size = 157, normalized size = 0.81

$$\frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left(-2a^3c^3\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(ac-bd)\left(ad(1+n)x(bd(-1+n)+a(-2c+dx))+\left(2a^2c^2-2abcdn+b^2d^2(-1+n)n\right){}_2F_1\left(1,1+n;2+n;1+\frac{b}{ax}\right)\right)\right)}{2a^3d^3(ac-bd)(1+n)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x)^n*x^2)/(c + d*x),x]
```

```
[Out] ((a + b/x)^n*(b + a*x)*(-2*a^3*c^3*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a
 + b/x))/(a*c - b*d)] + (a*c - b*d)*(a*d*(1 + n)*x*(b*d*(-1 + n) + a*(-2*c
+ d*x)) + (2*a^2*c^2 - 2*a*b*c*d*n + b^2*d^2*(-1 + n)*n)*Hypergeometric2F1[
1, 1 + n, 2 + n, 1 + b/(a*x)])))/(2*a^3*d^3*(a*c - b*d)*(1 + n)*x)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^n*x^2/(d*x+c),x)
```

```
[Out] int((a+1/x*b)^n*x^2/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n*x^2/(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(x^2*((a*x + b)/x)^n/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x**2/(d*x+c),x)
```

```
[Out] Integral(x**2*(a + b/x)**n/(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n*x^2/(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+\frac{b}{x}\right)^n}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b/x)^n)/(c + d*x),x)
```

```
[Out] int((x^2*(a + b/x)^n)/(c + d*x), x)
```

3.286 $\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx$

Optimal. Leaf size=131

$$\frac{\left(a+\frac{b}{x}\right)^{1+n}x}{ad}+\frac{c^2\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(ac-bd)(1+n)}-\frac{(ac-bdn)\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{b}{ax}\right)}{a^2d^2(1+n)}$$

```
[Out] (a+b/x)^(1+n)*x/a/d+c^2*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a
*c-b*d))/d^2/(a*c-b*d)/(1+n)-(-b*d*n+a*c)*(a+b/x)^(1+n)*hypergeom([1, 1+n],
[2+n],1+b/a/x)/a^2/d^2/(1+n)
```

Rubi [A]

time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {528, 382, 105, 162, 67, 70}

$$-\frac{\left(a+\frac{b}{x}\right)^{n+1}(ac-bdn)\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{a^2d^2(n+1)}+\frac{c^2\left(a+\frac{b}{x}\right)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(n+1)(ac-bd)}+\frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x)/(c + d*x),x]
```

```
[Out] ((a + b/x)^(1 + n)*x)/(a*d) + (c^2*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1
 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(d^2*(a*c - b*d)*(1 + n)) - ((a*c
- b*d*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])
/(a^2*d^2*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{d+\frac{c}{x}}\,dx\\
&= -\text{Subst}\left(\int \frac{(a+bx)^n}{x^2(d+cx)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad} + \frac{\text{Subst}\left(\int \frac{(a+bx)^n(ac-bdn-bcnx)}{x(d+cx)}\,dx, x, \frac{1}{x}\right)}{ad}\\
&= \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad} - \frac{c^2\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(ac-bdn)\text{Subst}\left(\int \frac{(a+bx)^n}{x}\,dx, x, \frac{1}{x}\right)}{ad^2}\\
&= \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad} + \frac{c^2\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1, 1+n; 2+n; \frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(ac-bd)(1+n)} - \frac{(ac-bdn)\left(a+\frac{b}{x}\right)^{1+n}\ {}_2\cdots}{a^2d^2(\cdots}\end{aligned}$$

Mathematica [A]

time = 0.16, size = 119, normalized size = 0.91

$$\frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left(a^2c^2\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(ac-bd)\left(ad(1+n)x+(-ac+bdn)\,{}_2F_1\left(1,1+n;2+n;1+\frac{b}{ax}\right)\right)\right)}{a^2d^2(ac-bd)(1+n)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x)^n*x)/(c + d*x),x]
```

```
[Out] ((a + b/x)^n*(b + a*x)*(a^2*c^2*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a +
b/x))/(a*c - b*d)] + (a*c - b*d)*(a*d*(1 + n)*x + (-(a*c) + b*d*n)*Hypergeo
metric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])))/(a^2*d^2*(a*c - b*d)*(1 + n)*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^n*x/(d*x+c),x)
```

```
[Out] int((a+1/x*b)^n*x/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n*x/(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(x*((a*x + b)/x)^n/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x/(d*x+c),x)
```

```
[Out] Integral(x*(a + b/x)**n/(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n*x/(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(a+\frac{b}{x}\right)}^n}{c+d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b/x)^n)/(c + d*x),x)
```

```
[Out] int((x*(a + b/x)^n)/(c + d*x), x)
```

3.287 $\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$

Optimal. Leaf size=101

$$-\frac{c\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(ac-bd)(1+n)}+\frac{\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{b}{ax}\right)}{ad(1+n)}$$

```
[Out] -c*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/d/(a*c-b*d)/
(1+n)+(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],1+b/a/x)/a/d/(1+n)
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {445, 457, 88, 67, 70}

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{ad(n+1)}-\frac{c\left(a+\frac{b}{x}\right)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(n+1)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(c + d*x),x]
```

```
[Out] -((c*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*
c - b*d)])/(d*(a*c - b*d)*(1 + n))) + ((a + b/x)^(1 + n)*Hypergeometric2F1[
1, 1 + n, 2 + n, 1 + b/(a*x)])/(a*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
```

```
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x}\,dx \\
&= -\text{Subst}\left(\int \frac{(a+bx)^n}{x(d+cx)}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\text{Subst}\left(\int \frac{(a+bx)^n}{x}\,dx, x, \frac{1}{x}\right)}{d} + \frac{c\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{d} \\
&= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(ac-bd)(1+n)} + \frac{\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{b}{ax}\right)}{ad(1+n)}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 97, normalized size = 0.96

$$
\frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left(ac\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(-ac+bd)\,{}_2F_1\left(1,1+n;2+n;1+\frac{b}{ax}\right)\right)}{ad(-ac+bd)(1+n)x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(c + d*x),x]
```

```
[Out] ((a + b/x)^n*(b + a*x)*(a*c*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x)
)/(a*c - b*d)] + (-(a*c) + b*d)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a
*x)]))/(a*d*(-(a*c) + b*d)*(1 + n)*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^n/(d*x+c),x)
```

```
[Out] int((a+1/x*b)^n/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/(d*x+c),x)
```

```
[Out] Integral((a + b/x)**n/(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(c + d*x),x)
```

```
[Out] int((a + b/x)^n/(c + d*x), x)
```

3.288 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)} \, dx$

Optimal. Leaf size=54

$$\frac{\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)(1+n)}$$

[Out] (a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/(a*c-b*d)/(1+n)

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {528, 455, 70}

$$\frac{\left(a+\frac{b}{x}\right)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^n/(x*(c + d*x)),x]

[Out] ((a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/((a*c - b*d)*(1 + n))

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x^2}\,dx\\ &= -\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)\\ &= \frac{\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 54, normalized size = 1.00

$$\frac{\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x*(c + d*x)),x]
[Out] ((a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c -
b*d)])/((a*c - b*d)*(1 + n))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^n/x/(d*x+c),x)
[Out] int((a+1/x*b)^n/x/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c),x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^n/(d*x^2 + c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{n}}{x \left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x/(d*x+c),x)
[Out] Integral((a + b/x)**n/(x*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^n/((d*x + c)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+\frac{b}{x}\right)}^n}{x\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(x*(c + d*x)),x)
[Out] int((a + b/x)^n/(x*(c + d*x)), x)
```

3.289 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(1+n)}-\frac{d\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(ac-bd)(1+n)}$$

```
[Out] -(a+b/x)^(1+n)/b/c/(1+n)-d*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)
/(a*c-b*d))/c/(a*c-b*d)/(1+n)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {528, 457, 81, 70}

$$-\frac{d\left(a+\frac{b}{x}\right)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(n+1)(ac-bd)}-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x^2*(c + d*x)),x]
```

```
[Out] -((a + b/x)^(1 + n)/(b*c*(1 + n))) - (d*(a + b/x)^(1 + n)*Hypergeometric2F1
[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c*(a*c - b*d)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x^3}\,dx\\ &= -\text{Subst}\left(\int \frac{x(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(1+n)} + \frac{d\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c}\\ &= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(1+n)} - \frac{d\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(ac-bd)(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 77, normalized size = 0.92

$$\frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left(ac-bd+bd\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)\right)}{bc(-ac+bd)(1+n)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x^2*(c + d*x)),x]
```

```
[Out] ((a + b/x)^n*(b + a*x)*(a*c - b*d + b*d*Hypergeometric2F1[1, 1 + n, 2 + n,
(c*(a + b/x))/(a*c - b*d)]))/(b*c*(-(a*c) + b*d)*(1 + n)*x)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^n/x^2/(d*x+c),x)
[Out] int((a+1/x*b)^n/x^2/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c),x, algorithm="maxima")
[Out] integrate((a + b/x)^n/((d*x + c)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c),x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^n/(d*x^3 + c*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x^2\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x**2/(d*x+c),x)
[Out] Integral((a + b/x)**n/(x**2*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^n/((d*x + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{x}\right)^n}{x^2\,\left(c + d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(x^2*(c + d*x)),x)
[Out] int((a + b/x)^n/(x^2*(c + d*x)), x)
```

3.290 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{(ac+bd)\left(a+\frac{b}{x}\right)^{1+n}}{b^2c^2(1+n)}-\frac{\left(a+\frac{b}{x}\right)^{2+n}}{b^2c(2+n)}+\frac{d^2\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(ac-bd)(1+n)}$$

```
[Out] (a*c+b*d)*(a+b/x)^(1+n)/b^2/c^2/(1+n)-(a+b/x)^(2+n)/b^2/c/(2+n)+d^2*(a+b/x)
^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/c^2/(a*c-b*d)/(1+n)
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {528, 457, 90, 70}

$$\frac{(ac+bd)\left(a+\frac{b}{x}\right)^{n+1}}{b^2c^2(n+1)}-\frac{\left(a+\frac{b}{x}\right)^{n+2}}{b^2c(n+2)}+\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(n+1)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x^3*(c + d*x)),x]
```

```
[Out] ((a*c + b*d)*(a + b/x)^(1 + n))/(b^2*c^2*(1 + n)) - (a + b/x)^(2 + n)/(b^2*
c*(2 + n)) + (d^2*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(
a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*d)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x^4}\,dx\\
&= -\text{Subst}\left(\int \frac{x^2(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{(-ac-bd)(a+bx)^n}{bc^2}+\frac{(a+bx)^{1+n}}{bc}+\frac{d^2(a+bx)^n}{c^2(d+cx)}\right)dx, x, \frac{1}{x}\right)\\
&= \frac{(ac+bd)\left(a+\frac{b}{x}\right)^{1+n}}{b^2c^2(1+n)}-\frac{\left(a+\frac{b}{x}\right)^{2+n}}{b^2c(2+n)}-\frac{d^2\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c^2}\\
&= \frac{(ac+bd)\left(a+\frac{b}{x}\right)^{1+n}}{b^2c^2(1+n)}-\frac{\left(a+\frac{b}{x}\right)^{2+n}}{b^2c(2+n)}+\frac{d^2\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(ac-bd)(1+n)}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 112, normalized size = 0.97

$$-\frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left((ac-bd)(-bc(1+n)+acx+bd(2+n)x)+b^2d^2(2+n)x\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)\right)}{b^2c^2(-ac+bd)(1+n)(2+n)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x^3*(c + d*x)),x]
```

```
[Out] -(((a + b/x)^n*(b + a*x)*((a*c - b*d)*(-(b*c*(1 + n)) + a*c*x + b*d*(2 + n)
*x) + b^2*d^2*(2 + n)*x*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a
*c - b*d)]))/(b^2*c^2*(-(a*c) + b*d)*(1 + n)*(2 + n)*x^2))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^n/x^3/(d*x+c),x)
[Out] int((a+1/x*b)^n/x^3/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c),x, algorithm="maxima")
[Out] integrate((a + b/x)^n/((d*x + c)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c),x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^n/(d*x^4 + c*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{n}}{x^{3}\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x**3/(d*x+c),x)
[Out] Integral((a + b/x)**n/(x**3*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^n/((d*x + c)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(x^3*(c + d*x)),x)
[Out] int((a + b/x)^n/(x^3*(c + d*x)), x)
```

3.291 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx$

Optimal. Leaf size=207

$$\frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{1+n}}{b^4c^4(1+n)}-\frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{2+n}}{b^4c^3(2+n)}+\frac{(3ac+bd)\left(a+\frac{b}{x}\right)^{3+n}}{b^4c^2(3+n)}-\frac{\left(a+\frac{b}{x}\right)^{4+}}{b^4c(4+n}$$

```
[Out] (a*c+b*d)*(a^2*c^2+b^2*d^2)*(a+b/x)^(1+n)/b^4/c^4/(1+n)-(3*a^2*c^2+2*a*b*c*
d+b^2*d^2)*(a+b/x)^(2+n)/b^4/c^3/(2+n)+(3*a*c+b*d)*(a+b/x)^(3+n)/b^4/c^2/(3
+n)-(a+b/x)^(4+n)/b^4/c/(4+n)+d^4*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*
(a+b/x)/(a*c-b*d))/c^4/(a*c-b*d)/(1+n)
```

Rubi [A]
time = 0.11, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {528, 457, 90, 70}

$$\frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{n+1}}{b^4c^4(n+1)}-\frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{n+2}}{b^4c^3(n+2)}+\frac{(3ac+bd)\left(a+\frac{b}{x}\right)^{n+3}}{b^4c^2(n+3)}-\frac{\left(a+\frac{b}{x}\right)^{n+4}}{b^4c(n+4)}+\frac{d^4\left(a+\frac{b}{x}\right)^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^4(n+1)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x^5*(c + d*x)),x]
```

```
[Out] ((a*c + b*d)*(a^2*c^2 + b^2*d^2)*(a + b/x)^(1 + n))/(b^4*c^4*(1 + n)) - ((3
*a^2*c^2 + 2*a*b*c*d + b^2*d^2)*(a + b/x)^(2 + n))/(b^4*c^3*(2 + n)) + ((3*
a*c + b*d)*(a + b/x)^(3 + n))/(b^4*c^2*(3 + n)) - (a + b/x)^(4 + n)/(b^4*c*
(4 + n)) + (d^4*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a
+ b/x))/(a*c - b*d)])/(c^4*(a*c - b*d)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x^6}\,dx$$

$$= -\text{Subst}\left(\int \frac{x^4(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{(ac+bd)\left(-a^2c^2-b^2d^2\right)(a+bx)^n}{b^3c^4} + \frac{\left(3a^2c^2+2abcd+b^2d^2\right)(a+bx)^{1+n}}{b^3c^3} +\right.\right.$$

$$= \frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{1+n}}{b^4c^4(1+n)} - \frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{2+n}}{b^4c^3(2+n)} + \frac{(3ac+bd)\,(a}{b^4c^2(3+}$$

$$= \frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{1+n}}{b^4c^4(1+n)} - \frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{2+n}}{b^4c^3(2+n)} + \frac{(3ac+bd)\,(a}{b^4c^2(3+}$$

Mathematica [A]
time = 0.26, size = 184, normalized size = 0.89

$$\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(\frac{(ac+bd)\left(a^2c^2+b^2d^2\right)}{b^4(1+n)} - \frac{c\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)}{b^4(2+n)} + \frac{c^2(3ac+bd)\left(a+\frac{b}{x}\right)^2}{b^4(3+n)} - \frac{c^3\left(a+\frac{b}{x}\right)^3}{b^4(4+n)} + \frac{d^4\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)(1+n)}\right)}{c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x^5*(c + d*x)),x]
```

```
[Out] ((a + b/x)^(1 + n)*(((a*c + b*d)*(a^2*c^2 + b^2*d^2))/(b^4*(1 + n)) - (c*(3
*a^2*c^2 + 2*a*b*c*d + b^2*d^2)*(a + b/x))/(b^4*(2 + n)) + (c^2*(3*a*c + b*
d)*(a + b/x)^2)/(b^4*(3 + n)) - (c^3*(a + b/x)^3)/(b^4*(4 + n)) + (d^4*Hype
```

```
rgeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/((a*c - b*d)*(1
+ n))))/c^4
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x^{5}\left(dx+c\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^n/x^5/(d*x+c),x)
[Out] int((a+1/x*b)^n/x^5/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^5/(d*x+c),x, algorithm="maxima")
[Out] integrate((a + b/x)^n/((d*x + c)*x^5), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^5/(d*x+c),x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^n/(d*x^6 + c*x^5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x^{5}\left(c+dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x**5/(d*x+c),x)
[Out] Integral((a + b/x)**n/(x**5*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^5/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(x^5*(c + d*x)),x)
```

```
[Out] int((a + b/x)^n/(x^5*(c + d*x)), x)
```

3.292 $\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2} \, dx$

Optimal. Leaf size=73

$$-\frac{\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{ax}\right)^{-n} x^{-1+m} F_1\left(1-m;-n,2;2-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{d^2(1-m)}$$

[Out] -(a+b/x)^n*x^(-1+m)*AppellF1(1-m,-n,2,2-m,-b/a/x,-c/d/x)/d^2/(1-m)/((1+b/a/x)^n)

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {528, 511, 140, 138}

$$-\frac{x^{m-1}\left(a+\frac{b}{x}\right)^n\left(\frac{b}{ax}+1\right)^{-n} F_1\left(1-m;-n,2;2-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{d^2(1-m)}$$

Antiderivative was successfully verified.

[In] Int[((a + b/x)^n*x^m)/(c + d*x)^2,x]

[Out] -(((a + b/x)^n*x^(-1 + m)*AppellF1[1 - m, -n, 2, 2 - m, -(b/(a*x)), -(c/(d*x))])/(d^2*(1 - m)*(1 + b/(a*x))^n))

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Dist[(-(e*x)^m)*(x^(-1))^m, Subst[Int[(a + b/x^n)^p*((
c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q},
 x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n x^{-2+m}}{\left(d+\frac{c}{x}\right)^2}\,dx\\
&= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\operatorname{Subst}\left(\int \frac{x^{-m}(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)\right)\\
&= -\left(\left(\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{ax}\right)^{-n}\left(\frac{1}{x}\right)^m x^m\right)\operatorname{Subst}\left(\int \frac{x^{-m}\left(1+\frac{bx}{a}\right)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)\right)\\
&= -\frac{\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{ax}\right)^{-n} x^{-1+m} F_1\left(1-m;-n,2;2-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{d^2(1-m)}
\end{aligned}$$

Mathematica [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b/x)^n*x^m)/(c + d*x)^2,x]
[Out] Integrate[((a + b/x)^n*x^m)/(c + d*x)^2, x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^n*x^m/(d*x+c)^2,x)
[Out] int((a+1/x*b)^n*x^m/(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^n*x^m/(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(x^m*((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(a + \frac{b}{x}\right)^n}{\left(c + dx\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x**m/(d*x+c)**2,x)
[Out] Integral(x**m*(a + b/x)**n/(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^m/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a + b/x)^n*x^m/(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,{\left(a+\frac{b}{x}\right)}^n}{{\left(c+d\,x\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b/x)^n)/(c + d*x)^2,x)
[Out] int((x^m*(a + b/x)^n)/(c + d*x)^2, x)
```

3.293 $\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=202

$$\frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{1+n}}{ad^2(ac-bd)\left(d+\frac{c}{x}\right)}+\frac{\left(a+\frac{b}{x}\right)^{1+n}x}{ad\left(d+\frac{c}{x}\right)}+\frac{c^2(2ac-bd(2-n))\left(a+\frac{b}{x}\right)^{1+n}\ {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^3(ac-bd)^2(1+n)}-\frac{(2ac$$

```
[Out] c*(2*a*c-b*d)*(a+b/x)^(1+n)/a/d^2/(a*c-b*d)/(d+c/x)+(a+b/x)^(1+n)*x/a/d/(d+
c/x)+c^2*(2*a*c-b*d*(2-n))*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)
/(a*c-b*d))/d^3/(a*c-b*d)^2/(1+n)-(-b*d*n+2*a*c)*(a+b/x)^(1+n)*hypergeom([1
, 1+n],[2+n],1+b/a/x)/a^2/d^3/(1+n)
```

Rubi [A]
time = 0.17, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {528, 382, 105, 156, 162, 67, 70}

$$-\frac{\left(a+\frac{b}{x}\right)^{n+1}(2ac-bdn)\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{a^2d^3(n+1)}+\frac{c^2\left(a+\frac{b}{x}\right)^{n+1}(2ac-bd(2-n))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^3(n+1)(ac-bd)^2}+\frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{n+1}}{ad^2\left(\frac{c}{x}+d\right)(ac-bd)}+\frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad\left(\frac{c}{x}+d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x^2)/(c + d*x)^2,x]
```

```
[Out] (c*(2*a*c - b*d)*(a + b/x)^(1 + n))/(a*d^2*(a*c - b*d)*(d + c/x)) + ((a + b
/x)^(1 + n)*x)/(a*d*(d + c/x)) + (c^2*(2*a*c - b*d*(2 - n))*(a + b/x)^(1 +
n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(d^3*(a*c
 - b*d)^2*(1 + n)) - ((2*a*c - b*d*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1
, 1 + n, 2 + n, 1 + b/(a*x)])/(a^2*d^3*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2}\,dx \\
&= -\text{Subst}\left(\int \frac{(a+bx)^n}{x^2(d+cx)^2}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad\left(d+\frac{c}{x}\right)} + \frac{\text{Subst}\left(\int \frac{(a+bx)^n(2ac-bdn+bc(1-n)x)}{x(d+cx)^2}\,dx, x, \frac{1}{x}\right)}{ad} \\
&= \frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{1+n}}{ad^2(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad\left(d+\frac{c}{x}\right)} + \frac{\text{Subst}\left(\int \frac{(a+bx)^n((ac-bd)(2ac-bdn)-bc(2ac-bd)nx)}{x(d+cx)}\,dx\right.}{ad^2(ac-bd)} \\
&= \frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{1+n}}{ad^2(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad\left(d+\frac{c}{x}\right)} - \frac{\left(c^2(2ac-bd(2-n))\right)\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \right.}{d^3(ac-bd)} \\
&= \frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{1+n}}{ad^2(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad\left(d+\frac{c}{x}\right)} + \frac{c^2(2ac-bd(2-n))\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n\right.}{d^3(ac-bd)^2(1+n)}
\end{aligned}
$$

Mathematica [A]
time = 0.24, size = 179, normalized size = 0.89

$$
\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(acd(ac-bd)(2ac-bd)(1+n)x+ad^2(ac-bd)^2(1+n)x^2+(c+dx)\left(a^2c^2(2ac+bd(-2+n))\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)-(ac-bd)^2(2ac-bdn)\,{}_2F_1\left(1,1+n;2+n;1+\frac{b}{ax}\right)\right)\right)}{a^2d^3(ac-bd)^2(1+n)(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x)^n*x^2)/(c + d*x)^2,x]
```

```
[Out] ((a + b/x)^(1 + n)*(a*c*d*(a*c - b*d)*(2*a*c - b*d)*(1 + n)*x + a*d^2*(a*c
- b*d)^2*(1 + n)*x^2 + (c + d*x)*(a^2*c^2*(2*a*c + b*d*(-2 + n))*Hypergeome
tric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)] - (a*c - b*d)^2*(2*a*c
- b*d*n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])))/(a^2*d^3*(a*c -
 b*d)^2*(1 + n)*(c + d*x))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{\left(dx+c\right)^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^n*x^2/(d*x+c)^2,x)
```

```
[Out] int((a+1/x*b)^n*x^2/(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^n*x^2/(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(x^2*((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+\frac{b}{x}\right)^n}{\left(c+dx\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x**2/(d*x+c)**2,x)
[Out] Integral(x**2*(a + b/x)**n/(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a + b/x)^n*x^2/(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+\frac{b}{x}\right)}^n}{{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b/x)^n)/(c + d*x)^2,x)
[Out] int((x^2*(a + b/x)^n)/(c + d*x)^2, x)
```

3.294 $\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{c\left(a+\frac{b}{x}\right)^{1+n}}{d(ac-bd)\left(d+\frac{c}{x}\right)}-\frac{c(ac-bd(1-n))\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(ac-bd)^2(1+n)}+\frac{\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1(1,1+n$$

```
[Out] -c*(a+b/x)^(1+n)/d/(a*c-b*d)/(d+c/x)-c*(a*c-b*d*(1-n))*(a+b/x)^(1+n)*hyperg
eom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/d^2/(a*c-b*d)^2/(1+n)+(a+b/x)^(1+n)
*hypergeom([1, 1+n],[2+n],1+b/a/x)/a/d^2/(1+n)
```

Rubi [A]
time = 0.08, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {528, 457, 105, 162, 67, 70}

$$-\frac{c\left(a+\frac{b}{x}\right)^{n+1}(ac-bd(1-n))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(n+1)(ac-bd)^2}-\frac{c\left(a+\frac{b}{x}\right)^{n+1}}{d\left(\frac{c}{x}+d\right)(ac-bd)}+\frac{\left(a+\frac{b}{x}\right)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b}{ax}+1\right)}{ad^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x)^n*x)/(c + d*x)^2,x]
```

```
[Out] -((c*(a + b/x)^(1 + n))/(d*(a*c - b*d)*(d + c/x))) - (c*(a*c - b*d*(1 - n))
*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c -
b*d)])/(d^2*(a*c - b*d)^2*(1 + n)) + ((a + b/x)^(1 + n)*Hypergeometric2F1[1
, 1 + n, 2 + n, 1 + b/(a*x)])/(a*d^2*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x}\,dx$$
$$= -\text{Subst}\left(\int \frac{(a+bx)^n}{x(d+cx)^2}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}}{d(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{\text{Subst}\left(\int \frac{(a+bx)^n(ac-bd-bcnx)}{x(d+cx)}\,dx, x, \frac{1}{x}\right)}{d(ac-bd)}$$
$$= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}}{d(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{\text{Subst}\left(\int \frac{(a+bx)^n}{x}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(c(ac-bd(1-n)))\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\right.}{d^2(ac-bd)}$$
$$= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}}{d(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{c(ac-bd(1-n))\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(ac-bd)^2(1+n)} + \frac{(a}{}$$

Mathematica [A]
time = 0.19, size = 120, normalized size = 0.80

$$\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(-\frac{cdx}{(ac-bd)(c+dx)} - \frac{c(ac+bd(-1+n))\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+n)} + \frac{{}_2F_1\left(1,1+n;2+n;1+\frac{b}{ax}\right)}{a(1+n)}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x)^n*x)/(c + d*x)^2,x]
```

```
[Out] ((a + b/x)^(1 + n)*(-((c*d*x)/((a*c - b*d)*(c + d*x))) - (c*(a*c + b*d*(-1
+ n))*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/((a*c
- b*d)^2*(1 + n)) + Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)]/(a*(1 +
 n))))/d^2
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^n*x/(d*x+c)^2,x)
```

```
[Out] int((a+1/x*b)^n*x/(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^n*x/(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(x*((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+\frac{b}{x}\right)^{n}}{\left(c+dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n*x/(d*x+c)**2,x)
[Out] Integral(x*(a + b/x)**n/(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n*x/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a + b/x)^n*x/(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(a+\frac{b}{x}\right)}^n}{{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b/x)^n)/(c + d*x)^2,x)
[Out] int((x*(a + b/x)^n)/(c + d*x)^2, x)
```

3.295 $\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2} \, dx$

Optimal. Leaf size=56

$$-\frac{b\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(2,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+n)}$$

```
[Out] -b*(a+b/x)^(1+n)*hypergeom([2, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/(a*c-b*d)^2/
(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {445, 455, 70}

$$-\frac{b\left(a+\frac{b}{x}\right)^{n+1} {}_2F_1\left(2,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(c + d*x)^2,x]
```

```
[Out] -((b*(a + b/x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (c*(a + b/x))/(a*
c - b*d)])/((a*c - b*d)^2*(1 + n)))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x^2}\,dx \\ &= -\text{Subst}\left(\int \frac{(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right) \\ &= -\frac{b\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(2, 1+n; 2+n; \frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 57, normalized size = 1.02

$$-\frac{b\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(2, 1+n; 2+n; -\frac{c\left(a+\frac{b}{x}\right)}{-ac+bd}\right)}{(-ac+bd)^2(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(c + d*x)^2,x]
[Out] -((b*(a + b/x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, -((c*(a + b/x))/(
-(a*c) + b*d))])/((-(a*c) + b*d)^2*(1 + n)))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^n/(d*x+c)^2,x)
[Out] int((a+1/x*b)^n/(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^n/(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/(d*x+c)**2,x)
[Out] Integral((a + b/x)**n/(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a + b/x)^n/(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(c + d*x)^2,x)
[Out] int((a + b/x)^n/(c + d*x)^2, x)
```

3.296 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2} dx$

Optimal. Leaf size=105

$$-\frac{d\left(a+\frac{b}{x}\right)^{1+n}}{c(ac-bd)\left(d+\frac{c}{x}\right)}+\frac{(ac-bd(1+n))\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(ac-bd)^2(1+n)}$$

```
[Out] -d*(a+b/x)^(1+n)/c/(a*c-b*d)/(d+c/x)+(a*c-b*d*(1+n))*(a+b/x)^(1+n)*hypergeo
m([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/c/(a*c-b*d)^2/(1+n)
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {528, 457, 79, 70}

$$\frac{\left(a+\frac{b}{x}\right)^{n+1}(ac-bd(n+1))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(n+1)(ac-bd)^2}-\frac{d\left(a+\frac{b}{x}\right)^{n+1}}{c\left(\frac{c}{x}+d\right)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x*(c + d*x)^2),x]
```

```
[Out] -((d*(a + b/x)^(1 + n))/(c*(a*c - b*d)*(d + c/x))) + ((a*c - b*d*(1 + n))*(
a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*
d)])/(c*(a*c - b*d)^2*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x^3}\,dx\\
&= -\text{Subst}\left(\int \frac{x(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{d\left(a+\frac{b}{x}\right)^{1+n}}{c(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{(ac-bd(1+n))\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c(ac-bd)}\\
&= -\frac{d\left(a+\frac{b}{x}\right)^{1+n}}{c(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{(ac-bd(1+n))\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(ac-bd)^2(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 88, normalized size = 0.84

$$\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(\frac{d(-ac+bd)x}{c+dx}+\frac{(ac-bd(1+n))\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{1+n}\right)}{c(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x*(c + d*x)^2),x]
```

```
[Out] ((a + b/x)^(1 + n)*((d*(-(a*c) + b*d)*x)/(c + d*x) + ((a*c - b*d*(1 + n))*H
ypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(1 + n)))/(c*
(a*c - b*d)^2)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x \left(d x +c \right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^n/x/(d*x+c)^2,x)
[Out] int((a+1/x*b)^n/x/(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^n/((d*x + c)^2*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^n/(d^2*x^3 + 2*c*d*x^2 + c^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{n}}{x \left(c + d x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x/(d*x+c)**2,x)
[Out] Integral((a + b/x)**n/(x*(c + d*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(x*(c + d*x)^2),x)
```

```
[Out] int((a + b/x)^n/(x*(c + d*x)^2), x)
```

3.297 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx$

Optimal. Leaf size=133

$$-\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc^2(1+n)}+\frac{d^2\left(a+\frac{b}{x}\right)^{1+n}}{c^2(ac-bd)\left(d+\frac{c}{x}\right)}-\frac{d(2ac-bd(2+n))\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(ac-bd)^2(1+n)}$$

```
[Out] -(a+b/x)^(1+n)/b/c^2/(1+n)+d^2*(a+b/x)^(1+n)/c^2/(a*c-b*d)/(d+c/x)-d*(2*a*c
-b*d*(2+n))*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/c^2
/(a*c-b*d)^2/(1+n)
```

Rubi [A]
time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {528, 457, 91, 81, 70}

$$\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}}{c^2\left(\frac{c}{x}+d\right)(ac-bd)}-\frac{d\left(a+\frac{b}{x}\right)^{n+1}(2ac-bd(n+2))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(n+1)(ac-bd)^2}-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x^2*(c + d*x)^2),x]
```

```
[Out] -((a + b/x)^(1 + n)/(b*c^2*(1 + n))) + (d^2*(a + b/x)^(1 + n))/(c^2*(a*c -
b*d)*(d + c/x)) - (d*(2*a*c - b*d*(2 + n))*(a + b/x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*d)^2*(1 + n)
)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x^4}\,dx \\
&= -\text{Subst}\left(\int \frac{x^2(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right) \\
&= \frac{d^2\left(a+\frac{b}{x}\right)^{1+n}}{c^2(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{\text{Subst}\left(\int \frac{(a+bx)^n(-d(ac-bd(1+n))+c(ac-bd)x)}{d+cx}\,dx, x, \frac{1}{x}\right)}{c^2(ac-bd)} \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc^2(1+n)} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+n}}{c^2(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{(d(2ac-bd(2+n)))\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c^2(ac-bd)} \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc^2(1+n)} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+n}}{c^2(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{d(2ac-bd(2+n))\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+\right.}{c^2(ac-bd)^2(1+n)}
\end{aligned}
$$

Mathematica [A]

time = 0.17, size = 125, normalized size = 0.94

$$-\frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left((ac-bd)(ac(c+dx)-bd(c+d(2+n)x))+bd(2ac-bd(2+n))(c+dx)\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)\right)}{bc^2(ac-bd)^2(1+n)x(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x^2*(c + d*x)^2),x]
[Out] -(((a + b/x)^n*(b + a*x)*((a*c - b*d)*(a*c*(c + d*x) - b*d*(c + d*(2 + n)*x
)) + b*d*(2*a*c - b*d*(2 + n))*(c + d*x)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (c*(a + b/x))/(a*c - b*d)]))/(b*c^2*(a*c - b*d)^2*(1 + n)*x*(c + d*x)))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{\left(a+\frac{b}{x}\right)^n}{x^2\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^n/x^2/(d*x+c)^2,x)
[Out] int((a+1/x*b)^n/x^2/(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^n/((d*x + c)^2*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^n/(d^2*x^4 + 2*c*d*x^3 + c^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+\frac{b}{x}\right)^n}{x^2\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x**2/(d*x+c)**2,x)
```

```
[Out] Integral((a + b/x)**n/(x**2*(c + d*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^n/((d*x + c)^2*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(x^2*(c + d*x)^2),x)
```

```
[Out] int((a + b/x)^n/(x^2*(c + d*x)^2), x)
```

3.298 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx$

Optimal. Leaf size=217

$$-\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(d(bd(2+n)(ac+bd(3+n))-ac(ac+bd(5+3n)))-\frac{c(ac-bd)(ac+bd(3+n))}{x}\right)}{b^2c^3(ac-bd)(1+n)(2+n)\left(d+\frac{c}{x}\right)}-\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(2+n)\left(d+\frac{c}{x}\right)}$$

```
[Out] -(a+b/x)^(1+n)*(d*(b*d*(2+n)*(a*c+b*d*(3+n))-a*c*(a*c+b*d*(5+3*n)))-c*(a*c-
b*d)*(a*c+b*d*(3+n))/x)/b^2/c^3/(a*c-b*d)/(1+n)/(2+n)/(d+c/x)-(a+b/x)^(1+n)
/b/c/(2+n)/(d+c/x)/x^2+d^2*(3*a*c-b*d*(3+n))*(a+b/x)^(1+n)*hypergeom([1, 1+
n],[2+n],c*(a+b/x)/(a*c-b*d))/c^3/(a*c-b*d)^2/(1+n)
```

Rubi [A]
time = 0.18, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {528, 457, 102, 151, 70}

$$-\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(d(bd(n+2)(ac+bd(n+3))-ac(ac+bd(3n+5)))-\frac{c(ac-bd)(ac+bd(n+3))}{x}\right)}{b^2c^3(n+1)(n+2)\left(\frac{c}{x}+d\right)(ac-bd)}+\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}(3ac-bd(n+3))\,{}_2F_1\left(1,n+1;n+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^3(n+1)(ac-bd)^2}-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc(n+2)x^2\left(\frac{c}{x}+d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^n/(x^3*(c + d*x)^2),x]
```

```
[Out] -(((a + b/x)^(1 + n)*(d*(b*d*(2 + n)*(a*c + b*d*(3 + n)) - a*c*(a*c + b*d*(
5 + 3*n))) - (c*(a*c - b*d)*(a*c + b*d*(3 + n)))/x))/(b^2*c^3*(a*c - b*d)*(
1 + n)*(2 + n)*(d + c/x))) - (a + b/x)^(1 + n)/(b*c*(2 + n)*(d + c/x)*x^2)
+ (d^2*(3*a*c - b*d*(3 + n))*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (c*(a + b/x))/(a*c - b*d)])/(c^3*(a*c - b*d)^2*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x^5}\,dx \\
&= -\mathrm{Subst}\left(\int \frac{x^3(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(2+n)\left(d+\frac{c}{x}\right)x^2} - \frac{\mathrm{Subst}\left(\int \frac{x(a+bx)^n(-2ad+(-ac-bd(3+n))x)}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)}{bc(2+n)} \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(d(bd(2+n)(ac+bd(3+n))-ac(ac+bd(5+3n))) - \frac{c(ac-bd)(ac+bd(3+n)}{x}\right.}{b^2c^3(ac-bd)(1+n)(2+n)\left(d+\frac{c}{x}\right)} \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(d(bd(2+n)(ac+bd(3+n))-ac(ac+bd(5+3n))) - \frac{c(ac-bd)(ac+bd(3+n)}{x}\right.}{b^2c^3(ac-bd)(1+n)(2+n)\left(d+\frac{c}{x}\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.29, size = 182, normalized size = 0.84

$$
\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(-\frac{1}{x(c+dx)}+\frac{-a^2c^2(c+dx)+b^2d^2(3+n)(c+d(2+n)x)-abcd(c(2+n)+d(3+2n)x)}{bc^2(-ac+bd)(1+n)(c+dx)}-\frac{bd^2(2+n)(-3ac+bd(3+n))\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(ac-bd)^2(1+n)}\right)}{bc(2+n)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^n/(x^3*(c + d*x)^2),x]
```

```
[Out] ((a + b/x)^(1 + n)*(-(1/(x*(c + d*x))) + (-(a^2*c^2*(c + d*x)) + b^2*d^2*(3
 + n)*(c + d*(2 + n)*x) - a*b*c*d*(c*(2 + n) + d*(3 + 2*n)*x))/(b*c^2*(-(a*
c) + b*d)*(1 + n)*(c + d*x)) - (b*d^2*(2 + n)*(-3*a*c + b*d*(3 + n))*Hyperg
eometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*d)^2
*(1 + n))))/(b*c*(2 + n))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\left(dx+c\right)^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x*b)^n/x^3/(d*x+c)^2,x)
```

```
[Out] int((a+1/x*b)^n/x^3/(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^n/((d*x + c)^2*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^n/(d^2*x^5 + 2*c*d*x^4 + c^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{n}}{x^{3} \left(c + dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**n/x**3/(d*x+c)**2,x)
[Out] Integral((a + b/x)**n/(x**3*(c + d*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^n/x^3/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a + b/x)^n/((d*x + c)^2*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\,\left(c+d\,x\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^n/(x^3*(c + d*x)^2),x)
[Out] int((a + b/x)^n/(x^3*(c + d*x)^2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```